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Solving Natural Convection Of Darcian Fluid In Porous Media Using Rational Chebyshev Collocation Method

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Abstract: In this paper, numerical technique is introduced for solving natural convection of Darcian fluid about a vertical full cone embedded in porous media with a prescribed wall temperature or surface heat flux boundary conditions. The power function of distance from the vertex of the inverted cone gives us nonlinear differential equation. Rational Chebyshev collocation method is used to solve the produced third order nonlinear differential equation with boundary conditions transformed to a system of nonlinear equations. The proposed base is specified by its ability of deal with boundary conditions with an independent variable that may tend to infinity with easy manner without divergent. Moreover, we compare the proposed method with other methods to investigate applicability and accuracy.

Keywords: Rational Chebyshev functions, natural convection of Darcian fluid, collocation method, nonlinear differential equations.

1 Introduction

The spectral methods have an important and significant role in approximate differential equations, which make it easy in treating many phenomena and models in physics, engineering, economic and many other fields. The most common distinctive feature for spectral methods is using bases in the form of polynomials or functions which are biorthogonal with respect to the weight functions, defined in bounded and unbounded domain. Various spectral methods can be applicable on finite domain [1–3], semi-infinite domain [4–6] and unbounded domain [7–13].

The choice of trial functions provides the spectral method a great distinguishing feature, where the choice of trial functions depended on the exact solution $f(x)$ and the values of x for the proposed equation. This means that if the solution $f(x)$ is polynomial in finite domain, use Chebyshev polynomials. However, if $f(x)$ is periodic, Fourier series is preferred. If $f(x)$ is defined in unbounded domain, use Hermit functions or exponential Chebyshev functions. Whereas, if the solution defined in semi-infinite interval $x \in [0,\infty)$, use Laguerre functions or rational Chebyshev functions, especially the solution $f(x)$in fraction or exponential form in semi-infinite domain the RC functions which are the best.

Many researchers have been publishing several papers on convection of porous media.

A porous medium is a material consisting of a solid matrix with interconnected pores. The solid matrix is rigid and undergoes simple deformation.The interconnectedness of the void allows the flow of one or more fluids through the material [14].

Prediction of natural convection heat transfer characteristics from heated bodies embedded in a porous media has a number of thermal engineering applications, such as ceramic processing, nuclear reactor cooling system, crude oil drilling, chemical reactor design, ground water pollution and filtration processes. Papers have addressed multiple problems associated with external natural convection in a porous medium adjacent to heated bodies in the form of flat plate [14, 15], cylinder [14, 16, 17], sphere [14, 17, 18][and cone [14, 19–24]. In all of these analyses, it is assumed that Darcy law and boundary layer approximations are applicable and the coupled set of governing equations was solved using numerical methods.

Thanks to their orthogonality and minimax properties, Chebyshev Polynomials have played a significant role in

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all areas of numerical analysis, including polynomial approximation, numerical integration, integral equations, and differential equation.

Using a transformation that maps an finite domain into an infinite domain, it is possible to generate a great variety of new basis sets for the infinite interval or semi-infinite interval that are the images under the change-of-coordinate of Chebyshev polynomials or Fourier series, where the new basis inherits most of the good numerical characteristics of the Chebyshev polynomials. Exploiting the mapping induced connection between Fourier cosine series and the new basis functions, defined in [25], is called rational Chebyshev (RC) functions. This strategy is a generalization of the particular map$x = \cos(\theta)$, which transforms the cosine functions into the Chebyshev polynomials. The rational Chebyshev functions are orthogonal on $[0,\infty)$ with the weight function $\frac{1}{(1+x)\sqrt{x}}$.

In recent works, the rational Chebyshev functions was introduced for solving high-order linear ordinary differential equations, system of high-order linear ordinary differential equations, and high-order integro-differential equations, defined in unbounded domains by transforming last equations and the given conditions to a system of nonlinear algebraic equations [26–34]. The solution is obtained in terms of RC functions.

The organization of this paper is, as follows. In Section 2, properties of the rational Chebyshev (RC) functions are presented. In Section 3, problem formulation is introduced. In Section 4, description of the solution method is presented. Section 5 involves numerical illustrations and the results that are compared with the exact solution, as well as other existing methods to demonstrate accuracy of the method.

2 Rational Chebyshev functions

In this section, we use an algebraic mapping from [-1,1] to $[0,\infty)$ with Chebyshev polynomials to provide a new set of basis functions, called rational Chebyshev functions, for solving differential equations on an semi-infinite interval. . We also present some properties of rational Chebyshev functions.

As a continuation of the papers conducted by John P. Boyd [35] as well as Grosch and Orszag [36], we show that these rational functions inherit most of the good numerical characteristics of the Chebyshev polynomials: orthogonality, completeness, exponential or "infinite order" convergence, matrix sparsity for equations with polynomial coefficients, and simplicity.

The basis functions, denoted by $TL_n(x)$, are defined by

$$TL_n(x) = T_n(y) = \cos n\theta, \qquad (1)$$

where L is a constant map parameter and the three coordinates are related by

$$x = \frac{L(1+y)}{1-y}, \quad y = \frac{x-L}{x+L}, \qquad (2)$$

$$x = L\cot^2(\theta/2), \ \ \theta = 2arc\cot([x/L]^{1/2}). \qquad (3)$$

To avoid confusion as we leap from one coordinate to another, we shall adopt the convention that $x \in [0,\infty)$ is the argument of the $TL_n(x)$, $y \in [-1,1]$ is the argument of the ordinary Chebyshev polynomials, and $\theta \in [0,\pi]$ is the argument of the cosines. We are free to estimate which of these three coordinates is the most convenient.

From (1) and (2), we get

$$TL_n(x) = T_n(y) = T_n\left(\frac{x-L}{x+L}\right). \qquad (4)$$

In this study, we take L=1 and relation (4) is obtained as

$$TL_n(x) = T_n\left(\frac{x-1}{x+1}\right) = R_n(x)$$

It symbolizes rational Chebyshev function as $R_n(x)$ instead of $TL_n(x)$. Consequently, we can define rational Chebyshev functions, as follows:

Definition 2.1

The rational Chebyshev functions $R_n(x)$ of the first kind are polynomials in x of degree n, defined by the relation

$$R_n(x) = T_n\left(\frac{x-1}{x+1}\right), \quad when \quad x = \cot^2(\theta/2)x \in [0,\infty) \qquad (5)$$

If the range of the variable x is the interval $[0,\infty)$, the range of the corresponding variable θ can be taken as $[0,\pi]$. These ranges are traversed in opposite directions, since $x = 0$ corresponds to $\theta = \pi$ and $x = \infty$ corresponds to $\theta = 0$.

For more details, (see, for instance, [25–30]).

Form the recurrence relation of the Chebyshev polynomials, we find that $R_n(x)$ satisfies the following recurrence formulae

$$R_{n+1}(x) = 2\left(\frac{x-1}{x+1}\right)R_n(x) - R_{n-1}(x), \quad \mathrm{n} \geq 1, \qquad (6)$$

which together with

$$R_0(x) = 1, \ \ R_1(x) = \frac{x-1}{x+1}. \qquad (7)$$

Rational Chebyshev functions are neither even nor odd for $n \geq 1$, while $R_0(x)$ is an even function.

It is useful and convenient in various applications to express rational Chebyshev functions explicitly in terms of powers of $v(x)=\frac{x-1}{x+1}$, and vice versa. We can state the following proposition:

Proposition 2.1 The power $v^n(x)$ can be expressed in terms of rational Chebyshev functions of degrees up to n *as:*

$$v^n(x) = 2^{1-n}\sum_{k=0}^{[n/2]}\binom{n}{k}R_{n-2k}(x) \tag{8}$$

Proposition 2.2 A simple formula for $R_n(x)$ in terms of powers of $v(x)$is

$$R_n(x) = \sum_{k=0}^{[n/2]} c_k^{(n)} v^{n-2k}(x), \tag{9}$$

where

$$c_k^{(n)} = (-1)^k 2^{n-2k-1}\frac{n}{n-k}\binom{n-k}{k}, \quad 2k \le n, \tag{10}$$

and

$$c_k^{(2k)} = (-1)^k, \ k \ge 0. \tag{11}$$

Since the set of RC functions is orthogonal and complete $f(x)$ defined over the interval $[0,\infty)$ can be expanded as:

$$f(x) = \sum_{n=0}^{\infty} a_n R_n(x),$$

where

$$a_n = \frac{2}{c_n\pi}\int_0^{\infty} R_n(x) f(x) w(x) dx. \tag{12}$$

with respect to the weight function$w(x) = 1/((x+1)\sqrt{x})$, and

$$c_n = \begin{cases} 2, & n=0 \\ 1, & n \ge 1 \end{cases}.$$

If $f(x)$ is truncated to $n < \infty$ in terms of RC functions take the form

$$[f_N(x)] = R(x)A \tag{13}$$

where $R(x)$ and A are vectors of the form:

$$R(x) = \left[R_0(x)\ R_1(x)\ \ldots\ R_N(x)\right], A = \left[a_0\ a_1\ \cdots\ a_N\right]^T,$$

and the derivative of $f(x)$ can be put in the matrix forms

$$[f_N^{(j)}(x)] = R^{(j)}(x)A\, j = 0,1,\ldots,n \le N. \tag{14}$$

where

$$R^{(j)}(x) = \left[R_0^{(j)}(x)\ R_1^{(j)}(x)\ \ldots\ R_N^{(j)}(x)\right]$$

3 Problem formulations

Consider an inverted cone with semi-angle γ and take axes in the manner indicated in Fig. 1(a). The stream function ψ defined as

$$u = \frac{1}{r}\frac{\partial\psi}{\partial y}, \upsilon = -\frac{1}{r}\frac{\partial\psi}{\partial x}. \tag{15}$$

In terms of the heated frustum $x = x_0$ is considered [14]. The boundary layer equations are:

$$\begin{array}{l} \frac{1}{r}\frac{\partial^2\psi}{\partial y^2} = \frac{g\beta K}{v}\frac{\partial T}{\partial y}, \\ \frac{1}{r}\left(\frac{\partial\psi}{\partial y}\frac{\partial T}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial T}{\partial y}\right) = \alpha\frac{\partial^2 T}{\partial y^2}. \end{array} \tag{16}$$

We have approximately $r = x\sin(\gamma)$ and suppose the temperature is power function of distance from the vertex of the inverted cone. Accordingly, the boundary conditions are:

$$\begin{array}{lll} u=0, & T=T_\infty & y\to\infty, \\ u=0, & & y=0,\ x_0\le x\le\infty, \\ T=T_\omega=T_\infty+A(x-x_0)^\lambda & & y=0,\ x_0\le x\le\infty. \end{array} \tag{17}$$

For the case of a full cone ($x_0 = 0$, *Fig.*1(b)) a similarity solution exists.
In the case of prescribed wall temperature, we let:

$$\begin{array}{l} \psi = \alpha r Ra_x^{1/2} f(\eta), \\ T - T_\infty = (T_\omega - T_\infty)\theta(\eta), \\ \eta = \frac{y}{x} Ra_x^{1/2}, \end{array} \tag{18}$$

where the Rayleigh number is

$$Ra_x = \frac{g\beta K\cos(\gamma)(T_\omega - T_\infty)x}{v\alpha}. \tag{19}$$

The governing equations are [14, 19] and [34]

$$\begin{array}{l} f' = \theta, \\ \theta'' + (\frac{\lambda+3}{2}) f\theta' - \lambda f'\theta = 0 \end{array} \tag{20}$$

with boundary conditions,

$$f(0) = 0, \quad \theta(0) = 1, \quad \theta(\eta) = 0, \quad where \quad \eta\to\infty \tag{21}$$

Finally, we have:

$$f''' + \left(\frac{\lambda+3}{2}\right) f f'' - \lambda(f')^2 = 0, \tag{22}$$

with boundry conditions

$$f(0) = 0, \quad f'(0) = 1, \quad f'(\eta) = 0, \quad where \quad \eta\to\infty. \tag{23}$$

It is of interest to obtain the value of the local Nusselt number which is defined as:

$$Nu_x = \frac{q_w x}{k(T_w - T_\infty)}. \tag{24}$$

Where $q_w$for the case of prescribed wall temperature can be computed from:

$$q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0}. \tag{25}$$

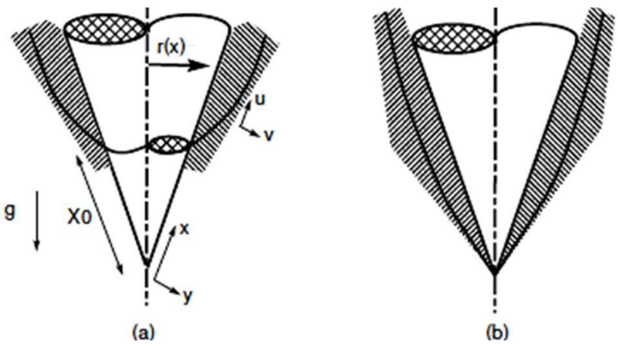

Fig. 1: (a) Coordinate system for the boundary layer on a heated frustum of a cone, (b) full cone, $x_0 = 0$

From Eqs. (18), (19), (24) and (25), it follows that the local Nusselt number is given by [14,34]:

$$Nu_x = Ra_x^{1/2}[-\theta'(0)].$$

Nomenclature
Aprescribed constant
f similarity function for stream function
$g$acceleration due to gravity
K permeability of the fluid-saturated porous medium
Nu local Nusselt number
$q_w$surface heat flux
r local radius of the cone
Ra_xlocal Raleigh number
T temperature
T_∞ ambient temperature
$T_w$wall temperature
$u,\ \upsilon$velocity vector along x, y axis
x, y Cartesian coordinate system
$x_0$distance of start point of cone from the vertex
Greek symbols
θ similarity function for temperature
η independent dimensionless parameter
λprescribed constants
β expansion coefficient of the fluid
αthermal diffusivity the fluid-saturated porous medium
ν kinematics viscosity of the fluid
ψ stream function

4 Method description

Consider the following nonlinear boundary value problem (22):

$$f''' + \left(\frac{\lambda+3}{2}\right) f f'' - \lambda (f')^2 = 0,$$

governed by the boundary conditions
$f(0) = 0, \quad f'(0) = 1, \quad f'(\eta) = 0.$ where $\eta \to \infty$

If f is approximated to f_N as in (13) and (14), the above-mentioned nonlinear equation (22) is translated to the following form in the unknown vector A:

$$R^{(3)}A + \left(\frac{\lambda+3}{2}\right) RA\left(R^{(2)}A\right) - \lambda\left(R^{(1)}A\right)^2 = 0, \quad (26)$$

an approximate solution f_N can be obtained by employing the typical collocation method. For this purpose, equation (26) is collocated at $(N+1)$ points. These points may be taken by

$$\eta_k = \left(\frac{1+\cos\left(\frac{k\pi}{n}\right)}{1-\cos\left(\frac{k\pi}{n}\right)}\right), \qquad (k = 1, \ldots, n-1). \qquad (27)$$

The same operation made with the conditions (23) produces extra equations. Thus, we replace the produced equations by the last three of the system. Hence, a set of $(N+1)$ nonlinear equations are generated in the expansion coefficients. This nonlinear system can be solved with the aid of a suitable solver. The well-known Newton iterative method can be used here with 100 iterations. Therefore, the corresponding approximate solution f_N can be obtained.

5 Numerical results

Consider governing equation of fluid flow and heat transfer of full cone embedded in porous medium expressed by Eq.

$$f''' + (\frac{\lambda+3}{2}) f f'' - \lambda (f')^2 = 0$$

for wall temperature boundary condition
$f(0) = 0, \quad f'(0) = 1, \quad f'(\eta) = 0,$ where $\eta \to \infty$.
We apply the present method to find the approximate solution $f_N(\eta)$ for $N = 26$ and 30 by the truncated rational Chebyshev series. Table1 shows the coefficients a_i of the rational Chebyshev series at different λ as $f_N(\eta) = \sum_{i=0}^{N} a_i R_i(\eta)$ at $N = 26$. Table 2 indicates, comparison of the approximate values for $\theta'(0)$ of the present method at N= 26 with the previous results obtained from different methods. Our obtained results are close to those of Runge-Kutta method, Homotopy analysis [34] and RCT method [32]. Table 3 involves a comparison between the residual errors with different volue for λ at N= 26 using the present method. The results for $\theta(0)$ are shown in Tables 4 and 5 with two selected λ as 0 and 1/2 and are compared with Runge-Kutta method, Homotopy analysis [34] and RCT method [32].

6 Conclusion

In this paper, rational Chebyshev collection method is applied to solve third order nonlinear differential equation

Table 1: A comparison between a_i the coefficient results for various λ at $N = 26$

a_i	$\lambda = 0$	$\lambda = 1/4$	$\lambda = 1/3$	$\lambda = 1/2$	$\lambda = 3/4$	$\lambda = 1$
a_0	0.554457225	0.525683296	0.517213395	0.501635582	0.481094861	0.463249716
a_1	0.508370705	0.473913467	0.463819425	0.445320709	0.421074189	0.400161055
a_2	-0.095862223	-0.096494523	-0.096625062	-0.096788466	-0.096833760	-0.096695227
a_3	-0.051888130	-0.045056784	-0.043064859	-0.039424946	-0.034674824	-0.030597502
a_4	0.006295945	0.0073669252	0.007653882	0.008143116	0.008703461	0.009102443
a_5	0.011799427	0.0104291547	0.010030844	0.009303263	0.008352501	0.007533551
a_6	0.002885521	0.0021411443	0.001932283	0.001561365	0.001100324	0.000727911
a_7	-0.001585738	-0.001533606	-0.001516993	-0.001483957	-0.001433941	-0.001382921
a_8	0.001530059	-0.001301434	-0.001236784	-0.001120733	-0.000972956	-0.000849070
a_9	-0.000421389	-0.000312750	-0.000282167	-0.000227773	-0.000160066	-0.000105371
a_{10}	0.000172862	0.000171915	0.000171797	0.000171489	0.000170452	0.000168478
a_{11}	0.000233196	0.000201207	0.000192247	0.000176179	0.000155607	0.000138101
a_{12}	0.000108172	0.0000867505	0.000080613	0.000069545	0.000055428	0.0000436674
a_{13}	5.57454×10^{-6}	-1.34199×10^{-7}	-1.85483×10^{-6}	-4.99523×10^{-6}	-8.96525×10^{-6}	-0.000012116
a_{14}	0.000031532	-0.000029057	-0.000028391	-0.000027186	-0.000025557	-0.000024014
a_{15}	-0.000028142	-0.000024123	-0.000022957	-0.000020819	-0.000017993	-0.000015515
a_{16}	-0.000013447	-0.000010742	-9.93711×10^{-6}	-8.45366×10^{-6}	-6.51028×10^{-6}	-4.85787×10^{-6}
a_{17}	-2.21395×10^{-6}	-1.15957×10^{-6}	-8.40112×10^{-7}	-2.54767×10^{-7}	4.89413×10^{-7}	1.08284×10^{-6}
a_{18}	2.51676×10^{-6}	2.53041×10^{-6}	2.53533×10^{-6}	2.53749×10^{-6}	2.51341×10^{-6}	2.45348×10^{-6}
a_{19}	2.7868×10^{-6}	2.42338×10^{-6}	2.31295×10^{-6}	2.10417×10^{-6}	1.8158×10^{-6}	1.55256×10^{-6}
a_{20}	1.33449×10^{-6}	1.00176×10^{-6}	9.0103×10^{-7}	7.14762×10^{-7}	4.71862×10^{-7}	2.69501×10^{-7}
a_{21}	8.99806×10^{-8}	-2.46998×10^{-7}	-2.93505×10^{-7}	-3.75983×10^{-7}	-4.72658×10^{-7}	-5.38649×10^{-7}
a_{22}	-8.4508×10^{-7}	-8.44258×10^{-7}	-8.42469×10^{-7}	-8.34568×10^{-7}	-8.10511×10^{-7}	-7.72498×10^{-7}
a_{23}	9.67181×10^{-7}	-8.86217×10^{-7}	-8.60356×10^{-7}	-8.08736×10^{-7}	-7.3028×10^{-7}	-6.50626×10^{-7}
a_{24}	-7.29713×10^{-7}	-6.40915×10^{-7}	-6.13196×10^{-7}	-5.59608×10^{-7}	-4.82972×10^{-7}	-4.10586×10^{-7}
a_{25}	-3.68448×10^{-7}	-3.15738×10^{-7}	-2.99426×10^{-7}	-2.68254×10^{-7}	-2.24696×10^{-7}	-1.84772×10^{-7}
a_{26}	-8.92647×10^{-8}	-7.55058×10^{-8}	-7.12647×10^{-8}	-6.32013×10^{-8}	-5.20492×10^{-8}	-4.19664×10^{-8}

Table 2: A comparison between present method and other existing methods with the values for $\theta'(0)(f''(0))$

λ	Runge-Kutta [34]	HAM [34]	RCT [32]	Method in [19]	Present method	
					$N = 26$	$N = 30$
0	-0.76854	-0.77363	-0.76600	-0.769	-0.768659	-0.76854
1/4	-0.88498	-0.88800	-0.88830	—	-0.885073	-0.88497
1/3	-0.92101	-0.92433	-0.92100	-0.921	-0.921085	-0.92099
1/2	-0.98956	-0.99382	-0.98581	-0.992	-0.989664	-0.98957
3/4	-1.08518	-1.08840	-1.08598	—	-1.08526	-1.08518
1	-1.17372	-1.17686	-1.17040	—	-1.17375	-1.17368

Table 3: A comparison between residual errors of present method at = 26 for different volumes forλ

η	$\lambda = 0$	$\lambda = 1/4$	$\lambda = 1/3$	$\lambda = 1/2$	$\lambda = 3/4$	$\lambda = 1$
0	0.0861367	0.0718359	0.0674466	0.0591442	0.0477793	0.0376464
0.5	3.61848×10^{-4}	2.99265×10^{-4}	2.80101×10^{-4}	2.43951×10^{-4}	1.94746×10^{-4}	1.51219×10^{-4}
1	2.77556×10^{-16}	3.33067×10^{-16}	3.88578×10^{-16}	4.44089×10^{-16}	2.22045×10^{-16}	3.33067×10^{-16}
2.4	2.08853×10^{-5}	1.66557×10^{-5}	1.53684×10^{-5}	1.29617×10^{-5}	9.74855×10^{-6}	6.98523×10^{-6}
5	-2.59245×10^{-7}	-1.90629×10^{-7}	-1.70096×10^{-7}	1.32374×10^{-7}	8.37343×10^{-8}	4.39285×10^{-8}
8.8	2.58985×10^{-7}	1.59259×10^{-7}	1.30012×10^{-7}	7.74002×10^{-8}	1.24571×10^{-8}	-3.72492×10^{-8}
10	3.89714×10^{-8}	2.27297×10^{-8}	1.79920×10^{-8}	9.51293×10^{-9}	8.46099×10^{-10}	-8.64873×10^{-9}

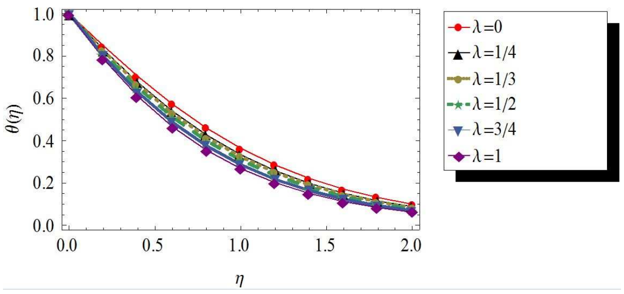

Fig. 2: RC collection approximation of $\theta(\eta)$ with different volume for λ

Table 4: Comparison between RC collection method and other existing methods for $\theta(0)$with $\lambda = 0$

η	Runge-Kutta [34]	RCT [32]	HAM [34]	RC collection . $N = 26$	$N = 30$
0	1.0000	1.0000	0.9999	1.0000	1.0000
0.2	0.8478	0.8477	0.8474	0.8477	0.8478
0.4	0.7036	0.7036	0.7028	0.7036	0.7036
0.6	0.5733	0.5732	0.5720	0.5732	0.5732
0.8	0.4599	0.4598	0.4583	0.4598	0.4598
1	0.3643	0.3641	0.3623	0.3641	0.3641
1.2	0.2855	0.2853	0.2833	0.2854	0.2854
1.4	0.2218	0.2218	0.2200	0.2218	0.2218
1.6	0.1713	0.1713	0.1698	0.1713	0.1713
1.8	0.1315	0.1316	0.1301	0.1316	0.1316
2	0.1007	0.1007	0.0991	0.1007	0.1007

Table 5: Comparison between RC collection method and other existing methods for $\theta(0)$ with $\lambda = 1/2$

η	Runge-Kutta [34]	RCT [32]	HAM [34]	RC collection $N = 26N = 30$	
0	1.0000	0.9999	1.0004	1.0000	1.0000
0.2	0.8130	0.8130	0.8125	0.8129	0.8129
0.4	0.6500	0.6500	0.6491	0.6499	0.6499
0.6	0.5124	0.5125	0.5111	0.5124	0.5124
0.8	0.3994	0.3994	0.3977	0.3993	0.3993
1	0.3084	0.3084	0.3063	0.3084	0.3084
1.2	0.2364	0.2364	0.2347	0.2364	0.2364
1.4	0.1802	0.1802	0.1785	0.1802	0.1802
1.6	0.1367	0.1367	0.1352	0.1368	0.1367
1.8	0.1034	0.1034	0.1020	0.1034	0.1034
2	0.0780	0.0779	0.0763	0.0780	0.0780

arising from similarity solution of inverted cone embedded in porous medium. Numerical results of the introduced technique are compared with rational Chebyshev tau method, homotopy analysis method, Runge-Kutta and method in [19]. The obtained solution in comparison with the numerical ones represents a remarkable accuracy.

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Competing interests

The authors declare that they have no competing interests.

Authors' contributions

First author revised the mathematical research point and performed the examples using Mathematica, and was a major contributor in writting the manuscript. Second author suggested the research point for this paper. Third author helped in coding using Mathematica program and modified the final draft of the manuscript.

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