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A Decomposition Approach based on Taylor Series for Solving Bi-Level Large Scale Quadratic Problems with Fuzzy Numbers

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Abstract: This paper presented a decomposition approach and TOPSIS approach to solve a bi-level large-scale quadratic programming problem (BLLSQPP) with fuzzy parameters in the objective function based on Taylor series and compare between a decomposition algorithm and TOPSIS approach. The basic idea of the proposed approach is to convert the fuzzy number nature of this problem into equivalent deterministic nature. Then the Taylor series will be combined with decomposition algorithm to obtain the satisfactory solution for problem under investigation. To demonstrate the power of the proposed approach, a numerical example is solved and compared with the solutions of the Technique for Order Preference by Similarity Ideal Solution (TOPSIS) approach.

Keywords: : Bi-level, Decomposition algorithm, Fuzzy numbers, Large scale, Quadratic programming,Taylor series, TOPSIS approach.

1 Introduction

Bi-level multi-objective programming problem (BLMOPP), an apparatus for modeling decentralized decisions, consists of the objectives of the leader level decision maker (LLDM) at its leader level and the objectives of the follower level decision maker (FLDM) at the follower level. The execution of decision is sequential from leader level to follower level.Each decision maker (DM) independently controls only a set of decision variables and is interested in optimizing his/her net benefits over a common feasible region. Emam et al. [1] presented a multi-level linear programming problem with random rough coefficients in objective functions. At the first phase of the solution approach and to avoid the complexity of this problem, the rough nature of this problem is converted into equivalent crisp problem. At the second phase, the concept of tolerance membership function is used at each level to solve a Tchebcheff problem till an optimal solution is obtained. Youness et al. [2] presented an algorithm to solve a bi-level multi-objective fractional integer programming problem involving fuzzy numbers in the right-hand side of the constraints. The suggested algorithm combined the method of Taylor series together with the Kuhn Tucker conditions to solve fuzzy bi-level multi-objective fractional integer programming problem then Gomory cuts was added till the integer solution is obtained. In the traditional approaches of large scale systems, parameters are fixed at some values in an experimental and/or subjective manner through the experts' understanding of the nature of the parameters. In practice, however, it is natural to consider that the possible values of these parameters are often ambiguously known to experts' understanding of the parameters as fuzzy numerical data, which can be represented by means of fuzzy subsets of the real line known as fuzzy numbers [3,4,5,6]. Abo-Sinna and Abou-El-Enien extended TOPSIS for solving interactive large scale multiple objective programming problems involving fuzzy parameters. These fuzzy parameters are characterized as fuzzy numbers. For such problems, the a-Pareto optimality is introduced by extending the ordinary Pareto optimality on the basis of the a-Level sets of fuzzy numbers. An interactive fuzzy decision making algorithm for generating a-Pareto optimal solution

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through TOPSIS approach is provided where the decision maker (DM) is asked to specify the degree and the relative importance of objectives. Abou-El-Enien [4] extended TOPSIS for solving large scale multi-objective nonlinear programming problems. As leader was developed by Hwang and Yoon [7] for solving a multiple attribute decision making problem, it is based upon the principle that the chosen alternative should have the shortest distance from the positive ideal solution (PIS) and the longest from the negative ideal solution (NIS). Baky and Abo-Sinna [8,9,10] extended the TOPSIS approach to solve Bi-Level Multi-objective decision making Problems. Then Baky et al. [11] solved a bi-level multi-objective programming problem with fuzzy demands using fuzzy goal programming algorithm. Osman et al. [12] presented a method for solving a special class of large scale fuzzy multi-objective integer problems depending on the decomposition algorithm. Furthermore, Abo-Sinna and Abou-Elenin extended TOPSIS approach to resolve large scale multiple objective programming problems involving fuzzy parameters [13]. Also an additional treatment for Multi-level Multi-objective Fractional Programming Problem using a fuzzy goal programming technique tackled by Osman et al. [14,15,16]. Abo-Sinna and Abou-El-Enien [17] introduced an algorithm for solving large scale multiple objective decision making (LSMODM) problems using TOPSIS. Sultan et al. [18] presented an algorithm for solving a three level large scale linear programming problem in which the objective functions at every level are to be maximized. Each level optimized its problem separately as a large scale programming problem using Dantzig and Wolfe decomposition method. Large-Scale three level fractional problem with a rough parameter in Constraints is considered in [5]. First, the intervals technique is used to convert rough parameters in constraints into equivalent crisp. Second, Taylor Series transformation is used to solve the fractional problem. Then, a proposed model has been constructed to solve the decision conflict of three-level problem. Finally, a decomposition technique is used to solve large-scale problem. In the following section, the formulation of BLLSQP problems will be presented. Also, converting the fuzzy nature of BLLSQPP in the objective functions into deterministic one is presented. The decomposition method based on Taylor series to solve BLLSQPP with fuzzy parameters is proposed in Section 3. A Brief notes about the TOPSIS approach is presented in Section 4. The decomposition algorithm for solving BLLSQPP with fuzzy numbers is presented in Section 5. The next section presents an illustrative numerical example and a comparative study to demonstrate the proposed decomposition approach. Section 6 is devoted to conclusion.

2 Problem Formulation and Solution Concept

Consider a large scale bi-level programming problem of maximization-type quadratic functions at each level. Let LLDM (The Leader-Level Decision-Maker) denote the decision maker at the first level that has control over the decision vector variable $x_1 = (x_{11}, x_{12}, \ldots, x_{1_{m1}}) \in R^{m_1}$, and let FLDM (The Follower-Level Decision-Maker) denote the decision maker at the second level that has control over the decision vector variable $x_2 = (x_{21}, x_{22}, \ldots, x_{2_{m2}}) \in R^{m_2}$, where $x = (x_1, x_2) \in R^m$ and $m = m_1 + m_2$ .furthermore, assume that $F_i : R^m \to R (i = 1, 2)$ are the first level quadratic objective function and the second level quadratic objective function that contain fuzzy numbers defined on R^m. Bi-level large scale quadratic programming problem (BLLSQPP) with fuzzy parameters in the objective function may be formulated as follows:
[Leader Level]

$$\max_{x_1} F_1(x, \tilde{u}) = \max_{x_1} [f_{11}(x, \tilde{u}_1), f_{12}(x, \tilde{u}_2), \ldots, f_{1n1}(x, \tilde{u}_{n_1})] \tag{1}$$

Where x_2 solves
[Follower Level]

$$\max_{x_2} F_2(x, u) = \max_{x_2} [f_{21}(x, u_1), f_{22}(x, u_2), \ldots, f_{2n2}(x, u_{1n1})] \tag{2}$$

Subject to

$$x \in G \tag{3}$$

Where

$$\begin{aligned} G = \{ & a_{0_{11}} x_{11} + a_{0_{12}} x_{12} \ldots a_{0_{1m1}} x_{1_{m1}} + a_{0_{21}} x_{21} + a_{0_{21}} x_{21} + a_{0_{12}} x_{22} \ldots a_{0_{1m2}} x_{1_{m2}} \le b_0 \\ & d_1 x_{11} \le b_1, d_2 x_{12} \le b_2 \ldots, d_{m_1} x_{1_{m_1}} \le b_{m_1} \\ & d_{m_1+1} x_{21} \le b_{m_1+1}, d_{m_1+2} x_{22} \le b_{m_1+2}, \ldots, d_{m_1+m_2} x_{2_{m_2}} \le b_{m_1+m_2}. \\ & x_{11}, x_{12}, \ldots, x_{1_{m_1}}, x_{21}, x_{22}, \ldots, x_{1_{m_2}} \ge 0 \} \end{aligned}$$

Where $\check{u}_i$ is m-dimensional row vector of fuzzy parameters in the objective functions, G is the large scale linear constraint set where, $b = (b_0, \ldots, b_m)^T$ is $(m+1)$ vector, and $a_{0_{11}}, a_{0_{12}}, \ldots, a_{0_{m_1}}, a_{0_{21}}, a_{0_{12}}, \ldots, a_{0_{1m2}}$ and $d_1, d_2, \ldots, d_m$ are constants.

Definition 1. For any $(x_1 \in G_1\{x_1|x \in G\})$ given by leader level decision maker, if the decision-making variable $(x_2 \in G_2\{x_2|x \in G\})$ is the optimal solution of the follower level decision maker, then x is a feasible solution of BLLSQPP.
Definition 2. If $x^* \in R^m$ is a feasible solution of the BLLSQP, no other feasible solution $x \in G$ exists, such that $F_1(x^*) \leq F_1(x)$, so x^* is the satisfactory solution of the BLLSQPP.

The following definitions is used to solve BLLSQPP with fuzzy numbers in the objective functions into crisp nature using linear ranking function:
Definition 3. [20] If $\tilde{A} = (a,v,c,d) \in F(R)$, then a linear ranking function is defined as:

$$\tilde{A} = a + b + \frac{1}{2}(d-c) \tag{4}$$

Definition 4. [20] let $\tilde{A}(a_1,b_1,c_1,d_1)$ and $\tilde{B}(a_2,b_2,c_2,d_2)$ be two trapezoidal fuzzy numbers and $x \in R$. A convenient method for comparing of the fuzzy numbers of ranking functions. A ranking function is a map from $F(R)$ into the real line. Thus, the orders on $F(R)$ are as follows:
$\tilde{A} \geq \tilde{B}$ If and only if $R(\tilde{A}) \geq R(\tilde{B})$.
$\tilde{A} > \tilde{B}$ If and only if $R(\tilde{A}) > R(\tilde{B})$.
$\tilde{A} = \tilde{B}$ If and only if $R(\tilde{A}) = R(\tilde{B})$.
where A and $\tilde{B}$ are in $F(R)$.
Then the problem can be understood as the corresponding deterministic bi-level large scale quadratic programming problem with fuzzy numbers in the objective functions as following problem:
[Leader Level]

$$\max_{x_1} F_1(x) = \max_{x_1}[f_{11}(x), f_{12}(x), \dots f_{1n1}(x)] \tag{5}$$

Where x_2 solves
[Follower Level]

$$\max_{x_2} F_2(x) = \max_{x_2}[f_{21}(x), f_{22}(x), \dots f_{2n2}(x)] \tag{6}$$

Subject to

$$x \in G \tag{7}$$

3 Decomposition Approach for the Bi-Level Large Scale Linear Programming Problem

To solve a bi-level large scale quadratic programming problem using the decomposition algorithm, Taylor series can overcome this problem by obtaining Polynomial objective functions which are equivalent to quadratic objective functions:

$$K_i(x) \cong F_i(x) = F_i(x_i^*) \sum_{j=1}^{n} (x_{jr} - x_{ijr}^*) \frac{\partial F_i(x_j^*)}{dx_{jr}}, (j = 1,2,\dots,m), (i,r = 1,2) \tag{8}$$

Hence, the bi-level large scale linear programming problem BLLSLPP can be written as:
[Leader Level]

$$\max_{x_1} F_1(x) = \text{Max} \sum_{j=1}^{m} \sum_{r=1}^{2} c_{1j} x_{jr} \tag{9}$$

Where x_2 solves
[Follower Level]

$$\max_{x_2} F_2(x) = \text{Max} \sum_{j=1}^{m} \sum_{r=1}^{2} c_{2j} x_{jr} \tag{10}$$

Subject to

$$x \in G \tag{11}$$

This bi-level large scale linear programming problem is solved by adopting the leader-follower Stackelberg strategy combined with Dantzig and Wolf decomposition method [13]. First, obtain the optimal solution that is acceptable to the leader level decision maker problem using the decomposition algorithm by breaking the large scale problem into n-sub problems that can be solved directly. The decomposition principle is based on representing the BLLSLPP in terms of the extreme points of the sets $d_j x_{1j} \leq b_j, j = 1,2,\dots,m_1, d_i x_{2i} \leq b_i, i = 1, m_1+2,\dots,m$ and $x_{k1} \geq 0, k = 1,2,\dots,m_k$. To do so, the solution space described is bounded and closed to seek the optimal solution using Dantzig and Wolf decomposition method that breaks the large scale problem into n-sub problems that can be solved directly and obtain the optimal solution for his/her problem which is the satisfactory solution to the BLLSPP.

The Leader-Level Decision-Maker (LLDM) Problem

The LLDM problem of the BLLSLPP is as follows:

$$\mathrm{Max} F_1(x) = \mathrm{Max} \sum_{j=1}^{m} \sum_{r=1}^{2} c_{ij} x_{jr} \tag{12}$$

Subject to

$x \in G$

To obtain the optimal solution of the leader level, suppose that the extreme points of $d_j x_{jr} \leq b_j, x_{jr} \geq 0$ are defined as $\hat{x}_{jkr} r, k = 1,2$ where x_{jr} defined by:

$$x_{jr} = \sum_{k=1}^{k_j} \beta_{jk} \hat{x}_{rjk}, j = 1,\ldots,mmr = 1,2 \tag{13}$$

And $\beta_{jk} \geq 0$, for all k and $\sum_{k=1}^{k_j} \beta_{jk} = 1$.
Now, the leader level problem in terms of the extreme points to obtain the following master problem of the leader Level are formulated as stated in 13:

$$\mathrm{Max} \sum_{k=1}^{k_1} c_{11} \hat{x}_{1rk} \beta_{1k} + \sum_{k=1}^{k_2} c_{12} \hat{x}_{2rk} \beta_{2k} + \cdots + \sum_{k=1}^{k_n} c_{1n} \hat{x}_{nrk} \beta_{nk} \tag{14}$$

Subject to

$$\sum_{k=1}^{k_1} a_{01} \hat{x}_{1rk} \beta_{1k} + \sum_{k=1}^{k_2} a_{02} \hat{x}_{2rk} \beta_{2k} + \cdots + \sum_{k=1}^{k_n} a_{0n} \hat{x}_{nrk} \beta_{nk} \leq b_0 \tag{15}$$

$$\sum_{k=1}^{k_1} \beta_{1k} = 1, \sum_{k=1}^{k_2} \beta_{2k} = 1, \sum_{k=1}^{k_n} \beta_{nk} = 1, \beta_{jk} \geq 0, \text{for all } j \text{ and } k.$$

The new variables in the leader level problem are β_{jk} which determined using Balinski's algorithm [23]. Once their optimal values β^*_{jk} are obtained, then the optimal solution to the original problem can be found by back substitution as follows:

$$x_{jr} = \sum_{k=1}^{k_j} \beta^*_{jk} \hat{x}_{jrk}, j, r = 1,2 \tag{16}$$

It may appear that the solution of the upper level problem requires prior determination of all extreme points $\hat{x}_{jrk}$. To solve the leader level problem by the revised simplex method, it must determine the entering and leaving variables at each iteration. Let us start leader with the entering variables. Given C_B and B^{-1} of the current basis of the Leader Level problem, then for non-basic

$$\beta_{jk} : z_{jk} - c_{jk} = C_B B^{-1} P_{jk} - c_{jk} \tag{17}$$

where

$$c_{jk} = c_j \hat{x}_{jrk} \,\mathrm{And}\, P_{jrk} \begin{bmatrix} a_j \hat{x}_{jrk} \\ 1 \\ 0 \end{bmatrix} \tag{18}$$

Now, to decide which of the variables β_{jk} should enter the solution it must determine:

$$z^*_{jk} - c^*_{jk} = \min\{z_{jk} - c_{jk}\} \tag{19}$$

Consequently, if$z^*_{jk} - c^*_{jk} \leq 0$, then according to the maximization optimality condition, β^*_{jk} must enter the solution.Otherwise, the optimal has been reached.

The Follower-Level Decision-Maker (FLDM) Problem

Finally, according to the mechanism of the BLLSLPP, the Leader Level variables x_{1r}^L should be passed to the Follower-Level, so the follower -level problem can be written as follows:

$$\text{Max}F_2(x) = \text{Max} \sum_{j=1}^{m} \sum_{r=1}^{2} c_{2j} x_{jr} \tag{20}$$

Subject to

$$(x_{1r}^L, \ldots, x_{mr} \in G \tag{21}$$

To obtain the optimal solution of the follower -level problem, the FLDM solves his master problem by the decomposition method 13 as the leader level. Now the optimal solution (x_{1r}^L, x_{2r}^F) of the follower level is satisfactory solution of the BLLSLPP.

4 TOPSIS Approach for the Bi Level Large Scale Quadratic Programming Problem

TOPSIS approach is a multiple criteria method that identifies solutions from a finite set of alternatives based upon simultaneous minimization of distance from an ideal point and maximization of distance from a nadir point. Baky and Abo-Sinna [10] extended the TOPSIS approach to solve Bi-Level Multi-objective decision making problems. Baky [20] also extended the concept of TOPSIS to develop a methodology for solving multi-level non-linear multi-objective decision-making. For more details about TOPSIS approach for the bi-level large-scale programming problem see [4,10,12,13,17,19,20,21,22].

The Decomposition Algorithm for Solving BLLSQPP with Fuzzy Numbers

Following the above-mentioned discussion, the proposed algorithm for solving BLLSQPP with Fuzzy Numbers in the objective functions is given as follows:
Step 1: Compute $R(\tilde{A})$ for all the coefficients of the problem (1)-(3), where A is trapezoidal fuzzy number.
Step 2: Convert from fuzzy to crisp formula.
Step 3: Formulate the equivalent bi-level large scale quadratic programming problem.
Step 4: Convert the bi-level large scale quadratic programming to bi-level large scale linear programming by using Taylor series approach as follows:

$$K_i(x) \cong \hat{F}_i(x) = F_i(x_i^*) + \sum_{j=1}^{n} (x_{jr} - x_{ijr}^*) \frac{\partial F_i(x_j^*)}{dx_{jr}}, (j = 1, 2, \ldots, m) m(i, r = 1, 2)$$

Step 5: Formulate the bi-level large scale linear programming problem.
Step 6: Start with the first level problem and convert the master problem in terms of extreme points of the sets $d_j x_{jr} \leq b_j, x_{jr} \geq 0, j = 1, 2, 3$.
Step 7: Determine the extreme points $x_{jr} = \sum_{k=1}^{k_j} \beta_{jk} \hat{x}_{rjk}$ using Balinski's algorithm 11
Step 8: Set $k = 1$.
Step 9: Compute $z_{jk} - c_{jk} = C_B B^{-1} P_{jk} - c_{jk}$.
Step 10: If $z_{jk}^* - c_{jk}^* \leq 0$, then go to Step 11. Otherwise, the optimal solution has been reached, go to Step 16.
Step 11: Determine $\hat{X}_{jrk}$ associated withmin $\{z_{jk}^* - c_{jk}^*\}$.
Step 12: β_{jk} Associated with extreme point $\hat{X}_{jrk}$ must enter the solution.
Step 13: Determine the leaving variable
Step 14: The new basis is determined by replacing the vector associated with leaving variable with the vectorβ_{jk}.
Step 15: Set $k = k + 1$, go to step 9.
Step 16: If the SLDM obtains the optimal solution go to Step 19. Otherwise, go to Step 17
Step 17: Set $(x_{1r}) = (x_{r1}^L)$ to the SLDM constraints.
Step 18: If the SLDM formulates the problem, go to Step 8.
Step 19: (x_{1r}^L, x_{2r}^F) is as an optimal solution for bi-level large scale linear programming problem, then stop.

5 An Illustrative Numerical Example

To demonstrate the solution for (BLLSQP) with fuzzy numbers, let us consider the following problem:
[Leader Level]

$$\max_{x_1,x_2} F_1(x_1,x_2,x_3,x_4) = \max_{x_1,x_2}[(1,2,4,2)x_1^2+(3,1,1,1)x_2^2+(2,1,3,1)x_3,(5,3,2,2)x_1^2+(2,1,5,3)x_2^2]$$

Where x_3,x_4 solves
[Follower Level]:

$$\max_{x_3,x_4} F_2(x_1,x_2,x_3,x_4) = \max_{x_1,x_2}[(3,5,2,2)x_1^2+(2,1,3,1)x_3^2+(2,1,3,1)x_3^2,(1,1,1,1)x_4^2+(4,1,5,3)x_32+(6,1,4,2)x_4]$$

Subject to:

$$x_1+x_2+x_3+x_4 \leq 80, x_1+3x_2 \leq 60, 4x_3+2x_4 \leq 20, x_3+x_4 \leq 12, x_1,x_2,x_3,x_4 \geq 0$$

First, apply ranking function $R(A) = a+b+1/2(d-c)$ to transform the fuzzy number into equivalent crisp form, so the problem reduces to:
[Leader Level]

$$\max_{x_1,x_2} F_1(x_1,x_2,x_3,x_4) = \max_{x_1,x_2}[2x_1^2+4x_2^2+2x_3,8x_1^2+2x_2^2]$$

Where x_3,x_4 solves
[Follower Level]:

$$\max_{x_3,x_4} F_2(x_1,x_2,x_3,x_4) = \max_{x_1,x_2}[8x_1^2+2x_3^2+2x_4^2,4x_3^2+6x_4]$$

Subject to:

$$x_1+x_2+x_3+x_4 \leq 80, x_1+3x_2 \leq 60, 4x_3+2x_4 \leq 20, x_3+x_4 \leq 12, x_1,x_2,x_3,x_4 \geq 0$$

Solving the Problem with the Proposed Decomposition Algorithm

Apply the first order Taylor series to convert the quadratic objectives function to linear objective functions as follows:
[Leader Level]

$$\max_{x_1,x_2} F_1(x_1,x_2,x_3,x_4) = \max_{x_1,x_2}(10x_1+4x_2+x_3-17)$$

Where x_3,x_4 solves
[Follower Level]

$$\max_{x_3,x_4} F_2(x_1,x_2,x_3,x_4) = \max_{x_3,x_4}(8x_1+5x_4-11)$$

Subject to:

$$x_1+x_2+x_3+x_4 \leq 80, x_1+3x_2 \leq 60, 4x_3+2x_4 \leq 20, x_3+x_4 \leq 12, x_1,x_2,x_3,x_4 \geq 0$$

First, identify solution space of each sub problem

Sub Problem 1	Sub Problem 2
$x_1+3x_2 \leq 60$ $x_1,x_2 \geq 0$ $C_1=(10,4)$ $A_1=(1,1)$ $X_1=(x_1,x_2)$	$4x_3+2x_4 \leq 20$ $x_3+x_4 \leq 12$ $x_3,x_4 \geq 0$ $C_2=(1,0)$ $A_2=(1,1)$ $X_2=(x_3,x_4)$

Second, slack variable x_5 convert common constraint into equation x_6,x_7 are artificial variables $x_1+x_2+x_3+x_4+x_5=80$

Iteraton 0

$$X_B = (x_5, x_6, x_7)^T, X_B = (80, 1, 1)^T, C_B = (0, -M, -M), B = 1, B^(-1) = 1.$$

Similarity, after four iterations the first level decision maker satisfactory solution is obtained as: $(x_1^F, x_2^F, x_3^F, x_4^F) = (60, 0, 5, 0)$ So $F_1 = 588$. Then set the decision variables of the first level decision maker $(x_1^F, x_2^F) = (60, 0)$ to the second level constraint. The second level decision maker will repeat the same steps as the first level decision maker until the second level decision maker gets the satisfactory solution, so

$$(x_3^s, x_4^s) = (0, 10)\,\text{So}\,(x_1^s, x_2^s, x_3^s, x_4^s) = (60, 0, 0, 10), F_1^* = 583, F_2^8 = 519.$$

Solving the Problem with TOPSIS Approach

Now the BLLSQP problem with fuzzy numbers in constraints can be understood as the following deterministic bi level large scale quadratic programming problem (BLLSQP)
[Leader Level]

$$\max_{x_1, x_2} F_1(x_1, x_2, x_3, x_4) = \max_{x_1, x_2}[2x_1^2 + 4x_2^2 + 2x_3, 8x_1^2 + 2x_2^2]$$

Where x_3, x_4 solves
[Follower Level]

$$\max_{x_3, x_4} F_2(x_1, x_2, x_3, x_4) = \max_{x_1, x_2}[8x_1^2 + 2x_3^2 + 2x_4^2, 4x_3^2 + 6x_4]$$

Subject to:

$$x_1 + x_2 + x_3 + x_4 \leq 80, x_1 + 3x_2 \leq 60, 4x_3 + 2x_4 \leq 20, x_3 + x_4 \leq 12, x_1, x_2, x_3, x_4 \geq 0$$

Calculate PIS and NIS payoff tables for the Leader Level of the BLLSQP problem as in Table 1 and Table 2 .

Table 1: PIS payoff table for The Leader Level of problem

	f_{11}	f_{12}	x_1	x_2	x_3	x_4
$\text{Max} f_{11}(x_1, x_2)$	1610	800	0	20	5	0
$\text{Max} f_{12}(x_1, x_2)$	7200	28800*	60	0	0	0

Table 2: NIS payoff table for The Leader Level of problem

	f_{11}	f_{12}	x_1	x_2	x_3	x_4
$\min f_{11}(x_1, x_2)$	0	0	0	0	0	0
$\text{Min} f_{12}(x_1, x_2)$	0	0	0	0	0	0

PIS: $f^* = (1610, 28800)$
NIS: $f^- = (0, 0)$
Next, compute the following equations:

$$dPIS_P^{LLDM} = \left[w_1^P \left(\frac{f_{11}^* - f_{11}(x)}{f_{11}^* - f_{11}^-} \right)^P + w_2^P \left(\frac{f_{12}^* - f_{12}(x)}{f_{12}^* - f_{12}^-} \right)^P \right]^{\frac{1}{P}}$$

$$dNIS_P^{LLDM} = \left[w_1^P \left(\frac{f_{11}(x) - f_{11}^-}{f_{11}^* - f_{11}^-} \right)^P + w_2^P \left(\frac{f_{12}(x) - f_{12}^-}{f_{12}^* - f_{12}^-} \right)^P \right]^{\frac{1}{P}}$$

To get numerical solution, assume that $w1 = 0.5, w2 = 0.5$ and

$$dPIS_2^{LLDM} = \left[(0.5)^2\left(\frac{1610-(2x_1^2+4x_2^2+2x_3)}{1610-0}\right)^2 + (0.5)^2\left(\frac{28800(8x_1^2+2x_1^2)}{28800-0}\right)^2\right]^{\frac{1}{2}}$$

$$dPIS_2^{LLDM} = \left[(0.5)^2\left(\frac{2x_1^2+4x_2^2+2x_3-0}{1610-0}\right)^2 + (0.5)^2\left(\frac{8x_1^2+2x_2^2-0}{28800-0}\right)^2\right]^{\frac{1}{2}}$$

Next, calculate PIS payoff table of problem (BLLSQP), when P=2 as in Table 3.

Table 3: PIS payoff table of problem (BLLSQP), when $P = 2$.

	d_2PIS^{LLDM}	d_2NIS^{LLDM}	x_1	x_2	x_3	4
Mind_2PIS^{LLDM} Maxd_2NIS^{LLDM}	1.7 0.71	1.12 0.25	60 0	0 0	5 5	0 0

$d_2^{*LLDM} = (0.71, 1.12), d_2^{-LLDM} = (1.7, 0.25)$
Now, it is easy to compute the Leader Level problem as:
Maxδ^{LLDM}
Subject to:

$$x_1 + x_2 + x_3 + x_4 \le 80, x_1 + 3x_2 \le 60, 4x_3 + 2x_4 \le 20, x_3 + x_4 \le 12$$

$$\left(\frac{d_2PIS^{LLDM}(x) - 0.71}{1.7-0.71}\right) \ge \delta^{\text{Leader Level}}, \left(\frac{1.12 - d_2PIS^{LLDM}(x)}{1.12-0.25}\right) \ge \delta^{\text{Leader Level}}, \delta^{LLDM} \in [0,1], x_1, x_2, x_3, x_4 \ge 0.$$

Thus, $\delta^{LLDM} = 0.55$ is achieved for the solution $(x_1^*, x_2^*, x_3^*, x_4^*) = (0, 20, 5, 0)$, and $(f_{11}, f_{12}) = (1610, 800)$. Let the Leader Level decide x_1^*, x_2^* with positive tolerance $t^R = 0.5$ (one sided membership function).
Obtain PIS and NIS payoff tables for the Follower Level of the BLLSQP problem as in Table 4 and Table 5.

Table 4: PIS payoff table for The Follower Level problem

	f_{21}	f_{22}	x_1	x_2	x_3	x_4
Max$f_{21}(x_1,x_2)$ Max$f_{22}(x_1,x_2)$	29000 50	60 100*	60 0	0 0	0 5	10 0

Table 5: NIS payoff table for The Follower Level problem

	f_{21}	f_{22}	x_1	x_2	x_3	x_4
$\min f_{21}(x_1,x_2)$	0	0	0	0	0	0
$\text{Min} f_{22}(x_1,x_2)$	0	0	0	0	5	0

$PIS : f^* = (29000, 100)$ $NIS : f^- = (0,0)$
Next, To get numerical solutions, assume that $w1 = 0.25, w2 = 0.25, w3 = 0.25, w4 = 0.25$ and $P = 2$

$$d\,PIS_P^{BL} = \left[w_1^P \left(\frac{f_{11}^* - f_{11}(x)}{f_{11}^* - f_{11}^-}\right)^P + w_2^P \left(\frac{f_{12}^* - f_{12}(x)}{f_{12}^* - f_{12}^-}\right)^P + w_3^P \left(\frac{f_{21}^* - f_{21}(x)}{f_{21}^* - f_{21}^-}\right)^P + w_4^P \left(\frac{f_{22}^* - f_{22}(x)}{f_{22}^* - f_{22}^-}\right)\right]^{\frac{1}{P}}$$

$$d\,VIS_P^{BL} = \begin{bmatrix} w_1^P \left(\frac{f_{11}^* - f_{11}(x)}{f_{11}^* - f_{11}^-}\right)^P + w_2^P \left(\frac{f_{12}^* - f_{12}(x)}{f_{12}^* - f_{12}^-}\right)^P + w_3^P \left(\frac{f_{21}^* - f_{21}(x)}{f_{21}^* - f_{21}^-}\right)^P + w_4^P \left(\frac{f_{22}^* - f_{22}(x)}{f_{22}^* - f_{22}^-}\right) \\ w_1^P \left(\frac{f_{11}(x) - f_{11}^-}{f_{11}^* - f_{11}^-}\right)^P + w_2^P \left(\frac{f_{12}(x) - f_{12}^-}{f_{12}^* - f_{12}^-}\right)^P + w_3^P \left(\frac{f_{21}(x) - f_{21}^-}{f_{21}^* - f_{21}^-}\right)^P + w_4^P \left(\frac{f_{22}(x) - f_{22}^-}{f_{22}^* - f_{22}^-}\right) \end{bmatrix}^{\frac{1}{P}}$$

$$dPIS_P^{BL} = \begin{bmatrix} (0.25)^2 \left(\frac{1610 - (2x_1^2 + 4x_2^2 + 2x_3)}{1610 - 0}\right)^2 + (0.25)^2 \left(\frac{28800 - (8x_1^2 + 2x_1^2)}{28800 - 0}\right)^2 + \\ (0.25)^2 \left(\frac{29000(8x_1^2 + 2x_3^2 + 2x_4^2)}{29000 - 0}\right)^2 + (0.25)^2 \left(\frac{100 - (4x_3^2 + 6x_4)}{100 - 0}\right)^2 \end{bmatrix}^{\frac{1}{2}}$$

Calculate PIS payoff table of problem (BLLSQP), when $P = 2$ as in Table 6.

Table 6: PIS payoff table of problem (BLLSQP), when $P = 2$.

	d_2PIS^{BL}	d_2NIS^{BL}	x_1	x_2	x_3	x_4
$\text{Min}d_2PIS^{BL}$	0.26^*	0.3	32.46	0	5	0
$\text{Max}d_2NIS^{BL}$	0.35	0.35^*	0	20	5	0

$d_2^{*BL} = (0.26, 0.35), d_2^{-BL} = (0.35, 0.3)$
Now, it is easy to compute the BL problem as:
$\text{Max}\delta^{BL}$
Subject to:

$$x_1 + x_2 + x_3 + x_4 \le 80, x_1 + 3x_2 \le 60, 4x_3 + 2x_4 \le 20, x_3 + x_4 \le 12$$

$$1 - \left[\frac{d_2PIS^{BL}(x) - 0.26}{0.35 - 0.26}\right] \ge \delta^{BL}, 1 - \left[\frac{0.35 - d_2PIS^{BL}(x)}{0.35 - 0.3}\right] \ge \delta^{BL}, \frac{(0 + 0.5) - x_1}{0.5} >= \delta^{BL}, \frac{(20 + 0.5) - x_2}{0.5} >= \delta^{BL}, \delta^{BL} \in [0,1], x_1, x_2, x_3, x_4 \ge 0$$

The maximum "satisfactory level" $\delta^{BL} = 0.9018$ is achieved for the solution $(x_1^*, x_2^*, x_3^*, x_4^*) = (0, 20, 5, 0)$, with objective function values $f_{11} = 1610, f_{12} = 800, f_{21} = 50$, and $f_{22} = 100$

A comparison given in Table 7 shows that the satisfactory solution of the proposed decomposition approach is more applicable than the satisfactory solution of the TOPSIS approach.

Table 7: The comparison of satisfactory solutions of the illustrative example based on the proposed TOPSIS approach and the decomposition algorithm

	(x_1, x_2, x_3, x_4)	f_{11}	f_{12}	f_{21}	f_{22}
TOPSIS approach	(0, 20, 5, 0)	1610	800	50	100
Decomposition Algorithm	(60, 0, 0, 10)	7200	28800	29000	60

Finally, the TOPSIS approach produces approximated inaccurate but fast solutions. These solutions can be used in fields such as agricultural decisions. The decomposition algorithm introduces accurate but slow solutions. These solutions can serve in fields such as medical and financial decisions.

6 Conclusion

This paper proposed a decomposition approach to solve a bi-level large-scale quadratic programming problem with fuzzy parameters in the objective function. The basic idea in treating the BLLSQPP is to convert the fuzzy number nature of the problem into an equivalent deterministic nature. Then, Taylor series approach is combined with the decomposition algorithm to obtain the satisfactory solution for the problem. A comparison between the proposed decomposition approach and TOPSIS approach was conducted to show that the satisfactory solution of the proposed decomposition approach is more applicable than the satisfactory solution of the TOPSIS approach.

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Conflict of Interest

The authors declare that they have no conflict of interest.

References

[1] O.E. Emam, M. El-Araby and M.A. Belal, ”On Rough Multi-Level Linear Programming Problem”, Information Sciences Letters, 4 (1) (2015), 41-49.
[2] M. Sakawa, ”Large Scale Interactive Fuzzy Multiobjective Programming”, PhysicaVerlag, a Springer-Verlag Company, New York, 2000.
[3] T.H.M. Abou-El-Enien, ”On the Solution of A Special Type of Large Scale Integer Linear Vector Optimization Problems with Uncertainty Data through TOPSIS Approach”, International Journal of Contemporary Mathematical Sciences, 6 (2011), 657-669.
[4] T.H.M. Abou-El-Enien, ”On the Solution of A Special Type of Large Scale Linear Fractional Multiple Objective Programming Problems with Uncertainty Data” Applied Mathematical Sciences, 4 (2010), 3095-3105.
[5] M.S. Osman, O.E. Emam and M.A. Elsayed, ”Interactive Approach for Multi-Level Multi-Objective Fractional Programming Problems with Fuzzy Parameters”, Journal of Basic and Applied Science, in press.
[6] M. Omran, L. Abd-Elatif and M. Thabet, ”Solving Large-Scale Three-Level Linear Fractional Programming Problem with Rough Coefficient in Constraints”, International Journal of Advanced Computer Technology, 175 (8) (2007), 14-19.
[7] C.L. Hwang, K. Yoon, ”Multiple Attribute Decision Making: Methods and Applications”, Springer-Verlag, Heidelberg, 1981.
[8] M.A. Abo-Sinna, ”Extensions of the TOPSIS for Multi-Objective Dynamic Programming Problems under Fuzziness”, Adv. Model, 4 (2000), 1-24.
[9] M.A. Abo-Sinna, A.H. Amer and A.S. Ibrahim, ”Extensions of TOPSIS for Large Scale Multi-Objective Non-Linear Programming Problems with Block Angular Structure”, Appl. Math. Model, 32 (2008), 292-302.
[10] I.A. Baky and M.A. Abo-Sinna, ”TOPSIS for Bi-Level MODM Problems”, Applied Mathematical Modelling, 37 (2013), 1004-1015.

[11] I.A. Baky, M.H. Eid and M.A. ElSayed, "Bi-Level Multi-Objective Programming Problem with Fuzzy Demands: A Fuzzy Goal Programming Algorithm", Operational Research Society of India, 51 (2014), 280-296.
[12] M.S. Osman, M.A. Abo-Sinna, A.H. Amer and O.E. Emam, "A Multi-Level Non-Linear Multi-Objective Decision-Making under Fuzziness", Applied Mathematics and Computation, 153 (2004), 239-252.
[13] M.A. Abo-Sinna and T.H.M. Abou-El-Enien, "An Interactive Algorithm for Large Scale Multiple Objective Programming Problems with Fuzzy Parameters through TOPSIS Approach", Yugoslav Journal of Operations Research, 21 (2011), 253-273.
[14] M.S. Osman, O.E. Emam and M.A.Elsayed, "On Parametric Multi-Level Multi-Objective Fractional Programming Problems with Fuzziness in the Constraints", British Journal of Mathematics & Computer Science 18(5) (2016) 1-19.
[15] M.S. Osman, O.E. Emam and M.A. Elsayed, "Stochastic Fuzzy Multi-level Multi-objective Fractional Programming Problem: A FGP Approach", Operational Research Society of India, 54 (2017), 816-840.
[16] E.A. Youness, O.E. Emam and M.S. Hafez, "Fuzzy Bi-Level Multi-Objective Fractional Integer Programming", Applied Mathematics & Information Sciences, 8 (6) (2014), 2857-2863.
[17] M.A. Abo-Sinna and T.H.M. Abou-El-Enien,"An Algorithm for Solving Large Scale Multiple Objective Decision Making Problems using TOPSIS Approach", Advances in Modelling and Analysis, 6 (2005), 31-48.
[18] T. Sultan, O.E. Emam and A.A. Abohany, "A Decomposition Algorithm for Solving A Three Level Large Scale Linear Programming Problem", Applied Mathematics and Information Sciences 8(5) (2014) 2217-2223.
[19] M.A. Abo-Sinna and T.H.M. Abou-El-Enien, "An Interactive Algorithm for Large Scale Multiple Objective Programming Problems with Fuzzy Parameters through TOPSIS Approach", Applied Mathematics and Computation, 177 (2006), 515-527.
[20] I.A. Baky, "Interactive TOPSIS Algorithms for Solving Multi-Level Non-Linear Multi-Objective Decision-Making Problems", Applied Mathematical Modelling, 38 (2014), 1417-1433.
[21] R.E. Bellman and L.A. Zadeh, "Decision-Making in A Fuzzy Environment", Manag. Sci., 17 (1970), 141-164.
[22] Y.J. Lai, T.J. Liu and C.L. Hwang, "TOPSIS for MODM", European Journal of Operational Research, 76 (1994), 486-500.
[23] M. Balinski, "An Algorithm for Finding All Vertices of Convex Polyhedral Sets", Journal of the Society for Industrial and Applied Mathematics, 1 (1961), 72-88.
[24] G. Dantzig and P. Wolfe, "The Decomposition Algorithm for linear Programs", Econometrics, 9 (1961), 767-778.