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Error Assessment Method of Ultra-precise Measurement in Reverse Engineering

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Abstract: In the process of high precision measure ,the error(positioning error) between geodesic data and actual data ,which are always caused by the deviation of the measured object placement ,will influence the analyzing of engineering model .According to the measuring mechanism of measured object , the mathematical model is created. Bisearch is used to chose simulation value of positioning error .Then positioning error is evaluated by fitting simulated and measured point cloud .This paper presents a practical program in error analysis and assessment ,which reveal the relationship between simulated data, restore image of point cloud and positioning error.

Keywords: high precision, bisearch, fitting, positioning error, point cloud

1. INTRODUCTION

The reverse engineering, also called nbspreverse engineering, is a technology based on the measuring technique which rebuilt the measured object using computer technology. It main contains data collection for models and the establish for the digital models. It`s a quickly developed digital technology grow out with the development of the computer technology.

With the development of related techniques like sensor technique, control technique, image processing and computer vision, objects reconstructed has already becoming a reserching issue and has already been used widly in progress and successfully used in culture relics and archaeological[1][2] and motor-car industry[3].The high-precision objects reconstructed bases on the high-precision point cloud data collection whose mode is mainly divided into touch collection and non-touch collection[4].As the constant improve of the apparatus precision，the former apparatus also appears some new application .For example ,the introduce in the literature 5 says it has already reached the effect of improving the high-precision by Integration of ATOS optical scanner and three-coordinates measuring machine .Although the latest measuring instruments and

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measuring means are almost maturity ,measuring error is inevitably in the practical process of high-precision measurement and ,which makes it impossible to reach the measuring standard because of the measuring instruments, the internal environment and many others problems' influence.

During the process of high-precision measuring, if the location of the measured object has some deviation, it always could not get a content result when objects were rebuilding. High-precision positioning error assessment programme, based on fitting and bisearch ,is able to indicate machines or artificial to adjust the location of the measured object by the measuring instrument location error which is found out by comparing the simulated point cloud data with the measured point cloud data .In the end, the measuring instrument positioning error of a globle model will be solved using the program.

2.HIGH-PRECISION POSITIONING ERROR ASSESSMENT PROGRAMME

2.1 Positioning error assessment algorithm processes

High-precision positioning error assessment programme is mainly divided into object modeling and positioning error assessment algorithm. Object modeling needs to complete the mathematical modeling of objects. Then the relationship between object positioning error and measured point cloud will be built ,which provides accurate mathematics basis for producing simulated data. According to the complexity of the measured object, object modeling could chose overall modelling or character modeling .

Positioning error assessment algorithm uses bisearch to find positioning error in a specific area and build the model with positioning error. Then the Simulated point cloud data will be acquired in this positioning error under the mathematical model of measured object. Finally, the simulated and measured point cloud will be compared. If the difference between the simulated and measured point is in the context of the admissible error, the estimated value of the positioning error is the value found by bisearch. The algorithm is as follows:

(1) Input the measured data of the object, $x_0 \ldots\ldots x_n$ 。

(2) Initialize positioning error interval of bisearch, $\left[x_s, x_d\right]$ 。

(3) Initialize the positioning offset value of the object $\sigma = \frac{\left(x_s + x_d\right)}{2}$ 。

(4)According to the mathematical modeling of measured object,the simulated data $x_0^{'} \ldots\ldots x_n^{'}$ with the positioning offset value σ are acquird .

(5)Compare the simulated data $x_0^{'}\ldots\ldots x_n^{'}$ and the measured data $x_0\ldots\ldots x_n$ and set variables average is $\text{average}=\frac{\sum_{i=0}^{n}\left(x_i - x_i^{'}\right)}{n}$ （The comparing could be with overall model or character model in different accuracy）。

(6) Set the upper limits of error as δ， if $|\text{average}| \le \delta$ then σ is the result.Or according to the concrete mathematical model,the next step could be：

If average>0 ,Set $x_s = \sigma$， goto (3);

If average<0,Set $x_d = \sigma$， goto (3).

Or

If average>0,Set $x_d = \sigma$， goto (3);

If average<0,Set $x_s = \sigma$， goto (3).

2.2 Technical analysis

(1) Data Compare

High-precision positioning error assessment programme compares data based on differentials.

Definition 1(interpolation function):Function $y = f(x)$ is meaningful in the interval $[a,b]$,it is known function values is $y_0, y_1, \ldots\ldots, y_n$ when variables x is $a \le x_0 < x_1 < \ldots\ldots < x_n \le b$ .If a simple function $P(x)$ meets the conditions of $P(x_i) = y_i \quad (i = 0,1,\cdots,n)$,it is the interpolation function of $f(x)$.

Set a curve $f(x)$ on a slice of measured object and the measured point cloud data (x_i,y_i) on $f(x)$,there is $P(x_i) = y_i \quad (i = 0,1,\cdots,n)$ .Then a interpolation function $P(x)$ could be found to approach the curve of measured object .So positioning error assessment algorithm could compare point cloud data instead of comparing the curve between simulated data and measured data .

(2) error analysis

Positioning error assessment algorithm uses bisearch to calculate errors and makes the interval of bisearch as the range of positioning errors.Then simulated point cloud data were acquired by the positioning error found by bisearch under the mathematical model of measured object. If the difference between the simulated and measured point is in the context of the admissible error, the estimated value of positioning error is the value found by bisearch. Positioning error assessment algorithm makes the most of computer 's ability in processing mass data, so it can reach high-precision which is impossible for traditional error measurement.

3. HIGH-PRECISION POSITIONING ERROR ASSESSMENT PROGRAMME APPLICATION

In this part ,high-precision positioning error assessment programme will be used to analysis the errors of high-precision sphere surface. Environment of the measured object will be introduced first .Then the model of the sphere is built in the concrete environment. Finally, positioning error will be calculated by positioning error assessment algorithm.

3.1 Measurement model analysis

The measurement model is as follows. Set a halfsqhere on a horizontal objective table .A measuring head pointed to the center of the objective table will measure the halfsqhere surface .The data precision is 10μm .Interactive data collection could remove noise and preprocess point cloud data as well as ensure the data's regular by eliminating redundant[6] .Then point cloud data will be preprocessed and registered to get the rational measured point cloud data .

The measurement process is as follows:

Set the measured object in the center of the objective table .The measuring head is set to be perpendicular to objective table plane and point to the center of the objective table .Then the measuring head revolves 90 degrees along the warp and collects a data every θ_1 degrees .When the measuring head has finished ,the objective table revolves θ_2 degrees and the measuring head will collect data along the other warp .the measuring system will work until it has measured the hole halfsqhere .Then it will measure another halfsqhere of the measured sphere .As shown in Figure 1 ,the points' trajectory is the measuring head's trajectory ,the ray is launched by the measuring head .The objective table revolves in the specified direction .But the measured object's location may not be in the center of objective table ,which would cause positioning errors.

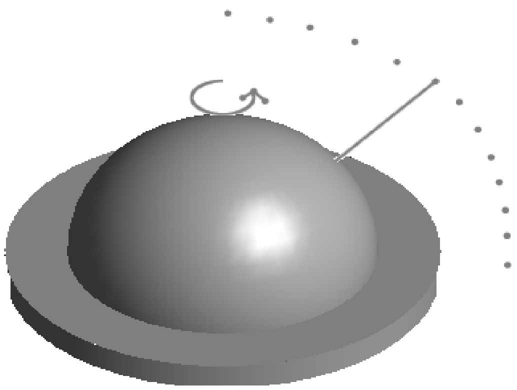


Figure 1 Commanding view of the measured sphere

3.2 Model analysis

According to the high-precision positioning error assessment programme based on curve fitting and bisearch ,the point cloud data of the sphere with positioning errors will be simulated by modeling the environment .These data ,the analog values of the measured data when measured object's location was not in the center of objective table ,will be the foundation of error comparing in the next step .

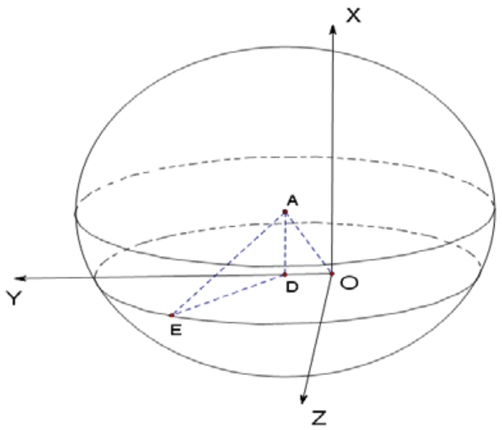


Figure 2 Side view of the measured object

The data of whole sphere can be simulated once without the measuring equipment limit .Suppose measuring head points to the centre of objective table and the centre of sphere shifts in the X-Y plane which is also the objective table plane .Set the X-Y plane for the objective table plane and the Y-Z plane for the plane in which the measuring head revolves .The centre of sphere has shifted in the XOY plane in Figure 2.

As shown in Figure 2 ,the center of measured sphere A makes the circular motion with O as the centre and OA as the radius .The sphere's projection in X-Y plane is shown in Figure 3 (a)~ (d).

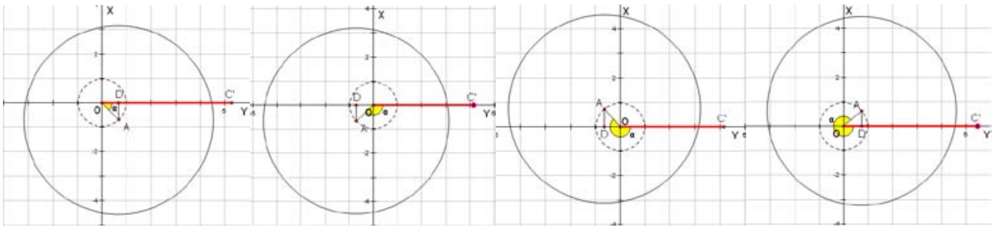
As shown in Figure 3(a)~ (d) ,the small circle is A's trajectory when objective table is revolving .α is the rotation angle of objective table .C'O is projection of the ray launched by measuring head in X-Y plane .As can be seen ,A's projection D is moving between intersections of Y axis and the small circle in Figure 3.AD length is as follows :

$$AD=\sin\alpha \times OA \quad (0^\circ \le \alpha < 90^\circ) \tag{3-1}$$

$$AD=\sin(\pi-\alpha) \times OA \quad (90^\circ \le \alpha < 180^\circ) \tag{3-2}$$

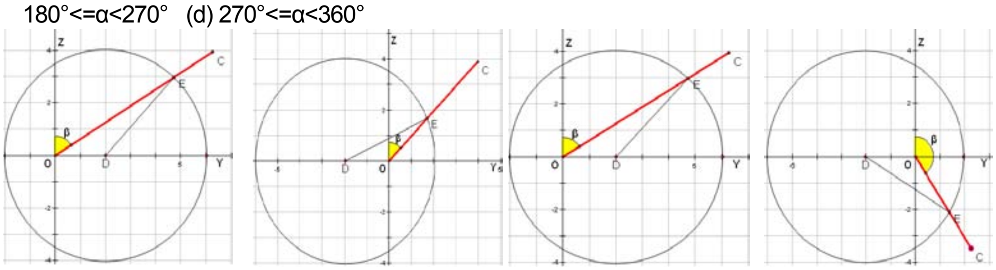
$$AD=\sin(\alpha-\pi) \times OA \quad (180^\circ \le \alpha < 270^\circ) \tag{3-3}$$

$$AD=\sin(2\pi-\alpha) \times OA \quad (270^\circ \le \alpha < 360^\circ) \tag{3-4}$$



(a) (b) (c) (d)

Figure 3 the sphere's projection in X-Y plane (a) 0°<=α<90° (b) 90°<=α<180° (c) 180°<=α<270° (d) 270°<=α<360°



(a) (b) (c) (d)

Figure 4 .the sphere's projection in Y-Z plane (a) D is in Y axis forward ,0° ≤β≤90° (b) D is in Y axis negative ,0° ≤β≤90°(c) D is in Y axis forward,90°<β ≤180° (d) D is in Y axis negative ,90°<β ≤180°

As shown in Figure 2 ,AE ,the distance between A and E ,equals to the radius of the measured sphere .So DE ,the radius of sphere 's projection in X-Y plane ,can be acquired.

$$DE = \sqrt{AE^2 - AD^2} \quad (3\text{-}5)$$

At the same time, D ,the center of the sphere 's projection in X-Y plane, is acquired.D's abscissa value is 0 .D's ordinate value Dy is acquired by the equation3-6 .

$$D_y = k \times \sqrt{OA^2 - AD^2} \quad (3\text{-}6)$$

$$(If\, 0^\circ \le \alpha < 90^\circ or 270^\circ \le \alpha < 360^\circ, k \ge 0 else k < 0)$$

Then as shown in Figure 4 ,the sphere 's projection in Y-Z plane is acquired .β is the rotation angle of measuring head .OC is the ray launched by measuring head .

Through the above description ,OE ,the value got by measuring head ,does not equal to the radius of measured sphere because of the displacement of measured sphere .As shown in Figure 4 (a)~ (d) ,the simulated data can be acquired by OE .OE can be acquired as follows :

When D is in Y axis forward and 0°≤β≤90° , triangle ODB follows equation $OE^2+OD^2-2\times\cos(\pi/2-\beta)\times OE\times OD=DE^2$.The solution results in OE ,

$$OE = \frac{2\times\cos\left(\frac{\pi}{2}-\beta\right)\times OD+\sqrt{\left(2\times\cos\left(\frac{\pi}{2}-\beta\right)\times OD\right)^2-4\times\left(OD^2-DE^2\right)}}{2} \quad (3\text{-}7)$$

When D is in Y axis negative and 0° ≤β≤90°,

$$OE = \frac{2\times\cos\left(\beta-\frac{\pi}{2}\right)\times OD+\sqrt{\left(2\times\cos\left(\beta-\frac{\pi}{2}\right)\times OD\right)^2-4\times(OD^2-DE^2)}}{2} \quad (3\text{-}8)$$

When D is in Y axis forward and 90°<β ≤180°,

$$OE = \frac{2\times\cos\left(\frac{\pi}{2}+\beta\right)\times OD+\sqrt{\left(2\times\cos\left(\frac{\pi}{2}+\beta\right)\times OD\right)^2-4\times(OD^2-DE^2)}}{2} \quad (3\text{-}9)$$

When D is in Y axis negative and 90°<β ≤180°,

$$OE = \frac{2\times\cos\left(\frac{3\pi}{2}-\beta\right)\times OD+\sqrt{\left(2\times\cos\left(\frac{3\pi}{2}-\beta\right)\times OD\right)^2-4\times(OD^2-DE^2)}}{2} \quad (3\text{-}10)$$

Then the simulated data can be shown like（OE,α ,β）as the form of polar coordinates.

3.3 Simulated results

Supports the environment is as follows : a 1mm-radius sphere shifts 300μm along the X axis .The scanning interval of measuring head is 1 degree . The scanning interval of objective table is 1 degree .

Set the error range $[x_s,x_d]$ of the positioning error assessment algorithm as [0μm，1000μm] and the upper limits of error is σ .According to the positioning error assessment algorithm , σ is initialized to 500μm and will be changed until |average|$\leq\delta$. The variables average is the average of error .

As shown in Table 1 , the measured data on the circumference of a circle., which were measured by the measuring head with rotation angle as 15 degrees ,is compared with the simulated data .The trend ,the simulated data gradual approach to the measured data ,is shown .In Table 1, coordinates is polar coordinates, α is the rotation angle of objective table , β is the rotation angle of measuring head , ρ is the measured data , ρ' is the simulated data .

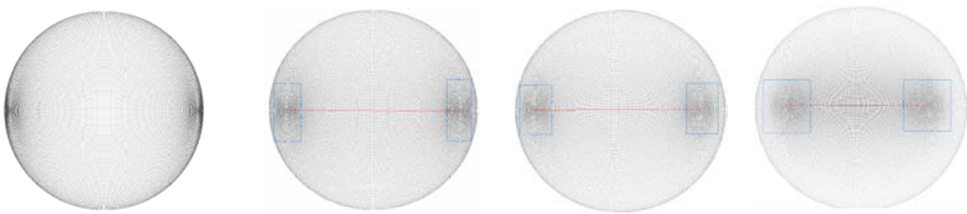
Table 1 .comparison between simulated data and measured data

（the unit of ρ and ρ' is mm）

α	B	ρ (300μm)	ρ (σ =500μm)	ρ' (σ =25μm)	ρ' (σ =375μm)	ρ' (σ =312.5μm)	…
0	15	1.034740	1.005050	1.035110	1.029149	1.034236	…
1	15	1.034727	1.005028	1.035100	1.029133	1.034222	…
2	15	1.034688	1.004960	1.035068	1.029084	1.034182	…
3	15	1.034625	1.004847	1.035016	1.029002	1.034116	…
4	15	1.034535	1.004689	1.034942	1.028888	1.034022	…
5	15	1.034420	1.004485	1.034848	1.028741	1.033902	…
6	15	1.034280	1.004237	1.034732	1.028562	1.033755	…
7	15	1.034114	1.003944	1.034596	1.028350	1.033582	…

8	15	1.033923	1.003606	1.034439	1.028106	1.033382	…
9	15	1.033707	1.003223	1.034261	1.027830	1.033156	…
10	15	1.033465	1.002796	1.034062	1.027522	1.032904	…
…	…	……	……	……	……	……	…
354	15	1.034280	1.004237	1.034732	1.028562	1.033755	…
355	15	1.034420	1.004485	1.034848	1.028741	1.033902	…
356	15	1.034625	1.004689	1.034942	1.028888	1.034022	…
357	15	1.034625	1.004847	1.035016	1.029002	1.034116	…
358	15	1.034688	1.004960	1.035068	1.029084	1.034182	…
359	15	1.034727	1.005028	1.035100	1.029133	1.034222	…
360	15	1.034740	1.005050	1.035110	1.029149	1.034236	…

The restore image of point cloud data with the positioning error σ as 0μm, 250μm, 300μm, 500μm are shown in Figure 5(a)~ (d) .The data in the rectangular box whose center are connect with a straight line are the most intensive parts of data in restore image



(a) (b) (c) (d)

Figure5 the restore image of point cloud data

(a) σ =0μm (b) σ =250μm (c) σ =300μm (d) σ =500μm

Comparison between Figure 5 (a) and Figure 5 (b) shows when the measuring head was collecting data ,the errors caused by the positioning error were appear .The errors caused the offset of the most intensive part of data in restore image which ought to be in the poles of the sphere .Because the measuring head and the objective table all rotate under a specified angle ,the deviation of the sphere 's location is certain to lead to the deviation of the data. These is consistent with the analysis of the experimental environment. Comparison between Figure 5 (a) and Figure 5 (d) shows a trend that the most intensive part of data in the restore image of point cloud data move to the center of sphere when the positioning error is get bigger .The trend shows the approximate range of positioning error intuitively. Comparison between Figure 5 (b) and Figure 5 (c) and comparison between Figure 5 (b) and Figure 5 (c) shows the closer the positioning error of the measured object is the more similar the restore image is ,which is also consistent with the data shown in Table 1.

4. CONCLUSION

The error assessment programming based on the research and fitting in the reverse engineering provides a program which can estimates the measured objects positioning error and indicate machines or artificial to adjust the instrument .The program makes good use of the measured point cloud data and has the features of enterprise data and highly precision. In a process of error analysis, it uses digitized means to improve the precision then reach the content results.

Through the instance of error analysis of the sphere model ,it shows the programme's feasibility in dealing with the similar model problems .On analysis positioning error ,the programme only need to modeling all or part of the measured objects for judging the positioning error in other practical problem .The programme behaves the feature of highly precision through the effect of the experiment, and it also fits in the detect and analysis of precise instrument .

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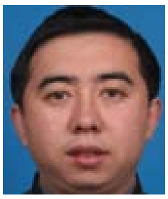

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