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Modeling User Equilibrium and the Day-to-day Traffic Evolution based on Cumulative Prospect Theory

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Abstract: With the rapid development of ATIS, the effect of information provision on travellers' route choice behaviour has received much attention. In this paper, we develop a dynamical traffic system within the frame of bounded rationality by an integration of the BNN dynamic and Cumulative Prospect Theory to characterize travellers' day-to-day route switching behaviours and investigate the process and conditions of achieving UE under ATIS. Under three mild constraints, we prove that no matter what the initial state is and whether travellers' reference points are fixed or updated under certain rules, the dynamical system can eventually evolve to UE, at which the link flow pattern is uniquely determined and globally asymptotical stable, while uniqueness and asymptotic stability of the route flow pattern cannot be ensured, which call for a further exploration of the network topology and link performance. Finally, we demonstrate the above conclusions and illustrate the process of achieving UE through a numerical experiment.

Keywords: User equilibrium, dynamical system, Cumulative Prospect Theory, the BNN dynamic.

1 Introduction

The user equilibrium (UE) principle has become a cornerstone for traffic assignment modeling ever since it was proposed by Wardrop [18], which implies that travelers have perfect information on traffic conditions and are entirely rational in their decision-makings. However, this is not the case in reality. Due to the uncertain nature of traffic conditions, it's impossible for travelers to have full access to the information and perform their travel decisions with entire rationality. UE is a method of equilibrium analysis, which primarily focuses on the issue of equilibrium existence and uniqueness. It's assumed that if equilibrium exists, then it will be bound to occur. Actually, absolute equilibrium is always difficult to achieve, while disequilibrium exists universally. Equilibrium and disequilibrium have achieved dialectical unity and can complement each other.

The traffic system is always in the process of evolving toward equilibrium which is usually hard to reach. Therefore, it's also essential to carefully examine the process and conditions of achieving UE from a disequilibrium state. Meanwhile, stability is an important performance in the sense that if equilibrium is unsustainable, even a small perturbation will lead the system to evolve away from the current equilibrium. Horowitz [8] demonstrated that even for a well-behaved system whose equilibrium solution was known to exist, the system might fail to converge to equilibrium, the achievement of which still depending on travelers' route switching process. In other words, equilibrium emerges from the long-term outcome of travelers' learning and decision-making based on their past experiences, which underlies the day-to-day traffic dynamics. The key question of traffic flow evolution modeling is how to characterize travelers' leaning procedures and decision rules more accurately.

Advanced traveler information system (ATIS) is generally believed to be efficient in improving travelers' trip planning, alleviating traffic congestion and enhancing network performance, etc. With the rapid development of ATIS, how to describe the trajectory of traffic flow evolution and forecast the final equilibrium state is becoming more and more important, which involves a further exploration of travelers' learning and decision-making behavior within the frame of bounded rationality. So far, the application of game theory in day-to-day traffic flow evolution modeling is still at a preliminary stage. Moreover, PT (or CPT) is usually found to be incorporated in equilibrium analysis of stochastic traffic networks. The BNN dynamic is a well-behaved evolutionary dynamic which can provide a plausible micro-foundation for describing human learning process in repeated games with incomplete information [14].

In this paper, we attempt to formulate a novel dynamical traffic system model by an integration of CPT and the BNN dynamic to characterize travelers' day-to-day route choice behavior based on information provided by ATIS. Under three mild constraints, we present a reasonable explanation to the process of achieving UE by the establishment of equivalence between UE and equilibrium point of the dynamical system, whose asymptotical stability is also analyzed.

2 UE based on CPT

2.1. Network representation and assumptions

Define $G = (N, A)$ as a strongly connected traffic network with N and A being the sets of nodes and links respectively. Let W be the set of origin-destination (OD) pairs of G and w be one of the OD pairs. Let R_w be the set of routes between OD pair w, and R be the set of routes of G. Let a, b, etc., denote the links, and p, q, etc., denote the routes. Let x_a be the flow on link a, and x be the vector of link flows. Let f_p^w be the flow on route p between OD pair w, $f^w = (\cdots, f_p^w, \cdots)$ be the vector of route flows between OD pair w, and $f = (\cdots, f^w, \cdots)$ be the vector of route flows of the network. Let d_w denote the travel demand between OD pair w. Let $\delta_{a,p}^w$ be the link-route incidence whose value is 1 if link a is on route p, and 0 otherwise. Let v_p^w denote the prospect value (PV) of route p between OD pair w, and v^w be the route PV vector between OD pair w, and $v = (\cdots, v^w, \cdots)$ be the route PV vector of the network. We assume that the traffic flow is continuous and the links possess the effects of separation and crowding: $\frac{\partial t_a(x_a)}{\partial x_b} = 0$ and $\frac{\partial t_a(x_a)}{\partial x_a} > 0$, $\forall a, b \in A, a \neq b$. According to flow conservation, there exist the following relations:

$$x_a = \sum_{w \in W} \sum_{p \in R_w} \delta_{a,p}^w f_p^w \tag{1}$$

$$T_p^w = \sum_{a \in A} \delta_{a,p}^w T_a \tag{2}$$

$$t_p^w = \sum_{a \in A} \delta_{a,p}^w t_a \tag{3}$$

$$\left(\sigma_p^w\right)^2 = \sum_{a \in A} \delta_{a,p}^w \left(\sigma_a\right)^2, \quad \forall p \in R_w, w \in W \tag{4}$$

If a route in the network is composed of many links, we can deduce that the travel time on the route follows a normal distribution according to the Central Limit Theorem:

$$T_p^w \sim N\left(t_p^w, \left(\sigma_p^w\right)^2\right), \quad \forall p \in R_w \tag{5}$$

where t_a^0 is the free flow travel time on link *a*, and c_a is the capacity of link *a*.

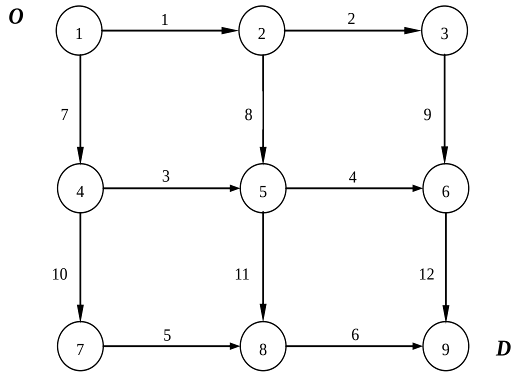

Figure 1 The test network

3 Conclusion

The UE Principle is a method of equilibrium analysis to describe a world with dust settled, while evolutionary game theory provides a method of evolutionary analysis to describe how the dust settled. In this paper, we establish a dynamical traffic system by an integration of CPT and the BNN dynamic to investigate the process of traffic flow evolution and achieving UE under ATIS. It's indicated that no matter what the initial state is and whether travelers' reference points are fixed or updated under certain rules, the dynamical system can evolve to UE eventually, at which the link flow pattern is uniquely determined and globally asymptotical stable, while uniqueness and asymptotic stability of the route flow pattern cannot be ensured, which call for a further exploration of the network topology and link performance. We also present a numerical experiment to validate our conclusions and illustrate the process of achieving UE from diverse initial states and by different reference point update rules. Equilibration and evolution are reference point dependent, implying that the traffic management authority can employ effective measures to manipulate travelers' reference points, so as to guide the traffic system evolution toward desirable direction. In addition, several directions for further research should be identified. Firstly, some conclusions need to be further verified with gradually relaxing the constraints in this paper. Secondly, parameter estimation for the PV function and analysis on reference point update rules with experimental or field data are required for real applications. Thirdly, seeking for more plausible evolutionary dynamics that conform to travelers' actual learning and decision-making behavior through empirical studies is also a research target. Lastly and most important of all, applying the models to real traffic network is a still bigger challenge in future.

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