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New Treatment of Fluid Mechanics with Heat and Mass Transfer: Theory of Diffusion

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Abstract: The analytical solutions of nonlinear Partial differential equations in fluid mechanics are considered as a strong obstacle up to date. In this paper, the nonlinear Navier-Stokes, Burger, and Korteweg-deVries equations are converted to a one linear diffusion equation based on the proposed linear velocity operator concept for the first time. The velocity operator is formulated in terms of a generalized new physical parameter (Mohammadein Parameter M*); which has a different physical meaning in fluid mechanics and heat mass transfer. The momentum and energy quantitative equations have been generalized in the form of one linear diffusion equation under different influences. Moreover, the present theory introduced a new point of views for a simplification of formulation and analytical solutions of many problems in the fields of physics**,** engineering, and biomedical sciences.

Keywords: Linear velocity operator concept. Heat and mass transfer. Linear Navier-Stokes equations. Mohammadein description. Linear and nonlinear convective terms. Nano fluids. The coupling of momentum and energy equations. Biomedical sciences.

1 Introduction

Many authors have developed their attention to study the nonlinear Partial differential equations (NLPDEns) using several methods [1-12]. The formulation of nonlinear Partial differential equations has a great role in various fields of Mathematics, Physics, Chemistry and Biology. The nonlinear partial differential equations may be used to describe the equations of weather, ocean currents, water flow in a channels, pipes and air flow around a wing of aircraft. But there is still the daunting problem of finding an analytical solution. So, it is important to obtain the exact solution of nonlinear Partial differential equations in physics and applied Mathematics.

Recently, many numerical, approximate and iteration variation methods are used for solving nonlinear problems such as Navier-Stokes, Burger, and Korteweg-deVries, (KDV) equations [4-12]. The nonlinear terms are still obstacles in the way of analytical solutions in different scales. Moreover, there is a list of nonlinear partial differential equations without analytical solutions up to date.

Most of problems in fluid mechanics are described by continuity, momentum, and energy equations. Some partial differential equations are formulated by vectors, tensors, and operators in mathematics and physics [1]. For example, Newton's law is derived in the vector form. Schrodinger equation is derived based on linear momentum operator [2]. Eldabe [4] introduced the peristaltic motion of nonnewtonian fluid flow in a vertical channel. Abo-Eldahab [5] introduced the free convective flow along a semi-infinite vertical plate with some effects.

Calogero and De Lillo [6] introduced a generalized Hopf-Cole transformation and solution of Burger equation with boundary conditions. The nonlinear partial differential equations of matrix Burgers equation is linearized by Hopf-Cole transformation [7].

Abourabia et al [8] introduced the exact solutions of the hierarchical Korteweg–de Vries equation of microstructure granular materials. The Korteweg-deVries (KDV) equations with some application in thermoplastic interaction in a half-space by pulsed laser heating and traveling wave are studied [9-11]. The solution of Nonlinear Navier-Stokes equation and its application is obtained for a growth problem under the effect of magnetic field [12].

The nonlinear Burger equation has the form

$$u_t + uu_x - \nu u_{xx} = 0, \qquad (1)$$

is linearized by using regular Hopf -Cole transformation [7] in the form.

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$$u(x,t) = \frac{-2\nu}{\phi(x,t)} \frac{\partial\phi(x,t)}{\partial x} . \qquad (2)$$

The above equation (1) transformed to a linear partial differential equation

$$\frac{\partial \phi}{\partial t} = \nu \frac{\partial^2 \phi}{\partial x^2} \qquad (3)$$

The linear diffusion equation (3) is solved by Picard method [11] with analytical way.

Up to date, the fluid state is described by Lagrange and Euler methods [1]. Euler proposed that, the total differentiation of any dependent physical function f can be written in the form

$$\underbrace{\frac{Df}{Dt}}_{total\ derivative} = \underbrace{\frac{\partial f}{\partial t}}_{local\ term} + \underbrace{(\hat{\underline{v}}.\underline{\nabla})f}_{convective\ term} . \qquad (4)$$

The parameter f represents some physical parameters (velocity, pressure, density, temperature, and enthalpy); which are very important in fluid mechanics and heat mass transfer. On the basic of the above equation, the acceleration of fluid flow in the point of view of Euler becomes

$$\underbrace{\frac{D\underline{v}}{Dt}}_{total\ acceleration} = \underbrace{\frac{\partial \underline{v}}{\partial t}}_{local\ acceleration} + \underbrace{(\hat{\underline{v}}.\underline{\nabla})\underline{v}}_{convective\ acceleration} \qquad (5)$$

where $\frac{\partial \underline{v}}{\partial t}$ is the local acceleration and $(\hat{\underline{v}}.\underline{\nabla})\underline{v}$ is the nonlinear term. The nonlinear convective acceleration stands as an obstacle against the analytical solutions. All mathematicians focus on solving the problem of that complex term since several years ago.

In this paper,. The (NLPDEns) are transformed to the linear ones by using a proposed linear velocity operator concept. For the first time, this treatment facilitates the solution of the non-linear problems by using the transformation of the nonlinear convective terms to a linear diffusion term. The linear velocity operator is formulated in terms of Mohammadein parameter M*. Moreover, the linear and nonlinear convective terms are transformed to the nano diffusion terms. This effort focus on the new point of nano view of fluid mechanics concepts, which not studied before. In section 1 the previous concepts of Euler, Lagrange and others are introduced. In section 2, the main theory of Mohammadein operator is proofed and Mohammadein parameter is classified in heat mass transfer in fluids and quantum mechanics. Moreover, the momentum and energy quantitative equations have been generalized in the form of one linear diffusion equation under different influences. In section 3, The nonlinear Navier-Stokes, Burger, Korteweg-deVries (KDV) are transformed to the linear diffusion equation. In the same way, the linear heat, concentration, are converted to the linear diffusion equation with different physical meaning of parameter M^*. In section 4, the discussion of present theory and its applications are introduced. In section 5, the concluded remarks of results and applications are tabulated.

2 Mohammadein Theories

The system of nonlinear (linear) partial differential equations that contains the nonlinear (linear) convective term $(\hat{\underline{v}}.\underline{\nabla})$ is transformed to the linear partial differential equations with diffusion terms $-M^*\underline{\nabla}^2$ .The total differentiation of any dependent physical function f in terms of parameter M^* (m^2/s) has the form

$$\underbrace{\frac{Df}{Dt}}_{total\ derivative} = \underbrace{\frac{\partial f}{\partial t}}_{local\ term} - \underbrace{M^*\,\underline{\nabla}^2 f}_{diffusion\ term} \qquad (6)$$

where M^* is proposed as Mohammadein parameter, which takes a different physical meaning in fluids, quantum mechanics, thermal and concentration diffusion in fluid follows

$$M^* = \left\{\begin{matrix} \frac{i\hbar}{m} & for\ \ quantum\ Mechanics \\ n\,\nu & for\ \ fluid\ mechanics \\ a_l & for\ \ thermal\ diffusion \\ D & for\ \ mass\ diffusion\ fraction \\ D_T & for\ \ concentration\ in\ bio\ tissues \\ zero & for\ Newton\ \ mechanics\ \ of\ solids \end{matrix}\right\}, \qquad (7)$$

Where M^* called Mohammadein parameter ($L^2\ T^{-1}$), $n \geq 1$ is constant, ħ is a reduced Plank constant, i is imaginary number, m is mass, ν is kinematic viscosity, a_l is thermal diffusivity coefficient, D is mass diffusivity of more volatile component, and D_T is concentration diffusion coefficient (m^2/s). The function f represents many physical parameters (velocity $\underline{v}$, temperature T, pressure P, density ρ, enthalpy h etc.) as in equation (7). The function f can be considered as vector or scalar amount.

Proof

A concept of the amount of linear motion equation in fluid mechanics [1] has the form

$$\underline{P} = m\underline{v} , \qquad (8)$$

and the linear momentum operator concept in quantum mechanics [2] has the form

$$\hat{\underline{p}} = -i\hbar\underline{\nabla}. \qquad (9)$$

Equation (8) can be rewritten in the operator form

$$\hat{\underline{p}} = m\hat{\underline{v}}. \qquad (10)$$

Now, from the above equations (9) and (10), the linear velocity operator takes the form

$$\hat{\underline{v}} = \frac{-i\hbar}{m}\underline{\nabla}. \qquad (11)$$

The linear velocity operator $\hat{\underline{v}}$ is a coupling of linear momentum operator in quantum and fluid mechanics. Then, the proposed linear velocity operator in terms of the new physical parameter M^* is formulated in the form

$$\hat{\underline{v}} = -M^*\underline{\nabla} . \qquad (12)$$

The total differentiation of any dependent physical function f can be written in the form

$$\frac{Df}{Dt} = \frac{\partial f}{\partial t} + (\underline{\hat{v}}.\underline{\nabla})f. \qquad (13)$$

The generalized form of total operator in terms of Mohammadein parameter M^*based on equations (12) and (13) becomes

$$\underbrace{\frac{D\ldots}{Dt}}_{total\ derivative} = \underbrace{\frac{\partial\ldots}{\partial t}}_{local\ derivative} - \underbrace{M^*\,\underline{\nabla}^2\ldots}_{diffusion\ derivative} \qquad (14)$$

The new definition of total operator for any physical function f (vector or scalar) with local and diffusion terms is called Mohammadein operator as follows

$$\frac{Df}{Dt} = \frac{\partial f}{\partial t} - M^*\underline{\nabla}^2 f, . \qquad (15)$$

where f represents density, velocity, temperature, pressure, density, mass diffusion in fluid mechanics, plasma, thermal, binary thermal and concentration of blood in the bio tissues. The theory introduces a new concept of nano fluid mechanics and some points of views in physics and mathematics. Moreover, equation (15) can be considered as a new description of operator f in fluid mechanics and heat mass transfer.

2.1 Mohammadein Description

The Euler and Lagrangian descriptions for a fluid state in fluid mechanics are studied only in terms of acceleration of fluid particles. In this section, the third description of fluid mechanics and heat mass transfer can be described by using equation (6) under some different effects in the form

$$\frac{\partial f}{\partial t} - M^*\underline{\nabla}^2 f = \textstyle\sum_i \Pi_i.$$

(16) where f, M^*, and Π_i are classified in Table 1 as in the appendix 2. Equation (16) indicates to the coupling of momentum and energy equations in a one linear partial differential diffusion equation. Table 1 shows the importance of diffusion equation (16) in fluid mechanics and heat mass transfer. Moreover, the parameter M^* takes different physical expressions (ν, a_l, D and D_T) with the same dimension ($L^2\ T^{-1}$)

3 Applications of New Treatment Theory

The importance of the present theory does not stop to transform the equations from nonlinear to a linear one only. But it, helps us to get the analytical solutions of these linear diffusion equations in unsteady state. Moreover, it gives us a new point of nano views in the different fields of physics**,** engineering, and biomedical sciences.

In the follows, the present theory can be applied for some famous equations that contains the convective nonlinear (linear) terms like Navier-Stokes, Burger, Korteweg-deVries (KDV), mass diffusion, and concentration distribution equations.

3.1 Navier – Stokes Equations

Navier–Stokes equations are very important for modelling many problems of physics, engineering, and biomedical sciences [4-12]. The nonlinear system of Navier–Stokes equations has the vector form

$$\frac{\partial \underline{v}}{\partial t} + (\underline{\hat{v}}.\underline{\nabla})\underline{v} = -\frac{1}{\rho}\nabla P + \frac{1}{\rho}\underline{\nabla}.\tau_{ij} + g\,\widehat{\underline{n}}\,. \qquad (17)$$

Applying the new operator (12) on the above equation, then

$$\frac{\partial \underline{v}}{\partial t} - \nu\,\underline{\nabla}^2\underline{v} = -\frac{1}{\rho}\nabla P + \frac{1}{\rho}\underline{\nabla}.\tau_{ij} + g\,\widehat{\underline{n}}. \qquad (18)$$

The above equation (18) called linear Navier-Stokes equations, which are obtained for the first time with linear convective diffusion terms. The total acceleration with local and linear convective diffusion terms has the form

$$\frac{D\underline{v}}{Dt} = \frac{\partial \underline{v}}{\partial t} - \nu\,\underline{\nabla}^2\underline{v}. \qquad (19)$$

The linear diffusion acceleration (19) can be considered as a third description of fluid mechanics. Moreover, it gives a new point of nano views in fluid mechanics.

3.2 Burger Equation

The Burger equation is one of the most important nonlinear Partial differential equations. It was originally derived for shallow water waves model with weak nonlinearities [6], but it has a wide variety of applications. The nonlinear Burger equation is written as

$$\frac{\partial u}{\partial t} + n\,u\frac{\partial u}{\partial x} - \nu\frac{\partial^2 u}{\partial x^2} = 0. \qquad (20)$$

Applying the new operator (12) on the above nonlinear equation (20) then, the linear Burger equation becomes

$$\frac{\partial u}{\partial t} = \nu(n+1)\frac{\partial^2 u}{\partial x^2} \qquad (21)$$

The above linear diffusion equation (21) represents the linear water wave of Burger equation with linear diffusion term in unsteady state.

3.3 Korteweg-deVries (KDV) Equation

The Korteweg-deVries (KDV) equation [8-10] is one of the most important nonlinear partial differential equations. It was originally derived for shallow water waves model with weak nonlinearities, but it has a wide variety of applications. The KDV equation is written as

$$\frac{\partial u}{\partial t} + 6u\frac{\partial u}{\partial x} + \frac{\partial^3 u}{\partial x^3} = 0. \qquad (22)$$

Applying the new operator (12) on the above nonlinear equation (22) then, the linear KDV equation becomes

$$\frac{\partial u}{\partial t} - 6\nu\frac{\partial^2 u}{\partial x^2} + \frac{\partial^3 u}{\partial x^3} = 0. \qquad (23)$$

The above diffusion equation (23) represents a linear water wave of KDV equation with diffusion term in unsteady state.

3.4 Heat Equation

The linear heat transfer equation in terms of temperature distribution with local and convective terms is

$$\frac{\partial T}{\partial t} + (\underline{\hat{v}}.\underline{\nabla})T = a_l\underline{\nabla}^2 T. \qquad (24)$$

Applying the new operator (12) for the above equation, then

$$\frac{\partial T}{\partial t} = 2a_l \underline{\nabla}^2 T, \tag{25}$$

where $a_l = \frac{k}{\rho c_p}$, k is a thermal conductivity, ρ is fluid density and c_p is specific heat at constant pressure.

The above heat equation (25) represents a linear diffusion equation with diffusion term in unsteady state of temperature.

3.5 Mass Diffusion Fraction Equation

The linear mass diffusion fraction in a superheated binary mixture state has the form

$$\frac{\partial \chi}{\partial t} + \left(\underline{\hat{v}}.\underline{\nabla}\right)\chi = D\underline{\nabla}^2 \chi, \tag{26}$$

where χ is the instantaneous mass fraction of more volatile component at any point in liquid. Applying the new operator (12) on the above equation, then

$$\frac{\partial \chi}{\partial t} = 2D\underline{\nabla}^2 \chi, \tag{27}$$

where D is a mass diffusivity of more volatile component in less volatile component, The above equation (27) represents a linear diffusion equation with diffusion term in unsteady state of instantaneous mass fraction.

3.6 Concentration Equation

The linear concentration distribution equation in the bio tissues has the form

$$\frac{\partial C}{\partial t} + \left(\underline{\hat{v}}.\underline{\nabla}\right)C = D_T\underline{\nabla}^2 C. \tag{28}$$

The above equation (28) has a wide applications in the field of bio medical sciences. Applying the new operator (12) on the above equation, then

$$\frac{\partial C}{\partial t} = 2D_T\underline{\nabla}^2 C. \tag{29}$$

The above equation (29) represents a linear concentration equation with diffusion term in unsteady state.

3.7 Schrödinger Equation

Schrödinger equation is very important in quantum mechanic's field. The main equation (16) in the present theory has the form

$$\frac{\partial f}{\partial t} + \left(\underline{\hat{v}}.\underline{\nabla}\right)f = \textstyle\sum_i \Pi_i. \tag{30}$$

In the follows, the parameters $f, \underline{\hat{v}}$, and Π_i are substituted by ψ, $\frac{-i\hbar}{2m}D_x$, and $\frac{1}{i\hbar}\mathrm{V(x)}\,\psi$ respectively, then, the above equation becomes

$$\left(\frac{\partial}{\partial t} - \frac{i\hbar}{2m}D_x^2\right)\psi = \frac{1}{i\hbar}\mathrm{V(x)}\,\psi. \tag{31}$$

The above equation can be simplified in the form

$$i\hbar\left(\frac{\partial}{\partial t} - \frac{i\hbar}{2m}D_x^2\right)\psi = \mathrm{V(x)}\,\psi\ . \tag{32}$$

The final form of Schrödinger equation can be written as follows

$$\left(-\frac{\hbar^2}{2m}D_x^2 + \mathrm{V(x)}\right)\psi = \left(i\hbar\frac{\partial}{\partial t}\right)\psi. \tag{33}$$

It is noted that based on the present theory (15), the above diffusion equation (33) represents Schrödinger equation in a simple way. Moreover, the linear velocity operator concept proves the validity of the present theory.

4 Discussion of Theory and Applications

The nonlinear convective term $\left(\underline{\hat{v}}.\underline{\nabla}\right)\underline{v}$ is converted to a linear diffusion term $-\nu\,\underline{\nabla}^2\underline{v}$ as performed in Navier-Stokes, Burger and KDV equations. On contrary, the linear convective term $\left(\underline{\hat{v}}.\underline{\nabla}\right)T$ is transformed to a linear diffusion term $-a_l\underline{\nabla}^2 T$ as performed in heat equation (24). The velocity of particles is considered in an unsteady state inside any point in the cases of rest and motion states.

The above simplest linear diffusion equations 18, 21, 23, 25, 27, 29 and 33 are obtained by applying new operator (12) and can be solved by analytical ways.

5 Conclusions

The linear velocity operator (12), the total operator (15) and the linear diffusion equation (16) represent the new treatment theory. Based on the previous discussions of Mohammadein theory and its applications the following remarks are concluded:

1. The total differentiation of the physical function f is defined in terms of new parameter M^*(Mohammadein parameter) in fluid mechanics and heat mass transfer.
2. A new parameter M^*play a real role in the modification of fluid state description in the range of macroscopic to a nanoscopic scales with a different physical meaning in different fields.
3. The dimension of parameter M^* is $((length)^2/second)$ in fluid mechanics and heat mass transfer.
4.The motion of particles in an unsteady state inside any point of physical medium is existed even when $\frac{\mathrm{D}f}{\mathrm{Dt}} = 0$.
5. The linear and nonlinear convective terms are converted into a linear diffusion term on the basis of equation (12).
6. The nonlinear Navier-Stokes, Burger, and Korteweg-deVries (KDV) equations are converted for a first time into a linear diffusion form based on the present theory.
7. The present theory is applied to a linear partial differential equation (heat, mass and concentration); which are transformed to a simple linear diffusion equation with nano diffusion terms.
8. The momentum and energy quantitative equations have been generalized in the form of the diffusion equation under different influences.
9. Equation (16) under different effects introduced the third description of fluid mechanics and heat mass transfer.
10. The derivation of *Schrödinger* equation is obtained based on the basis of the present theory (16). Moreover, the Schrödinger equation is derived by Hamiltonian definition as in Appendix 1.
11. Mohammadein description introduces a new point of nano view of the physical state of function f in fluid mechanics and heat mass transfer.

Moreover, there are still many NLPDEns without analytical solutions and mathematicians' peoples must thinking about the exact solutions as a future prospect.

Appendix 1

Derivation of Schrödinger Equation

Schrödinger equation for time dependent energy in quantum mechanics by using the proposed linear velocity operator concept (12) is obtained.

The Hamiltonian of the Eigen function ψ has the form

$$H\psi = \mathrm{E}\psi. \tag{A1}$$

The total energy equal to motion energy plus potential energy.

$$\left(\frac{\tilde{p}^2}{2m} + V(x)\right)\psi = E\psi. \tag{A2}$$

The linear momentum operator $\underline{\hat{p}}$ used in derivation of Schrödinger equation, moreover the relation between linear momentum and velocity can be written in the operator form $\underline{\hat{p}} = m\underline{\hat{v}}$ then equation (A2) becomes

$$\left(\frac{m\hat{v}^2}{2} + V(x)\right)\psi = E\psi. \tag{A3}$$

The operators $\hat{v}$ and E are as follows

$$E \rightarrow ih\, D_t\,,\ \hat{v} \rightarrow -\frac{i\hbar}{m} D_x \ and\ V(x) \rightarrow V(x). \tag{A4}$$

The final form of Schrödinger equation becomes

$$\left(-\frac{\hbar^2}{2m} D_x^2 + V(x)\right)\psi = ih\, D_t\, \psi. \tag{A5}$$

The above equation represents the time dependent Schrödinger equation under the effect of potential V(x).

Appendix 2

The relation between physical parameters $(f, M^*, \sum_i \Pi_i)$ in a main equation (16) are defined for some linear and nonlinear partial differential equations in the following Table

Table 1: The relation between physical parameters in main linear and nonlinear partial differential equations (30).

f	M^*	$\sum_i \Pi_i$	Kind of equation
$\underline{v}$	ν	Body and surface Forces	Navier-Stokes equations
n $\underline{v}$	$n\,\nu$	Without body and surface forces	Burger equation
T	a_l	Heating sink and Source	Heat equation
C	D_T	Suction and injection process	Concentration equation
Ψ	$\frac{i\hbar}{2m}$	Potential energy	Schrodinger equation
u	ν	Weak shallow water waves	Korteweg-deVries equation
u	Zero	Newton force	Newton equation

C is concentration field, u is a velocity of fluid or solid, and Ψ is the wave function.

Conflict of interest

The author has no conflicts of interest to disclose.

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References

[1] Landau, L. D. and E. M. Lifshitz,. Fluid Mechanics., **6**, (1987).

[2] Mahesh C. Jain. Quantum Mechanics (2011).

[3] Tyn Myint-U, and Lokenath Debnath. Linear Partial Differential Equations, Birkhäuser Boston (2007).

[4] N. T. M. Eldabe, M. O. Shaker, and Maha S. A., Peristaltic Flow of MHD Jeffrey Fluid Through Porous Medium in a Vertical Channel with Heat and Mass Transfer with Radiation. Journal of Nanofluids .,**7(3)**, 595-602(2018).

[5] Abo-Eldahab, E.M Emad M. The effects of temperature-dependent fluid properties on free convective flow along a semi-infinite vertical plate by the presence of radiation. Heat and Mass Transfer., **41(2)**,163-169 (2004).

[6] F. CALOGERO, S. DE LILLo. The burgers equation on the simi linear with general boundary condition at the origin. J. Math. Phys., **32**, 99-105(1991).

[7] K.T. Joseph, and A.S. Vasudeva Murthy. Hopf-Cole transformation to some systems of partial differential equations. Article in Nonlinear Differential Equations and Applications NoDEA · June 2001

[8] A. M. Abourabia, T.S. El-Danaf, A.M. Morad. Exact solutions of the hierarchical Korteweg–de Vries equation of microstructured granular materials. Chaos, Solitons and Fractals., **41**, 716–726 (2009)

[9] Saadi, F.; Azizpour, M.J.; Zahedi, S.A. Analytical solutions of Kortweg-de Vries (KdV) equation. World Acad. Sci. Eng. Technol. 2010., **69**, 171–175(2020).

[10] Hedli, R.; Kadem, A. Exact Traveling Wave Solutions to the Fifth-order KdV Equation Using the Exponential Expansion Method. IAENG Int. J. Appl. Math., **50**, 1–6. 18(2020).

[11] Ahmad, H.; Khan, T.A.; Stanimirovic, P.S.; Ahmad, I. Modified variational iteration technique for the numerical solution of fifth order KdV type equations. J. Appl. Comput. Mech. (2020).

[12] Mohammadein, S. A., R. A. GadEl-Rab, A. A Megahed, A. Mosbah. Solution of Navier-Stokes'equation and its application for a growth problem under the effect of magnetic field. Faculty of Education, Ain Shams University (2011).

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