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Comparison of Different Entropy Measures Using the Relative Loss Approach for the Lomax Distribution

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Abstract: The entropy measures are useful to measures the uncertainty or randomness in data. Their applications are found in Physics, electrical engineering, computer science, and geophysics, etc. In this paper, we considered different entropy measures to evaluate which one of these entropy measures is reliable while modeling the data by the Lomax distribution. For this purpose, the relative loss of each entropy is computed by using the truncated form of the Lomax distribution on $[0,t)$ instead of considering the Lomax distribution on $[0,\infty)$. For demonstration purposes, numerical results are obtained and it concluded that the Shannon, Renyi, and Sharma & Mittal entropy measures perform better than others.

Keywords: Entropies, Lomax distribution, truncated Lomax distribution, relative loss.

1 Introduction

The concept of entropy measures was first initiated by Shannon working in information theory [1]. Due to its wide applicability in various fields, for example, in Physics, it measures the disorders in a data, in statistics, it measures the randomness in a data; many researchers modified the Shannon entropy, for example, Renyi [2], Havrda & Charvat [3], Arimoto's [4] Sharma & Mittal [5].

The main focus of this paper is to delineate the relative significance of the above mentioned entropy measures on each other and also to decide which one of these entropies perform well as compared to the rest using the Lomax distribution.

Let X is a continuous nonnegative random variable having probability density function $f(x)$. The mathematical expression of the Shannon [1] entropy is:

$$H(X) = -\int_{R_X} f(x)\ln f(x)dx, \tag{1}$$

where $R_X = (x : f(x) \neq 0)$.

Renyi [2] studied the concept of uncertainty and randomness and formed new generalized entropy. The Renyi entropy is defined as:

$$H_\alpha(X) = \frac{1}{1-\alpha}\ln\int_{R_X} f(x)^\alpha dx; \quad \alpha > 0,\ \alpha \neq 1, \tag{2}$$

where, the constant α is conditional so that entropy gives a positive result.

The Havrda and Charvat [3] entropy is defined as

$$H^\alpha(X) = \frac{1}{2^{1-\alpha}-1}\left[\int_0^\infty \{f(x)\}^\alpha dx - 1\right]; \quad \alpha > 0,\ \alpha \neq 1. \tag{3}$$

The Arimoto [4] entropy is given by

$$A_\alpha(X) = \frac{1}{2^{1-\alpha}-1}\left[\left\{\int_0^\infty (f(x))^{\frac{1}{\alpha}} dx\right\}^\alpha - 1\right]; \ \alpha > 0,\ \alpha \neq 1. \tag{4}$$

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The mathematical expression of the Sharma and Mittals [5] entropy measure is defined by

$$H_\alpha(X) = \frac{1}{2^{1-\alpha}-1}\left[\exp\left\{(\alpha-1)\int_0^\infty f(x)\ln f(x)dx\right\}-1\right]; \tag{5}$$

$$\alpha>0,\ \alpha\neq 1.$$

2 Relative Loss

Let $H(X)$ and $H(Y)$ be the corresponding entropies over the complete range and a truncated form of the particular distribution. Then the general form of the relative loss of entropy in using Y instead of X is defined by

$$s_H(t) = \frac{H(X)-H(Y)}{H(X)}. \tag{6}$$

Many researchers have studied the relative importance of various distributions by using the above relationship. For example, Dey et. al [6] discussed the relative loss of different entropy measures using the truncated Rayleigh distribution, Basit et al. [7] also used the relative loss of entropy measures for the weighted and truncated weighted exponential (SBME, TSBME) distribution. Mir et. al [8] discussed the entropy measures of the size biased exponential distribution. Award and Alawneh [9] discussed the real application of the entropy measures to lifetime data.

In this paper, we consider various generalized entropy measures to calculate the relative loss of entropies when the lifetime distribution is truncated on $[0,t)$ instead of considering the Lomax distribution on $[0,\infty]$.

As a general rule, the entropy measure will perform better if it holds the natural phenomenon that is by increasing the truncation time, the relative loss must be decreases.

3 Preliminaries

Let X be a Lomax random variable having PDF $f(x;\theta,\lambda)$ and θ, λ be the unknown parameter. Then the PDF and cumulative distribution function (CDF) are given below

$$f(x,\theta,\lambda) = \frac{\theta}{\lambda}\left(1+\frac{x}{\lambda}\right)^{-(\theta+1)},\ 0\leq x\leq\infty,\ \theta,\lambda>0, \tag{7}$$

and

$$F(x;\theta,\lambda) = 1-\left(1+\frac{x}{\lambda}\right)^{-\theta}. \tag{8}$$

Let Y be a truncated Lomax random variable having the PDF and CDF with the following mathematical representation

$$f(y;\theta,\lambda) = \frac{\frac{\theta}{\lambda}\left(1+\frac{y}{\lambda}\right)^{-(\theta+1)}}{1-\left(1+\frac{t}{\lambda}\right)^{-\theta}},\ 0\leq y\leq t,\ \theta,\lambda>0, \tag{9}$$

and

$$F(y;\theta,\lambda) = \frac{1-\left(1+\frac{y}{\lambda}\right)^{-\theta}}{1-\left(1+\frac{t}{\lambda}\right)^{-\theta}}, \tag{10}$$

where t is a truncated random variable.

4 Different Entropy Measures

This section covers the entropy measures of the Lomax distribution and truncated Lomax distribution is given in Eq. (1-5).

1- The Shannon Entropy [1] of X and Y is defined by

$$H(X) = -\int_{R(X)} f(x)\ln f(x)dx,$$

$$= -\int_0^\infty \frac{\theta}{\lambda}\left[1+\frac{x}{\lambda}\right]^{-(\theta+1)}\cdot\ln\frac{\theta}{\lambda}\left[1+\frac{x}{\lambda}\right]^{-(\theta+1)}dx.$$

The result is obtained

$$H(X) = \ln\frac{\lambda}{\theta}+\frac{\theta+1}{\theta}. \tag{11}$$

Now, the truncated distribution is defined by

$$H(Y) = -\int_{R(Y)} f(y;t;\theta,\lambda)\ln f(y;t;\theta,\lambda)dy$$

$$= -\int_0^t \frac{\frac{\theta}{\lambda}\left[1+\frac{y}{\lambda}\right]^{-(\theta+1)}}{F(t;\theta,\lambda)}\ln\frac{\frac{\theta}{\lambda}\left[1+\frac{y}{\lambda}\right]^{-(\theta+1)}}{F(t;\theta,\lambda)}dy.$$

Finally, the result are obtained

$$H(Y) = \Bigg[-\ln\theta+\ln\lambda+\frac{\theta+1}{\theta}\left\{\frac{\left\{1-\left(1+\frac{t}{\lambda}\right)\right\}^{-\theta}\left\{1+\theta\ln\left(1+\frac{t}{\lambda}\right)\right\}}{F(t;\theta,\lambda)}\right\}+\ln F(t;\theta,\lambda)\Bigg]. \tag{12}$$

2- The Renyi [2] entropy of X and Y is given by

$$H_\alpha(X) = \frac{1}{1-\alpha}\ln\int_0^\infty f^\alpha(x;\theta,\lambda)dx;\ \ \alpha>0,\ \alpha\neq 1$$

$$= \frac{1}{1-\alpha}\ln\left[\int_0^\infty\left(\frac{\theta}{\lambda}\left[1+\frac{x}{\lambda}\right]^{-(\theta+1)}\right)^\alpha dx\right].$$

Solving the above integral, we have obtained

$$H_\alpha(X) = \frac{1}{1-\alpha}[\ln(1-\alpha-\alpha\theta)+\alpha\ln\theta-(\alpha-1)\ln\lambda]. \tag{13}$$

The truncated distribution in case of the Renyi entropy is defined by

$$H_\alpha(Y)=\frac{1}{1-\alpha}\ln\int_0^t f^\alpha(y;t;\theta,\lambda)dy;\ \ \alpha>0,\ \alpha\neq 1$$

$$\frac{1}{1-\alpha}\ln\int_0^t \frac{f^\alpha(y;\theta,\lambda)}{F^\alpha(t;\theta,\lambda)};\ \ \alpha>0,\ \alpha\neq 1.$$

The following results are obtained

$$=\frac{1}{1-\alpha}\left[\alpha\ln\theta+\ln\left(\left(1+\frac{t}{\lambda}\right)^{1-\alpha-\alpha\theta}-1\right)-(\alpha-1)\ln\lambda\right.$$
$$\left.-a\ln F(t;\theta,\lambda)-\ln(1-\alpha-\alpha\theta)\right]. \tag{14}$$

3- The Havrda & Charvat [3] entropy of X and Y is given by

$$H^\alpha(X)=\frac{1}{2^{1-\alpha}-1}\left[\int_0^\infty f^\alpha(x;\theta,\lambda)dx-1\right];\ \ \alpha>0$$
$$=\frac{1}{2^{1-\alpha}-1}\left[\int_0^\infty\left(\frac{\theta}{\lambda}\left[1+\frac{x}{\lambda}\right]^{-(\theta+1)}\right)^\alpha dx-1\right].$$

The result is obtained

$$=\frac{1}{2^{1-\alpha}-1}\left[\frac{-\theta^\alpha}{\lambda^{\alpha-1}(1-\alpha-\alpha\theta)}-1\right]. \tag{15}$$

The truncated PDF is given by

$$H^\alpha(Y)=\frac{1}{2^{1-\alpha}-1}\left(\int_0^t f^\alpha(y;t;\theta,\lambda)dy-1\right);\ \ \alpha>0$$
$$=\frac{1}{2^{1-\alpha}-1}\left[\int_0^t\frac{f^\alpha(y;\theta,\lambda)}{F^\alpha(t;\theta,\lambda)}dy-1\right].$$

Finally, we have

$$=\frac{1}{2^{1-\alpha}-1}\left[\frac{\theta^\alpha}{\lambda^{\alpha-1}F^\alpha(t;\theta,\lambda)}\left\{\frac{(1+\frac{t}{\lambda})^{1-\alpha-\alpha\theta}-1}{1-\alpha-\alpha\theta}\right\}-1\right]. \tag{16}$$

4- The Arimoto's [4] entropy of X and Y is given by

$$A_\alpha(X)=\frac{1}{2^{\alpha-1}-1}\left[\left\{\int_0^\infty f^{\frac{1}{\alpha}}(x;\theta,\lambda)dx\right\}^\alpha-1\right];\ \alpha>0,\alpha\neq 1,$$

$$A_\alpha(X)=\frac{1}{2^{\alpha-1}-1}\left[\left\{\int_0^\infty\left(\frac{\theta}{\lambda}\left[1+\frac{x}{\lambda}\right]^{-(\theta+1)}\right)^{\frac{1}{\alpha}}dx\right\}^\alpha-1\right].$$

By solving the above integral, we have

$$A_\alpha(X)=\frac{1}{2^{\alpha-1}-1}\left[\frac{\theta}{\lambda^{1-\alpha}}\left\{\frac{\alpha}{\theta+1-a}\right\}^\alpha-1\right]. \tag{17}$$

The truncated PDF is defined by

$$A_\alpha(Y)=\frac{1}{2^{\alpha-1}-1}\left[\left\{\int_0^t f^{\frac{1}{\alpha}}(y;t;\theta,\lambda)dy\right\}^\alpha-1\right];$$
$$\alpha>0,a\neq 1$$
$$=\frac{1}{2^{\alpha-1}-1}\left[\left\{\int_0^t\frac{f^{\frac{1}{\alpha}}(y;\theta,\lambda)}{F^{\frac{1}{\alpha}}(t;\theta,\lambda)}dy\right\}^\alpha-1\right].$$

We finally obtained the following result

$$=\frac{1}{2^{\alpha-1}-1}\left[\frac{\theta}{\lambda^{1-\alpha}F(t;\theta,\lambda)}\left\{\frac{(1+\frac{t}{\lambda})^{\frac{\alpha-\theta-1}{\alpha}}-1}{\frac{\alpha-\theta-1}{\alpha}}\right\}^\alpha-1\right]. \tag{18}$$

5- The Sharma & Mittel [5] entropy of X and Y are given by

$$H_\alpha(X)=\frac{1}{2^{1-\alpha}-1}\left[\exp\left\{(\alpha-1)\int_0^\infty f(x;\theta,\lambda)\right.\right.$$
$$\left.\left.\ln f(x;\theta,\lambda)dx\right\}-1\right];\ \ \alpha>0,\ \ a\neq 1.$$

Let

$$G(X)=\int_0^\infty f(x;\theta,\lambda)\ln f(x;\theta,\lambda)dx$$
$$=\int_0^\infty\frac{\theta}{\lambda}\left[1+\frac{x}{\lambda}\right]^{-(\theta+1)}\cdot\ln\frac{\theta}{\lambda}\left[1+\frac{x}{\lambda}\right]^{-(\theta+1)}dx.$$

By solving the above integral, we have

$$H_\alpha(X)=\frac{1}{2^{1-\alpha}-1}\left[\exp\left\{(\alpha-1)\left(\ln\frac{\theta}{\lambda}-\frac{\theta+1}{\theta}\right)\right\}-1\right]. \tag{19}$$

The truncated PDF is defined by

$$H_\alpha(Y)=\frac{1}{2^{1-\alpha}-1}\left[\exp\left\{(\alpha-1)\int_0^t f(y;t;\theta,\lambda)\right.\right.$$
$$\left.\left.\ln f(y;t;\theta,\lambda)dy\right\}-1\right];\ \ \alpha>0,\ \alpha\neq 1,$$

$$H_\alpha(Y)=\frac{1}{2^{1-\alpha}-1}\left[\exp\left\{(\alpha-1)\int_0^t\frac{\frac{\theta}{\lambda}\left[1+\frac{y}{\lambda}\right]^{-(\theta+1)}}{F(t;\theta,\lambda)}\right.\right.$$
$$\left.\left.\ln\frac{\frac{\theta}{\lambda}\left[1+\frac{y}{\lambda}\right]^{-(\theta+1)}}{F(t;\theta,\lambda)}dy\right\}-1\right]$$

$$H_\alpha(Y) = \frac{1}{2^{1-\alpha}-1}\left[\exp\left\{(\alpha-1)\left(\ln\theta - \ln\lambda \right.\right.\right.$$
$$- \frac{\theta+1}{\theta}\left\{\frac{\{1-(1+\frac{t}{\lambda})\}^{-\theta}\{1+\theta\ln(1+\frac{t}{\lambda})\}}{F(t;\theta,\lambda)}\right\}$$
$$\left.\left.\left. - \ln F(t;\theta,\lambda)\right)\right\} - 1\right]. \tag{20}$$

5 The Relative Loss of Entropy Measures

By using the relation given in Eq. (6), the relative loss of each entropy is computed as follows

By incorporating Eq. (11) and Eq. (12) in (6), we obtained the following relative loss of the Shannon entropy

$$S_H(t) = \frac{\frac{\theta+1}{\theta}\left[1-\left\{\frac{\{1-(1+\frac{t}{\lambda})\}^{-\theta}\{1+\theta\ln(1+\frac{t}{\lambda})\}}{F(t;\theta,\lambda)}\right\} - \ln F(t;\theta,\lambda)\right]}{\ln\lambda + \frac{\theta+1}{\theta} - \ln\theta}. \tag{21}$$

The relative loss of the Renyi entropy is obtained by putting Eq. (13) and Eq. (14) in (6), we have

$$S_{H\alpha}(t) = \left(\left[\alpha\ln F(t;\theta,\lambda) + 2\ln(1-\alpha-\alpha\theta)\right.\right.$$
$$\left.\left. - \ln\left(\left(1+\frac{t}{\lambda}\right)^{1-\alpha-\alpha\theta} - 1\right]\right)\right)$$
$$/[\ln(1-\alpha-\alpha\theta) + \alpha\ln\theta - (\alpha-1)\ln\lambda]. \tag{22}$$

The Relative loss of Havrda & Charvat entropy is obtained by using Eq. (15) and Eq. (16) in (6) as

$$S_{H^\alpha}(t) = \frac{\frac{\theta^\alpha}{\lambda^{\alpha-1}(1-\alpha-\alpha\theta)}\left[1+\left\{\frac{(1+\frac{t}{\lambda})^{1-\alpha-\alpha\theta}-1}{\{1-(1+\frac{t}{\lambda})^{-\theta}\}^{\alpha}}\right\}\right]}{\frac{\theta^\alpha}{\lambda^{\alpha-1}(1-\alpha-\alpha\theta)}+1} \tag{23}$$

By employing Eq. (17) and Eq. (18) in (6), we derived the following relative loss for the Arimoto entropy

$$S_{A\alpha}(t) = \left[\frac{\frac{\theta}{\lambda^{1-\alpha}}\left[\left\{\frac{\alpha}{\theta+1-\alpha}\right\}^{\alpha} - \frac{1}{1-(1+\frac{t}{\lambda})^{-\theta}}\left\{\frac{(1+\frac{t}{\lambda})^{\frac{\alpha-\theta-1}{\alpha}}-1}{\frac{\alpha-\theta-1}{\alpha}}\right\}^{\alpha}\right.}{\frac{\theta}{\lambda^{1-\alpha}\left\{\frac{\alpha}{\theta+1-\alpha}\right\}^{\alpha}-1}}\right]. \tag{24}$$

The relative loss by Sharma & Mittal entropy is defined by putting Eq. (19) and Eq. (20) in (6) as

$$S_{\alpha A}(t) = \left[\exp\left\{(\alpha-1)\left(\ln\frac{\theta}{\lambda} - \frac{\theta+1}{\theta}\right)\right\}\right.$$
$$- \exp\left\{(\alpha-1)\left(\ln\theta - \ln\lambda - \frac{\theta+1}{\theta}\right.\right.$$
$$\left\{\frac{\{1-(1+\frac{t}{\lambda})\}^{-\theta}\{1+\theta\ln(1+\frac{t}{\lambda})\}}{1-(1+\frac{t}{\lambda})^{-\theta}}\right\}$$
$$\left.\left.\left. - \ln\left(1-\left(1+\frac{t}{\lambda}\right)^{-\theta}\right)\right\}\right]$$
$$/\left[\exp\left\{(\alpha-1)\left(\ln\frac{\theta}{\lambda} - \frac{\theta+1}{\theta}\right)\right\} - 1\right] \tag{25}$$

6 Numerical Results and Discussions

This section illustrates the relative loss of each entropy measure by using a real data set. The real data set is taken from [8] presents the losses (in millions of dollars) due to wind catastrophes recorded in 1977. The numerical result of the relative loss of each entropy is given in Tables (1-6) for different values of α, t, θ and λ. Figures 1, and 2 summarized the results of each entropy. The data set values are Data set: 2, 2,2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 3, 3, 3, 3, 4, 4, 4, 5, 5, 5, 5, 6, 6, 6, 6, 8, 8, 9, 15, 17, 22, 23, 24, 25, 27, 32, 43 .

The parameters of the Lomax distribution are estimated by using the maximum likelihood method. The parameter values are $\theta = 9.44$ and $\lambda = 23.14$, by using these values, the relative loss of the different entropy measures are as follows

Table 1: Relative Loss of Shannon.

T	$\theta = 9.44, \lambda = 23.14$
	$S_H(t)$
0.5	5.57144
1.0	2.36590
1.5	1.11267
2.0	0.37137
2.5	−0.15572
3.0	−0.57087
3.5	−0.91916
4.0	−1.22374
4.5	−1.494781
5.0	−1.74950

Table 2: Relative Loss of Renyi and H & C Entropy.

T	$\theta = 9.44, \lambda = 23.14, a = 0.05$	
	$S_{H_\alpha}(t)$	$S_{H^\alpha}(t)$
0.5	−0.30543	0.98979
1.0	−0.31898	0.99999
1.5	−0.33192	1.00991
2.0	−0.34384	1.01965
2.5	−0.35483	1.02924
3.0	−0.36504	1.03871
3.5	−0.37458	1.04808
4.0	−0.38353	1.05735
4.5	−0.39197	1.06653
5.0	−0.39995	1.07563

Table 3: Relative Loss of Renyi and H & C Entropy

T	$\theta = 9.44, \lambda = 23.14, a = 0.06$	
	$S_{H_\alpha}(t)$	$S_{H^\alpha}(t)$
0.5	−0.62231	0.99197
1.0	−0.63834	0.99999
1.5	−0.65281	1.00775
2.0	−0.66576	1.01533
2.5	−0.67749	1.02278
3.0	−0.68824	1.03871
3.5	−0.69817	1.03736
4.0	−0.70740	1.04451
4.5	−0.71604	1.05158
5.0	−0.72415	1.05857

Table 4: Relative Loss of Arimoto Entropy

T	$\theta = 9.44, \lambda = 23.14, \alpha = 2$
	$S_{A\alpha}(t)$
0.5	2.04072
1.0	2.08119
1.5	2.12120
2.0	2.16054
2.5	2.19905
3.0	2.23659
3.5	2.27303
4.0	2.30829
4.5	2.34230
5.0	2.37501

Table 5: Relative Loss of Arimoto Entropy

T	$\theta = 9.44, \lambda = 23.14, \alpha = 2$
	$S_{A\alpha}(t)$
0.5	1.99925
1.0	1.99700
1.5	1.99329
2.0	1.98818
2.5	1.98173
3.0	1.97402
3.5	1.96515
4.0	1.95522
4.5	1.94432
5.0	1.93256

Table 6: Relative Loss of Sharma and Mittal Entropy

T	$\theta = 9.44, \lambda = 23.14, \alpha = 0.05$
	$S_{\alpha A}(t)$
0.5	0.00879
1.0	0.00592
1.5	0.00284
2.0	0.00044
2.5	−0.00142
3.0	−0.00295
3.5	−0.00423
4.0	−0.00535
4.5	−0.00634
5.0	−0.00722

Table 7: Relative Loss of Sharma and Mittal Entropy

T	$\theta = 9.44, \lambda = 23.14, \alpha = 0.06$
	$S_{\alpha A}(t)$
0.5	0.00909
1.0	0.00612
1.5	0.00294
2.0	0.00045
2.5	−0.00147
3.0	−0.00305
3.5	−0.00438
4.0	−0.00553
4.5	−0.00656
5.0	−0.00747

The following points were observed from Tables 1-7

1- Table 1 shows the numerical results of the relative loss of the Shannon entropy. The result shows that the behavior of the Shannon entropy measure is natural, i.e as the information t increases the relative loss tends to decrease.
2- The empirical results of the relative loss of Renyi and Havrda & Charvat entropy along with different t values, by considering a constant value of $\alpha = 0.05$ is given in Table 2. The results showed that the behavior of Renyi entropy is natural, while the natural phenomenon does not hold for Havrda & Charvat entropy measure, i.e, as the t increases the relative loss does get a decrease in the case of Renyi entropy but in the case of Havrda & Charvat entropy the relative loss increases as the t value increases. From the results it is clear that Renyi entropy measure is better than Havrda & Charvat entropy for the Lomax distribution.
3- Table 3 shows the empirical results of relative loss of Renyi and Havrda & Charvat entropy for an extra constant $\alpha = 0.06$. The result of the Reyni entropy is natural i.e the value of relative loss of Reyni entropy decreases as the t value increases. In the case of Havrda and Charvat entropy, the relative loss increases for increase in the t value. By comparing the results of Table 2 with that of the Shannon entropy in Table 1, it is concluded that, the Renyi entropy is not best for the Lomax distribution because of negative values of relative loss for all the t values.
4- The empirical results of the relative loss of the Arimoto entropy is given in Table 4. By considering a fixed value of $\alpha = 2$, the results showed that the natural phenomenon does not hold for Arimoto entropy. As t value increases the value of relative loss also increases. So comparatively Arimoto entropy could not be considered better than other entropy measures for the Lomax distribution.
5- The empirical results of the relative loss of Arimoto entropy for an extra constant $\alpha = 3$ is given in Table 5. It is clear that the value of the relative loss is positive but greater than one for all the values of t. So in this case the Arimoto entropy is better than the Havrda & Charvat entropy for the Lomax distribution.
6- The empirical results of the relative loss of Sharma and Mittal entropy are given in Table 6. The results showed that the natural phenomenon holds for Sharma and Mittal entropy. By taking the fixed value of $\alpha = 0.05$, the results showed that, the value of relative loss decreases as the t value increases. Some of the values lie between zero and one, while for some values of t the relative loss becomes negative. So comparatively the Sharma and Mittal entropy performs well than the Shannon and Renyi entropy for the Lomax distribution, because most of the values of relative loss of Sharma and Mittal entropy are positive and lie between zero and one.
7- Table 7 presents the empirical results of relative loss of Sharma and Mittal entropy along with different t values for an extra constant $\alpha = 0.06$. The result shows that natural phenomenon holds for Sharma and Mittal entropy measure i.e as the t value increases the relative loss decreases. Some of the values of relative loss are negative, while some are positive and also lie between zero and one. So from the results it can be concluded that Sharma and Mittal entropy is better than other entropy measures like Shannon, Renyi, Arimoto, and Havard & Charvat for Lomax distribution.

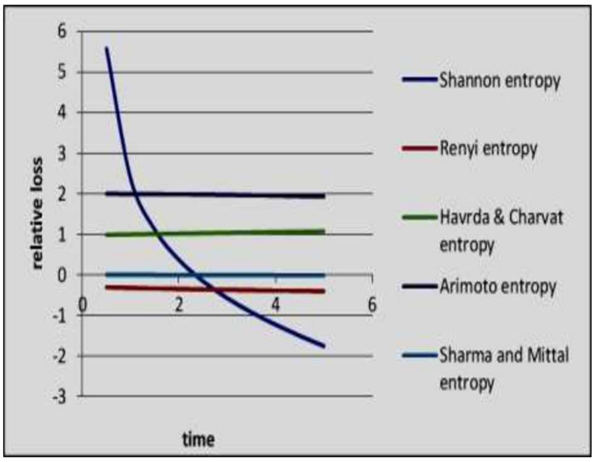

Fig. 1: Relative Loss of Various Entropy Measures

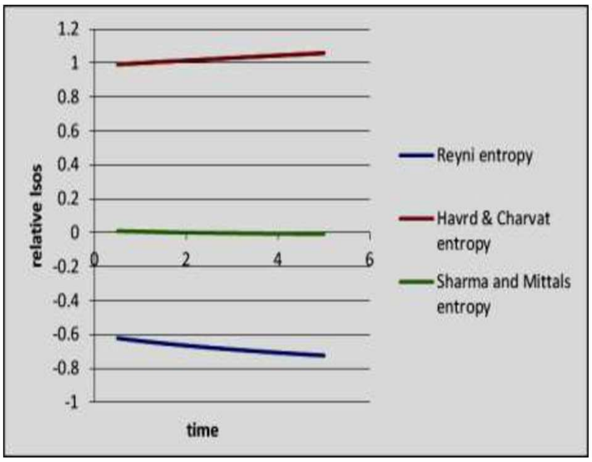

Fig. 2: Relative Loss of Various Entropy Measures

7 Conculsion

In this study, different entropy measures have been compared for the Lomax distribution in terms of the relative loss. It is concluded that the Shannon, Renyi, and Sharma & Mittal entropy measures following the natural phenomenon, i.e their relative losses decreases as the truncation time t increases. Hence, the Shannon, Renyi, and Sharma & Mittal entropy measures are comparatively better than other entropies for the Lomax distribution. A researcher involved in various fields of sciences must prefer the Shannon, Renyi, and Sharma & Mittal entropy measures while modelling the data using the Lomax distribution.

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Conflict of interest

The authors declare that there is no conflict regarding the publication of this paper.

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