

Applied Mathematics & Information Sciences An International Journal

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Performance Analysis of a Hybrid Codebook for Downlink Relay Systems

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Received: 9 Jan. 2012; Revised 8 May. 2012; Accepted 1 Jul. 2012 Published online: 1 January 2013

Abstract: In this paper, we propose a novel codebook for multiple-input multiple-output (MIMO) Rician fading channels. While the most conventional codebooks are designed in Rayleigh fading channels, we design a hybrid codebook which combines discrete Fourier transform matrix-based codebook for MIMO Rayleigh fading channels and array matrix-based codebook for MIMO line-of-sight channels. Especially, a backhaul link between base station and relay station can be a good target MIMO channels for the proposed codebook. Link-level simulations performed in a MIMO-OFDM system show that the proposed hybrid codebook outperforms the conventional codebooks when the *K* factor of Rician distribution is between 0.28 and 0.99. Moreover, the proposed codebook shows almost optimal link-level performances for MIMO Rayleigh fading channels.

Keywords: MIMO, Rician channels, limited feedback communication, codebook, line-of-sight, relay

1. Introduction

The channel dependent transmit-beamforming (TxBF) has been used to exploit diversity gain when multiple antennas are available at the transmitter [1]. Typically, TxBF requires the feedback of channel state information (CSI) from the receiver to the transmitter. However, such CSI feedback can potentially incur excessive overhead due to the multiplicity of channel coefficients [2], and thus a small number of feedback bits are sent via a feedback path for the transmitter to recreate the TxBF vector. These systems are known as limited feedback systems (see [3] and the references therein). To reduce the bandwidth requirement of the feedback systems, finite rate technique have been proposed for the case of the TxBF [4-6]. In these limited feedback systems, the receiver chooses the TxBF vector from a finite set of precoding matrices, called codebook, on the basis maximizing the effective signal-to-noise (SNR) after combining, and sends the corresponding indexing bits to the transmitter, and can retain the advantage of capacity enhancement or diversity gain as in the ideal TxBF scheme [7].

In general, most codebook design methods assume the wireless channels are Rayleigh distributed fading. Recently,

as downlink relay systems have become more influential, codebook that was designed for line-of-sight component has drawn considerable attention [8]. Especially, in the backhaul link, which is a wireless link between the base station and relay station, the direct path can be considered as Rician fading condition. In this paper, we consider that the direct path exists between the transmitter and receiver, and we propose a novel hybrid codebook that combines the discrete Fourier transform (DFT) matrix-based codebook for Rayleigh fading and the array matrix-based codebook for direct path. Also, a hybrid codebook design methodology is proposed for Rician fading channels. The proposed codebook shows that it outperforms the conventional codebooks in Rician fading channels and it still shows very good performances in Rayleigh fading channels.

The remainder of the paper is organized as follows. Section 2 reviews MIMO Rician channel for relay systems and conventional codebook for MIMO Rayleigh or lineof-sight channels. In Section 3, we propose a novel codebook for MIMO Rician fading channels. In Section 4, the simulation results are demonstrated and the related discussions are given. Finally, the concluding remarks are given in Section 5.

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2. System model

2.1. MIMO Rician Channel for Relay Systems

Consider the three node wireless network using the relay transmission in figure 1. Relay transmission can be seen as a collaborative communications, in which a relay station (RS) helps to forward user information from user equipment (UE) to a base station (BS). RS can effectively extend the signal and service coverage of BS and enhance the overall throughput performance of a wireless communication system. Besides, RS does not need a wired connection to the network; therefore RS can save system operators' backhaul deployment costs. Among many relay transmission schemes having been proposed to establish wireless communication between a BS and UEs located far away from the BS, it is common to employ two-hop transmission scheme which consists of the amplify and forward (AF) and the decode and forward (DF) between the BS and UE units via RSs [9-11]. AF generally has some kind of analog amplification circuit block to transmit signal while DF has modem and channel coding blocks to transmit signal.

The backhaul link has more stochastic line of sight (LOS) components rather than the access link, which is a wireless link between the RS and UE, in the relay transmission. In this regard, the backhaul link can be relatively considered to Rician distributed fading channels. Therefore, the MIMO fading channel in the presence of LOS can be modeled as the sum of a fixed component and a variable (or scattered, Rayleigh distributed) component [12]:

$$\mathbf{H} = \sqrt{\frac{K}{1+K}} \overline{\mathbf{H}} + \sqrt{\frac{1}{1+K}} \mathbf{H}_{w}, \qquad (1)$$

where $\sqrt{K/(1+K)}\overline{\mathbf{H}}$ is the fixed (or LOS) component of the channel and $\sqrt{1/(1+K)}\mathbf{H}_w$ is the fading component of the channel. The elements of $\overline{\mathbf{H}}$ are assumed to have unit power. *K* in (1) is the Rician factor of the channel and is the ratio of the total power in the fixed component of the channel to the power in the fading component. K = 0



Figure 1 Three node wireless network



Figure 2 Wavefront impinging across MIMO array

corresponds to pure Rayleigh fading, while the channel approaches a non-fading link as $K \to \infty$. Clearly \mathbf{H}_w dominates channel behavior for low values of K, while $\overline{\mathbf{H}}$ dominates the system behavior with an increasing degree of Rician fading. Generally, considering the array arrangement for MIMO channel in figure 2, $\overline{\mathbf{H}}$ is determined by the geometric characteristics as follows.

$$\overline{\mathbf{H}} = \begin{bmatrix} e^{j\theta} & e^{j2\theta} & \cdots & e^{jN_t\theta} \\ e^{j2\theta} & e^{j4\theta} & \cdots & e^{j2N_t\theta} \\ \vdots & \vdots & \ddots & \vdots \\ e^{jN_r\theta} & e^{jN_r2\theta} & \cdots & e^{jN_rN_t\theta} \end{bmatrix},$$
(2)

and

$$\theta = \frac{2\pi d\cos\Theta}{\lambda} = 2\pi\Delta\cos\Theta, \qquad (3)$$

where N_t/N_r is the number of transmitted/received antennas, d is the spacing between the antennas, λ is the wavelength of the carrier, and Θ is the angle of direct path to each user per relay. The parameter Δ is defined as the antenna spacing normalized by wavelength.

2.2. DFT matrix-based codebook for MIMO Rayleigh channels

The DFT matrix-based codebook, whose codewords are permuted columns of a DFT matrix, is considered as an effective design for spatially correlated channels. And the DFT matrix-based codebook well adapts to the uniform linear array of MIMO system. When the receiver estimates it MIMO channel, it chooses a M-dimensional DFT matrix-based precoding matrix $\mathbf{F} \in u(N_t, M)$ according to a specific codeword selection metric from a finite set of codebooks as follows [13].

$$\mathscr{F} = \{\mathbf{F}_1, \mathbf{F}_2, \cdots, \mathbf{F}_N\} = \{\mathbf{W}, \boldsymbol{\theta}\mathbf{W}, \cdots, \boldsymbol{\theta}^{N-1}\mathbf{W}\}, \quad (4)$$

where $u(N_t, M)$ denotes the set of $N_t \times M$ complex matrices with orthogonal columns, N represents the cardinality (or codebook length) of \mathscr{F} . The first codeword **W** is obtained by selecting M columns of $N_t \times N_t$ DFT matrix, of which



the (k,l)-th entry is given as $(1/\sqrt{N_t}) e^{j2\pi(k-1)(l-1)/N_t}$, k, $l = 1, 2, \dots, N_t$. Furthermore, θ is the diagonal matrix that is the codebook generation variable, as in (5).

$$\theta = diag\left(\left[e^{j2\pi u_1/N}e^{j2\pi u_2/N}\cdots e^{j2\pi u_t/N}\right]\right),\qquad(5)$$

where $\{u_i\}_{i=1}^{N_t} \left(0 \le \{u_i\}_{i=1}^{N_t} \le N-1\right)$ is an arbitrary constant. Given the first codeword **W**, the remaining (N-1) codewords are obtained by multiplexing **W** by θ^i , $i = 1, 2, \dots, N-1$. The vector $\mathbf{u} = [u_1, u_2, \dots, u_{N_t}]$ consist of $\{u_i\}_{i=1}^{N_t}$ is determined such that the minimum chordal distance is maximized, as follows.

$$\mathbf{u} = \underset{u_{1},\cdots,u_{N_{t}}}{\operatorname{arg\,max}} \min_{i=1,\cdots,N-1} d\left(\mathbf{W},\boldsymbol{\theta}^{i}\mathbf{W}\right), \quad (6)$$

and

$$d\left(\mathbf{W}, \boldsymbol{\theta}^{i}\mathbf{W}\right) = \sqrt{1 - |\mathbf{W}(\boldsymbol{\theta}^{i}\mathbf{W})^{H}|^{2}}, \qquad (7)$$

where A^H denotes conjugate transposition of a matrix A.

2.3. Array matrix-based codebook for MIMO line-of-sight channels

Generally, the *K* factor in Rician fading channel increases as the LOS component in the channel increases. If we consider this fact, then it is not spatial multiplexing but the TxBF that must be used. For this reason, the array matrixbased codebook has a 1-dimentsional codewords. Also, the array matrix-based codebook can be generated through an equal distance criterion [8]. In order to maximize the minimum distance among the codewords in the vector space, the angle differences between any two adjacent codewords must be equal, where the angle differences mean $|\theta_1 - \theta_2|$ of two codeword vectors $\mathbf{w}(\theta_1)$ and $\mathbf{w}(\theta_1)$. Using this criterion, the array matrix-based codebook \mathcal{W} can be obtained as follow.

$$\mathscr{W} = \{\mathbf{W}_1, \mathbf{W}_2, \cdots, \mathbf{W}_N\},\tag{8}$$

$$\mathbf{w}_{k}^{T} = \frac{1}{\sqrt{N_{t}}} \left[e^{-j\theta_{k}} e^{-j2\theta_{k}} \cdots e^{-jN_{t}\theta_{k}} \right], \qquad (9)$$

where where A^T denotes transposition of a matrix A and θ_k ($k = 1, 2, \dots, N$) is

$$\theta_k = \Phi_{diff} \left(\frac{k}{N} - \frac{1}{2} \right), \tag{10}$$

 Φ_{diff} denotes the difference between the upper and lower bounds of the quantization range Φ , which is given by $[-\min(\pi, 2\pi\Delta), \min(\pi, 2\pi\Delta)]$. N should be greater than or equal to $[N_t \Phi_{diff}/2\pi]$, otherwise the equal distance criterion does not guarantee the maximization of the minimum distance.

3. Proposed hybrid codebook

The proposed hybrid codebook is combined of the DFT matrix based codebook as in (4) for Rayleigh fading and the array matrix-based codebook as in (8) for direct path. When the number of transmit antennas is four, the proposed design step can be summarized as the following:

1) We determined the NDFT, which is the cardinality of DFT matrix-based codebook. Because there are almost similar performance in the case of $N_{DFT} \ge 8$, we set $N_{DFT} = 8$.

2) We determined the N_{LOS} , which is the cardinality of the array matrix-based codebook. In the same way as above, we set $N_{LOS} = 8$.

3) The codebook generated by steps 1) and 2) can duplicate some codewords. At that time, we change $N_{DFT} = 9$ and $N_{LOS} = 7$ to remove the duplicated one from array matrix-based codebook.

4) According to the determined cardinality of each codebook, the whole possible candidate DFT matrix-based codebooks are generated. Further, the array matrix-based codebook is then arranged in the front/rear of the whole candidate DFT matrix-based codebook. Consequently, many candidate hybrid codebooks $\mathcal{W}_{hyb} = \{\mathbf{w}_{hyb,1}\mathbf{w}_{hyb,2}\cdots\mathbf{w}_{hyb,16}\}$ are determined whose codebook size is 16.

5) The hybrid codebook is designed according to the Grassmannian beamforming criterion [6] as in (11) and (12) among the many candidate hybrid codebooks.

$$\delta\left(\mathscr{W}_{hyb}\right) = \min_{1 \le k < l \le N} \sqrt{1 - |\mathbf{w}_{hyb,k}^H \mathbf{w}_{hyb,l}|^2} \qquad (11)$$

and

$$\mathscr{W}_{hyb} = \operatorname*{arg\,max}_{X \in u^N_{N_t}} \delta(X), \tag{12}$$

where $u_{N_t}^N$ is the set of $N_t \times N$ complex matrices with unit vector columns. This criterion captures the essential point about quantized beamforming codebook design. Using the above design step, the proposed hybrid codebook is shown in table 1.

4. Simulation results

In this section, Monte-Carlo simulations are performed over 1 million iterations per SNR point to obtain symbol error rate (SER) probabilities of link-level performance. The average SER is characterized by the DFT / array matrixbased codebook and the proposed hybrid codebook. In the simulations, MIMO orthogonal frequency division multiplexing (OFDM) system having 4 transmit antennas in

Index	Port 1	Port 2	Port 3	Port 4
0	0.5000	0.5000	0.5000	0.5000
1	0.3830	0.0868	0.3830	-0.2500
	+0.3214i	+0.4924i	-0.3214i	-0.4330i
2	0.0868	-0.4698	0.0868	-0.2500
	+0.4924i	+0.1710i	-0.4924i	+0.4330i
3	-0.2500	-0.2500	-0.2500	0.5000
	+0.4330i	-0.4330i	-0.4330i	-0.0000i
4	-0.4698	0.3830	-0.4698	-0.2500
	+0.1710i	-0.3214i	-0.1710i	-0.4330i
5	-0.4698	0.3830	-0.4698	-0.2500
	-0.1710i	+0.3214i	+0.1710i	+0.4330i
6	-0.2500	-0.2500	-0.2500	0.5000
	-0.4330i	+0.4330i	+0.4330i	-0.0000i
7	0.0868	-0.4698	0.0868	-0.2500
	-0.4924i	-0.1710i	+0.4924i	-0.4330i
8	0.3830	0.0868	0.3830	-0.2500
	-0.3214i	-0.4924i	+0.3214i	+0.4330i
9	0.5000	0.3536	0.0000	-0.3536
		+0.3536i	+0.5000i	+0.3536i
10	0.5000	0.0000	-0.5000	-0.000
		+0.5000i	+0.0000i	-0.5000i
11	0.5000	-0.3536	-0.0000	0.3536
		+0.3536i	-0.5000i	+0.3536i
12	0.5000	-0.5000	0.5000	-0.5000
		+0.0000i	-0.0000i	+0.0000i
13	0.5000	-0.3536	0.0000	0.3536
		-0.3536i	+0.5000i	-0.3536i
14	0.5000	-0.0000	-0.5000	0.0000
		-0.5000i	+0.0000i	+0.5000i
15	0.5000	0.3536	-0.0000	-0.3536
		-0.3536i	-0.5000i	-0.3536i

Table 1 Proposed hybrid codebook generated for $N_t = 4$ and N = 16 (4 bits)

conjunction with 1, 2 and 4 receive antennas are considered. Moreover, 16-QAM (quadrature amplitude modulation) is assumed. Meanwhile, the channel model is assumed to be ITU-R Pedestrian-A model having 3 multi-paths [14]. Channel is a memoryless fading channel with independent and identically distributed (i.i.d.) entries according to CN(0,1), and additive white Gaussian noise (AWGN) is assumed with elements that are i.i.d. entries according to $CN(0,N_0)$. It is also assumed that the channel estimation and symbol synchronization are ideal, and there is no spatial correlation amongst antennas. The more detailed simulation parameters are shown in Table 2.

In the first experiment, a MIMO-OFDM system with K = 0 in the presence of pure Rayleigh fading is simulated with the DFT / array matrix-based codebook, and results of the proposed hybrid codebook are shown in Figure 3. When SER is at 10^{-3} , the proposed hybrid codebook is found to achieve 0.2dB better performance for 1 receive antenna than that of the array matrix-based codebook. Also, it achieves 0.3dB and 0.4dB better performances

Parameters	Values
Modulation/Demodulation	16-QAM
FFT/IFFT size	512
Number of occupied subcarrier	300
Cyclic prefix size	30
Number of transmit antennas	4
Number of receive antennas	1, 2, 4
Rician factor K	0, 4, 8
Channel bandwidth [MHz]	5
Sampling time [ns]	130
Number of multi-paths	3
Relative delay [ns]	[0 110 190]
Average Power [dB]	[0 -9.7 -19.21]
Feedback bits	4

 Table 2 Computer simulation parameters

than the array matrix-based codebook for 2 and 4 receive antennas.

In the second experiment, a MIMO-OFDM system with K = 4 is simulated as shown in figure 4. When SER is at 10^{-3} , the proposed hybrid codebook is found to achieve 0.3*dB* better performance for 1 receive antenna than the DFT matrix-based codebook. Also, it achieves 0.6*dB* and 0.7*dB* better performances than DFT matrix-based codebook for 2 and 4 receive antennas.

In the third experiment, a MIMO-OFDM system with K = 8 in the presence of non-fading link is simulated as shown in Figure 5. The proposed hybrid codebook is found to achieve 0.7dB better performance than DFT-matrix based codebook for 1 receive antenna. Further, it achieves 0.8dB and 0.9dB better performances than DFT matrix-based codebook for 2 and 4 receive antennas.

As shown in figure 4 and figure 5, it is quite clear that the more Rician factor K is increased, the more performance difference between the proposed and DFT matrix based codebook is increased. Figure 6 shows the $E_b N_o$ [dB] versus the Rician factor K for a MIMO-OFDM system with the DFT / array matrix-based codebook and the proposed hybrid codebook, which has four transmit and four receive antennas, when SER is at 10^{-3} . The array matrix-based codebook achieves better performance than DFT matrix-based codebook at over K = 0.56. The proposed hybrid codebook achieves better performance than DFT / array matrix-based codebook from K = 0.28 to K =0.99. Moreover, the array matrix-based codebook achieves the best performance among other codebooks at over K =0.99. For these reasons, the proposed hybrid codebook is optimally designed from K = 0.28 to K = 0.99.

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Figure 3 Comparison of SER performance of the proposed hybrid codebook for K = 0 ($N_t = 4, N_r = 1, 2, 4$)



Figure 4 Comparison of SER performance of the proposed hybrid codebook for K = 4 ($N_t = 4, N_r = 1, 2, 4$)

5. Conclusions

In this paper, we have investigated the codebook design problem for downlink relay systems, which has existed in the direct path between the BS and RS. We have proposed a novel hybrid codebook that combines the DFT matrixbased codebook for Rayleigh fading and the array matrix-



Figure 5 Comparison of SER performance of the proposed hybrid codebook for K = 8 ($N_t = 4, N_r = 1, 2, 4$)



Figure 6 Comparison of SER performance by Rician factor *K* when SER is 10^{-3} ($N_t = 4, N_r = 4$)

based codebook for direct path. Based on the simulation results, the advantage of the proposed codebook is that it can be applied to any fading condition and it also outperforms the conventional codebooks. Especially, outstanding gains in SER performance can be achieved rather than using the DFT matrix-based codebook when the Rician factor K is increased.



Acknowledgement

This research was supported by Basic Research Program through the National Research Foudation of Korea (NRF) funded by the Ministry of Education, Science and Technology (2012R1A1B6002111).

The authors are grateful to the anonymous referee for a careful checking of the details and for helpful comments that improved this paper.

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