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The Stability or Instability of Co-Even Domination in Graphs

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Abstract: The effect of deleting a vertex or (deleting or adding) an edge on co-even domination number on a graph that owns this number is discussed in this paper. Firstly, after deleting a vertex, the changes in this number were studied, which is the stability (unchanging) of the number or instability (changing) whether it was an increase or decrease. Secondly, the same study was done into how this number was affected when an edge was removed or added.

Keywords: Delete a vertex, delete an edge, add an edge, co-even domination number.

1 Introduction

In recent years, graph theory has become used to find alternative solutions in most sciences, as graphs $G = (V, E)$ depend on two sets, namely, the vertex set V, which are not empty, and the edge set E. Some aspects of the theory have been touched on in a simple, finite, undirected graph. The reader can be found all above concepts and every other concepts that[1,2] The concept of Domination in graphs is considered one of the most important research concepts, as it addresses most life problems by finding the lowest cost, the least time, the shortest path, and others to implement or reach any project. Therefore, many definitions appeared that address these problems in theory and then benefit from them in the appropriate application of them, as these definitions can find solutions to more than one problem as in[3,4,5]. Domination included in various fields in the graph as in fuzzy graph [6], labeled graph[7], and topological indices[8,9,10,11,12,13,14,15,16,17,18,19].

The first one who introduced the domination was C. Berge in[20] Ore was the first to use it in[21]. Throughout this paper, Our focus is on how the effect on domination number when deleting a vertex or an edge or adding an edge. Any definition of dominating set represents an integrated system for the workflow with a mechanism that mainly depends on specific points through which all parties to the work network are reached. As an engineering example, we study the effect of adding or removing roads leading to buildings such as schools or removing old buildings. After building a working system, the strength of this system must be examined in the event that it is damaged or the possibility of its development. We return to our example in the event of a fall of one of the buildings (delete a vertex) or a break in communication between any two buildings (deletion of an edge) or new connections between the buildings will add (an edge will add) in the future. The question here is the effect of these changes on the domination number, whether it is maintained, increased, or decreased. The notion of co-even domination number in graphs has been introduced by Omran and Shalaan [22]. In this work, the changing or unchanging for deleting a vertex on co-even domination number is studied. The condition for stability of the co-even domination number is presented. Also, the

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condition of the results in an increase or decrease or it is stable.

Definition 1.*[22] Let G be a graph and D is a dominating set, the set D is called co-even dominating set (CEDS) if, $d(v)$ is even number for all $v \in V - D$.*

Definition 2.*[22] Consider G be a graph and D is aCEDS, then D i9s called a minimal co-even dominating set if has no proper subset $D' \subseteq D$ is a co-even dominating of G Take us (MCEDS) refers to all minimal co- even dominating sets.*

Definition 3.*[22] The set $|D|$is called the co-even domination number if $|D| = min\{|D_i|, D_i \in MCEDS(G)\}$,and is denoted by $\gamma_{coe}(G)$*

Definition 4.*[1] The set $|D|$ is called the co-even domination number if $|D| = min/|D_i|, D_i \in MCEDS(G)/$, and is denoted by $\gamma_{coe}(G)$.*

Proposition 1*[22] Let $D = (n,m)$ be a graph and D is a CEDS,then*

1. *All vertices of odd or zero degrees belong to every co-even dominating set.*
2. *$deg(v) \geq 2$, for all $v \in V - D$.*

Let $G = (V,E)$ be graph hasγ_{coe},then let denoted the following

$$V^0 = \{u \in V; \gamma_{coe}(G) = \gamma_{coe}(G-u)\}.$$
$$V^+ = \{u \in V; \gamma_{coe}(G) < \gamma_{coe}(G-u)\}.$$
$$V^- = \{u \in V; \gamma_{coe}(G) > \gamma_{coe}(G-u)\}.$$
$$E^0_- = \{e \in E; \gamma_{coe}(G) = \gamma_{coe}(G-e)\}\ and$$
$$E^0_+ = \{e \in \bar{E}; \gamma_{coe}(G) = \gamma_{coe}(G+e)\}.$$
$$E^+_- = \{e \in E; \gamma_{coe}(G) < \gamma_{coe}(G-e)\}\ and$$
$$E^+_+ = \{e \in \bar{E}; \gamma_{coe}(G) < \gamma_{coe}(G+e)\}.$$
$$E^-_- = \{e \in E; \gamma_{coe}(G) > \gamma_{coe}(G-e)\}\ and$$
$$E^-_+ = \{e \in \bar{E}; \gamma_{coe}(G) > \gamma_{coe}(G+e)\}.N(v)$$
$$\cap (V-D) = N_{V-D}(v)$$

2 Main Results

2.1 Deleting a vertex

Theorem 2.*Let G be a graph has γ_{coe} and let a vertex $v \in V - D$, then$v \in V^0$ iff one of the two statements holds*

1. *If v is an isolated vertex in $V - D$, then each vertex in D that is adjacent to it has an odd degree or has a private neighborhood in $G - v$.*
2. *If v is not an isolated vertex in $V - D$, then for every adjacent vertex say u in $V - D$ there is a vertex say w in D such that w has an odd degree in G and $(D-w) \cup \{u\}$ is γ_{ce}-set in $G - v$.*

*Proof.*Let $v \in V^0$, then there are two cases as follow.
Case 1. If v is an isolated vertex in $V - D$, then there are at least two vertices in D that adjacent to the vertex v. So, these vertices still in D, since $v \in V^0$ by our assumption. Thus, there are two properties to occur that one of them that a vertex (*say u*) that adjacent with the vertex v has an odd degree in $G - v$, according to Proposition 1(as an example see, Figure 1; $v = v_1$ *and* $u = v_3$), or has a private neighborhood in $G-v$(as an example see, Figure 1; $v = v_1$ *and* $u = v_2$), where $D = \{v_2, v_3, v_8\}$.

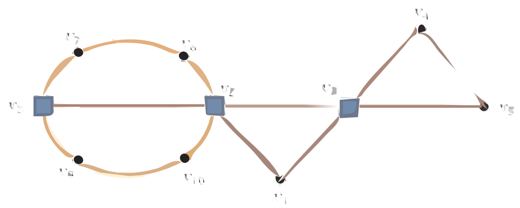
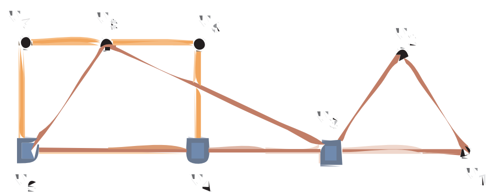


Fig. 1: Before deletion a vertex from $V - D$.

Case 2. If v is not an isolated vertex in $V - D$, then there is at least one vertex say $u \in V - D$ adjacent to it. It is obvious that u has an even degree in G, according to Definition 1. Thus, after deleting the vertex v, the vertex u would have an odd degree, then it must belong to every MCEDS, according to proposition1. Now, since $v \in V^0$, then it must leave one vertex say w alternative of the vertex u. Then there are two subcases as follows.

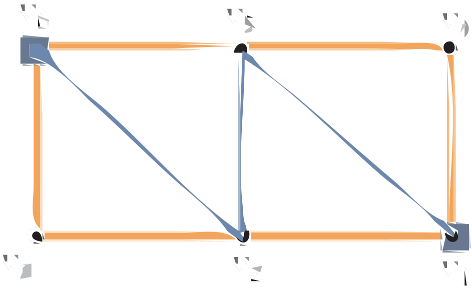
1. If $pn[w,D] = \{v\}$, and w has an even degree in $G - v$ and adjacent to at least one vertex in $D \cup \{u\}$, then $(D-w) \cup \{u\}$ is the $\gamma_{ce} - set$ in $G - v$ (as an example see, Figure 2(a); $v = v_6,\ u = v_8,\ and\ w = v_4$), where $D = \{v_3, v_4, v_5\}$.

2. If $pn[w,D] = \{u\}$ or $\emptyset$, and w has an even degree in $G - v$, then $(D-w) \cup \{u\}$ is the $\gamma_{ce}-$set in $G - v$ (as an example see, Figure 2; $v = v_4,\ u = v_5,\ and\ w = v_1$), where $D = \{v_1, v_6\}$.



(a)

Fig. 2: Before deletion a vertex from $V - D$.



((b)

Fig. 3: Before deletion a vertex from $V-D$.

Conversely, one can easily to check that in both cases the removal of the vertex v is not going to effect the current domination.
Therefore, from every case discussed above the proof is done

Theorem 3.*If $v \in D$ and is not an isolated in G, then $v \in V^0$ iff $N_{V-D}(v) = |S|+1$, where S is the set of vertices in D have even degree in $G-v$ and private neighborhood subset of $N_{V-D}(v)$, and each vertex of S is adjacent to at least one vertex in $(D-(S\cup\{v\}))\cup N_{V-D}(v)$.*

*Proof.*If $v \in V^0$ then there are three subcases depend on the number of vertices that are adjacent with v as follows.
Subcase 1. If v is not adjacent to any vertex in $V-D$, then $\gamma_{ce}(G-v) < \gamma_{ce}(G)$, and this is a contradiction with assumption
Subcase 2. If v is adjacent to t vertices in $V-D$(let T be the set of these vertices), then all these vertices belong to every *MCEDS*, according to Proposition 1. Since $v \in V^0$, then $t-1$ vertices from D must leave the set D and let S be the set of all these vertices . These vertices must have the following properties having even degree in $G-v$ and all private neighborhoods included in $N_{V-D}(v)$. Furthermore, each vertex in S adjacent to at least one vertex in $(D-(S\cup v))\cup N_{V-D}(v)$ to keep the co-even dominating set. Finally, it is clear that $|S \cup v| = |N_{V-D}(v)| = |T|$,then the set $(D-(S\cup v))\cup N_{V-D}(v)$ is $\gamma_{ce}-set$ (as an example see, Figure 4; $v=v_1$, $T=\{v_5,v_6\}$, and $S=v_2$), where $D=v_1,v_2,v_3,v_4$. Conversely, the proof is straightforward.
Therefore, from the above-discussed cases, the proof is done

Theorem 4.*The vertex $v \in V^+$ iff $|N_{V-D}(v)| > |S|$, if $v \in V-D$ and $|N_{V-D}(v)| > |S|+1$, if $v \in D$, where S is the set of all vertices of odd degree in D adjacent to v in G and have no private such that $(D-v)\cup N_{V-D}(v)-S$ is MCEDS.*

*Proof.*Let $v \in V^+$, then there are two position for the vertex v:

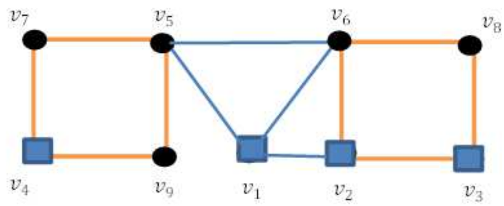


Fig. 4: Before deletion a vertex from D.

I) If $v \in V-D$, then all adjacent vertices in $V-D$ if exist have an odd degree in $G-v$. Thus, all these vertices belong to every *MCEDS*, according to Proposition 1. At the same time if there is a set S of adjacent vertices to v in D and have three properties. First, of them there are no private neighborhoods in $G-v$ and the second that they have even degree and finally, $D\cup N_{V-D}(v)-S$is *MCEDS*. Therefore, $|N_{V-D}(v)| \geqslant |S|$, since $v \in V^+$.
II) If $v \in D$, as the same manner in (I), but $|N_{V-D}(v)| > |S|$ to guaranty $v \in V^+$. Conversely, the proof is straightforward.

Theorem 5.*The vertex $v \in V^-$ iff one of the statements holds*
I) If $v \in D$ and it is an isolated vertex inG.
II) $|N_{V-D}(v)| < |S|$,if $v \in V-D$ and $|N_{V-D}(v)| \leq |S|$, if $v \in D$, where S is the set of all vertices of odd degree in D adjacent to v and have no private such that $(D-v)\cup N_{V-D}(v)-S$ is MCEDS.

*Proof.*Proof. I) It is obvious.
II) Two cases are discussed as the following.
Case 1: If $v \in D$, then all neighborhoods of v in $V-D$ have an odd degree in$G-v$. Thus, all these vertices belong to every MCEDS, according to Proposition 1. Now, let S be the set of all adjacent vertices to v in D such that carried the properties which have even degree, no private, and $(D-v)\cup N_{V-D}(v)-S$ is MCEDS with $\gamma-set$. Therefore, if $|N_{V-D}(v)| \leq |S|$, then $|(D-v)\cup N_{V-D}(v)-S| \leq |D|$, so $v \in V^-$.
Case 2: If $v \in V-D$, the same procedure in the previous case has been following except the next boundary $|N_{V-D}(v)| < |S|$ to guaranty that $|D\cup N_{V-D}(v)-S| < |D|$.

2.2 Deletion and adding an edge

Theorem 6.*Assume that G be a graph has $\gamma_{ce}-setD$. If $e \in E(<V-D>)$, then*

1. *$e=uv \in E^0_-$ iff there are $u_1,u_2 \in D$ have even degrees such that $pn[u_1,D]=v$ and $pn[u_2,D]=u$ or $u_1,u_2 \subseteq N[v],N[u]$ and u and v are not private neighborhood to any vertex in D and u_1 and u_2 have no private neighborhood, otherwise $e \in E^+_-$.*
2. *$e \in E^0_+$iff there are $u_1,u_2 \in D$ such that $pn[u_1,D]=v$ and $pn[u_2,D]=u$, and u_1 and u_2 are even degrees, otherwise $e \in E^+_+$.*

3.$E_{-}^{-} = E_{+}^{-} = \emptyset$.

*Proof.*Proof. 1) Let $e = uv \in E_{-}^{0}$, then the two vertices u and v have an odd degree in $G-e$, so they belong to D, according to Proposition 1. Now, there are two cases:
Case 1: If $pn[u_1,D] = v$ and $pn[u_2,D] = u$ and u_1 and u_2 are even degrees, then the two vertices u and v must leave the set D to keep the minimalist (as an example see, Figure 5; where $D = u_1,u_2$).
Case 2: If $u_1,u_2 \subseteq N[v],N[u]$ and u and v are not private neighborhood to any vertex in D and u_1 and u_2 have no private neighborhood, then again it is obvious that the two vertices u_1 and u_2 must leave the set D to keep the minimalist (as an example see, Figure 6; where $D = u_1,u_2$). If neither *case*1 nor case 2 hold, one can easily be concluded that the co-even domination increases, since if not satisfied any one of the conditions are mentioned in *case*1 or *case*2 will lead to join the two vertices u and v to any co-even dominating set and leave at most one vertex from the set D. Conversely, it is straightforward.
2) In the same technique in proof (1), one can conclusively prove this case.
3) From prove of the cases (1) and (2), there is no probability to obtain a vertex belongs to E_{-}^{-} or E_{+}^{-},then $E_{-}^{-} = E_{+}^{-} = \emptyset$.

Fig. 5: Before deletion an edge from $V-D$.

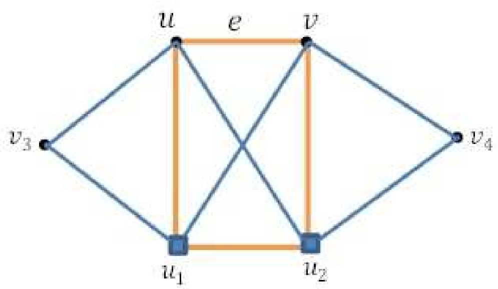


Fig. 6: Before deletion an edge from $V-D$.

Theorem 7.*Assume that G be a graph has $\gamma_{ce}-$ set D. If $e \in E < D >$,then*

1.*$e = uv \in E_{-}^{-}$ iff $pn[v,D] = \emptyset$ and v has even degree in $G-e$ or $pn[u,D] = \emptyset$ and u has even degree in $G-e$, otherwise $e \in E_{-}^{0}$*
2.*$e \in E_{+}^{-}$ iff $pn[v,D] = \emptyset$ and v has even degree in $G-e$ or $pn[u,D] = \emptyset$ and u has even degree in $G-e$, otherwise $e \in E_{+}^{0}$.*
3.*$E_{-}^{+} = E_{+}^{+} = \emptyset$.*

*Proof.*1) Let $e = uv \in V$, then there are two cases that depend on the private neighborhood of u and v.
Case 1: If $pn[v,D] = \emptyset$ or $pn[u,D] = \emptyset$, then
Subcase 1: Without a loss the generality assumes that $pn[v,D] = \emptyset$, then if the vertex v in $G-e$ is even, then the vertex v leave the set D to keep to the minimalist of *CEDS*, that means $e \in E_{-}^{-}$.
Subcase 2. If $pn[v,D] = \emptyset$ and $pn[u,D] = \emptyset$, then if the vertices v and u in $G-e$ are odd, then the vertex v still in the set D, according to Proposition 1. Thus, $e \in E_{-}^{0}$.
Conversely, it is straightforward. 2) The proof in the same manner in *case*1.
3) From prove of the cases (1) and (2), there is no probability to obtain a vertex belongs to E_{-}^{+}or E_{+}^{+},then $E_{-}^{+} = E_{+}^{+} = \emptyset$.

Theorem 8.*Let G be a graph has $\gamma_{ce}-$ set D. If $e \in uv; u \in D$ and $v \in V-D$, then*

1.*$e \in E_{-}^{0}, E_{+}^{0}$ if and only if there is just one vertex say w (maybe u) in D such that$pn[w,D] = v$ and w has even degree in $G-e$.*
2.*$e \in E_{-}^{-}, E_{+}^{-}$ if and only if there is more than one vertex say w_1 and w_2 in D at least such that $pn[w_1,D] = pn[w_2,D] = v$ and w_1 and w_2 have even degree in $G-e$.*
3.*$e \in E_{-}^{+}, E_{+}^{+}$ if and only if there is no vertex in D such that the private neighborhood of this vertex is v and u has a private neighborhood other than v or has an odd degree in $G-e$.*

Proof.

If $e \in E_{-}^{0}$, then after removing the edge e, the vertex v belongs to every co-even dominating set according to proposition 1. Now, since $e \in E_{-}^{0}$ by assumption, then there is just one vertex leave the co-even dominating set, an alternative to the vertex v and this vertex must have an even degree in $G - e$ according to the Definition 1 Additionally, this vertex has just a private neighborhood that is v. thus, the removal of the edge is not going to affect the current domination.
Conversely, it is straightforward. The same proof is followed if $e \in E_{+}^{0}$.
If $e \in E_{-}^{-}$, then in the same manner in proof (1), if there is more than one vertex say w_1 and w_2 in D at least such that $pn[w_1, D] = pn[w_2, D] = v$ and $w_1$andw_2 have even degree in $G - e$, then the verticesw_1 and w_2 must leave the dominating set after removal the edge e to keep minimalist. Therefore, the result is obtained. Conversely, it is straightforward. The same proof is followed if $e \in E_{+}^{-}$.
If $e \in E_{-}^{+}$, then after removing the edge e if there is no vertex in D such that the private neighborhood of this vertex is v, then all vertices in D except the vertex u are not affected by the removal of the vertex v. The remaining vertex must be discussed after removal the edge e that is the vertex u. Now, if u has an odd degree, then this vertex still in D according to Proposition 1. Therefore, in this case, the current domination is increasing and the result is obtained. Conversely, it is straightforward. The same proof is followed if $e \in E_{+}^{+}$.

3 Conclusions

The effect of removing a vertex on the co-even domination number depends on the location of this vertex. In this work, cases are clarified in which this effect is an increase, decrease, or stability. Also, the effect of a deletion or adding an edge is discussed. We have shown through this study that the effect on the domination number is increased or stabilizes when the edge does not belong to the co-even dominating set which has a co-even domination number. Furthermore, the stability or decreasing when the edge belongs to the co-even dominating set which has the co-even domination number.

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Conflicts of Interests

The authors declare that they have no conflicts of interests.

References

[1] F. Harary, Graph theory, Addison-Wesley, Reading, Mass.,1969.
[2]] T. W. Haynes, S. T. Hedetniemi, and P. J. Slater, Fundamentals of Domination in Graphs, Marcel Dekker, New York, NY, USA,1998.
[3] T. A. Ibrahim and A. A. Omran, Restrained Whole Domination in Graphs, J. Phys.: Conf. Ser.032029, **1897**,2021.
[4] T. A. Ibrahim and A. A. Omran, Upper whole domination in a graph, Journal of Discrete Mathematical Sciences and Cryptography,T. A. Ibrahim and A. A. Omran, Upper whole domination in a graph, Journal of Discrete Mathematical Sciences and Cryptography, DOI: 10.1080/09720529.2021.1939954.
[5] E. Kilic, B. Ayl1, Double Edge-Vertex Domination Number of Graphs, Advanced Mathematical Models and Applications,19-37, **5**, (2020).
[6] S. S. Kahat, A. A. Omran, and M. N. Al-Harere, Fuzzy equality co-neighborhood domination of graphs, Int. J. Nonlinear Anal. **12**, 537-545 , (2021).
[7] M.N. Al-Harere, A. A. Omran, On binary operation graphs, Boletim da Sociedade Paranaense de Matematica, **38**, 59-67,(2020).
[8] A. Alsinai, A. Alwardi, and N. D. Soner. ”Topological Properties of Graphene Using ψ_k polynomial.” In Proceedings of the Jangjeon Mathematical Society,**24(3)**. 375-388,2021.
[9] A. Alsinai, A. Alwardi, H. Ahmed, and N. D. Soner. ”Leap Zagreb indices for the Central graph of graph.” Journal of Prime Research in Mathematics**17(2)**,73-78,2021.
[10] A. Alsinai, H. Ahmed, A. Alwardi, and Soner, N. D. HDR Degree Bassed Indices and Mhr-Polynomial for the Treatment of COVID-19, Biointerface Research in Applied Chemistry,**12(6)**, 7214-7225.2021.
[11] F. Afzal, A. Alsinai, S.Hussain, D. Afzal, F. Chaudhry, and M. Cancan, On topological aspects of silicate network using M-polynomial. Journal of Discrete Mathematical Sciences and Cryptography,**24(7)**, 1-11,2021.
[12] A. Alsinai, A. Alwardi,and Soner, N. D. On Reciprocals Leap indices of graphs. International Journal of Analysis and Applications, **19(1)**, 1-19,2021.
[13] S. Javaraju,A. Alsinai,H. Ahmed, A. Alwardi,and Soner, N. D. Reciprocal leap indices of some wheel related graphs. Journal of Prime Research in Mathematics, **17(2)**, 101-110,2021.
[14] A. Hasan, M. H. A. Qasmi,A. Alsinai, M. Alaeiyan,M. R. Farahani,and M. Cancan. Distance and Degree Based Topological Polynomial and Indices of X-Level Wheel Graph. Journal of Prime Research in Mathematics,**17(2)**, 39-50, 2021.
[15] H. Ahmed, M. R. Salestina,and A. Alwardi.Domination topological indices and their polynomials of a firefly graph. Journal of Discrete Mathematical Sciences and Cryptography, **24(2)**, 325-341,2021.
[16] A. Alsinai, A. Saleh, H. Ahmed, L.N. Mishra,and N.D. Soner.On fourth leap Zagreb index of graphs, Discrete Mathematics, Algorithms and Applications,2022, https://doi.org/10.1142/S179383092250077X.

[17] A. Alsinai, A. A. Omran, T. A. A. Ibrahim,and H. A. Othman. Some Properties of Whole Edge Domination in Graphs. Appl. Math, **16(3)**, 435-440.2022
[18] A. Alsinai, A. Alwardi,and Soner, N. D. On Reciprocal leap function of graph. Journal of Discrete Mathematical Sciences and Cryptography, **17(2)**, 307-324.2021
[19] A. Alsinai, Hafiz Mutee ur Rehman, Yasir Manzoor, Murat Cancan, Ziyattin Taş & Moahmmad Reza Farahani, Sharp upper bounds on forgotten and SK indices of cactus graph, Journal of Discrete Mathematical Sciences and Cryptography,(2022) DOI:10.1080/09720529.2022.2027605
[20] C. Berge, The theory of graphs and its applications, Methuen and Co, London,1962.
[21] O. Ore, Theory of Graphs, Amer. Math. Soc. Colloq. Publ.**38**,(1962).
[22] M.M.Shalaan and A. A. Omran, Co-even Domination in Graphs, International Journal of Control and Automation,**13** ,330-334,2020.

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