Applied Mathematics & Information Sciences
An International Journal

http://dx.doi.org/10.18576/amis/150608

Decomposition Matrices for the Spin Characters of the Symmetric Group $23 \leq n \leq 24$

AbdulKareem A. Yaseen[1,*] *and Marwa M. Jawad*[2]

[1]Department of Mathematics and Sciences, College of Humanities and Sciences, Ajman University, Ajman, UAE
[2]Department of Mathematics, College of Sciences, Basrah University, Basrah, Iraq

Received: 19 Jul. 2021, Revised: 20 Sep.2021, Accepted: 24 Sep.2021
Published online: 1 Nov. 2021

Abstract: In this paper, we compute the Brauer trees of the symmetric group S_n, $23 \leq n \leq 24$. It gives the decomposition matrices of spin characters of S_n, $23 \leq n \leq 24$, *modulo* $p = 17$. We conclude that all blocks of defect zero or one and the decomposition numbers are all zero or one.

Keywords: Brauer trees, decomposition matrix for the spin characters, Modular representations and characters

1 Introduction

The Symmetric group S_n has a representation group $\bar{S}_n$ of order $2(n!)$ and it has a central subgroup $Z = \{-1,1\}$ such that $\bar{S}_n/Z \approx S_n$, then the irreducible representations or characters of $\bar{S}_n$ fall into two classes [1,2]. The irreducible representations and characters are indexed by partitions λ of n and the character is denoted by $[\lambda]$. And the irreducible spin representations are indexed by partitions of n with distinct parts which are called bar partitions of n and denoted by $\langle\lambda\rangle$, see [1,2].

In fact, if $\lambda = (\lambda_1, \lambda_2, ..., \lambda_n)$, $\lambda \mapsto n$ and if $n-m$ is even, then there is one irreducible spin character denoted by $\langle\lambda\rangle$ which is self-associate, and if $n-m$ is odd, then there are two associate spin characters denoted by $\langle\lambda\rangle$ and $\langle\lambda\rangle'$. The decomposition matrix gives the relationship between the irreducible spin characters and projective indecomposable spin characters of S_n.

In this paper, we determined the decomposition matrices of spin characters of S_n, $23 \leq n \leq 24$, *modulo* $p = 17$ by using method of (r,r')-inducing (restricting) in [2] to distribute the spin characters into p-blocks [2,3]. Decomposition matrices for the spin characters of the symmetric group S_n, $17 \leq n \leq 22$ *modulo* $p = 17$ are found by Yaseen and Tahir [4]. For $p = 13$, the symmetric group S_n, $13 \leq n \leq 20$, the symmetric group S_{21} and the symmetric group S_{22}, see [5,6,7]. For $p = 11$, the symmetric group S_{26} see [8].

2 Preliminaries

The following general results are useful in determining the decomposition matrices and modular characters:

1. The degree of spin characters $\langle\lambda\rangle$, $\lambda = (\lambda_1, \lambda_2, ..., \lambda_n)$ is [1,2]:

$$deg\langle\lambda\rangle = 2^{\left[\frac{(n-m)}{2}\right]} \frac{n!}{\prod_{i=1}^{m} \lambda!} \prod_{1 \leq i < j \leq m} \frac{\lambda_i - \lambda_j}{\lambda_i + \lambda_j}. \quad (1)$$

2. Every spin (modular, projective) character of S_n can be written as a linear combination with non-negative integer coefficients of the irreducible spin (irreducible modular, projective indecomposable) characters respectively [9].
3. Let H be a subgroup of the group G, then [10]:
   a. If φ is a modular (principle) character of a subgroup H of G, then $\varphi \uparrow G$ is a modular (principal) character of G, (where $\uparrow$ denotes inducing).
   b. If ψ is a modular (principal) character of group G, then $\psi \downarrow H$ is a modular (principal) character of a subgroup H, (where $\downarrow$ denotes the restricting).
4. Let B be a p-block G of defect one and let b be the number of p-conjugate characters to the irreducible ordinary character χ of G, then [11]:
   a. There exists a positive integer number N such that the irreducible ordinary characters lying in the

* Corresponding author e-mail: a.yaseen@ajman.ac.ae

block B can be partitioned into two disjoint classes: $B_1 = \{x \in B \mid b\ deg\ x \equiv N\ mod\ p^a\}$, $B_2 = \{x \in B \mid b\ deg\ x \equiv -N\ mod\ p^a\}$.

b. Each coefficient of the decomposition matrix of the block B is 1 or 0.

c. If α_1 and α_2 are not p-conjugate characters and belong to the same partition class B_1 or B_2 above, then they have no irreducible modular character in common.

d. For every irreducible ordinary character χ in B_1, there exists irreducible ordinary character φ in B_2 such that they have one irreducible modular character in common with multiplicity one.

5.Let λ and μ be bar partitions such that $\lambda \neq \mu$, then $\langle\lambda\rangle$ and $\langle\mu\rangle$ are in the same p-block if and only if $\lambda(\bar{p}) = \mu(\bar{p})$, where p is an odd prime. The associative irreducible spin characters $\langle\lambda\rangle$ and $\langle\lambda\rangle'$ are in the same p-block if $\lambda(\bar{p}) \neq \lambda$, see [2].

6. Let G be a group of order $m = m_0 p^a$, where $(p, m_0) = 1$. If c is a principal character of H, then $deg\ c \equiv 0\ mod\ p^a$, see [12].

7. If c is a principal character of G for an odd prime p and all entries in c are divisible by positive integer q, then c/q is a principal character of G, see [9].

8. Let p be odd and n be even, then from [13]:

a. If $p \nmid n$, then $\langle n\rangle = \varphi\langle n\rangle$ and $\langle n\rangle' = \varphi\langle n\rangle'$ are distinct irreducible modular spin characters of degree $2^{(n-2)/2}$.

b. If $p \nmid n$ and $p \nmid (n-1)$, then $\langle n-1,1\rangle = \varphi\langle n-1,1\rangle^*$ is an irreducible modular spin characters of degree $2^{(n-2)/2}(n-2)$.

9. Let $\alpha = (\alpha_1, \alpha_2, ..., \alpha_m)$ be a bar partition of n, not a p-bar core, and B be the block containing $\langle\alpha\rangle$, then:

a. If $n - m - m_0$ is even, then all irreducible modular spin characters in B are double.

b. If $n - m - m_0$ is odd, then all irreducible modular spin characters in B are associate, (where m_0 is the number of parts of α divisible by p), see [14]. For more details, see [15,16].

We shall use the following notations next: Irreducible modular spin characters (I.M.S), Modular spin characters (M.S), Principal indecomposable spin character (P.I.S), and Principal spin character (P.S).

3 Decomposition Matrices for the Spin Characters of the symmetric groups S_n, $23 \leq n \leq 24$ for the prime $p = 17$

In the following sections, we calculate the decomposition matrices of the spin characters of the symmetric group S_n, when $23 \leq n \leq 24$ for the prime $p = 17$. In each we find the irreducible spin characters and $(17,\ \alpha)$–regular classes of S_n, $23 \leq n \leq 24$ when $p = 17$. All blocks in these sections are 17-blocks.

3.1 Decomposition Matrix for the Spin Characters of S_{23}

The symmetric group S_{23} has 155 irreducible spin characters and has 149 of $(17,\ \alpha)$-regular classes, then the decomposition matrix of the spin characters for S_{23}, $p = 17$ has 155 rows and 149 columns. By using Preliminary 5, there are 106 blocks of S_{23} four of them B_1, B_2, B_3, B_4 of defect 1. All the 102 remaining characters form their own blocks $B_5, B_6, ..., B_{106}$ of defect zero [10]. In the principal block B_1, all I.M.S. of the decomposition matrix are double (Preliminary 9) and $\langle\beta\rangle \neq \langle\beta\rangle'$. The principal block B_1 contains the irreducible spin characters: $\{\langle 23\rangle, \langle 17,6\rangle, \langle 17,6\rangle', \langle 16,6,1\rangle^*, \langle 15,6,2\rangle^*, \langle 14,6,3\rangle^*, \langle 13,6,4\rangle^*, \langle 12,6,5\rangle^*, \langle 10,7,6\rangle^*, \langle 9,8,6\rangle^*\}$ with 17-bar core $\langle 6\rangle$.

In the block B_2, all I.M.S. of the decomposition matrix are associate (Preliminary 9) and $\langle\beta\rangle \neq \langle\beta\rangle'$.

The block B_2 contains the irreducible spin characters: $\{\langle 22,1\rangle, \langle 22,1\rangle', \langle 18,5\rangle, \langle 18,5\rangle', \langle 17,5,1\rangle^*, \langle 15,5,2,1\rangle, \langle 15,5,2,1\rangle', \langle 14,5,3,1\rangle, \langle 14,5,3,1\rangle', \langle 13,5,4,1\rangle, \langle 13,5,4,1\rangle', \langle 11,6,5,1\rangle, \langle 11,6,5,1\rangle', \langle 10,7,5,1\rangle, \langle 10,7,5,1\rangle', \langle 9,8,5,1\rangle, \langle 9,8,5,1\rangle'\}$ has 17-bar core $\langle 5,1\rangle$.

In the block B_3, all I.M.S. of the decomposition matrix are associate (Preliminary 9) and $\langle\beta\rangle \neq \langle\beta\rangle'$. The block B_3 contains the irreducible spin characters: $\{\langle 21,2\rangle, \langle 21,2\rangle', \langle 19,4\rangle, \langle 19,4\rangle', \langle 17,4,2\rangle^*, \langle 16,4,2,1\rangle, \langle 16,4,2,1\rangle', \langle 14,4,3,2\rangle, \langle 14,4,3,2\rangle', \langle 12,5,4,2\rangle, \langle 12,5,4,2\rangle', \langle 11,6,4,2\rangle, \langle 11,6,4,2\rangle', \langle 10,7,4,2\rangle, \langle 10,7,4,2\rangle', \langle 9,8,4,2\rangle, \langle 9,8,4,2\rangle'\}$ has 17-bar core $\langle 4,2\rangle$.

In the block B_4, all I.M.S. of the decomposition matrix are associate (Preliminary 9) and $\langle\beta\rangle \neq \langle\beta\rangle'$. The block B_4 contains the irreducible spin characters: $\{\langle 20,2,1\rangle^*, \langle 19,3,1\rangle^*, \langle 18,3,2\rangle^*, \langle 17,3,2,1\rangle, \langle 17,3,2,1\rangle', \langle 13,4,3,2,1\rangle^*, \langle 12,5,3,2,1\rangle^*, \langle 11,6,3,2,1\rangle^*, \langle 10,7,3,2,1\rangle^*, \langle 9,8,3,2,1\rangle^*\}$ has 17-bar core $\langle 3,2,1\rangle$.

Proposition 3.1. The Brauer tree for the principal block B_1 is:

$\langle 23^*\rangle$ — $\langle 17,6\rangle = \langle 17,6\rangle'$ — $\langle 16,6,1\rangle^*$ — $\langle 15,6,2\rangle^*$ — $\langle 14,6,3\rangle$ — $\langle 13,6,2\rangle^*$ — $\langle 12,6,5\rangle^*$ — $\langle 10,7,6\rangle^*$ — $\langle 9,8,6\rangle^*$

Proof.
$deg\ \{\langle 23^*\rangle, \langle 16,6,1\rangle^*, \langle 14,6,3\rangle^*, \langle 12,6,5\rangle^*, \langle 9,8,6\rangle^*, \} \equiv 8\ mod 17$;
$deg\ \{\langle 17,6\rangle = \langle 17,6\rangle', \langle 15,6,2\rangle^*, \langle 13,6,4\rangle^*, \langle 10,7,6\rangle^*, \} \equiv -8\ mod 17$. (Preliminary 4)

By using (r, r')-inducing of P.i.s of S_{22} [see Appendix $D_{22,17}$ in Table 5] to S_{23}, we have:

$$G_1 \uparrow (6,12) S_{23} = \langle 23\rangle^* + \langle 17,6\rangle + \langle 17,6\rangle' = d_1.$$

$$G_3 \uparrow (6,12) S_{23} = \langle 17,6\rangle + \langle 17,6\rangle' + \langle 17,6\rangle^* = d_2.$$

$$G_5 \uparrow (6,12) S_{23} = \langle 16,6,1\rangle^* + \langle 15,6,2\rangle^* = d_3.$$

$$G_7 \uparrow (6,12)S_{23} = \langle 15,6,2\rangle^* + \langle 14,6,3\rangle^* = d_4.$$

$$G_9 \uparrow (6,12)S_{23} = \langle 14,6,3\rangle^* + \langle 13,6,4\rangle^* = d_5.$$

$$G_{11} \uparrow (6,12)S_{23} = \langle 13,6,4\rangle^* + \langle 12,6,5\rangle^* = d_6.$$

$$G_{13} \uparrow (6,12)S_{23} = \langle 12,6,5\rangle^* + \langle 10,7,6\rangle^* = d_7.$$

$$G_{15} \uparrow (6,12)S_{23} = \langle 10,7,6\rangle^* + \langle 9,8,6\rangle^* = d_8.$$

So, we have the Brauer tree for B_1 and the decomposition matrix for this block $D^{(1)}_{23,17}$ in Table 1

Table 1: $D^{(1)}_{23,17}$

The spin characters	The decomposition matrix for the block B_1							
$\langle 23\rangle^*$	1							
$\langle 17,6\rangle$	1	1						
$\langle 17,6\rangle'$	1	1						
$\langle 16,6,1\rangle^*$		1	1					
$\langle 15,6,2\rangle^*$			1	1				
$\langle 14,6,3\rangle^*$				1	1			
$\langle 13,6,4\rangle^*$					1	1		
$\langle 12,6,5\rangle^*$						1	1	
$\langle 10,7,6\rangle^*$							1	1
$\langle 9,8,6\rangle^*$								1
	d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8

Proposition 3.2. The Brauer tree for the block B_2 is:

$\langle 22,1\rangle$—$\langle 18,5\rangle$ $\langle 15,5,2,1\rangle$—$\langle 14,5,3,1\rangle$—$\langle 13,5,4,1\rangle$—$\langle 11,6,6,1\rangle$—
—$\langle 10,7,5,1\rangle$—$\langle 9,8,5,1\rangle$

$\langle 17,5,1\rangle^*$

$\langle 22,1\rangle'$—$\langle 18,5\rangle'$ $\langle 15,5,2,1\rangle'$—$\langle 14,5,3,1\rangle'$—$\langle 13,5,4,1\rangle'$—$\langle 11,6,6,1\rangle'$—
—$\langle 10,7,5,1\rangle'$—$\langle 9,8,5,1\rangle'$

Proof.
$deg\{\langle 22,1\rangle, \langle 22,1\rangle', \langle 17,5,1\rangle^*, \langle 14,5,3,1\rangle, \langle 14,5,3,1\rangle', \langle 11,6,5,1\rangle\langle 11,6,5,1\rangle', \langle 9,8,5,1\rangle, \langle 9,8,5,1\rangle'\} \equiv 1 \ mod 17$.
$deg\{\langle 18,5\rangle, \langle 18,5\rangle', \langle 15,5,2,1\rangle + \langle 15,5,2,1\rangle', \langle 13,5,4,1\rangle, \langle 13,5,4,1\rangle'\langle 10,7,5,1\rangle', \langle 10,7,5,1\rangle\} \equiv 1 \ mod 17$
(Preliminary 4).
By using (1,0)-inducing of P.I.S of S_{22} [see Appendix $D^{(1)}_{22,17}$ in Table 10] to S_{23}, we have:

$$G_1 \uparrow (1,0)S_{23} = \langle 22,1\rangle + \langle 18,5\rangle + \langle 18,5\rangle' + \langle 17,5,1\rangle^* = K_1$$
$$G_2 \uparrow (1,0)S_{23} = \langle 22,1\rangle' + \langle 18,5\rangle + \langle 18,5\rangle' + \langle 17,5,1\rangle^* = K_2$$
$$G_3 \uparrow (1,0)S_{23} = \langle 18,5\rangle + \langle 18,5\rangle' + 2\langle 17,5,1\rangle^* = K_3$$
$$G_5 \uparrow (1,0)S_{23} = \langle 17,5,1\rangle^* + \langle 15,5,2,1\rangle = d_{13}$$
$$G_6 \uparrow (1,0)S_{23} = \langle 17,5,1\rangle^* + \langle 15,5,2,1\rangle' = d_{14}$$
$$G_7 \uparrow (1,0)S_{23} = \langle 15,5,2,1\rangle + \langle 14,5,3,1\rangle = d_{15}$$
$$G_8 \uparrow (1,0)S_{23} = \langle 15,5,2,1\rangle' + \langle 14,5,3,1\rangle' = d_{16}$$
$$G_9 \uparrow (1,0)S_{23} = \langle 14,5,3,1\rangle + \langle 13,5,4,1\rangle = d_{17}$$
$$G_{10} \uparrow (1,0)S_{23} = \langle 14,5,3,1\rangle' + \langle 13,5,4,1\rangle' = d_{18}$$
$$G_{11} \uparrow (1,0)S_{23} = \langle 13,5,4,1\rangle + \langle 11,6,5,1\rangle = d_{19}$$
$$G_{12} \uparrow (1,0)S_{23} = \langle 13,5,4,1\rangle' + \langle 11,6,5,1\rangle' = d_{20}$$
$$G_{13} \uparrow (1,0)S_{23} = \langle 11,6,5,1\rangle + \langle 10,7,5,1\rangle = d_{21}$$
$$G_{14} \uparrow (1,0)S_{23} = \langle 11,6,5,1\rangle' + \langle 10,7,5,1\rangle' = d_{22}$$
$$G_{15} \uparrow (1,0)S_{23} = \langle 10,7,5,1\rangle + \langle 9,8,5,1\rangle = d_{23}$$
$$G_{16} \uparrow (1,0)S_{23} = \langle 10,7,5,1\rangle' + \langle 9,8,5,1\rangle' = d_{24}$$

$\langle 18,5,1\rangle \downarrow (1,0)S_{23} = \langle 18,5\rangle + \langle 17,5,1\rangle^* = d_{11}$ since $\langle 18,5,1\rangle$ i.ms in S_{24}
$\langle 18,5,1\rangle' \downarrow (1,0)S_{24} = \langle 18,5\rangle' + \langle 17,5,1\rangle^* = d_{12}$ since $\langle 18,5,1\rangle'$ i.m.s in S_{24} Then K_3 split to d_{11} and d_{12}.

Since $h_1 \uparrow (5,13)S_{23} = \langle 21,1\rangle^* + \langle 18,4\rangle^* \uparrow (5,13)S_{23}$
$= \langle 22,1\rangle + \langle 22,1\rangle' + \langle 18,5\rangle + \langle 18,5\rangle'$
$= K_4 = K_1 + K_2 - d_{11} - d_{12}$ and $\langle 22,1\rangle \neq \langle 22,1\rangle'$ and $\langle 18,5\rangle \neq \langle 18,5\rangle'$ on (17α)-regular classes then $K_1 - d_{12} = d_9$, $K_2 - d_{11} = d_{10}$

So, we have the Brauer tree for B_2 and the decomposition matrix for the block $D^{(2)}_{23,17}$ in Table 2.

Table 2: $D^{(2)}_{23,17}$

The spin characters	The decomposition matrix for the block B_2															
$\langle 22,1\rangle$	1															
$\langle 22,1\rangle'$		1														
$\langle 18,5\rangle$	1		1													
$\langle 18,5\rangle'$		1		1												
$\langle 17,5,1\rangle^*$			1	1	1	1										
$\langle 15,5,2,1\rangle$					1		1									
$\langle 15,5,2,1\rangle'$						1		1								
$\langle 14,5,3,1\rangle$							1		1							
$\langle 14,5,3,1\rangle'$								1		1						
$\langle 13,5,4,1\rangle$									1		1					
$\langle 13,5,4,1\rangle'$										1		1				
$\langle 11,6,5,1\rangle$											1		1			
$\langle 11,6,5,1\rangle'$												1		1		
$\langle 10,7,5,1\rangle$													1		1	
$\langle 10,7,5,1\rangle'$														1		1
$\langle 9,8,5,1\rangle$															1	
$\langle 9,8,5,1\rangle'$																1
	d_9	d_{10}	d_{11}	d_{12}	d_{13}	d_{14}	d_{15}	d_{16}	d_{17}	d_{18}	d_{19}	d_{20}	d_{21}	d_{22}	d_{23}	d_{24}

Proposition 3.3. The Brauer tree for the block B_3 is:

$\langle 21,2\rangle$—$\langle 19,4\rangle$ $\langle 16,4,2,1\rangle$—$\langle 14,4,3,2\rangle$—$\langle 12,5,4,2\rangle$—$\langle 11,6,4,2\rangle$—
—$\langle 10,7,4,2\rangle$—$\langle 9,8,4,2\rangle$

$\langle 17,4,2\rangle^*$

$\langle 21,2\rangle'$—$\langle 19,4\rangle'$ $\langle 16,4,2,1\rangle'$—$\langle 14,4,3,2\rangle'$—$\langle 12,5,4,2\rangle'$—$\langle 11,6,4,2\rangle'$—
—$\langle 10,7,4,2\rangle'$—$\langle 9,8,4,2\rangle'$

Proof.
$deg\{\langle 21,2\rangle,\langle 21,2\rangle',\langle 17,4,2\rangle^*+\langle 14,4,3,2\rangle,\langle 14,4,3,2\rangle',\langle 11,6,4,2\rangle\langle 11,6,4,2\rangle',\langle 9,8,4,2\rangle,\langle 9,8,4,2\rangle'\}\equiv 3\ mod 17$.
$deg\{\langle 19,4\rangle,\langle 19,4\rangle',\langle 16,4,2,1\rangle,\langle 16,4,2,1\rangle',\langle 12,5,4,2\rangle,\langle 12,5,4,2\rangle'\langle 10,7,4,2\rangle',\langle 10,7,4,2\rangle\}\equiv -3\ mod 17$ (Preliminary 4).
By using $(2,16)$-inducing of P.I.S of S_{22} [see Appendix $D^{(1)}_{22,17}$ in Table 11] to S_{23}, we have:
$h_1\uparrow(2,16)S_{23}=\langle 21,2\rangle+\langle 21,2\rangle'+\langle 19,4\rangle'+\langle 19,4\rangle'$
$=K_1=d_{25}+d_{26}$
$h_2\uparrow(2,16)S_{23}=\langle 19,4\rangle+\langle 19,4\rangle'+2\langle 17,4,2\rangle^*$
$=K_2=d_{27}+d_{28}$
$h_3\uparrow(2,16)S_{23}=2\langle 17,4,2\rangle^*+\langle 16,4,2,1\rangle+2\langle 16,4,2,1\rangle'$
$=K_3=d_{29}+d_{30}$
$h_4\uparrow(2,16)S_{23}=\langle 16,4,2,1\rangle+\langle 16,4,2,1\rangle'+\langle 14,4,3,2\rangle+\langle 14,4,3,2\rangle'=K_4=d_{31}+d_{32}$
$h_5\uparrow(2,16)S_{23}=\langle 14,4,3,2\rangle+\langle 14,4,3,2\rangle'+\langle 12,5,4,2\rangle+\langle 12,5,4,2\rangle'=K_5=d_{33}+d_{34}$
$h_6\uparrow(2,16)S_{23}=\langle 12,5,4,2\rangle+\langle 12,5,4,2\rangle'+\langle 11,6,4,2\rangle+\langle 11,6,4,2\rangle'=K_6=d_{35}+d_{36}$
$h_7\uparrow(2,16)S_{23}=\langle 11,6,4,2\rangle+\langle 11,6,4,2\rangle'+\langle 10,7,4,2\rangle+\langle 10,7,4,2\rangle'=K_7=d_{37}+d_{38}$
$h_8\uparrow(2,16)S_{23}=\langle 10,7,4,2\rangle+\langle 10,7,4,2\rangle'+\langle 9,8,4,2\rangle+\langle 9,8,4,2\rangle'=K_8=d_{39}+d_{40}$
Since $\langle 21,2\rangle\neq\langle 21,2\rangle'$, $\langle 19,4\rangle\neq\langle 19,4\rangle'$, $\langle 16,4,2,1\rangle\neq\langle 16,4,2,1\rangle'$, $\langle 14,4,3,2\rangle\neq\langle 14,4,3,2\rangle'$, $\langle 12,5,4,2\rangle\neq\langle 12,5,4,2\rangle'$, $\langle 11,6,4,2\rangle\neq\langle 11,6,4,2\rangle\prime$, $\langle 10,7,4,2\rangle\neq\langle 10,7,4,2\rangle'$, $\langle 9,8,4,2\rangle\neq\langle 9,8,4,2\rangle'$ on $(17,\alpha)$-regular classes then $K_1,K_2,K_3,K_4,K_5,K_6,K_7,K_8$ are split respectively. So, we have the Brauer tree for B_3 and the decomposition matrix for the block $D^{(3)}_{23,17}$ in Table 3.

Proposition 3.4. The Brauer tree for the block B_4 is:
$\langle 20,2,1\rangle^*$—$\langle 19,3,1\rangle^*$—$\langle 18,3,2\rangle$—$\langle 17,3,2,1\rangle=$
$\langle 17,3,2,1\rangle'$—$\langle 13,4,3,2,1\rangle^*$—
$\langle 12,5,3,2,1\rangle^*$—$\langle 11,6,3,2,1\rangle^*$—$\langle 10,7,3,2,1\rangle^*$—$\langle 9,8,3,2,1\rangle^*$

Proof.
$deg\ \{\langle 20,2,1\rangle^*,\langle 18,3,2\rangle,\langle 13,4,3,2,1\rangle',\langle 11,6,3,2,1\rangle^*,\langle 9,8,3,2,1\rangle^*\}\equiv 8\ mod 17$.
$deg\ \{\langle 19,31\rangle^*,(\langle 17,3,2,1\rangle+\langle 17,3,2,1\rangle'),\langle 12,5,3,2,1\rangle^*,\langle 10,7,3,2,1\rangle^*\}\equiv -8\ mod 17$. (Preliminary 4)
We apply (r,r')-inducing of P.I.S of S_{22} to S_{23}, [see Appendix $D^{(3)}_{22,17}$ in Table 12], we have:
$H_1\uparrow(0,1)S_{23}=\langle 20,2,1\rangle^*+\langle 19,3,1\rangle^*=d_{41}$
$H_2\uparrow(0,1)S_{23}=\langle 19,3,1^*\rangle+2\langle 18,3,2\rangle+\langle 17,3,2,1\rangle'=K_1$
$H_3\uparrow(0,1)S_{23}=2\langle 18,3,2\rangle+2\langle 17,3,2,1\rangle+2\langle 17,3,2,1\rangle'=2d_{43}$
$H_4\uparrow(0,1)S_{23}=\langle 17,3,2,1\rangle^*+\langle 13,4,3,2,1\rangle^*=d_{44}$
$H_5\uparrow(0,1)S_{23}=\langle 13,4,3,2,1\rangle^*+\langle 12,5,3,2,1\rangle^*=d_{45}$
$H_6\uparrow(0,1)S_{23}=\langle 12,5,3,2,1\rangle^*+\langle 11,6,3,2,1\rangle^*=d_{46}$
$H_7\uparrow(0,1)S_{23}=\langle 11,6,3,2,1\rangle^*+\langle 10,7,3,2,1\rangle^*=d_{47}$
$H_8\uparrow(0,1)S_{23}=\langle 10,7,3,2,1\rangle^*+\langle 9,8,3,2,1\rangle^*=d_{48}$
Since $\langle 19,3,2\rangle^*\downarrow(2,16)S_{23}=\langle 19,3,1\rangle^*+\langle 18,3,2\rangle^*=$

Table 3: $D^{(2)}_{23,17}$

The spin characters	The decomposition matrix for the block B_3															
$\langle 21,2\rangle$	1															
$\langle 21,2\rangle'$		1														
$\langle 19,4\rangle$	1		1													
$\langle 19,4\rangle'$		1		1												
$\langle 17,4,2\rangle^*$			1	1	1	1										
$\langle 16,4,2,1\rangle$					1		1									
$\langle 16,4,2,1\rangle'$						1		1								
$\langle 14,4,3,2\rangle$							1		1							
$\langle 14,4,3,2\rangle'$								1		1						
$\langle 12,5,4,2\rangle$									1		1					
$\langle 12,5,4,2\rangle'$										1		1				
$\langle 11,6,4,2\rangle$											1		1			
$\langle 11,6,4,2\rangle'$												1		1		
$\langle 10,7,4,2\rangle$													1		1	
$\langle 10,7,4,2\rangle'$														1		1
$\langle 9,8,4,2\rangle$															1	
$\langle 9,8,4,2\rangle'$																1
	d_{25}	d_{26}	d_{27}	d_{28}	d_{29}	d_{30}	d_{31}	d_{32}	d_{33}	d_{34}	d_{35}	d_{36}	d_{37}	d_{38}	d_{39}	d_{40}

$d_{42}=K_1-d_{43}$ and $\langle 19,3,2\rangle^*$ i.m.s in S_{24}. So, we have the Brauer tree for B_4, and the decomposition matrix for this block $D^{(4)}_{23,17}$ in Table (4).

Table 4: $D^{(4)}_{23,17}$

The spin characters	The decomposition matrix for the block B_4							
$\langle 20,2,1\rangle^*$	1							
$\langle 19,3,1\rangle^*$	1	1						
$\langle 18,3,2\rangle^*$		1	1					
$\langle 17,3,2,1\rangle$			1	1				
$\langle 17,3,2,1\rangle'$			1	1				
$\langle 13,4,3,2,1\rangle^*$				1	1			
$\langle 12,4,3,2,1\rangle^*$					1	1		
$\langle 11,6,3,2,1\rangle^*$						1	1	
$\langle 10,7,3,2,1\rangle^*$							1	1
$\langle 9,8,3,2,1\rangle^*$								1
	d_{41}	d_{42}	d_{43}	d_{44}	d_{45}	d_{46}	d_{47}	d_{48}

(3.2) Decomposition Matrix for the spin Characters of S_{24}

The Symmetric group S_{24} has 183 irreducible spin characters and 175 of $(17,\alpha)$-regular classes, then the decomposition matrix of the spin characters for S_{24}, p=17 has 183 rows and 175 columns. By using (Preliminary 5), there are 124 blocks of S_{24} five of them B_1,B_2,B_3,B_4,B_5 of defect 1. All the 119 remaining characters form their own blocks $B_6,\cdots,B_{124}$ of defect zero [James and kerber,1981] [10]. In the block B_1 all i.m.s. of the decomposition matrix are double (Preliminary 9) and $<\beta>\neq<\beta>'$.

The block B_1 contains the irreducible spin characters $\{<23,1>^*,<18,6>^*,<17,6,1>,<17,6,1>'$

$< 15,6,2,1 >^*, < 14,6,3,1 >^*, < 13,6,4,1 >^*, < 12,6,4,1 >^*, < 10,7,6,1 >^*, < 9,8,6,1 >^*\}$ with 17-bar core $< 6,1 >$

In the block B_2 of the i.m.s of the decomposition matrix are double (Preliminary 9) and $< \beta > \neq < \beta >'$.

The block B_2 contains the irreducible spin characters $\{< 22,2 >^*, < 19,5 >^*, < 17,5,2 >, < 17,5,2 >'$ $< 16,5,2,1 >^*, < 14,5,3,2 >^*, < 13,5,4,2 >^*, < 11,6,5,2 >^*, < 10,7,5,2 >^*, < 9,8,5,2 >^*\}$ with 17-bar core $< 5,2 >$

In the block B_3 of the i.m.s of the decomposition matrix are double (Preliminary 9) and $< \beta > \neq < \beta >'$.

The block B_3 contains the irreducible spin characters $\{< 21,3 >^*, < 20,4 >^*, < 17,4,3 >, < 17,4,3 >'$ $< 16,4,3,1 >^*, < 15,4,3,2 >^*, < 12,5,4,3 >^*, < 11,6,4,3 >^*, < 10,7,4,3 >^*, < 9,8,4,3 >^*\}$ with 17-bar core $< 4,3 >$

In the block B_4 of the i.m.s of the decomposition matrix are double (Preliminary 9) and $< \beta > \neq < \beta >'$.

The block B_4 contains the irreducible spin characters $\{< 21,2,1 >, < 21,2,1 >', < 19,4,1 >, < 19,4,1 >', < 18,4,2 >, < 18,4,2 >', < 17,4,2,1 >^*, < 14,4,3,2,1 >, < 14,4,3,2,1 >', < 12,5,4,2,1 >, < 12,5,4,2,1 >', < 11,6,4,2,1 >, < 11,6,4,2,1 >', < 10,7,4,2,1 >, < 10,7,4,2,1 >', < 9,8,4,2,1 >, < 9,8,4,2,1 >'\}$ with 17-bar core $< 4,2,1 >$.

In the principal block B_3 of the i.m.s of the decomposition matrix are double (Preliminary 9) and $< \beta > \neq < \beta >'$.

The block B_5 contains the irreducible spin characters $\{< 24 >^*, < 24 >', < 17,7 >^*, < 16,7,1 >, < 16,7,1 >'$ $< 15,7,2 >, < 15,7,2 >', < 13,7,4 >, < 13,7,4 >', < 12,7,5 >, < 12,7,5 >', < 11,7,6 >, < 11,7,6 >', < 9,8,7 >, < 9,8,7 >'\}$ with 17-bar core $< 7 >$

Proposition 3.5. The Brauer tree for the block B_1 is:
$\langle 23,1\rangle^*$—$\langle 18,6\rangle^*$—$\langle 17,6,1\rangle = \langle 17,6,1\rangle'$—$\langle 15,6,2,1\rangle^*$—$\langle 14,6,3,1\rangle^*$—$\langle 13,6,4,1\rangle^*$—$\langle 12,6,5,1\rangle^*$—$\langle 10,7,6,1\rangle^*$—$\langle 9,8,6,1\rangle^*$

Proof.

$deg\,\{\langle 23,1\rangle^*, \langle 17,6,1\rangle = \langle 17,6,1\rangle', \langle 12,6,5,1\rangle^*, \langle 9,8,6,1\rangle^*$ $\langle 8,6,5,3\rangle^*\} \equiv 6\ mod\,17$.
$deg\ \{\langle 18,6\rangle^*, \langle 15,6,2,1\rangle^*, \langle 13,6,4,1\rangle^*, \langle 10,7,6,1\rangle^*\} \equiv -6\ mod\,17$. (Preliminary 4)

By using (r,r')-inducing of P.I.S of S_{23} to S_{24}, we have:
$d_9 \uparrow (6,12)S_{24} = D_1$
$d_{11} \uparrow (6,12)S_{24} = D_2$
$d_3 \uparrow (6,12)S_{24} = D_3$
$d_4 \uparrow (6,12)S_{24} = D_4$
$d_5 \uparrow (6,12)S_{24} = D_5$
$d_6 \uparrow (6,12)S_{24} = D_6$
$d_7 \uparrow (6,12)S_{24} = D_7$
$d_8 \uparrow (6,12)S_{24} = D_8$
So, we have the Brauer tree for B_1 and the decomposition matrix for this block $D^{(1)}_{24,17}$ in Table 5

Proposition 3.6. The Brauer tree for the block B_2 is:
$\langle 22,2\rangle^*$—$\langle 19,5\rangle^*$—$\langle 17,5,2\rangle = \langle 17,5,2\rangle'$—$\langle 16,5,2,1\rangle^*$—$\langle 14,5,3,2\rangle^*$—$\langle 13,5,4,2\rangle^*$—$\langle 11,6,5,2\rangle^*$—$\langle 10,7,5,2\rangle^*$—$\langle 9,8,5,2\rangle^*$

Table 5: $D^{(1)}_{24,17}$

The spin characters	The decomposition matrix for the block B_1							
$\langle 23,1\rangle^*$	1							
$\langle 18,6\rangle^*$	1	1						
$\langle 17,6,1\rangle$		1	1					
$\langle 17,6,1\rangle'$		1	1					
$\langle 15,6,2,1\rangle^*$			1	1				
$\langle 14,6,3,1\rangle^*$				1	1			
$\langle 13,6,4,1\rangle^*$					1	1		
$\langle 12,6,4,1\rangle^*$						1	1	
$\langle 10,7,6,1\rangle^*$							1	1
$\langle 9,8,6,1\rangle^*$								1
	D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8

Proof.

$deg\,\{\langle 22,2\rangle^*, \langle 17,5,2\rangle = \langle 17,5,2\rangle', \langle 14,5,3,2\rangle^*, \langle 11,6,5,2\rangle^*$ $\langle 9,8,5,2\rangle^*\} \equiv 4\ mod\,17$.
$deg\ \{\langle 19,5\rangle^*, \langle 16,5,2,1\rangle^*, \langle 13,5,4,2\rangle^*, \langle 10,7,5,2\rangle^*\} \equiv -4\ mod\,17$. (Preliminary 4)

By using (r,r')-inducing of P.I.S of S_{23} to S_{24}, we have:
$d_9 \uparrow (6,12)S_{24} = D_9$
$d_{11} \uparrow (6,12)S_{24} = D_{10}$
$d_{13} \uparrow (6,12)S_{24} = D_{11}$
$d_{15} \uparrow (6,12)S_{24} = D_{12}$
$d_{17} \uparrow (6,12)S_{24} = D_{13}$
$d_{19} \uparrow (6,12)S_{24} = D_{14}$
$d_{21} \uparrow (6,12)S_{24} = D_{15}$
$d_{23} \uparrow (6,12)S_{24} = D_{16}$
So, we have the Brauer tree for B_2 and the decomposition matrix for this block $D^{(2)}_{24,17}$ in Table 6

Table 6: $D^{(2)}_{24,17}$

The spin characters	The decomposition matrix for the block B_2							
$\langle 22,2\rangle^*$	1							
$\langle 19,5\rangle^*$	1	1						
$\langle 17,5,2\rangle$		1	1					
$\langle 17,5,2\rangle'$		1	1					
$\langle 16,5,2,1\rangle^*$			1	1				
$\langle 14,5,3,2\rangle^*$				1	1			
$\langle 13,5,4,2\rangle^*$					1	1		
$\langle 11,6,5,2\rangle^*$						1	1	
$\langle 10,7,5,2\rangle^*$							1	1
$\langle 9,8,5,2\rangle^*$								1
	D_9	D_{10}	D_{11}	D_{12}	D_{13}	D_{14}	D_{15}	D_{16}

Proposition 3.7. The Brauer tree for the block B_3 is:
$\langle 21,3\rangle^*$—$\langle 20,4\rangle^*$—$\langle 17,4,3\rangle = \langle 17,4,3\rangle'$—$\langle 16,4,3,1\rangle^*$—

$\langle 15,4,3,2\rangle^{*}$—$\langle 12,5,4,3\rangle^{*}$—$\langle 11,6,4,3\rangle^{*}$—$\langle 10,7,4,3\rangle^{*}$—$\langle 9,8,4,3\rangle^{*}$

Proof.

$deg\,\{\langle 21,3\rangle^{*},\langle 17,4,3\rangle{=}\langle 17,4,3\rangle',\langle 15,4,3,2\rangle^{*},\langle 11,6,4,3\rangle^{*}\ \langle 9,8,4,3\rangle^{*}\}\equiv 6\ mod 17.$
$deg\ \ \{\langle 20,4\rangle^{*},\langle 16,4,3,1\rangle^{*},\langle 12,5,4,3\rangle^{*},\langle 10,7,4,3\rangle^{*}\}\equiv -6\ mod 17.$ (Preliminary 4)

By using (r,r')-inducing of P.I.S of S_{23} to S_{24}, we have:
$d_{25}\uparrow(6,12)S_{24}=D_{17}$
$d_{27}\uparrow(6,12)S_{24}=D_{18}$
$d_{29}\uparrow(6,12)S_{24}=D_{19}$
$d_{31}\uparrow(6,12)S_{24}=D_{20}$
$d_{33}\uparrow(6,12)S_{24}=D_{21}$
$d_{35}\uparrow(6,12)S_{24}=D_{22}$
$d_{37}\uparrow(6,12)S_{24}=D_{23}$
$d_{39}\uparrow(6,12)S_{24}=D_{24}$
So, we have the Brauer tree for B_3 and the decomposition matrix for this block $D^{(3)}_{24,17}$ in Table 7

Table 7: $D^{(3)}_{24,17}$

The spin characters	The decomposition matrix for the block B_3							
$\langle 21,3\rangle^{*}$	1							
$\langle 20,4\rangle^{*}$	1	1						
$\langle 17,4,3\rangle$		1	1					
$\langle 17,4,3\rangle'$		1	1					
$\langle 16,4,3,1\rangle^{*}$			1	1				
$\langle 15,4,3,2\rangle^{*}$				1	1			
$\langle 12,5,4,3\rangle^{*}$					1	1		
$\langle 11,6,4,3\rangle^{*}$						1	1	
$\langle 10,7,4,3\rangle^{*}$							1	1
$\langle 9,8,4,3\rangle^{*}$								1
	D_{17}	D_{18}	D_{19}	D_{20}	D_{21}	D_{22}	D_{23}	D_{24}

Proposition 3.8. The Brauer tree for the block B_4 is:

$\langle 21,2,1\rangle$—$\langle 19,4,1\rangle$—$\langle 18,4,2\rangle$ $\quad$ $\langle 14,4,3,2,1\rangle$—$\langle 12,5,4,2,1\rangle$—$\langle 11,6,4,2,1\rangle$—$\langle 10,7,4,2,1\rangle$—$\langle 9,8,4,2,1\rangle$

$\langle 17,4,2,1\rangle^{*}$

$\langle 21,2,1\rangle'$—$\langle 19,4,1\rangle'$—$\langle 18,4,2\rangle'$ $\quad$ $\langle 14,4,3,2,1\rangle'$—$\langle 12,5,4,2,1\rangle'$—$\langle 11,6,4,2,1\rangle'$—$\langle 10,7,4,2,1\rangle'$—$\langle 9,8,4,2,1\rangle'$

Proof.

$deg\{\langle 19,4,1\rangle,\langle 19,4,1\rangle',\langle 17,4,2,1\rangle^{*}{+}\langle 12,5,4,2,1\rangle,\ \langle 12,5,4,2,1\rangle',\langle 10,7,4,2,1\rangle\langle 10,7,4,2,1\rangle'\}{\equiv}6\ mod 17.$
$deg\{\langle 21,21\rangle,\langle 21,21\rangle',\langle 18,4,2\rangle,\langle 18,4,2\rangle',\langle 14,4,3,2,1\rangle,\ \langle 14,4,3,2,1\rangle',\langle 11,6,4,2,1\rangle\}{\equiv}-6\ \ mod 17$ (Preliminary 4).

By using (r,r')-inducing of P.I.S of S_{23} to S_{24}, we have:
$d_{25}\uparrow(0,1)S_{24}=D_{25}$
$d_{26}\uparrow(0,1)S_{24}=D_{26}$
$d_{27}\uparrow(0,1)S_{24}=K_1=D_{27}+D_{28}$
$d_{29}\uparrow(0,1)S_{24}=K_2=D_{29}+D_{30}$
$d_{31}\uparrow(0,1)S_{24}=D_{31}$
$d_{32}\uparrow(0,1)S_{24}=D_{32}$
$d_{33}\uparrow(0,1)S_{24}=D_{33}$
$d_{34}\uparrow(0,1)S_{24}=D_{34}$
$d_{35}\uparrow(0,1)S_{24}=D_{35}$
$d_{36}\uparrow(0,1)S_{24}=D_{36}$
$d_{37}\uparrow(0,1)S_{24}=D_{37}$
$d_{38}\uparrow(0,1)S_{24}=D_{38}$
$d_{39}\uparrow(0,1)S_{24}=D_{39}$
$d_{40}\uparrow(0,1)S_{24}=D_{40}$
$\langle 18,4,2,1\rangle\downarrow S_{24}=\langle 18,4,2\rangle+,\langle 17,4,2,1\rangle^{*}=D_{29}$, since $\langle 18,4,2,1\rangle$ i.m.s in S_{25}
$\langle 18,4,2,1\rangle'\downarrow S_{24}=\langle 18,4,2\rangle'+,\langle 17,4,2,1\rangle^{*}=D_{30}$, since $\langle 18,4,2,1\rangle'$ i.m.s in S_{25}
Then K_2 split to D_{29} and D_{30}.
Since $\langle 19,4,1\rangle\neq\langle 19,4,1\rangle'$ and $\langle 18,4,2\rangle\neq\langle 18,4,2\rangle'$ on $(17,\alpha)$-regular classes then K_1 split to D_{27} and D_{28}
So, we have the Brauer tree for B_4 and the decomposition matrix for the block $D^{(4)}_{24,17}$ in Table 8.

Table 8: $D^{(4)}_{24,17}$

The spin characters	The decomposition matrix for the block B_4															
$\langle 21,2,1\rangle$	1															
$\langle 21,2,1\rangle'$		1														
$\langle 19,4,1\rangle$	1		1													
$\langle 19,4,1\rangle'$		1		1												
$\langle 18,4,2\rangle$			1		1											
$\langle 18,4,2\rangle'$				1		1										
$\langle 17,4,2,1\rangle^{*}$					1	1	1	1								
$\langle 14,4,3,2,1\rangle$							1		1							
$\langle 14,4,3,2,1\rangle'$								1		1						
$\langle 12,5,4,2,1\rangle$									1		1					
$\langle 12,5,4,2,1\rangle'$										1		1				
$\langle 11,6,4,2,1\rangle$											1		1			
$\langle 11,6,4,2,1\rangle'$												1		1		
$\langle 10,7,4,2,1\rangle$													1		1	
$\langle 10,7,4,2,1\rangle'$														1		1
$\langle 9,8,4,2,1\rangle$															1	
$\langle 9,8,4,2,1\rangle'$																1
	D_{25}	D_{26}	D_{27}	D_{28}	D_{29}	D_{30}	D_{31}	D_{32}	D_{33}	D_{34}	D_{35}	D_{36}	D_{37}	D_{38}	D_{39}	D_{40}

Proposition 3.9. The Brauer tree for the block B_5 is:

$\langle 24\rangle \quad \langle 16,7,1\rangle$—$\langle 15,7,2\rangle$—$\langle 14,7,3\rangle$—$\langle 13,7,4\rangle$—$\langle 12,7,5\rangle$—$\langle 11,7,6\rangle$—$\langle 9,8,7\rangle$

$\langle 17,7\rangle^*$

$\langle 24\rangle' \quad \langle 16,7,1\rangle'$—$\langle 15,7,2\rangle'$—$\langle 14,7,3\rangle'$—$\langle 13,7,4\rangle'$—$\langle 12,7,5\rangle'$—$\langle 11,7,6\rangle'$—$\langle 9,8,7\rangle'$

Proof.
$deg\{\langle 24\rangle,\langle 24\rangle',\langle 16,7,1\rangle,\langle 16,7,1\rangle',\langle 14,7,3\rangle,\langle 14,7,3\rangle'$
$\langle 12,7,5\rangle,\langle 12,7,5\rangle',\langle 9,8,7\rangle\langle 9,8,7\rangle'\}\equiv 8\ mod 17$.
$deg\{\langle 24\rangle,\langle 17,7\rangle',\langle 15,7,2\rangle,\langle 15,7,2\rangle',\langle 13,7,4\rangle,\langle 13,7,4\rangle'$
$\langle 11,7,6\rangle,\langle 11,7,6\rangle'\}\equiv -8\ mod 17$. (Preliminary 4).
By using (r,r')-inducing of P.I.S of S_{23} to S_{24}, we have:
$d_1 \uparrow (7,11)S_{24} = K_1 = D_{41} + D_{42}$
$d_2 \uparrow (7,11)S_{24} = K_2 = D_{43} + D_{44}$
$d_3 \uparrow (7,11)S_{24} = K_3 = D_{45} + D_{46}$
$d_4 \uparrow (7,11)S_{24} = K_4 = D_{47} + D_{48}$
$d_5 \uparrow (7,11)S_{24} = K_5 = D_{49} + D_{50}$
$d_6 \uparrow (7,11)S_{24} = K_6 = D_{51} + D_{52}$
$d_7 \uparrow (7,11)S_{24} = K_7 = D_{53} + D_{54}$
$d_8 \uparrow (7,11)S_{24} = K_8 = D_{55} + D_{56}$

Since $\langle 24\rangle \neq \langle 24\rangle'$ are distinct irreducible modular spin characters Property (8) and $\langle 17,7\rangle^*$ contains $\langle 24\rangle,\langle 24\rangle'$ with the same multiplicity [12] then K_1 split D_{41}, D_{42}.
$\langle 16,7\rangle \uparrow (1,0)S_{20} = \langle 17,7\rangle^* + \langle 16,7,1\rangle = D_{43}$ since $\langle 16,6\rangle$ i.m.s in S_{23} and
$\langle 16,7\rangle' \uparrow (1,0)S_{20} = \langle 17,7\rangle^* + \langle 16,7,1\rangle' = D_{44}$ since $\langle 16,6\rangle'$ i.m.s in S_{23}
Also, we have $\langle 15,7,2\rangle \neq \langle 15,7,2\rangle'$, $\langle 14,7,3\rangle \neq \langle 14,7,3\rangle'$,$\langle 13,7,4\rangle \neq \langle 13,7,4\rangle'$, $\langle 12,7,5\rangle \neq \langle 12,7,5\rangle'$,$\langle 11,7,6\rangle \neq \langle 11,7,6\rangle'$, $\langle 9,8,7\rangle \neq \langle 9,8,7\rangle'$ on $(17,\alpha)$-regular classes then K_3,K_4,K_5,K_6,K_7,K_8 are split respectively.
So, we have the Brauer tree for B_5 and the decomposition matrix for the block $D^{(5)}_{24,17}$ in Table 9.

Table 9: $D^{(5)}_{24,17}$

The spin characters	The decomposition matrix for the block B_5															
$\langle 24\rangle$	1															
$\langle 24\rangle'$		1														
$\langle 17,7\rangle^*$	1	1	1	1												
$\langle 16,7,1\rangle$			1		1											
$\langle 16,7,1\rangle'$				1		1										
$\langle 15,7,2\rangle$					1		1									
$\langle 15,7,2\rangle'$						1		1								
$\langle 14,7,3\rangle$							1		1							
$\langle 14,7,3\rangle'$								1		1						
$\langle 13,7,4\rangle$									1		1					
$\langle 13,7,4\rangle'$										1		1				
$\langle 12,7,5\rangle$											1		1			
$\langle 12,7,5\rangle'$												1		1		
$\langle 11,7,6\rangle$													1		1	
$\langle 11,7,6\rangle'$														1		1
$\langle 9,8,7\rangle$															1	
$\langle 9,8,7\rangle'$																1
	D_{41}	D_{42}	D_{43}	D_{44}	D_{45}	D_{46}	D_{47}	D_{48}	D_{49}	D_{50}	D_{51}	D_{52}	D_{53}	D_{54}	D_{55}	D_{56}

Discussion and Conclusion

In this work, motivated by previous results given in the papers [2,4,6,12,16], we conclude that all blocks of defect one and the decomposition numbers are zero or one. Also we compute the Brauer trees of the symmetric group S_n, $23 \leq n \leq 24$ modulo $P = 17$. Finally, all the 17-decomposition matrices of spin characters of S_n, $23 \leq n \leq 24$, are found. The new results are very useful to find the decomposition matrices for the spin characters of large $n > 24$, which is equivalent to find irreducible modular spin characters of the symmetric group by using (r,r')-inducing(restricting) method.

Acknowledgement

The authors are grateful to the reviewers for a careful checking of the details and for helpful comments that improved this paper.

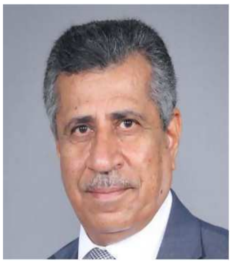
Conflict of interest

The authors declare that they have no conflict of interest.

References

[1] A.O. Morris, The spin representation of the symmetric group, *proc. London Math. Soc.*, **12**, 55-76 (1962).
[2] A.O. Morris and A.K. Yaseen, Decomposition matrices for spin characters of symmetric group, *Proc. Of Royal Society of Edinburgh*, **108**A, 145–164 (1988).
[3] J.F. Humphreys, Blocks of the projective representations of symmetric groups, *J. London math. Society*, **33**, 441-452 (1986).
[4] A.K. Yaseen and M.B. Tahir, 17-Decomposition matrices for the spin characters of symmetric groups S_n, $17 \leq n \leq 22$, *Adv Math Sci Journal*, **9**, 10541-10561 (2020).
[5] S.A. Taban and M.M. Jawad, Brauer trees for spin characters of S_n, $13 \leq n \leq 20$ *modulo* $p = 13$, *Basrah Journal of Science*, **35**, 106-112 (2017).
[6] A.K. Yaseen, The Brauer trees of the symmetric groups S_{21}, *modulo* $p = 13$, *Basrah Journal of Science*, **37**, 126-140 (2019).
[7] A.K. Yaseen and M. B. Tahir, 13-Brauer trees of the symmetric group S_{22}, *Appl. Math. & Inf. Sci.*, **14**, 327-334 (2020).
[8] A. H. Jassim and M. M. Jawad, Decomposition matrices for the spin characters of S_{26} *modulo* 11, *Basrah Journal of Science*, **37**, (2019).
[9] L. Dornhoff, *Group Representation Theory*, Marcel Dekker Inc, New York, (1971).
[10] G.D. James, *The Representation Theory of the Symmetric Groups*, Springer, Berlin, Heidelberg, (1978).
[11] B.M. Puttaswamaiah and J. D. Dixon, *Modular Representation of Finite Groups*, Academic press, London, (1977).

[12] G. James, The modular characters of Mathew groups, *J. Algebra*, **27**, 57-111 (1973).
[13] D.B. Wales, Some projective representations of S_n, *J. Algebra*, **61**, 37-57 (1979).
[14] C. Bessenrodt, A.O. Morris and J. B. Olsson, Decomposition matrices for spin characters of symmetric groups at characteristic 3, *J. Algebra*, **164**, 146-172 (1994).
[15] C.W. Curtis and I. Reiner, *Representation Theory of Finite Groups and Associative Algebras*, AMS Chelsea Publishing, American Mathematical Society, Rhode Island, (1966).
[16] L. Mass. *Modular Spin Characters of Symmetric Groups*. PhD thesis, Universitat Duisburg–Essen, Essen, (2011).



AbdulKareem A.Yaseen He received his Ph.D degree in Mathematics, Modern Algebra, at the University of Wales, Aberystwyth, UK. His research area is the computation of the modular spin characters of the symmetric group. He has published many research papers in reputed national and international scientific journals.

Marwa Mohammed Jawad Al-musawi She received here master degree at University of Basrah in Iraq. She interested in researching about decomposition matrices of the symmetric group and she has published papers in this area of research

Appendix

The decomposition matrix for the spin characters of the symmetric group S_{22}, p=17.

Table 10: $D^{(1)}_{22,17}$

The spin characters	The decomposition matrix for the block B_1															
$\langle 22\rangle$	1															
$\langle 22\rangle'$		1														
$\langle 17,5\rangle^*$	1	1	1	1												
$\langle 16,5,1\rangle$			1		1											
$\langle 16,5,1\rangle'$				1		1										
$\langle 15,5,2\rangle$					1		1									
$\langle 15,5,2\rangle'$						1		1								
$\langle 14,5,3\rangle$							1		1							
$\langle 14,5,3\rangle'$								1		1						
$\langle 13,5,4\rangle$									1		1					
$\langle 13,5,4\rangle'$										1		1				
$\langle 11,6,5\rangle$											1		1			
$\langle 11,6,5\rangle'$												1		1		
$\langle 10,7,5\rangle$													1		1	
$\langle 10,7,5\rangle'$														1		1
$\langle 9,8,5\rangle$															1	
$\langle 9,8,5\rangle'$																1
	G_1	G_2	G_3	G_4	G_5	G_6	G_7	G_8	G_9	G_{10}	G_{11}	G_{12}	G_{13}	G_{14}	G_{15}	G_{16}

Table 11: $D^{(2)}_{22,17}$

The spin characters	The decomposition matrix for the block B_2							
$\langle 21,1\rangle^*$	1							
$\langle 18,4\rangle^*$	1	1						
$\langle 17,4,1\rangle$		1	1					
$\langle 17,4,1\rangle'$		1	1					
$\langle 15,4,2,1\rangle^*$			1	1				
$\langle 14,4,3,1\rangle^*$				1	1			
$\langle 12,5,4,1\rangle^*$					1	1		
$\langle 11,6,4,1\rangle^*$						1	1	
$\langle 10,7,4,1\rangle^*$							1	1
$\langle 9,8,4,1\rangle^*$								1
	h_1	h_2	h_3	h_4	h_5	h_6	h_7	h_8

Table 12: $D^{(3)}_{22,17}$

The spin characters	The decomposition matrix for the block B_3							
$\langle 20,2\rangle^*$	1							
$\langle 19,3\rangle^*$	1	1						
$\langle 17,3,2\rangle$		1	1					
$\langle 17,3,2\rangle'$		1	1					
$\langle 16,3,2,1\rangle^*$			1	1				
$\langle 13,4,3,2\rangle^*$				1	1			
$\langle 12,5,3,2\rangle^*$					1	1		
$\langle 11,6,3,2\rangle^*$						1	1	
$\langle 10,7,3,2\rangle^*$							1	1
$\langle 9,8,3,2\rangle^*$								1
	H_1	H_2	H_3	H_4	H_5	H_6	H_7	H_8