Applied Mathematics & Information Sciences
An International Journal

http://dx.doi.org/10.18576/amis/140413

Extreme Analysis of Maxima Rainfall in the Upper East Region of Ghana: A Case Study of Navrongo Municipality

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Received: 3 Jan. 2020, Revised: 2 Apr. 2020, Accepted: 18 Apr. 2020
Published online: 1 Jul. 2020

Abstract: Climate change represents great concern to the world as a result of extreme weather conditions, such as floods, droughts and heat waves, which are observed all over the world. The sustainanble development of developing nations is greatly affected because their economies are sensitive to these conditions. Extreme weather conditions have had devastating effect on human lives, infrastructure and the environment. Therefore, the present paper investigates the extremal behaviour of yearly maximum rainfall data of the Upper East Region of Ghana with Navrongo Municipality as a case study, using generalized extreme value distribution considering both stationary and non-stationary models. Data consist of yearly maximum rainfalls from January 1983 to December 2018 of Navrongo Municipality Municipality. The least and highest yearly maximum rainfalls recorded in Navrongo Municipality over the period under study are 173.5 mm and 455.5 mm respectively. The data are rightly skewed, leptokurtic and randomly distributed. The results suggests that the behaviour of the limiting distribution of the yearly maximum rainfall data follows a non-stationary Gumbel distribution. Return levels are estimated and are observed to be increasing over a period of time. This means that policy makers and the general public should consider implementing efficient flood mitigating strategies.

Keywords: Maxima, extreme weather, generalized extreme value distribution, rainfall, randomness, return period, stationarity, trend.

1 Introduction

Climate change represents great concern to developing nations because it causes increasing uncertainties about the environment and associated risks. It also presents a significant risk to the sustainable development of developing nations, including Ghana, because the growth of these nations is sensitive to climate change [1]. The economies of these nations are susceptible to changes of climate as critical elements of the economy, such as crop and energy production, are affected. Climate change has affected the reliability of rainfall because it has changed the onset and duration of rainfall [2], including its intensity. Moreover, it has influenced several aspects of human lives, such as the supply of food or crop production, health, water supply, and the environment [5]. All these concerns are associated with climate change that occurs as a result of the extreme weather conditions, such as floods, droughts and heat waves that are observed throughout the world.

Extreme weather conditions have had a devastating effect on human lives, infrastructure, and the environment. In Ghana, flooding has become an annual event, resulting in loss of lives and property. Therefore, it is essential for policy makers and the general public to understand the pattern and predict extreme rainfall. In this paper, the probability of occurrence, the period of time, it will take on average, to observe extreme rainfall, and the likelihood of future occurrence of extreme rainfall in Navrongo Municipality, are investigated using generalized extreme value (GEV) distribution.

The study was conducted in the Navrongo Municipality in the Upper East Region which is located in the northern part of Ghana. The climatic condition of Navrongo Municipality is an indication of the climatic condition of the Upper East Region. Like most parts of Ghana, the people of Upper East Region rely on rainfall for crop production. The region is covered by Sahel and Sudan-Savannah type of vegetation. This is characterized by irregular and erratic rainfall [6] resulting in persistent

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flooding. Houses are mostly built with mud and the rate of poverty in the area is high. Hence, extreme rainfalls always come with a higher impact on the affected individuals and the society. Ghana is characterised by two seasons: rainy and dry seasons. In the northern part of Ghana, the rainy season may last for about five (5) to six (6) months. Hence, a distraction during this short rainy season usually results in low crop production which affects the livelihood of the affected individuals. Consequently, it is essential to understand the pattern of extreme rainfall in Navrongo Municipality.

The concern of extreme weather conditions has encouraged a large number of researchers to adopt extreme value theory for the analysis of extreme weather events. In modelling extreme events, some researchers have assumed a staionarity process for the data [7,8,9] using the classical GEV distribution. That is, a GEV with constant parameters. However, a false assumption of stationarity of the process may lead to an underestimation of the probability of a disastrous extreme event [11]. Hasan and Yeong [12] and Okorie *et al.* [13], in addition to modelling extreme events with a stationary process, also considered non-stationary models using a covariate approach. That is, they assumed that one or more of the parameters of the GEV have a time trend. In this study, several non-stationary GEV models are also considered introducing covariates in the form of time trends with linear, quadratic, cyclical and exponential trends.

The paper is organised, as follows: Section 2 presents methodology. Section 3 addresses the preliminary analysis of the data and investigates the nature of the data. Section 4 presents further analysis of the data by fitting the GEV distributions to the data. Section 5 is devoted to conclusion.

2 Methodology

2.1 Study Area

The study was conducted in Navrongo Municipality (as a case study) in the Upper East Region of Ghana. Fig. 1 shows a map of the location. It is covered by Sahel and Sudan-Savannah type of vegetation. The study area is also characterized by two seasons: the rainy season and the dry season. The rainy season mostly starts in April and ends in November. These seasons are mainly influenced by the North-East Trade winds and the Tropical Maritime winds. Temperatures in Navrongo Municipality get up to 42^0C and go down to 18^0C [14].

2.2 Generalized Extreme Value (GEV) Distribution

Suppose that $X_1, X_2, X_3, \ldots, X_n$ are independent and identically distributed random variables and let

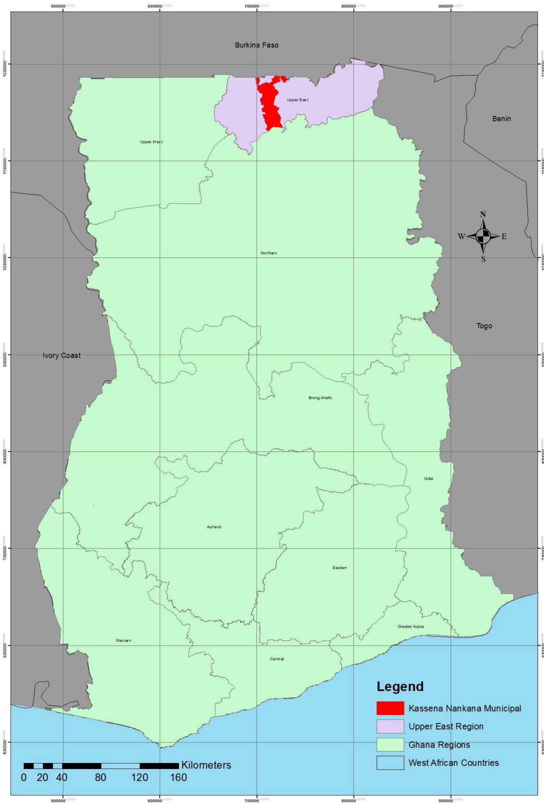

Fig. 1: Map of Ghana showing the location of Navrongo Municipality

$M_n = \max(X_1, X_2, X_3, \ldots, X_n)$. If there exist normalizing constants $\{a_n > 0\}$ and $\{b_n \in \mathbb{R}\}$ such that

$$\lim_{n\to\infty}\left[\Pr\left(\frac{M_n - b_n}{a_n} \leq X\right)\right] = \lim_{n\to\infty}[\Pr(X_n \leq x)] \to H(x) \tag{1}$$

where H is a non-degenerate distribution, then the limiting distribution of M_n belongs to either Gumbel, Fréchet or Weibull distributions. These three distributions are combined to form the GEV distribution with cumulative distribution function

$$GEV(\mu,\sigma,\xi) = \begin{cases} \exp\left\{-\left[1+\xi\left(\frac{x-\mu}{\sigma}\right)\right]^{-\frac{1}{\xi}}\right\}, \xi \neq 0, \\ \exp\left\{-\exp\left[-\left(\frac{x-\mu}{\sigma}\right)\right]\right\}, \quad \xi = 0 \end{cases} \tag{2}$$

where μ is the location parameter, σ is the scale parameter and ξ is the shape parameter. In equation (2), when $\xi = 0$, we obtain the Gumbel distribution; when $\xi > 0$, we obtain the Fréchet distribution and when $\xi < 0$ we obtain the Weibull distribution.

In modeling extreme events, the dataset is put into n blocks consisting of a group of observations of k each. Let $M_i = \max(X_1, X_2, X_3, \ldots, X_k)$, $i = 1, 2, 3, \ldots, n$ be yearly maximums for the period under study, then a GEV distribution is used to model or examine the distribution of M_i.

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Equation (2) is the classical GEV distribution which assumes a stationary process for the data with time independent parameters. When the data is not stationary, the non-stationary GEV is used [10,5].

2.3 Tentative Generalized Extreme Value Distributions

In this study, both classical and non-stationary models are considered because the parameters of the GEV distribution for the yearly maximums are not constant, but they change or are affected by a time trend t. Time trend will capture any variations in the data that may be increasing or decreasing. Also, climatic conditions in the study area, such as rainfall, may exhibit some yearly or seasonal pattern. Hence, a quadratic, cyclical and exponential trends are introduced. Therefore, the following distributions are considered in this study:
Model 1: $GEV(\mu,\sigma,\xi)$.
Model 2: $GEV(\mu(t)=\alpha_0+\alpha_1 t,\sigma,\xi)$.
Model 3: $GEV(\mu(t)=\alpha_0+\alpha_1 t+\alpha_2 t^2,\sigma,\xi)$.
Model 4: $GEV(\mu,\sigma(t)=exp(\beta_0+\beta_1 t),\xi)$.
Model 5: $GEV(\mu(t)=\alpha_0+\alpha_1 \sin\left(\frac{2\pi t}{12}\right),\sigma,\xi)$.
Model 6: $GEV(\mu(t)=\alpha_0+\alpha_1 \cos\left(\frac{2\pi t}{12}\right),\sigma,\xi)$.
Model 7: $GEV(\mu(t)=\alpha_0+\alpha_1 \sin\left(\frac{2\pi t}{12}\right)+\alpha_2 \cos\left(\frac{2\pi t}{12}\right),\sigma,\xi)$.
Model 8: $GEV(\mu(t)=\alpha_0+\alpha_1 \cos\left(\frac{2\pi t}{12}\right),\sigma,\xi(t)=\delta_0+\delta_1 t)$.

The method of maximum likelihood is used to estimate the parameters of the various models. Given that the model consists of m parameters, we define the set of parameters as $\Theta=(\theta_1,\theta_2,\theta_3,\cdots,\theta_m)'$. The data are fitted to the models and the set of parameters which maximizes the log-likelihood function is defined as

$$\ell(\Theta)=\sum_{i=1}^{n}\log f(z_i;\Theta) \tag{3}$$

and obtained using the R software via the ismev package by Heffernan and Stephenson [15].

2.4 Model Selection

To select the best model, several goodness-of-fit measures are considered. These include Akaike information criteria (AIC), Bayesian information criteria (BIC), Cramér von Mises (CVM), Anderson-Darling (A-D) and Kolmogrov-Smirnov (K-S) measures. The best model for the data is the model which gives the least statistic in terms of each of the various goodness-of-fit measures.

2.5 Return Levels

Return levels play a vital role in extreme value analysis and are considered measures of risk. Return level denoted R_T is the level which, on average, is expected to be equal to or surpassed once every time interval (t), with a given probability. Given the classical GEV distribution, the return level is defined as

$$R_T=\begin{cases}\mu+\frac{\sigma}{\xi}\left(\left[-\ln\left(1-\frac{1}{T}\right)\right]^{-\xi}-1\right), & \xi\neq 0,\\ \mu+\sigma\ln\left[\ln\left(1-\frac{1}{T}\right)\right]^{-1}, & \xi=0.\end{cases} \tag{4}$$

Another important measure is the return period. The mean return period denoted T is defined as the amount of time (t) that is expected to pass on average before a new extreme event, such as flood with the same or increased intensity, is observed. Given that the likelihood of recurrence of events with a specified intensity past a certain level or threshold is denoted p, then its corresponding return period is defined as $T=\frac{1}{p}$.

The confidence intervals of the return levels are constructed to assess the suitability of the return level estimates. The profile likelihood confidence intervals are constructed in this study.

Let $\Theta=(\mu,\sigma,\xi)'$ be the set of parameters of the GEV distribution. To obtain the profile likelihood confidence intervals, the GEV distribution is re-parametrized to have R_T as one of its parameters with a new set of parameters written as $\Psi=(R_T,\sigma,\xi)'$. This is achieved by making μ the subject in equation (4) and substituting it into equation (2). Using the re-parametrized GEV distribution, the log-likelihood function; known as the profile likelihood function of R_T, is obtained as

$$\ell_{R_T}(\Psi)=\sum_{i=1}^{n}\log f(z_i;\Psi). \tag{5}$$

Let $\hat{\Psi}_{R_T^*}=(R_T^*,\hat{\sigma},\hat{\xi})'$, where R_T^* is a fixed return level. A $100(1-\alpha)\%$ confidence interval for R_T is the set of all values of R_T^* such that the test hypothesis $H_0: R_T=R_T^*$ is not rejected at α level of significance. We reject H_0 at α level if the likelihood ratio statistic $2\left[\ell_{R_T}(\hat{\Psi})-\ell_{R_T}(\hat{\Psi}_{R_T^*})\right]$ exceeds $100(1-\alpha)$th percentile of the $\chi^2_{1-\alpha}(1)$ distribution. Specifically, $\chi^2_{0.05}(1)=3.84$ and the 95% confidence interval would consist of all values of R_T^* for which $2\left[\ell_{R_T}(\hat{\Psi})-\ell_{R_T}(\hat{\Psi}_{R_T^*})\right]\leq 3.84$. Equivalently, the 95% confidence interval would consist of all values of R_T^* for which $\ell_{R_T}(\hat{\Psi}_{R_T^*})\geq\ell_{R_T}(\hat{\Psi})-1.92$ [3,4].

3 Preliminary Analysis

The data used for this study consist of the monthly total rainfall data of Navrongo Municipality, from January 1993 to December 2018. They are obtained from the Navrongo Meteorological Agency. Considering each year, the yearly maximum rainfalls are obtained and analyzed. Scatter plots of both the monthly total rainfall data and the yearly maximum rainfalls are shown in Fig. 2.

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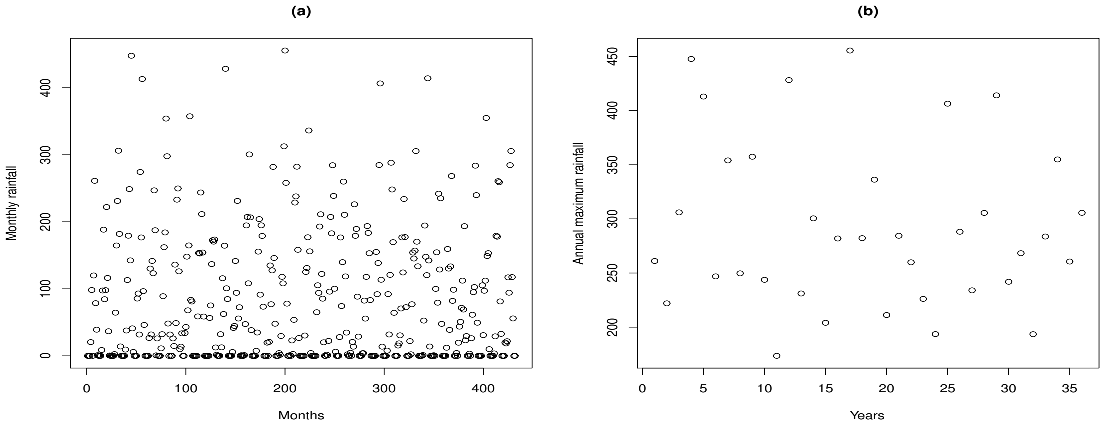

Fig. 2: Plot of (a) monthly total rainfalls and (b) yearly maximum rainfalls

The summary statistics of both data are shown in Table 1. A total of 432 months are observed within the period under study. Table 1 shows that the mean monthly total rainfall recorded is 81.48 mm, and the highest monthly total rainfall is 455.5 mm and was recorded in August, 1999. A minimum monthly total rainfall of 0.00 mm was recorded for several months during the period under consideration. The monthly total rainfall is observed to be rightly skewed and leptokurtic in nature. Again, Table 1 indicates 36 yearly maximum rainfalls. The highest yearly maximum rainfall was recorded in 1999 as 455.5 mm with a least yearly maximum of 173.5 mm recorded in 1993. The yearly maximum rainfall data is observed to be right skewed and platykurtic in distribution.

Table 1: Summary statistics of rainfall data

Statistic	Monthly	Yearly
No.	432	36
Minimum	0.00	173.50
Maximum	455.50	455.50
Mean	81.48	292.40
Median	38.55	282.05
Standard Deviation	97.63	76.23
Skewness	1.29	0.63
Kurtosis	1.24	-0.64

According to Fig. 2, the outliers in the yearly maximum rainfall data are not very evident. Hence, a boxplot of the monthly total rainfall data is obtained and shown in Figure 3. It indicates some extreme yearly maximum rainfall over the period under consideration.

The stationarity, hence the randomness, of the yearly maximum data is very essential for further analysis of the data as this is a basic requirement for proceeding to fit a GEV distribution. In this study, the autocorrelation function (ACF) and partial autocorrelation function (PACF) plots are employed for this purpose. Fig. 4 shows that all the spikes are within the confidence interval except at lag 6 for both ACF and PACF plots. This suggests that there exists no autocorrelation at the lags except at lag 6. Therefore, several tests of randomness are carried out to further examine the randomness of the yearly maximum rainfalls. Table 2 shows the results of various tests.

The null hypothesis of all the test is stated as H_0 : the yearly maximum data is random and alternative is H_1 : the yearly maximum data is not random. The results of these tests are shown in Table 2. It exhibits that all the test results indicate evidence of randomness at 5% significance level as we fail to reject the null hypothesis for all the tests.

Table 2: Test of randomness of the yearly maximum rainfall data

Test	Statistic	p-values
Cox Stuart test	9.0000	0.5927
Difference-Sign test	-0.2848	0.7758
Mann-Kendall Rank test	-0.3269	0.7437
Wald-Wolfowitz Runs test	1.0146	0.3103
Turning point test	0.9465	0.3439
Bartels Ratio test	1.0821	0.2792

Based on the analysis, we conclude that the data satisfies the assumption of a classical GEV distribution. Thus, we are set to fit a GEV distribution to the data. However, in fitting the data, non-stationary models are also considered because climatic data may exhibit some seasonality.

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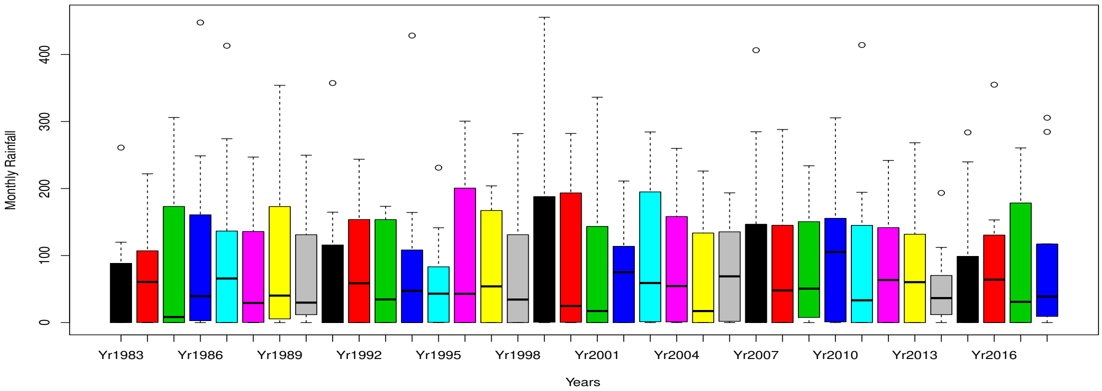

Fig. 3: Boxplot of yearly rainfall

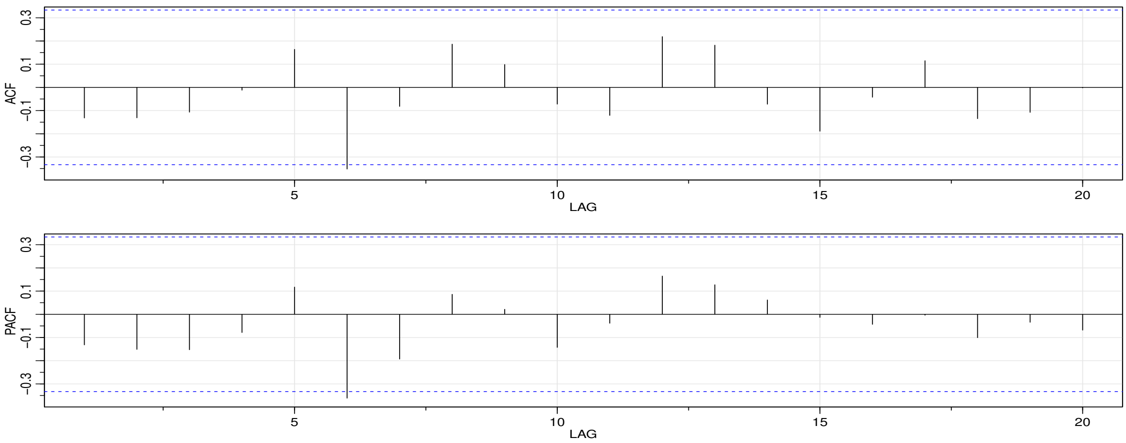

Fig. 4: Plots of ACF (above) and PACF (below)

4 Results and Discussion

The models stated in section 2.3 are fitted to the yearly maximum rainfall data of Navrongo Municipality. Table 3 shows the fitted models indicating the estimated parameters of the models and some goodness-of-fit statistics for the models. These are used for selection of the best model.

Table 3 reveals that model 1 has the least measure in terms of BIC and K-S criteria. Model 6 has the least in terms of AIC and CVM. Furthermore, model 8 has the least in terms of A-D measure. Since the goodness-of-fit measures are inconclusive in selecting the best model, the models are ranked in terms of model performance. This is shown in Table 4. For each measure, the model with the least statistic is ranked with the least value ranging from 1 to 8. This is done for all models and goodness-of-fit measures. Thus, a smaller value indicates a better fit and a larger value indicates a worse fit to the data. The total rank for each model is computed and the model with the least total rank is considered the best model for the extreme of the yearly maximum rainfall and the worst model has the highest total rank.

The results of the ranking in Table 4 indicate that model 6 is the best model that provides a better fit to the data, followed by model 1. However, model 4 provides the worst fit to the data. We select the two best models, model 1 and 6, for further analysis.

The results in Table 3 manifest that the parameter estimates are substituted into model 1 and model 6, respectively, to obtain the following:

$$F(x;\mu,\sigma,\xi) = \exp\left\{-\left[1+0.0106\left(\frac{x-257.1969}{59.1983}\right)\right]^{-\frac{1}{0.0106}}\right\}$$

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Table 3: Estimated Parameters of the Model and Selection Criteria

		Model							
		1	2	3	4	5	6	7	8
Parameter estimates	$\hat{\mu}$	257.1969	-	-	266.3884	-	-	-	-
	$\hat{\sigma}$	59.1983	59.2819	57.7399	-	58.7052	56.7319	55.5349	56.2612
	$\hat{\xi}$	0.0106	0.0074	0.0375	8.8152	0.00477	0.0311	0.0394	-
	$\hat{\alpha}_0$	-	259.1952	277.3796	-	257.7867	257.5529	257.8332	257.5088
	$\hat{\alpha}_1$	-	-0.101	-3.223	-	12.1919	-20.4164	13.8518	-22.0784
	$\hat{\alpha}_2$	-	-	0.0862	-	-	-	-22.2744	-
	$\hat{\beta}_0$	-	-	-	6.8349	-	-	-	-
	$\hat{\beta}_1$	-	-	-	-0.0115	-	-	-	-
	$\hat{\delta}_0$	-	-	-	-	-	-	-	0.34734
	$\hat{\delta}_1$	-	-	-	-	-	-	-	-0.0163
Model selection measures	AIC	414.3358	416.3264	417.6298	509.588	415.5618	**414.114**	414.9456	415.3083
	BIC	**419.0863**	422.6605	425.5474	515.9221	421.8959	420.4481	422.8632	423.2259
	KS	**0.0869**	0.0905	0.103	0.5775	0.1319	0.1493	0.2077	0.1589
	CVM	0.0318	0.0303	0.0482	2.8509	0.047	0.0213	0.0412	**0.0188**
	A-D	0.2566	0.2502	0.3243	13.1655	0.3422	**0.2268**	0.3342	0.1918

Note: Least measures of model selection criteria are boldened.

Table 4: Ranking of the tentative models

No.	Model	AIC	BIC	KS	CVM	AD	Total rank	Rank
1	$GEV(\mu,\sigma,\xi)$	2	1	1	4	4	12	2nd
2	$GEV(\mu=\alpha_0+\alpha_1 t,\sigma,\xi)$	6	4	2	3	3	18	3rd
3	$GEV(\mu=\alpha_0+\alpha_1 t+\alpha_2 t^2,\sigma,\xi)$	7	7	3	7	5	29	7th
4	$GEV(\mu,\sigma=exp(\beta_0+\beta_1 t),\xi)$	8	8	8	8	8	40	8th
5	$GEV\left(\mu=\alpha_0+\alpha_1\sin\left(\frac{2\pi t}{12}\right),\sigma,\xi\right)$	5	3	4	6	7	25	5th
6	$GEV\left(\mu=\alpha_0+\alpha_1\cos\left(\frac{2\pi t}{12}\right),\sigma,\xi\right)$	1	2	5	1	2	11	1st
7	$GEV\left(\mu=\alpha_0+\alpha_1\sin\left(\frac{2\pi t}{12}\right)+\alpha_1\cos\left(\frac{2\pi t}{12}\right),\sigma,\xi\right)$	3	5	7	5	6	26	6th
8	$GEV\left(\mu=\alpha_0+\alpha_1\cos\left(\frac{2\pi t}{12}\right),\sigma,\xi=\delta_0+\delta_1 t\right)$	4	6	6	2	1	19	4th

and

$$F\left(x;\mu(t)=\alpha_0+\alpha_1\cos\left(\frac{2\pi t}{12}\right),\sigma,\xi\right)=$$
$$\exp\left\{-\left[1+0.0311\left(\frac{x-257.5529+20.4164\cos\left(\frac{2\pi t}{12}\right)}{56.7319}\right)\right]^{-\frac{1}{0.0311}}\right\}.$$

Model 6 indicates that the location parameter is described by a cosine function in time (t). To further ascertain the best model, the probability-probability (P-P) plots of the two best models are obtained and shown in Fig. 5. It shows that model 6 provides a better fit to the yearly maximum rainfall data of Navrongo Municipality compared to model 1 because most of the points lie on the diagonal line. Thus, model 6 is classified as the best model to describe the annual maximum rainfall of Navrongo Municipality.

We now test the significance of $\hat{\xi}$ in the model. This enables us to classify the model as one of the GEV distributions. Hence, the null hypothesis $H_0:\hat{\xi}=0$ is tested against the alternative $H_1:\hat{\xi}\neq 0$ at a 5% level of significance. The t-statistic is defined as $\frac{\hat{\xi}}{SE(\hat{\xi})}$ where $SE(\hat{\xi})$ is the standard error of $\hat{\xi}$. This is given as 0.1874. The corresponding p-value is also obtained as $2[1-P(Z\leq t-\text{statistic})]$, where Z is asymptotically standard normally distributed. The p-value is given as 0.8514. Hence, at 5% significance level, the result indicates that $\hat{\xi}=0$ as $H_0:\hat{\xi}=0$ can not be rejected. This indicates that the Gumbel extreme value distribution is suggested as the limiting distribution for the yearly maximum rainfall data of Navrongo Municipality. Hence, the model is given as

$$F\left(x;\mu(t)=\alpha_0+\alpha_1\cos\left(\frac{2\pi t}{12}\right),\sigma\right)=$$
$$\exp\left\{-\exp\left[-\left(\frac{x-257.5529+20.4164\cos\left(\frac{2\pi t}{12}\right)}{56.7319}\right)\right]\right\}.$$

Using model 6, the return periods for the yearly maximums are all computed. These are shown in Table 7 in the Appendix. It can generally be observed that higher yearly maximum rainfalls correspond to lower probabilities and longer return periods. This can be attributed to the fact that higher yearly maximum rainfalls

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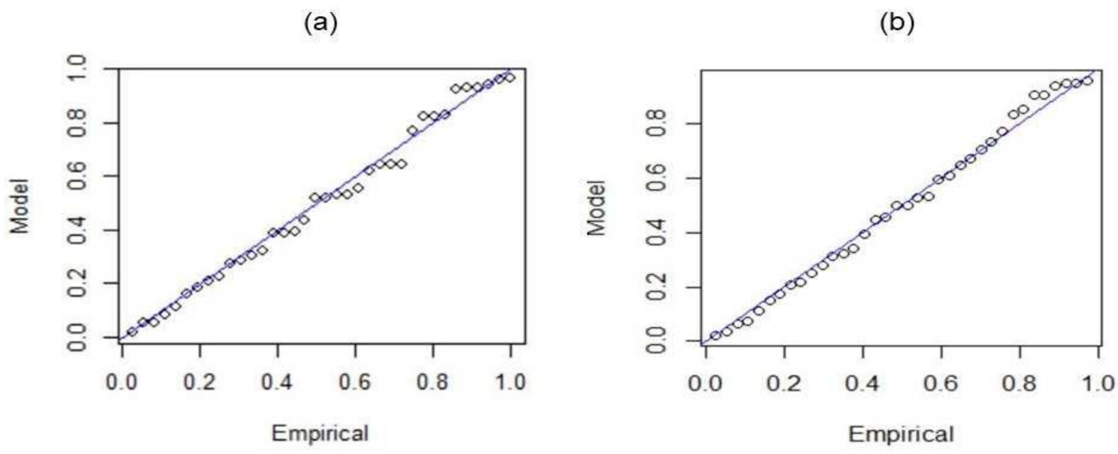

Fig. 5: P-P Plots of (a) model 1 and (b) model 6

rarely occur. That is, with lower probabilities and longer intervals of occurrence in this location. Their occurrence devastate the society. Therefore, these higher yearly maximum rainfalls are of much interest.

Since the yearly maximum rainfall data is observed to be skewed to the right, further observation of yearly maximum rainfall with probabilities less than the median (0.4989) are extracted from Table 7. These are shown in Table 5, which shows the reference years with observed yearly maximum rainfall and their corresponding return periods. With the return periods, the corresponding observed return levels are also shown.

Table 5 reveals that although some return levels are higher than the rainfall levels in the reference year, others are below. This means that policy makers and the society would have to implement proper flood mitigating strategies to address this pattern.

The return levels for selected return periods up to a 100 year return period are obtained. These are shown in Table 6 and illustrated in Fig. 6. The corresponding 95% profile likelihood confidence intervals are also shown in Table 6. It is noticeable that the yearly maximum rainfall has an increasing trend over the period considered.

Policy makers and the society have to be prepared for the increasing trend of rainfall. They have to employ mitigation and adaptation measures to address this trend. Since the study area experiences shorter periods of rainfall and with the increasing trend predicted over a long period of time, rainwater harvesting should be adopted as a mitigation strategy. This could include the construction of facilities to serve as reservoirs for small and large scale agricultural purposes, to harvest water during heavy rains, and to combat drought. Also, the reafforestation of depleted land of the area should be considered to reduce the effects of heavy rainfall. Again, rivers and drainage systems should be restored and new ones should be constructed because they can control flooding when heavy rainfalls occur. Finally, people should be educated on the effects of climate change, the predicted trend of rainfall, as well as mitigation and adaptation strategies that could be employed.

Table 5: Yearly maximum rainfall with probabilities below the median probability

	Reference		**Approximate**	Return	
No.	**Year**	**Maximum Rainfall**	**Return Period**	**Year**	**Level**
1	1983	261.1	3	1986	447.7
2	1985	306.0	3	1988	246.9
3	1986	447.7	22	2008	288.1
4	1987	413.0	11	1998	281.9
5	1989	354.0	5	1994	428.2
6	1991	357.4	7	1998	281.9
7	1994	428.2	26	2020	-
8	1996	300.5	4	2000	282.2
9	1999	455.5	22	2021	-
10	2001	336.2	4	2005	226.1
11	2003	284.4	3	2006	193.6
12	2007	406.4	18	2025	-
13	2008	288.1	3	2011	414.1
14	2010	305.5	3	2013	268.3
15	2011	414.1	12	2023	-
16	2015	283.7	3	2018	305.6
17	2016	354.9	7	2023	-
18	2018	305.6	4	2022	-

Note: - denotes return levels for return years that are unavailable within the data.

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Table 6: Return level estimates and 95% confidence intervals

Return Period	Return Level	Confidence Interval
10	379.6	(366.5,406.6)
20	444.3	(408.1,452.9)
30	480.4	(424.5,485.6)
40	488.7	(452.5,497.3)
50	482.7	(469.6,509.7)
60	484.3	(475.3,526.1)
70	504.6	(491.4,531.5)
80	533.7	(497.5,542.3)
90	551.6	(495.6,556.8)
100	548.3	(512.1,556.9)

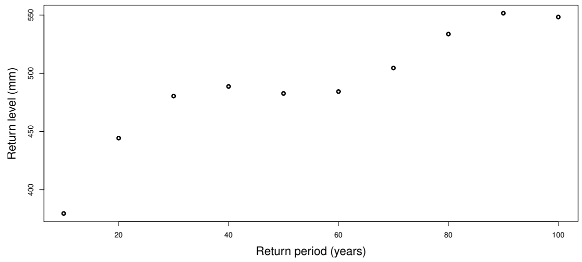

Fig. 6: Return level plot

5 Conclusion

The study investigated the yearly maximum rainfall data of Navrongo Municipality within the Upper East Region of Ghana using extreme value distribution. The data was observed to be random satisfying the requirements of a stationary GEV distribution. However, non-stationary distributions were also investigated. It was detected that a non-stationary model with a cyclical trend best described the behaviour of the yearly maximum rainfall. Significance of the shape parameter was tested to classify the model. The parameter was insignificant and suggested that the behaviour of the limiting distribution of the yearly maximum rainfall data of Navrongo Municipality followed the Gumbel distribution. Return periods with their corresponding return levels and profile likelihood confidence intervals were obtained. The estimates showed that the monthly maximum rainfall would continue to increase for a long period of time. This means that policy makers and the general public must be prepared for the increasing trend by employing mitigation and adaptation measures.

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Appendix

Table 7: Return probabilities and return periods of annual maximum rainfall

No.	Year	Maximum Rainfall	$P(X \geq x)$	T (years)
1	1983	261.1	0.49809	2.00767
2	1984	222.0	0.79156	1.26332
3	1985	306.0	0.34981	2.85871
4	1986	447.7	0.04740	21.09910
5	1987	413.0	0.09173	10.90110
6	1988	246.9	0.82402	1.21357
7	1989	354.0	0.22651	4.41478
8	1990	249.7	0.74768	1.33747
9	1991	357.4	0.16492	6.06367
10	1992	243.7	0.65576	1.52494
11	1993	173.5	0.96285	1.03859
12	1994	428.2	0.03982	25.11120
13	1995	231.1	0.68890	1.45159
14	1996	300.5	0.32773	3.05129
15	1997	204.0	0.92622	1.07966
16	1998	281.9	0.54167	1.84616
17	1999	455.5	0.04715	21.20750
18	2000	282.2	0.60474	1.65359
19	2001	336.2	0.29353	3.40687
20	2002	211.2	0.93628	1.06806
21	2003	284.4	0.46481	2.15140
22	2004	259.9	0.55160	1.81290
23	2005	226.1	0.72083	1.38729
24	2006	193.6	0.88631	1.12827
25	2007	406.4	0.05848	17.10080
26	2008	288.1	0.38823	2.57577
27	2009	234.0	0.78101	1.28040
28	2010	305.5	0.40409	2.47471
29	2011	414.1	0.09017	11.09000
30	2012	241.9	0.85052	1.17576
31	2013	268.3	0.67706	1.47699
32	2014	193.5	0.97774	1.02276
33	2015	283.7	0.46889	2.13269
34	2016	354.9	0.14656	6.82315
35	2017	260.6	0.50111	1.99557
36	2018	305.6	0.26349	3.79522

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