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Soliton Propagation in a Left-handed Four-Level Atomic System

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Abstract: In this paper we analyze and discuss the solitons propagation in a left-handed four-level atomic system coupled to three laser fields. The system is analyzed through the density matrix formalism. We show that by varying the external probe field amplitude in time, the soliton propagation is obtained in the left-handed medium . Our scheme allows to engineer under certain conditions, the soliton propagation in the four- level atomic system while maintaining the negative refractive index.

Keywords: Solitons, density matrix, negative refractive index, four level system

1 Introduction

Considerable attention was paid to the formation of localized electromagnetic waves propagating without spreading shape, called solitons in nonlinear optical media [1, 2]. Ultra slow optical solitons were extensively studied in electromagnetically induced transparency (EIT) media [3–6] where the wave propagation of a weak probe field displays large suppression of optical absorption and a significant reduction of group velocity. This leads to important physical features [7, 8]. In particular, the spatio-temporal solitons known as STSs [9] are multidimensional pulses which maintain their shape in the longitudinal (temporal) and transverse (spatial) directions. This is due to the balance between the group-velocity dispersion (GVD), diffraction, and nonlinear self-phase modulation. On the other hand, non linear optical media, known as left-handed materials have shown interesting nonlinear optical phenomena such as the harmonic generation, parametric amplification, sub-wavelength imaging, and solitary wave propagation [10, 11]. These materials, known as Metamaterials (MMs), are artificially engineered materials that have unique unnatural properties. In fact such media are able to focus light into a spot much smaller than the wavelength, which lead to realize a perfect lens [12]. Moreover, the electromagnetic properties of such materials can be controlled by modulating the design of the cell and bringing several promising applications [13, 14]. These laters are of great significance for developing applications in meta-materials due to their properties [15, 16]. In this paper it is assumed that the material is a four level system which exhibits Left-handed properties under certain conditions and in a range of frequency. Our aim is to define how the meta-material properties can be associated with soliton propagation and to investigate how this behavior can be controlled. This paper is organized as follows: In the first section we present the model of a four level atomic system interacting with three electromagnetic fields. In the second section, we analyze and describe our system in terms of the density matrix formalism. Then we present the results followed by a discussion and concluding remarks.

2 The model

In this model, the different levels denoted by (k,l) where $(k,l = 1,4)$ are coupled to the corresponding (k,l) fields as follows: the level $|3\rangle$ is coupled to the level $|4\rangle$ through the field E_c while the level $|2\rangle$ is coupled by the weak probe field E_p to the level $|3\rangle$. The electric and magnetic fields of E_p are coupled to the level pairs $|3\rangle$ to $|2\rangle$ and $|2\rangle$ to $|1\rangle$, respectively, while the weak signal field Es drives the transition $|1\rangle$ to $|3\rangle$. The Rabi frequencies of the corresponding fields are Ω_c, Ω_p, Ω_s, also denoted by, the Ω_{kl}, corresponding to the (k,l) levels. The frequency detuning δ_{kl} corresponds to Δ_c, Δ_p,

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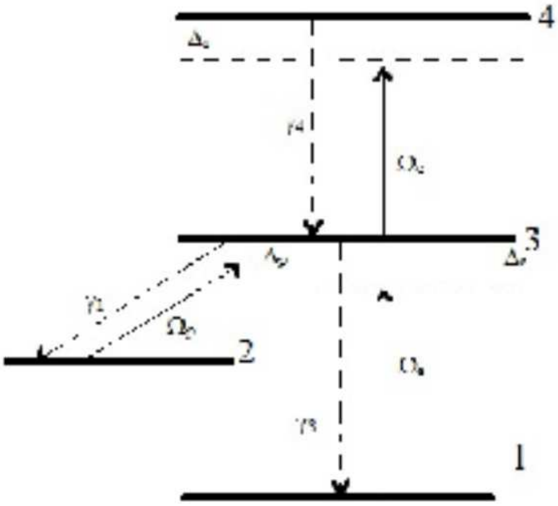

Fig. 1: A schematic diagram of a four level atomic system interacting with three electromagnetic fields.

Δ_s as $\Delta_c = \omega_c - \omega_{34}$, $\Delta_p = \omega_p - \omega_{23}$ and $\Delta_s = \omega_s - \omega_{13}$, and the decay rates γ_{kl} are $\gamma_{4,2,3}$ from the higher level $|4\rangle$ to $|3\rangle$, $|3\rangle$ to $|2\rangle$ and $|3\rangle$ to $|1\rangle$. This is illustrated by Figure 1. This four-level system is irradiated by a light beam propagating along an arbitrary direction, with polarization adequate to couple the optical transitions, and containing three monochromatic fields.$\bar{E}_p$ and $\bar{E}_c$ and $\bar{E}_s$ are assumed to be slowly varying functions in the following sense [11–14]:

$$\frac{1}{\omega_i}\left|\frac{\partial E_i}{\partial t}\right| << \left|\bar{E}_i\right|$$

The Hamiltonian describing the interaction of the four level atom with the fields has the expression:

$$H = H_0 + H_i$$
$$H_0 = \sum_i \varepsilon_i a_i^+ a_i$$
$$\langle k|H_i|l\rangle = -\frac{1}{2}\hbar\Omega_{kl} e^{-i(\omega_k - \omega_l)t}.$$

Where H_0 is the field free Hamiltonian and H_i is the interaction of the atom with the electromagnetic fields. a_i and a_i^+ are the annihilation and creation fermions operators of the atomic level i. a_i and a_i^+ verify the anticommutation relation $\left[a_i, a_j^+\right]_+ = \delta_{ij}$. The levels $|2\rangle$ and $|3\rangle$ have opposite parity.

Thus we can denote by d_{23} the electric dipole element $\langle 2|d_{23}|3\rangle \neq 0$. In the same way, μ_{23} denotes the magnetic dipole element $\langle 1|\mu_{23}|2\rangle \neq 0$ between levels $|1\rangle$ and $|2\rangle$. Moreover, we have:

$$D = \varepsilon\omega\mathbf{E}\mathbf{B} = \mu\omega\mathbf{H} \quad (1)$$

We recall the Fourier transform of the curl equation for electric field in Maxwell's equations by:

$$\mathbf{B} = \frac{\mathbf{k} \wedge \mathbf{E}}{\omega} \quad (2)$$

To study the evolution of interaction between the atom and fields we use the density matrix formalism under rotating-wave approximation. The density matrix operator is defined by

$$\rho = \sum_{k,l} \rho_{kl} e^{i(\omega_k - \omega_l)t} \langle k|\rangle. \quad (3)$$

The time-varying off-diagonal density matrix elements verify the evolution equations:

$$\frac{d}{dt}\rho_{kl} = (-i\delta_{kl} - \gamma_{kl}) - i\sum_{m=1,4}(\Omega_{ml}\rho_{km} - \Omega_{km}\rho_{ml}). \quad (4)$$

We neglect the irreversible decay part in the system which corresponds to the incoherent processes. In term of the basis set of the bare atom $\{|1\rangle, |2\rangle, |3\rangle, |4\rangle\}$ we have:

$$a_k^+ a_l = |k\rangle\langle l| \text{ and } \rho = \sum_{k,l=0}^{4} |k\rangle \rho_{kl} \langle l| \quad (5)$$

The coupling constants are denoted by $g_i = \frac{-id_i}{\hbar}$ as well as the $m_{ij} = \frac{-if_{ij}}{\hbar}$. The off-diagonal elements describe the atomic coherences. The ρ_{34}, ρ_{23} as well as ρ_{13} terms oscillate at the respective driving field frequency and the all others oscillate with frequency differences of the two light fields. Hence, we can define the slowly varying amplitudes of the off-diagonal density matrix elements through the rotating wave frame [12]. The signal field $E_j for j = 1$ is described by the Maxwell equations for a slowly varying approximation (SVA) by [6, 14, 16, 17]

$$\frac{\partial E_j}{\partial t} + c\frac{\partial E_j}{\partial z} = ig' - \rho_j.$$

In general, the propagation constants of the fields g'_i are given by $g'_i = 2\Pi\varepsilon_0 N g_i(\omega_k + \delta_k)$. ε_0 is the vacuum electric constant, N is the atomic dipole density and c is the velocity of light. v_g represents the group velocity of the soliton. One can vary more than one field signal to obtain a soliton pair propagation [6]. Here, we limit our analysis to one variable field amplitude in time, since we want to maintain the negative refractive index property in the medium. In addition, the condition for soliton propagation is expressed as:

$$E_j(x,t) = E_j(x - v_g t) \quad (6)$$

We introduce a moving coordinate $z = x - v_g(t)$ which propagates with the pulse velocity. In this new moving coordinate we have $\frac{\partial}{\partial t} = -v_g\frac{\partial}{\partial z}$ Thus, the motion equations of the density matrix off diagonal elements in the dipole and RWA for this system can be written as follows:

$$\frac{d}{dz}\rho_{11} = \frac{-1}{v_g}(2g_3\rho_{33}+2g_1\rho_{22}+i\Omega_s(\rho_{13}-\rho_{31}))$$
$$\frac{d}{dz}\rho_{22} = \frac{-1}{v_g}(2g_2\rho_{33}-2g_1\rho_{22}+i\Omega_p(\rho_{23}-\rho_{32}))$$
$$\frac{d}{dz}\rho_{33} = \frac{-1}{v_g}(-2(g_2+g_3)\rho_{33}+2g_4\rho_{44}-i\Omega_s(\rho_{13}-\rho_{31}) -i\Omega_p(\rho_{23}-\rho_{32})+i\Omega_c(\rho_{34}-\rho_{43}))$$
$$\frac{d}{dz}\rho_{12} = \frac{-1}{v_g}(-2(g_1-i(\Delta_s-\Delta_p))\rho_{12}+i\Omega_p\rho_{13}-i\Omega_s\rho_{32})$$
$$\frac{d}{dz}\rho_{13} = \frac{-1}{v_g}(-2(g_2+g_3-i\Delta_s)\rho_{13}+i\Omega_p\rho_{12}+i\Omega_c\rho_{14} +i\Omega_s(\rho_{11}-\rho_{33}))$$
$$\frac{d}{dz}\rho_{14} = \frac{-1}{v_g}(-(g_4-i(\Delta_s+\Delta_c))\rho_{14}+i\Omega_c\rho_{13}-i\Omega_s\rho_{34})$$
$$\frac{d}{dz}\rho_{23} = \frac{-1}{v_g}(-(g_2+g_3-i\Delta_p)\rho_{23}+i\Omega_s\rho_{21}+i\Omega_c\rho_{24} +i\Omega_p(\rho_{22}-\rho_{33}))$$
$$\frac{d}{dz}\rho_{24} = \frac{-1}{v_g}(-(g_1+g_4-i(\Delta_p+\Delta_c))\rho_{24}+i\Omega_c\rho_{23}-i\Omega_p\rho_{34})$$
$$\frac{d}{dz}\rho_{34} = \frac{-1}{v_g}(-(g_2+g_3+g_4-i\Delta_c)\rho_{34}-i\Omega_s\rho_{14}-i\Omega_p\rho_{24} +i\Omega_c(\rho_{33}-\rho_{44}))$$
$$\frac{d}{dz}\Omega_p = \frac{-d_2'g_2}{v_g(c-v_g)}\rho_{32}$$

To analyze the system we use the Mathematica Software to numerically solve the density matrix equations. The following experimental parameters were taken into consideration:$\Omega_s = 0.15\Gamma$, $\Gamma_1 = 0.05\Gamma$, $\Gamma_2 = 0.03\Gamma$, $\Gamma_3 = 0.01\Gamma$, $\Gamma_4 = 0.1\Gamma$,$\Omega_c = 2\Gamma$;Δ_c= -1.5; Δ_s= 1.5;

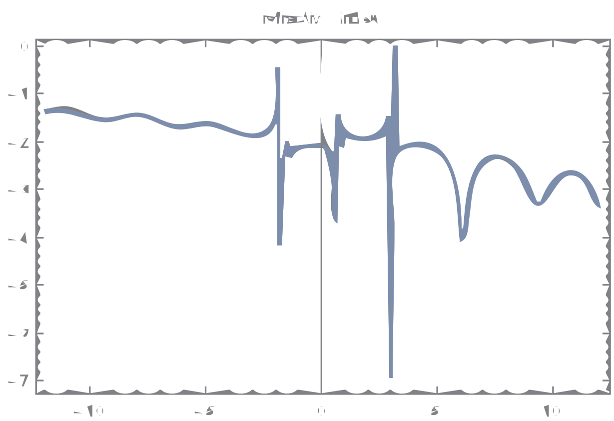

Fig. 2: The refractive index in the considered fourlevel atomic media.

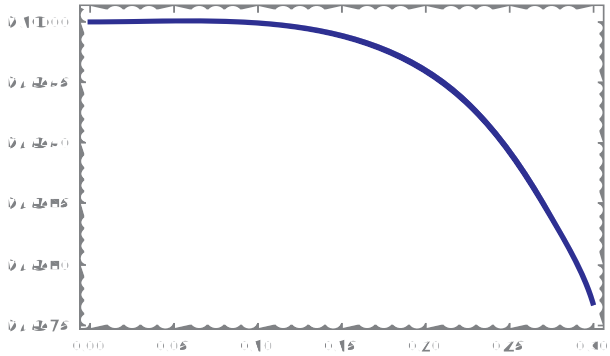

Fig. 3: The soliton in the negative refractive index four-level atomic media

3 Perspective

In this paper we investigated the soliton propagation in a left-handed atomic four-level system coupled with three fields of laser. The stokes strong field was considered as a constant while the probe field was variable. The system was described by means of the density matrix formalism under the rotating wave approximation. Varying the external probe field amplitude in time, the propagation of soliton was simulated in the four-level atomic system while maintaining a negative refractive medium. This was achieved in a large detuning interval under suitable experimental conditions. In the future, we aim to investigate an analytical analysis of the system to examine the effect of fields amplitudes, the detunings, and the probe field on the existence of the solitons. This can be achieved by deriving the full analytical solutions of the density matrix.

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