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Propagation of Rayleigh Waves in Layered Elastic Half-Space with Finite Sliding Contact

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Abstract: The propagation of Rayleigh waves in an orthotropic elastic half-space underlying an orthotropic elastic layer is analysed. The layer and the half-space are considered in finite sliding contact, for which a parameter $\xi, (0 \leq \xi \leq 1)$ has been introduced to represent the sliding contact interfaces. The extreme values of ξ correspond to smooth and perfect contact interface, respectively. It was found that the general dispersion equation exhibiting finite sliding contact between the layer and the half-space depends on the sliding parameter. Frequency equations derived by Vinh et al. [1] and Vinh and Anh [2] have been recovered as particular cases of the present formulation. Numerically the effect of sliding parameter on the speed of Rayleigh wave has been examined for orthotropic half-space (Topaz) underlying an orthotropic layer (Barytes), (ii) Uniform half-space (Granite) underlying an elastic layer (Sandstone). The comparison of Rayleigh wave speed behavior corresponding to smooth sliding and perfect contact has been shown graphically.

Keywords: Exact secular equation, Orthotropic, Phase speed, Rayleigh wave, Sliding contact.

1 Introduction

Surface waves are the elastic waves that confine themselves along the boundary surface of an elastic body. They have been theoretically and practically important because of their enormous applications in several fields, such as, geophysics, engineering, communication and terrestrial radio broadcast. They also have technical applications in non-destructive material testing and electro-mechanical transducers. Rayleigh [3] was the first to discover and characterize " Rayleigh waves", which were named after him in the literature. They are a combination of dilatational and distortional waves and propagate with speed slightly less than that of distortional waves. Several problems pertaining to Rayleigh type waves have been explored by the researchers of elastic half-space having different properties by deriving secular equation with the help of appropriate boundary conditions in accordance with the considered model. This frequency equation provides the information concerning the characteristics of Rayleigh wave propagation.

Elastic half-space can be spotted frequently in nature, the possibility of a model consisting of an elastic layer over an elastic half space can not be ruled out even in natural settings. The existence of various anisotropic layered elastic media in geophysical settings and various composite materials and structures, such as crystals, motivated us to study the propagation characteristics of Rayleigh type surface waves. A layer of earth material may be found resting on an elastic half-space of different material in different geophysical settings; for instance, a layer of sandstone may be in contact with elastic rock of larger dimension. This situation may be modeled as a layer resting over an elastic half-space.

A huge volume of literature pertaining to Rayleigh wave propagation currently addresses different problems of waves and vibrations in the half-space and layered half-space. For the treatment of these research problems, one can refer to the papers by Achenbach and Keshava [4] and Tiersten [5]. Chandrasekharaiah [6] explored the effect of surface stresses and voids on Rayleigh wave propagation in homogeneous and isotropic linear elastic half-space with voids. The dispersive nature of the Rayleigh wave has been characterized using the phase-velocity equation. The author has observed that the presence of voids and surface stresses in the medium is the reason for the dispersive nature of the wave. Vinh and Linh in [7] have studied Rayleigh wave propagation in an orthotropic thin layered elastic half-space and established

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an approximate secular equation of third-order employing the effective boundary condition method. Using effective boundary conditon method Vinh and his coworkers [8]-[10] have also examined the propagation of Rayleigh waves in anisotropic elastic half-spaces coated by a thin elastic layer. For this problem the contact between the layer and the half-space is supposed to be sliding. Authors have obtained an approximate secular equations of third-order for the case when the layer and the half-space are both orthotropic, whereas fourth-order approximate secular equation is derived when both the layer and the half space are isotropic. These approximate secular equations have high accuracy.

Singh et al. [11] investigated the propagation of Rayleigh waves in an incompressible visco-elastic material under the effect of initial stress and defined the dispersion equation to study the effect of parameters and incompressibility on the Rayleigh-type wave propagation. Kaur et al. [12] have derived dispersion relation for Rayleigh-type surface wave in an isotropic homogeneous nonlocal elastic solid half-space with voids. They have detected that the Rayleigh-type wave travels with complex speed and is dispersive and attenuating. Authors have noticed that the dispersive nature of the wave results from the presence of voids and nonlocality in the medium. Recently, Tomar and Kaur [13] have investigated the role of sliding contact interface on torsional waves in a layered medium. Singh and Tochhawng [14] have analysed the surface waves (Stoneley and Rayleigh waves) propagation in thermoelastic materials with voids. They considered two dissimilar half-spaces of thermoelastic materials with voids and obtained secular equations of the Stoneley waves at the bonded and unbonded interfaces.

The present paper aims to establish a general frequency equation for Rayleigh waves propagating in an orthotropic elastic half-space overlaid by an orthotropic elastic layer of arbitrary thickness, which is in finite sliding contact with the orthotropic half-space. This general frequency equation provides us with the frequency equation earlier derived by Vinh and his group for the corresponding problems. A parameter $\xi, (0 \le \xi \le 1)$ is introduced for this purpose. For $\xi = 0$, the general frequency equation for Rayleigh waves propagating in an orthotropic elastic half-space overlaid by an orthotropic elastic layer obtained in the present study reduces to the frequency equation of Rayleigh wave propagtioning in smooth sliding contact interface as obtained in equation no. (35) by Vinh and Anh [2]. However, for $\xi = 1$, the general frequency equation reduces to the frequency equation of Rayleigh wave propagation in welded contact interface given in equation no. (36) by Vinh et al. [1]. The effect of sliding contact on the speed of Rayleigh wave propagation in question has been studied numerically. A special case of isotropic layered structure has been reduced from the present formulation and studied numerically. To analyse the effect of sliding parameter on Rayleigh wave speed numerical computaions are performed for four different models. In Model-I & Model-II, the values of relevant elastic parameters for layered orthotropic half-space are taken from Vinh et al. [1] and Vinh and anh [2], respectively. For Model-III, the orthotropic half-space (Topaz) underlying an orthotropic layer (Barytes) is considered. Lastly, in Model-IV, an isotropic elastic half-space (Granite) underlying an isotropic elastic layer (Sandstone) is considered. The parameter values corresponding to different models are set depending on their respective geophysical settings. The numerically computed results have been depicted graphically and discussed.

2 Model formulation

We consider an elastic half-space in contact with an elastic layer of uniform thickness h. Under rectangular cartesian coordinate system, let the plane of contact between the half-space and the layer be $x_2 = 0$ such that the half-space occupies the region $x_2 \ge 0$ and the layer occupies the region $-h \le x_2 \le 0$. The layer and the half-space are both homogeneous, orthotropic and in finite sliding contact with each other. The entities related to the layer are denoted with an over bar in the half-space and without an over bar.

For a plane strain problem, we shall take the components of displacement vector u_i in the half-space and in the layer as

$$u_i = u_i(x_1, x_2, t), \quad \bar{u}_i = \bar{u}_i(x_1, x_2, t), \qquad (1)$$
$$u_3 = \bar{u}_3 \equiv 0, \quad (i = 1, 2), \quad \frac{\partial(.)}{\partial x_3} \equiv 0,$$

where x_1, x_2 and x_3 are the spatial variables and t is the time variable.

2.1 Wave propagation

(i) In the layer:

The stress-strain relation in the orthotropic elastic layer is given by

$$\bar{\sigma}_{11} = \bar{c}_{11}\bar{u}_{1,1} + \bar{c}_{12}\bar{u}_{2,2}, \quad \bar{\sigma}_{22} = \bar{c}_{12}\bar{u}_{1,1} + \bar{c}_{22}\bar{u}_{2,2}, \qquad (2)$$
$$\bar{\sigma}_{12} = \bar{c}_{66}(\bar{u}_{1,2} + \bar{u}_{2,1}),$$

where $\bar{\sigma}_{ij}$ are the components of stress tensor, a comma before an index indicates the differentiation with respect to spatial variable x_k and $\bar{c}_{11}, \bar{c}_{22}, \bar{c}_{12}$ and $\bar{c}_{66}$ are the material constants. To make strain energy function positive definite, the following inequalities must hold among the material constants

$$\bar{c}_{kk} > 0, \quad \bar{c}_{11}\bar{c}_{22} - \bar{c}_{12}^2 \ge 0, \quad (k = 1, 2, 6), \qquad (3)$$

The equations of motion in the absence of body forces are given, as follows,

$$\bar{c}_{11}\bar{u}_{1,11} + \bar{c}_{66}\bar{u}_{1,22} + (\bar{c}_{12} + \bar{c}_{66})\bar{u}_{2,12} = \bar{\rho}\ddot{\bar{u}}_1, \qquad (4)$$

$$(\bar{c}_{12}+\bar{c}_{66})\bar{u}_{1,12}+\bar{c}_{66}\bar{u}_{2,11}+\bar{c}_{22}\bar{u}_{2,22}=\bar{\rho}\ddot{\bar{u}}_2. \tag{5}$$

Here, $\bar{\rho}$ represents the mass density of the layer material and an over dot signifies temporal derivative.

Following Vinh et al. [1] and Vinh and Anh [2] for the Rayleigh wave traveling along the interface between the layer and the half-space with speed c in the positive x_1- direction, we take the displacement components satisfying (4) and (5) as

$$\{\bar{u}_1,\bar{u}_2\}=\{\bar{U}_1(y),\bar{U}_2(y)\}\exp\{\iota k(x_1-ct)\}, \tag{6}$$

where

$$\begin{aligned}\bar{U}_1(y)&=A_1ch(\bar{b}_1y)+A_2sh(\bar{b}_1y)\\&\quad+A_3ch(\bar{b}_2y)+A_4sh(\bar{b}_2y),\\\bar{U}_2(y)&=\iota\left[\bar{\alpha}_1A_1sh(\bar{b}_1y)+\bar{\alpha}_1A_2ch(\bar{b}_1y)\right]\\&\quad+\iota\left[\bar{\alpha}_2A_3sh(\bar{b}_2y)+\bar{\alpha}_2A_4ch(\bar{b}_2y)\right].\end{aligned} \tag{7}$$

A_1, A_2, A_3 and A_4 are constants, k is the wavenumber and $y=kx_2$. Using (6) into (2), the relevant components of stresses in the layer can be written as

$$\{\bar{\sigma}_{12},\bar{\sigma}_{22}\}=k\{\bar{\Sigma}_1(y),\bar{\Sigma}_2(y)\}\exp\{\iota k(x_1-ct)\}, \tag{8}$$

where

$$\begin{aligned}\bar{\Sigma}_1(y)&=\bar{\beta}_1A_1sh(\bar{b}_1y)+\bar{\beta}_1A_2ch(\bar{b}_1y)\\&\quad+\bar{\beta}_2A_3sh(\bar{b}_2y)+\bar{\beta}_2A_4ch(\bar{b}_2y),\\\bar{\Sigma}_2(y)&=\iota\left[\bar{\gamma}_1A_1ch(\bar{b}_1y)+\bar{\gamma}_1A_2sh(\bar{b}_1y)\right]\\&\quad+\iota\left[\bar{\gamma}_2A_3ch(\bar{b}_2y)+\bar{\gamma}_2A_4sh(\bar{b}_2y)\right].\end{aligned} \tag{9}$$

Substituting $x_2=0$ into equations (7) and (9), one obtains a system of four equations in A_1, A_2, A_3 and A_4, which on solving yields

$$\begin{aligned}A_1&=\frac{\bar{\gamma}_2}{[\bar{\gamma}]}\bar{U}_1(0)+\frac{\iota}{[\bar{\gamma}]}\bar{\Sigma}_2(0),\\A_2&=\frac{\iota\bar{\beta}_2}{[\bar{\alpha};\bar{\beta}]}\bar{U}_2(0)+\frac{\bar{\alpha}_2}{[\bar{\alpha};\bar{\beta}]}\bar{\Sigma}_1(0),\\A_3&=-\frac{\bar{\gamma}_1}{[\bar{\gamma}]}\bar{U}_1(0)-\frac{\iota}{[\bar{\gamma}]}\bar{\Sigma}_2(0),\\A_4&=-\frac{\iota\bar{\beta}_1}{[\bar{\alpha};\bar{\beta}]}\bar{U}_2(0)-\frac{\bar{\alpha}_1}{[\bar{\alpha};\bar{\beta}]}\bar{\Sigma}_1(0).\end{aligned} \tag{10}$$

where the notations are defined as

$$[f;g]=f_2g_1-f_1g_2,\quad\text{and}\quad[f]=f_2-f_1. \tag{11}$$

(ii) In the half-space:

The stress-strain relations in the orthotropic elastic half-space are given earlier by (1), but without an overbar, that is

$$\begin{aligned}&\sigma_{11}=c_{11}u_{1,1}+c_{12}u_{2,2},\quad\sigma_{22}=c_{12}u_{1,1}+c_{22}u_{2,2},\\&\sigma_{12}=c_{66}(u_{1,2}+u_{2,1}).\end{aligned} \tag{12}$$

Following Vinh et al. [1], the relevant displacement components for the Rayleigh surface wave traveling with speed c and wavenumber k in the positive x_1- direction are given as

$$\{u_1,u_2\}=\{U_1(y),U_2(y)\}\exp\{\iota k(x_1-ct)\}, \tag{13}$$

where

$$\begin{aligned}U_1(y)&=B_1e^{-b_1y}+B_2e^{-b_2y},\\U_2(y)&=\iota\left(\alpha_1B_1e^{-b_1y}+\alpha_2B_2e^{-b_2y}\right).\end{aligned} \tag{14}$$

B_1 and B_2 are constants. As in Vinh and Ogden [11], the existence of Rayleigh wave will be ensured if the following inequality holds

$$0<X<\min\{c_{11},c_{66}\}. \tag{15}$$

Employing (13) into (12), the stress-strain relations are given by

$$\{\sigma_{12},\sigma_{22}\}=k\{\Sigma_1(y),\Sigma_2(y)\}\exp\{\iota k(x_1-ct)\}, \tag{16}$$

where

$$\begin{aligned}\Sigma_1(y)&=\beta_1B_1e^{-b_1y}+\beta_2B_2e^{-b_2y},\\\Sigma_2(y)&=\iota\left(\gamma_1B_1e^{-b_1y}+\gamma_2B_2e^{-b_2y}\right).\end{aligned} \tag{17}$$

Substituting $x_2=0$ in (14) and (17), we obtain

$$\begin{aligned}&U_1(0)=B_1+B_2,\quad U_2(0)=\iota(\alpha_1B_1+\alpha_2B_2),\\&\Sigma_1(0)=\beta_1B_1+\beta_2B_2,\quad\Sigma_2(0)=\iota(\gamma_1B_1+\gamma_2B_2).\end{aligned} \tag{18}$$

3 Boundary conditions and secular equation

The present study is performed by introducing a sliding parameter ξ $(0\le\xi\le1)$ such that when $\xi=0$, the layer and the half-space are in smooth contact and when $\xi=1$, the layer and the half-space are in perfect contact with each other. While for $0<\xi<1$, the contact between the layer and the half-space is in finite sliding contact. The appropriate boundary conditions will be set up on the displacements and stresses at $x_2=0$ and at the surface of the layer $x_2=-h$. We shall assume that the top surface of the layer is mechanically stress free. Mathematically, these boundary conditions are expressed as:

At the interface $x_2=0$:

$$\begin{aligned}\bar{\sigma}_{12}=\xi\sigma_{12},\quad\bar{\sigma}_{22}=\sigma_{22},\\(1-\xi)\sigma_{12}+Fk\xi u_1=Fk\xi\bar{u}_1,\quad u_2=\bar{u}_2.\end{aligned} \tag{19}$$

F is a constant quantity having the dimension of force per unit area.

At the boundary surface $x_2=-h$:

$$\bar{\sigma}_{12}=\bar{\sigma}_{22}=0, \tag{20}$$

When $\xi=1$, the boundary conditions given in (19) reduce to those given by Vinh et al. [1] for the corresponding problem at welded contact interface. Whereas, when $\xi=0$, they reduce to those given by Vinh

and Anh [2] for the corresponding problem at smooth contact interface.
Boundary conditions given in (19) can be written as

$$\bar{\Sigma}_1(0) = \xi\Sigma_1(0), \quad \bar{\Sigma}_2(0) = \Sigma_2(0),$$
$$(1-\xi)\Sigma_1(0) + F\xi U_1(0) = F\xi\bar{U}_1(0), \quad U_2(0) = \bar{U}_2(0). \tag{21}$$

Using equations (8) and (9) in (20) gives

$$\bar{\beta}_1 A_1 sh(\varepsilon_1) - \bar{\beta}_1 A_2 ch(\varepsilon_1) + \bar{\beta}_2 A_3 sh(\varepsilon_2) - \bar{\beta}_2 A_4 ch(\varepsilon_2) = 0, \tag{22}$$

$$\bar{\gamma}_1 A_1 ch(\varepsilon_1) + \bar{\gamma}_1 A_2 sh(\varepsilon_1) + \bar{\gamma}_2 A_3 ch(\varepsilon_2) + \bar{\gamma}_2 A_4 sh(\varepsilon_2) = 0, \tag{23}$$

where $\varepsilon_j = \bar{b}_j\varepsilon$ and $\varepsilon = kh$.
Introducing (10) into equations (22) and (23), we have

$$a_{11}\bar{\Sigma}_1(0) - \iota a_{12}\bar{\Sigma}_2(0) + b_{11}\bar{U}_1(0) - \iota b_{12}\bar{U}_2(0) = 0, \tag{24}$$

$$a_{21}\bar{\Sigma}_1(0) - \iota a_{22}\bar{\Sigma}_2(0) + b_{21}\bar{U}_1(0) - \iota b_{22}\bar{U}_2(0) = 0, \tag{25}$$

where

$$\begin{aligned} a_{11} &= \frac{[\bar{\alpha}; \bar{\beta} ch\varepsilon]}{[\bar{\alpha}; \bar{\beta}]}, \quad a_{12} = -\frac{[\bar{\beta} sh\varepsilon]}{\bar{\gamma}}, \\ a_{21} &= \frac{[\bar{\gamma} sh\varepsilon; \bar{\alpha}]}{[\bar{\alpha}; \bar{\beta}]}, \quad a_{22} = \frac{[\bar{\gamma} ch\varepsilon]}{\bar{\gamma}}, \\ b_{11} &= \frac{[\bar{\beta} sh\varepsilon; \bar{\gamma}]}{[\bar{\gamma}]}, \quad b_{12} = \frac{\bar{\beta}_1\bar{\beta}_2[ch\varepsilon]}{[\bar{\alpha}; \bar{\beta}]}, \\ b_{21} &= -\frac{\gamma_1\gamma_2[ch\varepsilon]}{\bar{\gamma}}, \quad b_{22} = \frac{[\bar{\beta}; \bar{\gamma} sh\varepsilon]}{[\bar{\alpha}; \bar{\beta}]}. \end{aligned} \tag{26}$$

Now using (21) into (24) and (25), we obtain

$$\left[a_{11} + \frac{b_{11}(1-\xi)}{F\xi}\right]\Sigma_1(0) - \iota a_{12}\Sigma_2(0) + b_{11}U_1(0) - \iota b_{12}U_2(0) = 0, \tag{27}$$

$$\left[a_{21} + \frac{b_{21}(1-\xi)}{F\xi}\right]\Sigma_1(0) - \iota a_{22}\Sigma_2(0) + b_{21}U_1(0) - \iota b_{22}U_2(0) = 0. \tag{28}$$

Using (18) into (27) and (28), we obtain two homogeneous linear equations in two unknowns B_1 and B_2. For nontrivial solution of this system of equations, the determinant of its coefficient matrix must vanish, which yields

$$\begin{aligned} &\left[(a_{11}a_{22} - a_{12}a_{21})\xi + (a_{22}b_{11} - a_{12}b_{21})\left(\frac{1-\xi}{F\xi}\right)\right][\gamma; \beta] \\ &+ \left[(a_{11}b_{22} - a_{21}b_{12})\xi + (b_{11}b_{22} - b_{12}b_{21})\left(\frac{1-\xi}{F\xi}\right)\right][\alpha; \beta] \\ &- (a_{12}b_{21} - b_{11}a_{22})[\gamma] + (a_{12}b_{22} - a_{22}b_{12})[\alpha; \gamma] \\ &+ (b_{11}b_{22} - b_{12}b_{21})[\alpha] - \xi(a_{11}b_{21} - a_{21}b_{11})[\beta] = 0, \end{aligned} \tag{29}$$

where

$$\begin{aligned} [\alpha] &= (X - c_{11} - c_{66}b_1b_2)\theta, \\ [\gamma; \beta] &= c_{66}[c_{12}^2 - c_{22}(c_{11} - X)]b_1b_2 + X(c_{11} - X)\theta, \\ [\beta] &= [\alpha; \gamma], \quad [\alpha; \beta] = c_{66}(c_{11} - X)(b_1 + b_2)\theta, \\ [\gamma] &= c_{22}c_{66}b_1b_2(b_1 + b_2)\theta, \\ [\alpha; \gamma] &= c_{66}(c_{11} - X - c_{12}b_1b_2)\theta, \\ \theta &= [b][(c_{12} + c_{66})b_1b_2]^{-1}. \end{aligned}$$

Employing (26) into (29), we obtain

$$\begin{aligned} A_1 + B_1 ch\varepsilon_1 ch\varepsilon_2 + C_1 sh\varepsilon_1 sh\varepsilon_2 + D_1 ch\varepsilon_1 sh\varepsilon_2 \\ + E_1 ch\varepsilon_2 sh\varepsilon_1 = 0, \end{aligned} \tag{30}$$

where the coefficients A_1, B_1, C_1, D_1 and E_1 are given by

$$\begin{aligned} A_1 &= -\xi(\bar{\alpha}_1^*\bar{\beta}_2^*\bar{\gamma}_2^* + \bar{\alpha}_2^*\bar{\beta}_1^*\bar{\gamma}_1^*) \\ &\quad \left[\{e_3^2 - e_2(e_1 - x)\}\sqrt{P} + x(e_1 - x)\right] \\ &\quad + 2\bar{\beta}_1^*\bar{\beta}_2^*\bar{\gamma}_1^*\bar{\gamma}_2^*\bar{c}_f\frac{(1-\xi)}{\xi}\left[(e_1 - x)\sqrt{S + 2\sqrt{P}}\right] \\ &\quad - \bar{\beta}_1^*\bar{\beta}_2^*(\bar{\gamma}_1^* + \bar{\gamma}_2^*)\left(e_1 - x - e_3\sqrt{P}\right) \\ &\quad + 2\bar{\beta}_1^*\bar{\beta}_2^*\bar{\gamma}_1^*\bar{\gamma}_2^*\left(x - e_1 - \sqrt{P}\right) \\ &\quad - \xi\bar{\gamma}_1^*\bar{\gamma}_2^*(\bar{\alpha}_2^*\bar{\beta}_1^* + \bar{\alpha}_1^*\bar{\beta}_2^*)\left(e_1 - x - e_3\sqrt{P}\right), \\ B_1 &= -A_1 + \xi[\bar{\gamma}^*][\bar{\alpha}^*, \bar{\beta}^*]\left(e_3^2 - e_1e_2 + e_2x\right)\sqrt{P} + x(e_1 - x), \\ C_1 &= -\xi\left(\bar{\alpha}_1^*\bar{\beta}_1^*\bar{\gamma}_2^* + \bar{\alpha}_2^*\bar{\beta}_2^*\bar{\gamma}_1^*\right) \\ &\quad \left[\{e_3^2 - e_2(e_1 - x)\}\sqrt{P} + x(e_1 - x)\right] \\ &\quad + (\bar{\gamma}_1^{*2}\bar{\beta}_2^{*2} + \bar{\gamma}_2^{*2}\bar{\beta}_1^{*2})\bar{c}_f\frac{(1-\xi)}{\xi}\left[(e_1 - x)\sqrt{S + 2\sqrt{P}}\right] \\ &\quad + (\bar{\gamma}_1^{*2}\bar{\beta}_2^{*2} + \bar{\gamma}_2^{*2}\bar{\beta}_1^{*2})\left(x - e_1 - \sqrt{P}\right) \\ &\quad - \xi\left(\bar{\gamma}_2^{*2}\bar{\alpha}_1^*\bar{\beta}_1^*, + \bar{\gamma}_1^{*2}\bar{\alpha}_2^*\bar{\beta}_2^*\right)[e_1 - x - e_3\sqrt{P}] \\ &\quad - (\bar{\beta}_1^{*2}\bar{\gamma}_2^* + \bar{\beta}_2^{*2}\bar{\gamma}_1^*)\left(e_1 - x - e_3\sqrt{P}\right), \\ D_1 &= \bar{\beta}_2^*\bar{\gamma}_1^*[\bar{\alpha}^*; \bar{\beta}^*]\bar{c}_f\frac{(1-\xi)}{\xi} \\ &\quad \left[\{e_3^2 - e_2(e_1 - x)\}\sqrt{P} + x(e_1 - x)\right] \\ &\quad + \left[\xi\bar{\beta}_1^*\bar{\gamma}_2^*(\bar{\gamma}_2^* - \bar{\gamma}_1^*)(x - e_1) + \bar{\beta}_2^*\bar{\gamma}_1^*e_2\sqrt{P}(\bar{\alpha}_2^*\bar{\beta}_1^* - \bar{\alpha}_1^*\bar{\beta}_2^*)\right] \\ &\quad \sqrt{S + 2\sqrt{P}}, \\ E_1 &= -\bar{\beta}_1^*\bar{\gamma}_2^*[\bar{\alpha}^*; \bar{\beta}^*]\bar{c}_f\frac{(1-\xi)}{\xi} \\ &\quad \left[\{e_3^2 - e_2(e_1 - x)\}\sqrt{P} + x(e_1 - x)\right] \\ &\quad + \left[\xi\bar{\beta}_2^*\bar{\gamma}_1^*(\bar{\gamma}_2^* - \bar{\gamma}_1^*)(e_1 - x) - \bar{\beta}_1^*\bar{\gamma}_2^*e_2\sqrt{P}(\bar{\alpha}_2^*\bar{\beta}_1^* - \bar{\alpha}_1^*\bar{\beta}_2^*)\right] \\ &\quad \sqrt{S + 2\sqrt{P}}. \end{aligned}$$

Here the quantities $\bar{\alpha}_k^*$, $\bar{\beta}_k^*$, $\bar{\gamma}_k^*$, S and P are given by

$$\bar{\alpha}_k^* = \frac{\bar{b}_k^2 + r_v^2 x - \bar{e}_1}{(1+\bar{e}_3)\bar{b}_k}, \quad \bar{\beta}_k^* = r_\mu(\bar{b}_k - \bar{\alpha}_k^*),$$
$$\bar{\gamma}_k^* = r_\mu\left(\bar{e}_3 + \frac{\bar{\alpha}_k^* \bar{b}_k}{\bar{e}_2}\right), (k=1,2)$$
$$S = \frac{e_2(e_1-x)+1-x-(e_3+1)^2}{e_2},$$
$$P = \frac{(e_1-x)(1-x)}{e_2}$$
$$x = \frac{X}{c_{66}}, \quad e_1 = \frac{c_{11}}{c_{66}}, \quad e_2 = \frac{c_{22}}{c_{66}}, \quad e_3 = \frac{c_{12}}{c_{66}},$$
$$\bar{e}_1 = \frac{\bar{c}_{11}}{\bar{c}_{66}}, \quad \bar{e}_2 = \frac{\bar{c}_{66}}{\bar{c}_{22}}, \quad \bar{e}_3 = \frac{\bar{c}_{12}}{\bar{c}_{66}}, \quad \bar{c}_f = \frac{\bar{c}_{66}}{F},$$
$$c_2 = \sqrt{\frac{c_{66}}{\rho}}, \quad \bar{c}_2 = \sqrt{\frac{\bar{c}_{66}}{\bar{\rho}}},$$
$$r_\mu = \frac{\bar{c}_{66}}{c_{66}}, \quad r_v = \frac{c_2}{\bar{c}_2}.$$

Note that the expressions for $\bar{b}_1$ and $\bar{b}_2$ are the same as what stated earlier. However, but with $\bar{S}$ and $\bar{P}$, they are given

$$\bar{S} = (\bar{e}_1 - r_v^2 x) + \bar{e}_2[1 - r_v^2 x - (\bar{e}_3+1)^2],$$
$$\bar{P} = \bar{e}_2(\bar{e}_1 - r_v^2 x)(1 - r_v^2 x).$$

Equation (30) is the secular equation of Rayleigh waves in an orthotropic half-space overlaid an orthotropic layer in contact with the half-space. It can be seen that the coefficients of this secular equation depend on the sliding parameter ξ. Hence the speed of Rayleigh waves will certainly depend upon the sliding parameter ξ. It is general secular equation that can provide frequency equation of Rayleigh wave propagation in the following cases: (i) when the half-space and the layer are in smooth sliding contact ($\xi = 0$), (ii) when the half-space and the layer are in smooth sliding contact ($\xi = 1$), and (iii) when the half-space and the layer are in finite sliding contact ($0 < \xi < 1$). The following section addresses the two extreme cases of the sliding parameter.

4 Limiting cases

Case 1: (**Smooth contact interface**) Substituting $\xi = 0$ into (30), the various coefficients of this equation take the following form

$$\begin{aligned}
A_1 &= 2\bar{\beta}_1^*\bar{\beta}_2^*\bar{\gamma}_1^*\bar{\gamma}_2^*(e_1-x)\sqrt{S+2\sqrt{P}} = -B_1,\\
C_1 &= (\bar{\beta}_2^{*2}\bar{\gamma}_1^{*2} + \bar{\beta}_1^{*2}\bar{\gamma}_2^{*2})(e_1-x)\sqrt{S+2\sqrt{P}},\\
D_1 &= \bar{\beta}_2^*\bar{\gamma}_1^*[\bar{\alpha}^*;\bar{\beta}^*]\left[\{e_3^2 - e_2(e_1-x)\}\sqrt{P} + x(e_1-x)\right],\\
E_1 &= -\bar{\beta}_1^*\bar{\gamma}_2^*[\bar{\alpha}^*;\bar{\beta}^*]\left[\{e_3^2 - e_2(e_1-x)\}\sqrt{P} + x(e_1-x)\right].
\end{aligned} \tag{31}$$

With these coefficients, the equation (30) is the exact secular equation of Rayleigh wave in an orthotropic elastic half-space overlaid by an orthotropic layer, which is in smooth sliding contact with the half-space. These coefficients are exactly the same coefficients (apart from notations) obtained in equation (35) by Vinh and Anh [1] for the corresponding problem of smooth sliding contact interface between the layer and the half-space.

Case 2:(Welded contact interface) Substituting $\xi = 1$ into (30), the various coefficients of this equation take the following form

$$\begin{aligned}
A_1 &= -\left(\bar{\alpha}_1^*\bar{\beta}_2^*\bar{\gamma}_2^* + \bar{\alpha}_2^*\bar{\beta}_1^*\bar{\gamma}_1^*\right)\\
&\quad \left[\{e_3^2 - e_2(e_1-x)\}\sqrt{P} + x(e_1-x)\right]\\
&\quad - \left[\bar{\beta}_1^*\bar{\beta}_2^*(\bar{\gamma}_1^* + \bar{\gamma}_2^*) + \bar{\gamma}_1^*\bar{\gamma}_2^*(\bar{\alpha}_2^*\bar{\beta}_1^* + \bar{\alpha}_1^*\bar{\beta}_2^*)\right]\\
&\quad \left(e_1 - x - e_3\sqrt{P}\right)\\
&\quad + 2\bar{\beta}_1^*\bar{\beta}_2^*\bar{\gamma}_1^*\bar{\gamma}_2^*\left(x - e_1 - \sqrt{P}\right),\\
B_1 &= -A_1 + [\bar{\gamma}^*][\bar{\alpha}^*;\bar{\beta}^*]\left(e_3^2 - e_1e_2 + e_2x\right)\sqrt{P} + x(e_1-x),\\
C_1 &= -(\bar{\alpha}_1^*\bar{\beta}_1^*\bar{\gamma}_2^* + \bar{\alpha}_2^*\bar{\beta}_2^*\bar{\gamma}_1^*)\\
&\quad \left[\{e_3^2 - e_2(e_1-x)\}\sqrt{P} + x(e_1-x)\right]\\
&\quad + (\bar{\beta}_2^{*2}\bar{\gamma}_1^{*2} + \bar{\beta}_1^{*2}\bar{\gamma}_2^{*2})\left(x - e_1 - \sqrt{P}\right)\\
&\quad - \left[\bar{\gamma}_2^{*2}\bar{\alpha}_1^*\bar{\beta}_1^* + \bar{\gamma}_1^{*2}\bar{\alpha}_2^*\bar{\beta}_2^* + \bar{\beta}_1^{*2}\bar{\gamma}_2^* + \bar{\beta}_2^{*2}\bar{\gamma}_1^*\right]\\
&\quad \left(e_1 - x - e_3\sqrt{P}\right),\\
D_1 &= \left[\bar{\beta}_1^*\bar{\gamma}_2^*[\bar{\gamma}^*](x-e_1) + \bar{\beta}_2^*\bar{\gamma}_1^*e_2\sqrt{P}[\bar{\alpha}^*;\bar{\beta}^*]\right]\sqrt{S+2\sqrt{P}},\\
E_1 &= \left[\bar{\beta}_2^*\bar{\gamma}_1^*[\bar{\gamma}^*](e_1-x) - \bar{\beta}_1^*\bar{\gamma}_2^*e_2\sqrt{P}[\bar{\alpha}^*;\bar{\beta}^*]\right]\sqrt{S+2\sqrt{P}}.
\end{aligned} \tag{32}$$

These coefficients are exactly the same as what was obtained in equation (37) by Vinh et al. [1] for the corresponding problem of welded contact interface between the layer and the half-space. Note that the symbol $\bar{\alpha}_k$ should be $\bar{\alpha}_k^*$ $(k=1,2)$ in equation (36) of Vinh et al. [1].

5 Particular case

If we neglect the presence of layer, the problem reduces to the problem of Rayleigh wave propagation in orthotropic elastic half-space. Thus, to recover the frequency equation of Rayleigh wave in orthotropic half-space, we shall set $h = 0$ (or $\varepsilon = 0$) into equation (37). Then, one can obtain the relevant frequency equation as follows:

$$\begin{aligned}
&(c_{66} - X)[c_{12}^2 - c_{22}(c_{11} - X)]\\
&+X\sqrt{c_{22}c_{66}}\sqrt{(c_{11}-X)(c_{66}-X)} = 0.
\end{aligned} \tag{33}$$

This is the same secular equation for Rayleigh wave propagation as obtained by Vinh and Ogden [11] in the relevant half-space and given in (2.17) apart from notations.

Remark: Diminishing thickness of the top layer, one can recover the frequency of Rayleigh wave propagation in an orthotropic half-space coated with thin layer of orthotropic material. Thus, making $h \to 0$ (or $\varepsilon \to 0$) into equation (37), one can obtain the frequency equation containing the parameter ξ. It can be verified that on setting either $\xi = 0$ or $\xi = 1$, the equation (37) reduces to

$$[c_{12}^2 - c_{22}(c_{11} - X)]\sqrt{P} + (c_{11} - X) = 0. \quad (34)$$

This equation is the same as equation (36) obtained by Vinh and Anh [2] or equation (34) of Vinh et al. [1] for the corresponding problems. The question arising is why equation (37) reduces to the same equation for two different values of ξ. This occurs because when the presence of the layer over the half-space is neglected, the concept of smooth or smooth sliding contact interface between the layer and the half-space is meaningless. Thus, equation (37) reduces to equation (34) in either cases of ξ.

6 Special case: Isotropic case

When both the layer and the half-space are isotropic, the secular equation for the Rayleigh wave propagation can be obtained from (30). For this case, the various coefficients reduce to

$$c_{11} = c_{22} = \lambda + 2\mu, \quad c_{12} = \lambda, \quad c_{66} = \mu,$$

$$\bar{c}_{11} = \bar{c}_{22} = \bar{\lambda} + 2\bar{\mu}, \quad \bar{c}_{12} = \bar{\lambda}, \quad \bar{c}_{66} = \bar{\mu},$$

where λ and μ are the Lame's constants. With the coefficients given in (35), equation (30) reduces to the frequency equation of Rayleigh wave in an isotropic half-space overlaid with an isotropic elastic layer in finite sliding contact. With $\xi = 1$ (perfect contact), it can be verified that the relevant frequency equation is consistent with those earlier obtained by Vinh et al. [1] for the corresponding problem. However, when $\xi = 0$ (smooth sliding contact), the frequency equation (30) reduces to the secular equation of Rayleigh wave propagation in isotropic half-space overlaid with isotropic layer in smooth sliding contact, where the various coefficients can be reduced from (31) as

$$\begin{aligned}
A_1 &= 8(2-\bar{x})x\bar{b}_1(\bar{b}_2^2+1)(b_1+b_2)b_1^2 r_\mu = -B_1,\\
C_1 &= x\frac{(b_1+b_2)b_1^2}{\bar{b}_2}\left[8\bar{b}_1^2\bar{b}_2^2 + (\bar{b}_2^2+1)^2(2-\bar{x})^2\right] r_\mu,\\
D_1 &= (2-\bar{x})\bar{x}\frac{(\bar{b}_2^2+1)\bar{b}_1}{\bar{b}_2}\left[\{(2-x)^2 - 4b_1^2\}b_1 b_2 + x^2 b_1^2\right],\\
E_1 &= -4\bar{x}\bar{b}_1^2\left[\{(2-x)^2 - 4b_1^2\}b_1 b_2 + x^2 b_1^2\right].
\end{aligned} \quad (35)$$

With the coefficients given in equation (35), equation (30) represents the secular equation of Rayleigh wave propagation in an isotropic half-space overlaid an isotropic layer with smooth sliding contact. This result is new and has not been derived hitherto.

Table I: Material Constants (dimensionless)

Symbol (Layer)	Value (Model-I)	Value (Model-II)	Symbol (Half-space)	Value (Model-I)	Value (Model-II)
$\bar{e}_1$	2.2	2.5	e_1	2.5	3.5
$\bar{e}_2$	1.8	1.2	e_2	3.0	2.8
$\bar{e}_3$	0.5	0.5	e_3	1.5	1.0
r_μ	1.0	0.5	r_ν	1.2	2.8

Table II: Material Constants ($\times 10^{11}\, dynes/cm^2$)

Baryte (Layer)		Topaz (Half-space)	
Symbol	Value	Symbol	Value
$\bar{c}_{11}$	8.62	c_{11}	28.1
$\bar{c}_{12}$	5.23	c_{12}	12.13
$\bar{c}_{22}$	9.17	c_{22}	35.03
$\bar{c}_{66}$	2.74	c_{66}	13.14

7 Numerical Results

To study the effect of sliding parameter on the phase speed of Rayleigh wave propagation, we have performed numerical computations for the following specific models. For Model-I and Model-II, the values of relevant parameters have been borrowed from Vinh et al. [1] and Vinh and Anh [2], respectively. For Model-III, Topaz has been considered as orthotropic elastic half space, while Barytes is taken as the overlaid orthotropic layer. The relevant values of various elastic constants for Topaz and Barytes have been taken from Love ([15], pp-164. The corresponding density for Barytes and Topaz is taken as 4.48 gm/cm^3 and 3.55 gm/cm^3 respectively. With these parametric values, the non-dimensional phase speed x has been computed for different values of non-dimensional wavenumber ε at different values of sliding parameter ξ. The value $\xi = 0.000000001$ ($\to 0$) approximately corresponds to smooth contact interface, while $\xi = 1$ corresponds to welded contact interface and any value of ξ lying between 0 and 1, *VIZ* $\xi = 0.5$ corresponds to finite sliding contact interface. The reason for choosing $\xi = 0.000000001$ instead of $\xi = 0$ for smooth contact interface is that the coding of the program could not allow us to use the value $\xi = 0$ as the quantity ξ is present in the denominator of all the coefficients of secular equation (30). Moreover, the value of F is set equal to $\bar{c}_{66}$.

The results obtained from Model-I and Model-II have been depicted in Figures 1(a) and 1(b) respectively. We notice that the Rayleigh modes are dispersive for welded contact (Black curve), finite sliding (Red curve) and smooth sliding contact (Blue curve) interface. Figure 1(a) shows that the effect of sliding parameter ξ is significant enough on the fundamental mode of Rayleigh propagation in the low range of ε, while in the high range

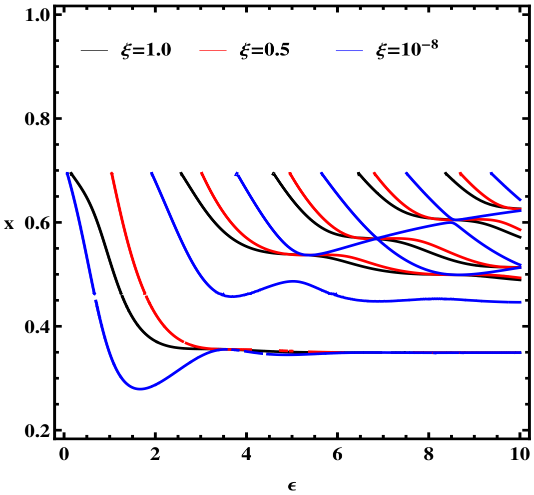

Fig. 1: (Model-I) Variation of non-dimensional Rayleigh wave phase speed 'x' versus wavenumber 'ε'.

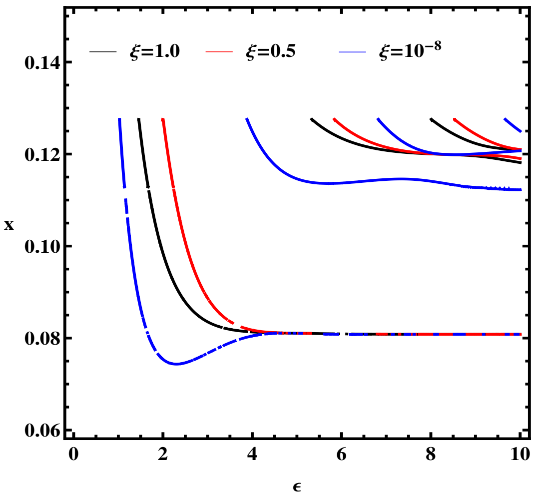

Fig. 2: (Model-II) Variation of non-dimensional Rayleigh wave phase speed 'x' versus wavenumber 'ε'.

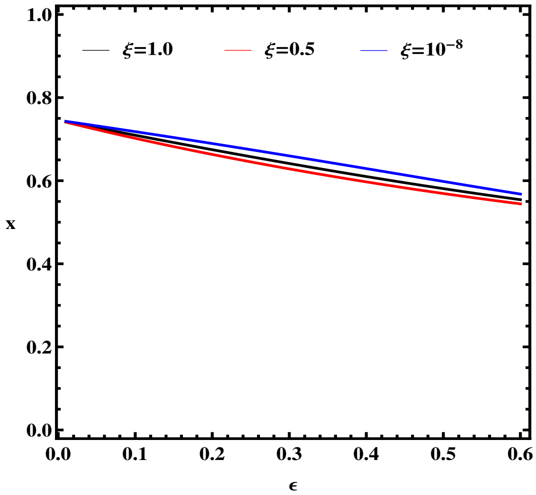

Fig. 3: (Model-III: Topaz half-space overlaid with Barytes layer) Variation of phase speed 'x' versus wavenumber 'ε' (Zoom version of Fundamental mode).

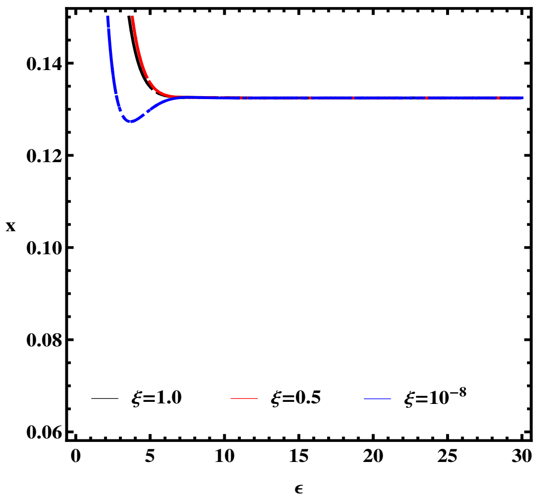

Fig. 4: (Model-III: Topaz half-space overlaid with Barytes layer) Variation of phase speed 'x' versus wavenumber 'ε'.

of ε, the effect of ξ is hardly seen. This shows that the fundamental mode of Rayleigh waves with large wavenumber is non-dispersive. However, the higher modes are seen to be affected most by the sliding parameter in comparison to fundamental mode. The values of the speed of Rayleigh waves in case of smooth sliding contact interface are less than those in case of finite sliding contact interface, but higher than those in case of smooth contact interface. Figure 1(b) follows similar trend as what was observed for Figure 1(a), but with different set of parameter values that are mentioned in Table-I corresponding to Model-II.

For Model-III, we have computed the Rayleigh modes for orthotropic elastic solid half-space underlying an orthotropic elastic solid layer. Here, Barytes is taken as layer and Topaz is taken as half-space, whose relevant numerical values of elastic parameters are presented in Table-II. The graphical illustration corresponding to Model-III has been plotted through Figures 2(a) and 2(b). In Figure 2(a), fundamental modes have been compared to three different types of interfaces in the range $0 \leq \varepsilon \leq 0.6$. For the range $0 \leq \varepsilon \leq 30$, the plot of

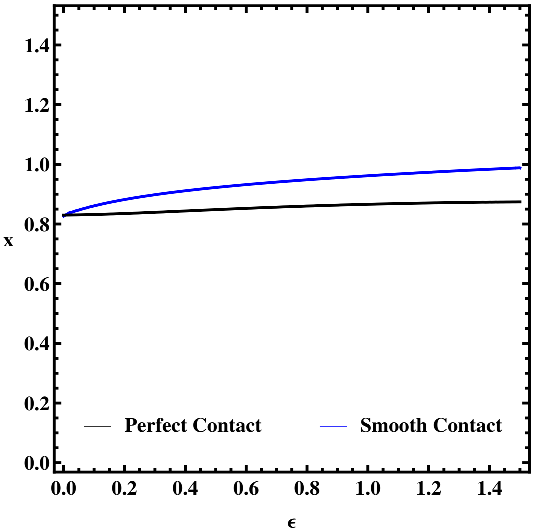

Fig. 5: (Model-VI: Granite half-space overlaid with Sandstone layer) Comparison and variation of phase speed 'x' versus wavenumber 'ε'for perfect contact ($\xi = 1$) and smooth sliding contact ($\xi = 10^{-8}$).

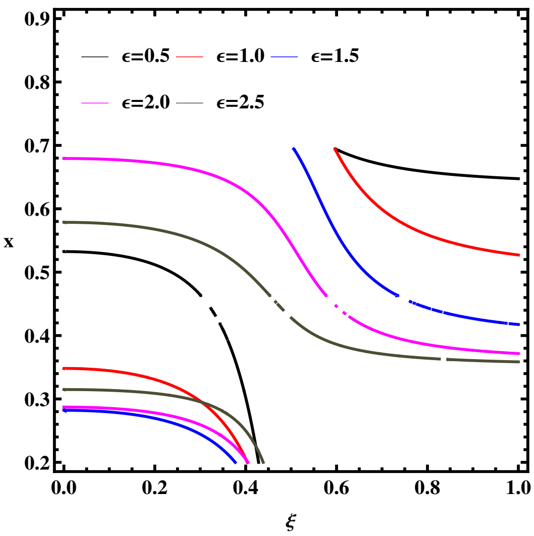

Fig. 6: (Model-III :) Variation of '(ξ)' against phase speed 'x'.

fundamental modes of Model-III has been depicted in Figure 2(b). It is noticeable that in the small range of ε, the fundamental mode of Rayleigh waves corresponding to welded contact lies below that of the smooth sliding contact and above the finite sliding contact interface. For higher values of ε, the sliding/nonsliding contact interface is irrelevant, the fundamental mode remains non-dispersive.

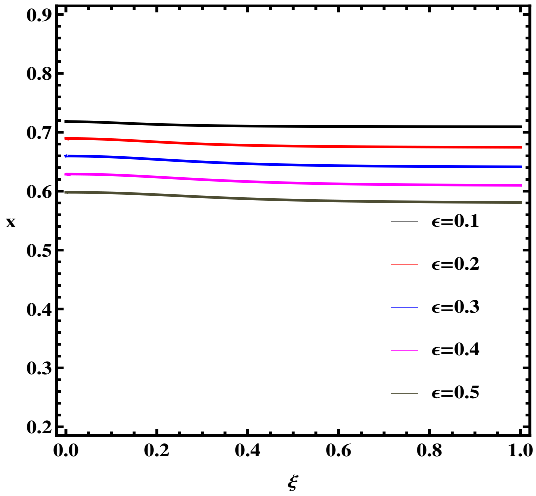

Fig. 7: (Model-III :) Variation of '(ξ)' against phase speed 'x'.

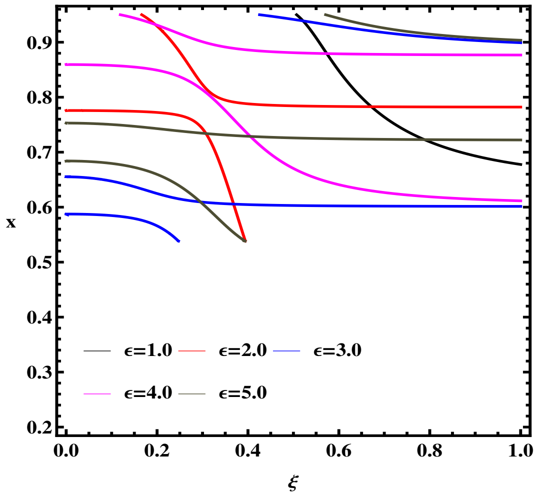

Fig. 8: (Model-I:) Variation of '(ξ)' against phase speed 'x'.

Table III: Material Constants

Symbol	Sandstone (Layer)	Granite (Half-space)
Density (ρ)	2000 kg/m^3	2700 km/m^3
Young's Modulus (E)	1.0–20 GPa	10–70 GPa
Poisson's Ratio (ν)	0.21–0.38	0.1–0.3

Apart from the above models, another model called Model-IV has been considered. This model illustrates the isotropic elastic solid half-space underlying an isotropic elastic solid layer. It can be viewed in geophysical settings, such as Sandstone layer lying over a Granite elastic half-space. The elastic parameters for Granite and Sandstone are given below in Table-III.

To compute the phase speed of Rayleigh modes, the frequency equation (44) for welded contact interface earlier derived by Vinh et al. [1] has been considered. Whereas, for the smooth contact interface the frequency equation (35) derived in the present work has been considered. The results obtained from Model-IV are shown in Figure 3, where a comparison between the Rayleigh modes occurring at smooth sliding and welded contact interfaces has been presented.

Figures 4(a)and 4(b) represent the phase speed corresponding to Model-III, plotted against the sliding parameter $'\xi'$ for different values of ε. For $\varepsilon = 0.5(0.5)2.5$ the plots are depicted in Figure 4(a), while for $\varepsilon = 0.1(0.1)0.5$, the plots are depicted in Figure 4(b). Figures 4(a) and 4(b) indicate that the mode shifts downward with increasing value of ε in the considered range. Figure 4(c) represents the phase speed corresponding to Model-I, plotted against the sliding parameter $'\xi'$, for $\varepsilon = 1.0(1.0)5.0$. The figure asserts that the modes follow different trends for the opted values for ε.

8 Conclusion

In this paper, the explicit exact secular equation for Rayleigh wave propagating in an orthotropic half-space overlaid by an orthotropic layer of arbitrary thickness has been obtained. The layer and the half-space are assumed to be in sliding contact at the interface. The sliding contact between the layer and the half-space has been defined through a parameter ξ $(0 \leq \xi \leq 1)$. The exact and explicit secular equation has been derived using effective boundary condition method, which would be highly potential for evaluating the associated material parameter of the layer and the half-space. The two extreme values of ξ, i.e. 0 and 1, correspond to smooth sliding contact interface and perfectly bonded contact interface, respectively. The obtained secular equation reduced to the secular equations previously obtained for orthotropic materials in smooth sliding contact and that for perfect contact interface. Numerical computations for investigating the exact secular equation for four different models were presented. Equation (30) was solved to compute and explicate the behaviors of Rayleigh modes against wave number. The Rayleigh waves were found to be dispersive in nature for the considered models and the results were depicted graphically. The effect of sliding parameter on dispersiveness of Rayleigh waves has also been observed and shown graphically. The figure exhibiting the behavior of Rayleigh wave propagation in orthotropic half-space layer with orthotropic layer for different values of ξ with real physical data corresponding to topaz as the half-space coated with barytes layer was presented. For isotropic case consisting granite as a half-space and sandstone as a layer, a comparison of behavior of Rayleigh wave speed corresponding to smooth sliding and perfect contact was conducted. The manuscript attempted to present general secular equation for Rayleigh wave propagation for various materials comprising definite sliding contact between the layer and the half-space.

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Conflicts of Interest

The authors declare that there is no conflict of interest regarding the publication of this article.

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