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Value-at-Risk Estimation of Precious Metal Returns using Long Memory GARCH Models with Heavy-Tailed Distribution

Knowledge Chinhamu[1,*], *Retius Chifurira*[1] *and Edmore Ranganai*[2]

[1]School of Mathematics, Statistics and Computer Science, University of KwaZulu-Natal, Private Bag X54001, Durban 4000, South Africa
[2]Department of Statistics, University of South Africa, Florida, South Africa

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Abstract: It is essential for financial institutions and regulators to implement an effective risk management system against market risk. Value-at-Risk (VaR) is the most popular tool to measure such risk. It is thus important to model the volatility of precious metal prices and develop more robust approaches in the estimation of VaR. There is a gap in literature in terms of VaR models that can capture all the empirical properties of precious metals. The main aim of the study was to propose a modelling framework using Long memory models coupled with normal-inverse Gaussian (NIG), the variance-gamma (VG) and the Pearson type-IV (PIV) distributions which can be used in precious metal market for carrying out accurate risk management or assessment. In this study, we evaluate the relative performances of long memory (LM) generalized autoregressive conditional heteroscedasticity (GARCH) models, under a number of conditional assumptions, in estimating VaR for daily returns from three precious metal (platinum, gold, silver) prices. Such models aim at jointly capturing the volatility clustering, unconditional and conditional heavy-tailed, asymmetrical distributions and LM inherent in the data series. In particular, the conditional variance and LM are modeled by nonlinear GARCH models, while the NIG, the VG and the PIV distributions are applied to the extracted standardized residuals so as to capture the heavy tail behavior in metal returns. Anderson-Darling (AD) test is utilized to check for model adequacy while Kupiec likelihood ratio test is used in this study to objectively compare relative performances of the VaR models. The backtesting results confirm that the LM GARCH-heavy-tailed distribution models are adequate methods in improving risk management assessments and hedging strategies in the highly volatile metals market. The main findings indicate that ARFIMA-FIGARCH, ARFIMA-HYGARCH and ARFIMA-FIAPARCH models with PIV, VG and NIG error distributions are suitable for depicting the extreme risk of precious metal prices and can be used for the estimation of VaR. The accuracy of the volatility model is essential in forecasting volatility of future returns in which the predictability of volatility plays an integral role in risk management and portfolio management.

Keywords: Long memory, normal-inverse Gaussian, Pearson type-IV, precious metals. Value-at Risk, variance-gamma.

1 Introduction

Major precious metals such as gold, palladium, platinum and silver emerged as natural desirable classes eligible for portfolio diversification. Investigating the price dynamics of precious metals is of great interest to investors, traders and policy makers. Recent studies have examined the issue of price volatility modeling and information transmission for a broader set of precious metals, oil and industrial commodities.

There has been a growing interest in precious metal markets by agents that incorporate metals in production processes, such as metallurgic companies, and the jewelry industry where metals such as gold, platinum and silver are clearly dominant. These characteristics imply that there has been a strong demand in these markets. Most research in literature focused on the analysis of the gold market. The main interest has been the role of this precious metal as a hedge against inflation. Little research has been done in regard to other precious metals (e.g., silver, platinum, palladium). Hiller *et al*. [1] found financial portfolios that contain precious metals to perform significantly better than standard equity

* Corresponding author e-mail: Chinhamu@ukzn.ac.za

portfolios. They also found that precious metals exhibit some hedging capability during periods of abnormal volatility. Sari *et al*. [2] examined the co-movements and information transmission among the spot prices of four precious metals (gold, silver, platinum and palladium), oil price and the US Dollar/Euro exchange rate. They found evidence of a weak long-run equilibrium relationship and strong feedbacks in the short run.

Various modelling methodologies have been employed in literature. Krężłek [3] used stable distributions to measure the investment risk of precious non-ferrous metals. Their results confirmed the validity in using stable distributions to assess the risk on the precious non-ferrous metals market. Precious metal return series exhibit volatility clustering, heavy-tails and the dependence amongst distant observations, a phenomenon referred to as long-range dependence or long memory (LM). LM GARCH models have been applied to modelling precious metal returns. Extensive work on the application of nonlinear GARCH models (LM models) in modelling precious metals were carried out by Arouri *et al.* [4], Cochran *et al.* [5], Chkili *et al*. [6], Bentes [7], and Ranganai and Khubeka [8], among others. According to Chkili *et al*. [6], the FIGARCH model is best suited for estimating Value-at-Risk (VaR) forecasts of commodity prices which included daily spot and three-month futures of gold and silver. Cochran *et al.* [5] found that the FIGARCH (1,d,1) appropriately describes the volatility processes of metal returns. They used daily log returns of prices of gold, silver, platinum and palladium from January 2000 to June 2011. Bentes [7] compared the relative performances of GARCH, IGARCH and FIGARCH on daily gold price returns. Using daily price returns from 2 August 1976 to 6 February 2015 her results were similar to Cochran *et al.* [5].

LM has been observed in both the mean and volatility of precious metal returns, i.e. the dual LM phenomenon. Arouri *et al.* [4], Diaz [9] and Ranganai and Khubeka [8] have accomplished commendable research on the dual LM phenomenon. The LM in the mean is handled by the autoregressive fractional integrated moving average (ARFIMA) models and the LM inherent in volatility by nonlinear GARCH models. Arouri *et al.* [4]) found that ARFIMA-FIGARCH is the best model for describing daily spot and 3-month futures prices of gold, silver, platinum and palladium. However, they did not address the issue of heavy-tailed error distributions. Diaz [9] addressed the issue of dual LM and asymmetry phenomena. Ranganai and Khubeka [8] addressed the issue of structural breaks and heavy-tailed error distributions. Their results suggest that platinum returns are mean reverting while its volatility exhibited strong LM. They also found that for platinum returns, ARFIMA-FIEGARCH model under the Skewed Student-*t* distribution (SSTD) and ARFIMA-HYGARCH under the normal distribution were able to capture the ARCH-effect. The best model for platinum return was the AFRIMA-FIAPARCH under the Student-*t* distribution (STD), based on the Alkaike information criterion (AIC).

The development of a more robust approach in estimating VaR is crucial, since regulators accept it as a basis for setting capital requirements for market risk exposure. We extend the work of Arouri *et al.* [4], Ranganai and Khubeka [8], Youssef *et al.* [10], Mabrouk and Saadi [11], Cochran *et al*. [5], McNeil and Frey [12], Bhattacharyya *et al.* [13], Bhattacharyya and Madhav [14] by evaluating the relative performances of LM GARCH models in estimating VaR of three precious metal prices using a number of conditional assumptions. We intend to improve LM models by relaxing the normality assumption of the return innovations by allowing negative skewness and heavy-tails. Since all precious metal return series exhibit volatility clustering and long-range dependence (LM), we propose the LM GARCH models under Pearson type-IV (PIV), normal-inverse Gaussian (NIG) and variance-gamma (VG) error distributions. The aim of the study was to improve estimations using appropriate underlying distributions to capture extreme tails of the profit and loss distribution and, as a result, improve the estimation of VaR, a suitable measure of market risk. The main contribution is to improve the existing models for precious metals by coupling long memory models with heavy-tailed distributions (NIG, VG and PIV).

We fit ARFIMA-FIGARCH, ARFIMA-HYGARCH and ARFIMA-FIAPARCH in the presence of three alternative innovation distributions, namely normal, Student-*t* and skewed Student-*t.* We further propose three other innovation distributions: PIV, VG and NIG. Thus we compute VaR for the three models, FIGARCH, FIAPARCH and HYGARCH, with six different innovation distributions.

The rest of the paper is organized as follows: in Section 2, the basic definitions and theoretical properties of long memory "LM" models and heavy-tailed distributions are presented. The Anderson-Darling (AD) test and backtesting methods are also discussed in this Section. Section 3 discusses precious metal data sets and their properties. The empirical results of model estimation and backtesting results are summarized in Section 4. Section 5 concludes the study.

2 Methodology

In this section, the long memory processes and the long memory-GARCH-type models are discussed. We also discuss the heavy tailed distributions used to account for conditional heavy tails of the precious metal returns.

2.1 Long Memory (LM) processes

A stationary process X_t is a LM process if there exists a real number $0 < H < 1$, such that the ACF $\rho(k)$ has the following hyperbolic decay: $\lim_{k\to\infty} \rho(k) = C^{2H-1}$, $C > 0$, where C is a finite constant and H is the well-known Hurst exponent. Such a process has auto-covariance function $\gamma(k)$ which is not summable, i.e., $\sum_{\forall k} |\gamma(k)| = \infty$.

The long-range dependence parameter d of a stationary LM process has the following relationship with the Hurst exponent: $d = H - \frac{1}{2}$. Thus we can deduce the equivalent interval for the Hurst exponent as $\frac{1}{2} < H < 1$.

Equivalently, in the frequency domain LM can be defined in terms of the behavior of the spectrum:

$$f(\omega) \approx C|\omega|^{-2d},\ 0 < d < \frac{1}{2};\ \text{for } d > 0, \tag{1}$$

the ACF decays very slowly and the spectrum typically diverges to infinity at frequency $\omega = 0$, i.e., $\lim_{\omega\to 0} f(\omega) = \infty$ indicating strong long-range dependence; for $d < 0$, the spectrum of the series is equal to zero at $\omega = 0$ indicating intermediate-range dependence.

A simple example of LM is a fractionally integrated noise process, $I(d)$ with $0 < d < 1$, given by

$$(1-L)^d r_t^2 = \mu_t \tag{2}$$

for a precious metal return series r_t, where L is the lag operator and $\mu_t \to iid(0, \sigma^2)$. Thus this model includes the traditional extremes of a stationary process, $I(0)$ and generalizes the $I(1)$ process to include fractional values of d.

LM Mean and Volatility Models

We shall model the metal price returns using ARFIMA-FIGARCH, ARFIMA-HYGARCH and ARFIMA-FIAPARCH processes, starting with ARFIMA model for the mean. Without loss of generality we denote the LM parameters for the mean and volatility models as d_m and d_v, respectively.

Consider an ARFIMA(p,d,q) model of the form

$$\phi(L)(1-L)^{d_m} r_t^2 = \theta(L)\varepsilon_t, \tag{3}$$

where ε_t is a white noise process, $\phi(L) = \phi_1 L + \phi_2 L^2 + \ldots + \phi_p L^p$ and $\theta(L) = \theta_1 L + \theta_2 L^2 + \ldots + \theta_q L^q$.

The ARFIMA model is said to be stationary when $-0.5 < d_m < 0.5$, where the effect of shocks to ε_t decays at a gradual rate to zero. The model becomes nonstationary when $d_m \geq 0.5$ and is a stationary but noninvertible process when $d_m \leq -0.5$, implying that the data time series is impossible to be modelled by any AR process. The ARFIMA(p,d,q) model has a positive dependence among distant observations (i.e. LM process) when $0 < d_m < 0.5$. It has an anti-persistent property (an intermediate memory) if $-0.5 < d_m < 0$.

Baillie *et al.* [15] introduced a new class of models called the fractional integrated GARCH model i.e., the FIGARCH. The FIGARCH(p,d,q) model allows us to model a slow decay of volatility (LM behavior), as well as to distinguish the LM and short memory in conditional variance. The FIGARCH(p,d,q) can be expressed as

$$[\phi(L)(1-L)^{d_v}]a_t^2 = \omega + [1-\beta(L)](a_t^2 - \sigma_t^2), \tag{4}$$

where d represents a fractional integration parameter, a_t is a white noise residual process; $(1-L)^{d_v}$ represents fractional differencing operator; $\phi(L)$ denotes an infinite summation which has to be truncated. This model provides greater flexibility for modelling the conditional variance, since it accommodates the covariance stationary GARCH(p,q) model when $d_v = 0$ and the integrated generalised autoregressive conditional heteroscedasticity (IGARCH) model when $d_v = 1$, as special cases. That is, If $d_v = 0$, the model has a short memory and reduces to the GARCH(p,q) model. If $d_v = 1$, model becomes an IGARCH(p,q).

It has a LM when $0 < d_v < 1$ and this allows more flexibility in modelling conditional variance rather than the mean. For the FIGARCH(p,d,q) process the persistence of shocks to the conditional variance or the degree of LM is measured by the fractional differencing parameter d_v. The advantage of this methodology is that for $0 < d_v < 1$, it is sufficiently flexible to allow for an intermediate range of LM because the shocks in variance decrease at a hyperbolic rate [16].

Tse [17] extended the FIGARCH(p,d,q) model to take into account of asymmetry and the LM feature of the process of the conditional variance. He proposed the fractional integrated asymmetric power ARCH (FIAPARCH) model to explicitly accommodate for both LM and asymmetric effects in the conditional volatility. He introduced the function $(|a_t| - \gamma a_t)^\delta$ of the APARCH process. The FIAPARCH(p,d,q) model can be written as:

$$\sigma_t^\delta = \omega[1-\beta(L)]^{-1} + \{1-[1-\beta(L)]^{-1}\phi(L)(1-L)^{d_v}\}(|a_t| - \gamma a_t)^\delta, \quad (5)$$

where $\omega > 0,\ \delta > 0,\ \beta < 1$. The parameter γ refers to the asymmetric parameter satisfying the condition $-1 < \gamma < 1$. The FIAPARCH process can explain various stylized facts inherent in the volatility of financial and commodity prices. If $\gamma > 0$, negative shocks will have more impact on the commodity return volatility than positive shocks of equal magnitude. If $0 < d_v < 1$, the FIAPARCH model captures the patterns of LM property in the conditional variance process. i.e. volatility exhibits LM property. The FIAPARCH process nests the FIGARCH process when $\gamma = 0$ and $\delta = 2$. As a result, the FIAPARCH process is superior to the FIGARCH as it takes into account the asymmetry and LM in the conditional variance behavior. The parameter δ is a power term in the volatility structure and should be specified by the data.

The HYGARCH model by Davidson [18] is obtained by extending the conditional variance of the FIGARCH model by introducing weights in the difference operator. The conditional variance of the HYGARCH is expressed as follows:

$$\sigma_t^2 = \omega[1-\beta(L)]^{-1} + \{1-[1-\beta(L)]^{-1}\rho[1+\alpha\{(1-L)^{d_v}\}]\}a_t^2, \quad (6)$$

where L is the log operator, $\omega > 0,\ \delta > 0,\ \beta < 1$, and $0 < d_v < 1$. Davidson [18] showed that the HYGARCH permits the existence of the second moment at more extreme magnitudes than the simple IGARCH and FIGARCH models do. According to Conrad [19], the HYGARCH process can obtained by modifying equation (6) to

$$\phi(L)[(1-\alpha) + \alpha(1-L)^d]a_t^2 = \omega + \beta(L)(a_t^2 - \sigma_t^2), \quad (7)$$

by incorporating additional parameter $\alpha \geq 0$. Clearly, the HYGARCH model nests the stable GARCH and FIGARCH when $\alpha = 0$ and $\alpha = 1$, respectively. When $d = 1$ the parameter α becomes an autoregressive root and the HYGARCH reduces to either a stationary GARCH$(\alpha < 1)$, an IGARCH$(\alpha = 1)$ or an explosive GARCH$(\alpha > 1)$.

In our methodology, the parameters of the volatility models are estimated using an approximate quasi-maximum likelihood estimation technique, as advocated by Bollerslev and Woodridge [20].

2.2 *Heavy-tailed distributions*

The Generalized Hyperbolic Distribution and its subclasses

The generalized hyperbolic distribution (GHD) and its subclasses are continuous probability distributions defined as variance-mean mixtures of generalized inverse Gaussian distributions. The class owes its name to the fact that the logarithm of the densities is of the hyperbolic shape, whereas the logarithmic values of the normal distribution are parabolic. GHDs are five parameter continuous distributions and are useful in modeling a variety of data mainly due to the fact that they cater for asymmetry, heavy and semi-heavy tailed properties [21,22,23].

We follow Prause [22] for the parameterization of the univariate GHD. Suppose X is a random variable following the GHD, then its probability density function (pdf) can be defined as

$$f_{GHD}(x) = \frac{(\alpha^2-\beta^2)^{\lambda/2} K_{\lambda-1/2}(\alpha\sqrt{\delta^2+(x-\mu)^2})\exp(\beta(x-\mu))}{\sqrt{2\pi}\alpha^{\lambda-1/2}\delta^\lambda K_\lambda(\delta\sqrt{\alpha^2-\beta^2})(\sqrt{\delta^2+(x-\mu)^2})^{1/2-\lambda}}, \quad (8)$$

where K_j is the modified Bessel function of the third kind of order j [24] and the following conditions apply to the parameters: $\delta \geq 0$, $|\beta| < \alpha$, if $\lambda > 0$; $\delta > 0$, $|\beta| < \alpha$, if $\lambda = 0$; $\delta > 0$, $|\beta| \leq \alpha$, if $\lambda < 0$. Sub-classes of the GHDs are obtained via different parameter choices, some of which arise as limiting distributions. The special cases used in this study are given below.

The NIG distribution is a subclass of the GHD with $\lambda = -1/2$. Its pdf is

$$f_{NIG}(x) = \frac{\alpha\delta}{\pi} e^{\delta\sqrt{\alpha^2-\beta^2}+\beta(x-\mu)} \frac{K_1(\alpha\sqrt{\delta^2+(x-\mu)^2})}{\sqrt{\delta^2+(x-\mu)^2}}, \tag{9}$$

where K_1 denotes the Bessel function of the third kind with index 1. The two tails of NIG are semi-heavy and non-identical. This makes the NIG attractive for financial applications. However, it is only appropriate when the two tails are not too heavy [22].

For $\lambda > 0$ and $\delta \to 0$ in equation (8), we obtain the pdf of the VG distribution,

$$f_{VG}(x) = \frac{(\alpha^2-\beta^2)^{\lambda}|x-\mu|^{\lambda-1/2}K_{\lambda-1/2}(\alpha|x-\mu|)}{\sqrt{\pi}\Gamma(\lambda)(2\alpha)^{\lambda-1/2}} e^{\beta(x-\mu)}, \tag{10}$$

where $K_{\lambda-1/2}$ denotes the Bessel function of the third kind with index $\lambda - 1/2$. The tails of VG decreases more slowly than that of the normal distribution, making it a suitable model for phenomena where extreme values are more probable than in the case of the normal distribution. Returns from financial assets exhibit this phenomenon. We utilize the maximum likelihood estimation (MLE) for parameter estimates of the GHDs.

Pearson type-IV (PIV) distribution

The pdf of the PIV distribution is given by

$$f_{PIV}(x) = k\left[1+\left(\frac{x-\lambda}{a}\right)^2\right]^{-m} \times \exp\left[-\upsilon\tan^{-1}\left(\frac{x-\lambda}{a}\right)\right], \tag{11}$$

where $m > \frac{1}{2}$, υ, $a > 0$, $-\infty < x < \infty$, λ are real valued parameters and $k = \dfrac{\left[\Gamma(m+\frac{\upsilon}{2}i)\right]^2}{aB\left(m-\frac{1}{2},\frac{1}{2}\right)}$ is a normalization constant that depends on m, υ and a. The pdf of the PIV distribution is invariant under simultaneous change (a to $-a$, υ to $-\upsilon$). We specify $a > 0$ so that the curve is always bell-shaped. λ and a are the location and scale parameters respectively and υ is the skewness parameter. If $\upsilon > 0$ then the distribution is positive while if $\upsilon < 0$, then the distribution is negative. Parameter m controls the tail thickness and can thus be regarded as a kurtosis parameter. If m is decreased, the kurtosis is increased and for smaller values of m, the tails of PIV distribution are much heavier than those of a Gaussian distribution. The PIV distribution is essentially an asymmetric version of the STD, i.e. when $\upsilon = 0$.

2.3 ***Anderson Darling (AD) Goodness-of-fit Test (GoF)***

In this study, we use the AD statistic which is a tail-weighted statistic, i.e. a statistic that gives more weight to the tails and less weight to the center of the distribution. The AD test is a general test to compare the fit of an observed cumulative distribution function to an expected cumulative distribution function [25]. This test gives more weight to the tails than the Kolmogorov-Smirnov test. Further, the AD test assumes that there are no parameters to be estimated in the distribution being tested. The test statistic is given by:

$$A^2 = -n - \frac{1}{n}\sum_{i=1}^{n}(2i-1)\cdot[\ln F(X_i) + \ln(1-F(X_{n-i+1}))] \tag{12}$$

(Stephens, 1974).

2.4 *Value-at-Risk (VaR) and Backtesting Procedures*

The amount of asset risk capital, reserved by financial institutions as per the Basel agreement, is directly associated to the portfolio risk level and two of the most common benchmark measures for evaluating such risk are value-at-risk (VaR) and expected shortfall (ES). VaR is intended to assess the maximum possible loss of a portfolio over a given time period, given a certain risk level, and its calculations focus on the tails of a distribution. Hence, the accuracies of VaR estimation depend on how well a selected model portrays the extreme data observations [26].

Value-at-Risk

Value-at-Risk (VaR) has become a benchmark for evaluating market risks. There are two main approaches to calculating VaR for financial data series: the parametric method and the non-parametric method [27]. For a random variable X (usually for modeling the return distribution of some risky financial portfolio) with distribution function F over a specified time period, the VaR (for a given probability p) can be defined as the p-th quantile of F, i.e.

$$VaR_p = F^{-1}(1-p), \tag{13}$$

where F^{-1} is the quantile function.

Having estimated the unknown parameters of a model, the VaR for the $p-$quantile of the assumed distribution can be calculated using the equation

$$VaR(p) = \mu + F^{-1}(1-p)\sigma \tag{14}$$

[28].

The VaR of the $p-$quantile for long and short trading positions are computed as follows;

$$\begin{aligned} VaR_{long} &= \mu + F^{-1}(1-p)\sigma, \\ VaR_{short} &= \mu + F^{-1}(p)\sigma. \end{aligned} \tag{15}$$

Backtesting

Financial risk model evaluation or backtesting is a key part of the internal model's approach to market risk management as laid out by the Basel Committee on Banking Supervision [29]. VaR models are useful only if they predict future risks accurately. In order to evaluate the quality of the estimates, the models should always be backtested with appropriate methods. Backtesting is a statistical procedure where actual profits and losses are systematically compared to corresponding VaR estimates. In the backtesting process, we could statistically examine whether the frequency of exceptions over some specified time interval is in line with the selected confidence level.

To examine the adequacy and to evaluate the performance of model-based VaR estimates, we utilize the Kupiec LR unconditional coverage test [30]. The Kupiec test exploits the fact that an adequate model ought to have its proportion of violations of VaR estimates close to the corresponding tail probability level. The method consists of calculating the number of times x^p the observed returns fall below (for long positions) or above (for short positions) the VaR estimate at level p, i.e., $r_t < VaR^p$ or $r_t > VaR^p$, and compare the corresponding failure rates to p. The null hypothesis is that the expected proportion of violations is equal to p. Under this null hypothesis, the Kupiec statistic, which is given by

$$LR_{UC} = 2\ln\left(\left(\frac{x^p}{N}\right)^{x^p}\left(1-\frac{x^p}{N}\right)^{N-x^p}\right) - 2\ln\left(\alpha^{x^p}(1-p)^{N-x^p}\right), \tag{16}$$

is asymptotically distributed according to a chi-square distribution with one degree of freedom.

3 Data

We make use of daily data for three major precious metal prices (i.e., gold, silver and platinum) from 2 April 1990 to 18 September 2014. All of these prices are internationally regarded as pricing mechanism for a variety of precious metals transactions and products. We downloaded daily gold, platinum and silver P.M. fixing prices from London Bullion Market Association (LBMA) database quandl.com. The return series for each index are calculated as the first backward-differences of the natural logarithm of the index values. For day t, the daily log return r_t is defined as

$$r_t = \ln(P_t) - \ln(P_{t-1}), \tag{17}$$

where P_t is the price at day t.

Descriptive statistics, correlation, normality, heteroscedasticity, unit root and stationary tests are reported in Table 1. From Panel A, it is observed that all returns have a positive mean, i.e., indicating that the overall returns were increasing over the period under investigation. We can also see that all precious metal returns are skewed towards the left. The excess kurtosis value indicates the leptokurtotic behaviour of these return series. This implies that the empirical distribution of the daily precious metal returns are much heavier than the normal distribution.

From Panel B, the AD GoF test and Jarque-Bera test for normality give p-values less than 0.001 for all three precious metals returns, i.e., rejecting the normality assumption at all levels of significance. The Ljung-Box test, at $Q(5)$, $Q(10)$ and $Q^2(5)$, $Q^2(10)$ indicates that autocorrelation is insignificant for returns but highly significant in squared returns. These results show signs of high degree of persistence in the volatility process of precious metals. The ARCH-LM test shows the presence of conditional heteroscedasticity in all daily precious metal return series. Therefore, the GARCH-type models are appropriate for modeling and forecasting their time varying conditional volatility.

In Panel C, we present the results of the augmented Dickey-Fuller (ADF) Test [31], the Phillips and Perron [32] unit root (PP) tests and the Kwiatkowski *et al.* [33] stationarity test. The ADF and the PP tests reject the hypothesis of unit root for all precious metal return series studied. The KPSS test reveals that we cannot reject the stationarity null hypothesis for all precious metal return time series. We can conclude that precious metal price returns are stationary in mean.

Table 1: Descriptive statistics and unit root tests of precious metal price returns

	Gold		*Platinum*		*Silver*	
Panel A: Descriptive Statistics						
Minimum	-0.0797		-0.1728		-0.1869	
Maximum	0.07001		0.1178		0.1828	
Mean	0.0003		0.0002		0.0003	
Std.dev	0.0101		0.0138		0.0196	
Skewness	-0.1052		-0.5253		-0.3643	
Excess Kurtosis	6.2030		10.1959		9.6716	
Panel B: Testing for correlation, normality and heteroscedasticity						
	Statistic	*p-value*	*Statistic*	*p-value*	*Statistic*	*p-value*
$Q(5)$	5.4356	0.3651	9.9475	0.07674	42.252	<0.0001
$Q(10)$	12.987	0.2244	10.954	0.3611	47.054	<0.0001
Jarque-Bera test	9670.8	<0.0001	22686	<0.0001	21165	<0.0001
Anderson Darling test	80.093	<0.0001	67.823	<0.0001	71.102	<0.0001
$Q^2(5)$	489.95	<0.0001	554.012	<0.0001	721.953	<0.0001
$Q^2(10)$	796.36	<0.0001	799.762	<0.0001	1055.31	<0.0001
ARCH LM test	421.56	<0.0001	504.85	<0.0001	607.1	<0.0001
Panel C :Unit root and stationary tests						
ADF test	*Statistic*	*p-value*	*Statistic*	*p-value*	*Statistic*	*p-value*
	-17.274	<0.01	-15.5335	<0.01	-17.4005	<0.01
PP test	*Statistic*	*p-value*	*Statistic*	*p-value*	*Statistic*	*p-value*
	-5166.2	<0.01	-4923.12	<0.01	-5597.22	<0.01
KPSS test	*Statistic*	*p-value*	*Statistic*	*p-value*	*Statistic*	*p-value*
	0.3119	>0.1	0.0815	> 0.1	0.1122	>0.1

Further analysis was done using Q-Q and box plots of returns and the results are reported in Figures 1-3.

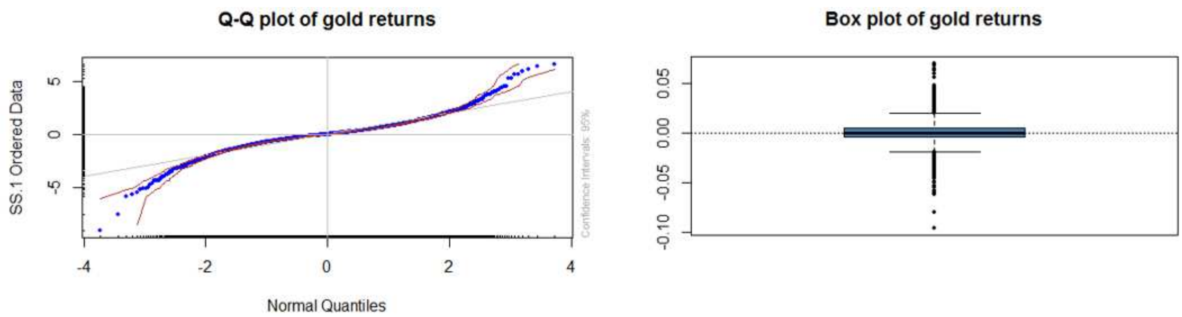

Fig. 1: Q-Q plot and Box plot for the daily gold returns

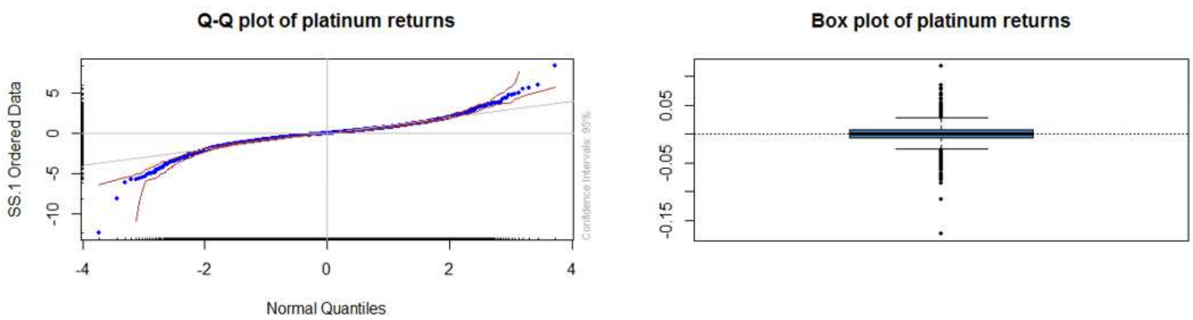

Fig. 2: Q-Q plot and Box plot for the daily platinum returns

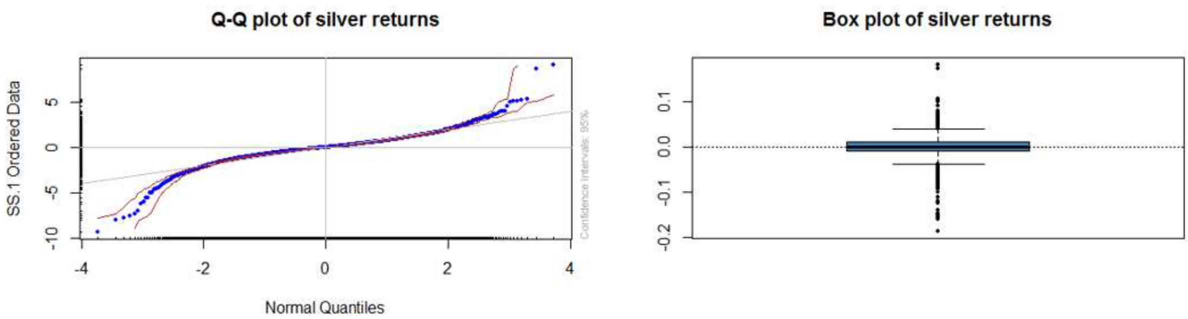

Fig. 3: Q-Q plot and Box plot for the daily silver returns

Figures 1, 2 and 3 show the Q-Q plots and box plots for gold, platinum and silver returns. The Q-Q plots illustrate that the tails of daily precious metal returns are heavier than the normal distribution. The negative skewness as well as the heavy tails are mirrored from their quantitative values in this graph. It can be seen from the box plots that daily precious metal returns are skewed to the left and heavy-tails are evident.

3.1 Testing LM in metal price volatility

In LM testing, we firstly carried out LM tests to the squared log returns of gold, platinum and silver returns based on the heuristic hurst estimator. The Hurst exponent results of the LM tests are shown in Table 2 for precious metal squared returns.

Table 2: LM tests R/S method

Returns	Hurst	Standard Error	t-value	p-value
Gold	0.6246	0.03272	19.0909	< 0.0001
Platinum	0.5511	0.02025	27.2105	< 0.0001
Silver	0.5435	0.02589	20.9978	< 0.0001

For all precious metal squared returns, the test suggests LM and all p-values are less than 0.0001, rejecting the null hypothesis of no persistence. Secondly, the GPH test was also carried out over the in-sample period and results are shown in Table 3. Based on d-values for $m = T^{0.5}$, the GPH test confirms the existence of LM in the squared returns.

Table 3: GPH LM test for returns and squared returns

	Returns	*Squared returns*
Gold	*-0.1345*	*0.5363*
Platinum	*0.0074*	*0.2291*
Silver	*-0.0686*	*0.3626*

4 Empirical Results

4.1 Estimating GARCH-type models

Tables 4 to 12 provide the parameter estimation results of ARFIMA-FIGARCH, ARFIMA-HYGARCH and ARFIMA-FIAPARCH models assuming normal (N), STD and SSTD innovation distributions for all three precious metal returns. For all models, the long range dependence parameter of the ARFIMA model (d_m) is negative for silver and platinum returns, indicating anti-persistence. This shows that the log returns of both platinum and silver are mean revering. However for gold returns d_m was statistically insignificant for all models.

As shown in Tables 4 to 12, all ARFIMA-FIGARCH models under different innovation distributions are able to capture LM phenomenon for gold, platinum and silver returns volatilities. We used several diagnostic tests to evaluate the adequacy of model specifications. The Ljung-Box test on both standardized and squared standardized residuals show no serial correlation and no ARCH effect. The insignificance of ARCH LM(10) and $Q^2(10)$ statistics show that no ARCH effect are observed in the residuals. All ARFIMA-HYGARCH and all ARFIMA-FIAPARCH models under different error distributions are adequate for all three precious metals as show by diagnostics test in Tables 4 to 12.

Finally, we employ Akaike information criterion (AIC) and Schwartz information criterion (SIC) to select the best model to describe the conditional dependence in the volatility process. The most appropriate model to describe the data is the one that minimizes SIC and AIC criteria. Based on the AIC and SIC, the best model to capture the dependence in the conditional variance for gold returns is the FIGARCH under the STD. For platinum returns, both AIC and SIC selected the ARFIMA-HYGARCH under the STD as the best model. In the case of silver, both AIC and SIC selected the ARFIMA-HYGARCH under the STD. Since ARFIMA-FIGARCH and ARFIMA-FIAPARCH were adequate for all precious metals, our results agree with the results of Arouri *et al.* [4] and Diaz [9].

ARFIMA-FIGARCH models

Table 4: ARFIMA-FIGARCH parameter estimation with different error distributions and Diagnostic tests (Gold returns)

Parameters	*Normal*		*Student-t*		*Skewed Student-t*	
	Statistic	*p-value*	*Statistic*	*p-value*	*Statistic*	*p-value*
Cst(M)	-		-		-	
d-ARFIMA (d_m)	-		-		-	
AR(1)	0.9414	<0.0001	-		-	
MA(1)	-0.9482	<0.0001	-		-	
Cst(V)	-	-	0.3070	0.0031	0.3071	0.0030
d-FIGARCH (d_v)	0.4551	<0.0001	0.4359	<0.0001	0.4342	<0.0001
ARCH(α_1)	0.3757	<0.0001	0.2950	<0.0001	0.2948	0.0419
GARCH(β_1)	0.7144	<0.0001	0.6662	<0.0001	0.6647	0.0542
Q (10)	13.2902	0.1022	9.1882	0.5150	9.1733	0.5157
Q(15)	17.4321	0.1803	14.8394	0.4630	14.8337	0.4635
Q(20)	37.0810	0.0051	30.0215	0.0695	30.0125	0.0697
Q^2*(10)*	5.7269	0.6778	13.9073	0.0842	13.9726	0.0825
Q^2*(15)*	9.4998	0.7342	18.0398	0.1560	18.0824	0.1544
Q^2*(20)*	13.3652	0.7694	21.4726	0.2562	21.5023	0.2548
ARCH LM(5)	0.5151	0.7651	2.1576	0.0559	2.1737	0.0542
ARCH LM (10)	0.5591	0.8482	1.3437	0.2006	1.3503	0.1972
AIC	2.6402		2.5330		2.5330	

Table 5: ARFIMA-FIGARCH parameter estimation with different error distributions and Diagnostic tests (Platinum returns

Parameters	*Normal*		*Student-t*		*Skewed Student-t*	
	Statistic	*p-value*	*Statistic*	*p-value*	*Statistic*	*p-value*
Cst(M)	-		0.0201	0.0814	0.0272	0.0122
d-ARFIMA (d_m)	-		-0.03268	0.0244	-0.0333	0.0221
AR(1)	-		-0.5561	0.0084	-0.5667	0.0087
MA(1)	-		0.5947	0.0030	0.6061	0.0031
Cst(V)	1.2430	0.0009	1.0931	0.0020	1.1089	0.0002
d-FIGARCH (d_v)	0.4050	<0.0001	0.3823	<0.0001	0.3801	<0.0001
ARCH(α_1)	0.3378	0.0001	0.2600	0.0002	0.2626	0.0003
GARCH(β_1)	0.5885	<0.0001	0.5391	<0.0001	0.5365	<0.0001
Q (10)	13.6255	0.1908	11.9830	0.1520	12.1821	0.1433
Q(15)	17.3142	0.3004	16.3808	0.2292	16.5966	0.2184
Q(20)	28.6010	0.0959	29.0951	0.0472	29.4207	0.0435
Q^2*(10)*	7.3393	0.5005	9.6251	0.2923	9.3193	0.3161
Q^2*(15)*	13.8802	0.3823	18.0289	0.1564	17.7681	0.1665
Q^2*(20)*	16.0976	0.5857	20.3276	0.3147	20.0317	0.3310
ARCH LM(5)	0.7775	0.5658	1.1099	0.3527	1.0445	0.3895
ARCH LM (10)	0.7441	0.6833	0.9788	0.4594	0.9476	0.4878
AIC	3.2147		3.1598		3.1601	

Table 6: FIGARCH parameter estimation with different error distributions and Diagnostic tests (Silver returns)

Parameters	*Normal*		*Student-t*		*Skewed Student-t*	
	Statistic	*p-value*	*Statistic*	*p-value*	*Statistic*	*p-value*
Cst(M)	-		-		-	
d-ARFIMA (d_m)	-0.062379	<0.0001	0.0642	<0.0001	-0.0639	<0.0001
AR(1)	-		-		-	
MA(1)	-		-		-	
Cst(V)	3.539744	0.0113	-		2.869921	0.0023
d-FIGARCH (d_v)	0.452067	<0.0001	0.491001	<0.0001	0.450248	<0.0001
ARCH(α_1)	0.349151	<0.0001	0.399324	<0.0001	0.425372	<0.0001
GARCH(β_1)	0.696679	<0.0001	0.774474	<0.0001	0.752825	<0.0001
Q (10)	31.5091	0.0004	32.1970	0.0004	31.8889	0.0004
Q(15)	36.2849	0.0017	37.5668	0.0010	36.8180	0.0013
Q(20)	42.9962	0.0021	44.5951	0.0013	43.6307	0.0017
Q^2*(10)*	4.4048	0.8189	8.03816	0.4298	6.5431	0.5866
Q^2*(15)*	5.9916	0.9465	10.0508	0.6898	8.2025	0.8302
Q^2*(20)*	10.1211	0.9279	14.0482	0.7259	12.5310	0.8186
ARCH LM(5)	0.6016	0.6988	1.3592	0.2365	1.0521	0.3851
ARCH LM (10)	0.4411	0.9268	0.8126	0.6166	0.6506	0.7710
AIC	3.9204		3.8390		3.8347	

ARFIMA-HYGARCH models

Table 7: ARFIMA-HYGARCH parameter estimation with different error distributions and Diagnostic tests (Gold returns)

Parameters	*Normal*		*Student-t*		*Skewed Student-t*	
	Statistic	*p-value*	*Statistic*	*p-value*	*Statistic*	*p-value*
Cst(M)	-		-		-	
d-ARFIMA (d_m)	-		-		-	
AR(1)	-0.700426	<0.0001	-		-	
MA(1)	0.694912	<0.0001	-		-	
Cst(V)	-		-		-	
d-FIGARCH (d_v)	0.4245	<0.0001	0.5762	<0.0001	0.5724	<0.0001
ARCH(α_1)	0.3223	0.0186	0.2444	<0.0001	0.2448	<0.0001
GARCH(β_1)	0.6337	<0.0001	0.7631	<0.0001	0.7607	<0.0001
Q (10)	10.5705	0.2272	10.2080	0.4224	10.1961	0.4235
Q(15)	16.8240	0.2075	15.4740	0.4178	15.4682	0.4182
Q(20)	32.9035	0.0171	30.1939	0.0668	30.1897	0.0668
Q^2*(10)*	3.9160	0.8646	14.8211	0.0627	14.8370	0.0624
Q^2*(15)*	7.1238	0.8957	20.4539	0.0845	20.4292	0.0850
Q^2*(20)*	10.9321	0.8972	24.5492	0.1378	24.4969	0.1394
ARCH LM(5)	0.3228	0.8995	2.1800	0.0536	2.1875	0.0522
ARCH LM (10)	0.3908	0.9513	1.4199	0.1645	1.4221	0.1635
AIC	2.6332		2.5348		2.5346	

Table 8: ARFIMA-HYGARCH parameter estimation with different error distributions and Diagnostic tests (Platinum returns)

Parameters	*Normal*		*Student-t*		*Skewed Student-t*	
	Statistic	*p-value*	*Statistic*	*p-value*	*Statistic*	*p-value*
Cst(M)	-		-			
d-ARFIMA (d_m)	-0.0367	0.0553	-0.0274	0.0403	-0.02403	0.0678
AR(1)	0.0650	0.0080	-0.5845	0.0048	-0.6049	0.0054
MA(1)	-		0.6196	0.0017	0.6382	0.0021
Cst(V)	-		-			
d-FIGARCH (d_v)	0.4002	<0.0001	0.3737	<0.0001	0.3770	<0.0001
ARCH(α_1)	0.3059	0.0008	0.2651	0.0002	0.2691	0.0002
GARCH(β_1)	0.57777	<0.0001	0.5487	<0.0001	0.5520	<0.0001
Q (10)	7.3660	0.5991	11.0008	0.2017	10.4932	0.2321
Q(15)	11.0353	0.6833	15.0318	0.3054	14.5651	0.3353
Q(20)	23.3377	0.2228	26.8208	0.0824	26.0900	0.0977
$Q^2(10)$	6.8445	0.5535	8.7407	0.3646	8.7124	0.3671
$Q^2(15)$	13.2234	0.4307	16.8605	0.2058	16.9849	0.1999
$Q^2(20)$	15.3730	0.6362	18.8428	0.4016	19.0697	0.3875
ARCH LM(5)	0.7914	0.5557	1.0293	0.3984	0.9993	0.4165
ARCH LM (10)	0.6878	0.7369	0.8763	0.5548	0.8746	0.5564
AIC	3.2023		3.1605		3.1494	

Table 9: HYGARCH parameter estimation with different error distributions and Diagnostic tests (Silver returns)

Parameters	*Normal*		*Student-t*		*Skewed Student-t*	
	Statistic	*p-value*	*Statistic*	*p-value*	*Statistic*	*p-value*
Cst(M)	-		-		-	
d-ARFIMA (d_m)	-		-		-	
AR(1)	-0.1038	<0.0001	-		-0.1161	<0.0001
MA(1)	-		-0.1166	<0.0001	-	
Cst(V)	-		-		-	
d-FIGARCH (d_v)	0.4105	<0.0001	0.4382	<0.0001	0.4386	<0.0001
ARCH(α_1)	0.3889	<0.0001	0.4611	<0.0001	0.4609	<0.0001
GARCH(β_1)	0.6997	<0.0001	0.7767	<0.0001	0.7762	<0.0001
Q (10)	14.6218	0.1019	19.3259	0.0226	16.2763	0.0613
Q(15)	20.8588	0.1053	25.6994	0.0283	22.7416	0.0646
Q(20)	26.9518	0.1058	31.6694	0.0340	28.8287	0.0687
$Q^2(10)$	4.4855	0.8109	6.4937	0.5921	6.5187	0.5893
$Q^2(15)$	6.1174	0.9418	8.2545	0.8266	8.2755	0.8252
$Q^2(20)$	9.9368	0.9339	11.7819	0.8583	11.7768	0.8585
ARCH LM(5)	0.5832	0.7129	1.0161	0.4063	1.0159	0.4064
ARCH LM (10)	0.4483	0.9229	0.6433	0.7776	0.6436	0.7773
AIC	3.9166		3.8294		3.8291	

ARFIMA-FIAPARCH models

Table 10: ARFIMA-FIAPARCH parameter estimation with different error distributions and Diagnostic tests (Gold returns)

Parameters	*Normal*		*Student-t*		*Skewed Student-t*	
	Statistic	*p-value*	*Statistic*	*p-value*	*Statistic*	*p-value*
Cst(V)	-		-		-	
d-ARFIMA (d_m)	-		-		-	
AR(1)	-		-		-	
MA(1)	-		-		-	
Cst(V)	-		-		-	
d-FIGARCH (d_v)	0.4609	<0.0001	0.5758	<0.0001	0.5721	<0.0001
ARCH(α_1)	0.3041	0.0090	0.2330	<0.0001	0.2336	<0.0001
GARCH(β_1)	0.6461	<0.0001	0.7456	<0.0001	0.7427	<0.0001
APARCH(γ_1)	-0.0741	0.0305	-0.1087	0.0215	-0.1118	0.0185
APARCH (δ)	2.1436	<0.0001	2.0663	<0.0001	2.0667	<0.0001
Q (10)	9.2958	0.5043	9.9909	0.4413	9.9682	0.4433
Q(15)	15.4851	0.4171	15.3839	0.4241	15.3733	0.4249
Q(20)	30.9463	0.0559	29.7761	0.0736	29.7603	0.0738
Q^2*(10)*	4.0528	0.8523	13.5152	0.0953	13.4541	0.09715
Q^2*(15)*	7.4254	0.8789	18.8831	0.1268	18.7631	0.1306
Q^2*(20)*	10.9112	0.8981	22.2555	0.2008	22.0963	0.2277
ARCH LM(5)	0.3009	0.9126	1.9109	0.0891	1.9039	0.0903
ARCH LM (10)	0.4049	0.9451	1.3116	0.2176	1.3066	0.2204
AIC	2.6331		2.5350		2.5349	

Table 11: ARFIMA-FIAPARCH parameter estimation with different error distributions and Diagnostic tests (Platinum returns)

Parameters	*Normal*		*Student-t*		*Skewed Student-t*	
	Statistic	*p-value*	*Statistic*	*p-value*	*Statistic*	*p-value*
Cst(M)	0.02740	0.0125	0.0294	0.0056	-	
d-ARFIMA (d_m)	-0.04845	0.0255	-0.0387	0.0255	-0.0272	0.0444
AR(1)	0.0766	0.0038	0.0411	0.0604	-0.5738	0.0059
MA(1)	-				0.6084	0.0022
Cst(V)	-				-	
d-FIGARCH (d_v)	0.3971	<0.0001	0.3818	<0.0001	0.4396	<0.0001
ARCH(α_1)	0.2900	0.0013	0.2528	0.0002	0.2509	<0.0001
GARCH(β_1)	0.5615	<0.0001	0.5397	<0.0001	0.5946	<0.0001
APARCH(γ_1)	-0.1023	0.0159	-0.1133	<0.0001	-0.1059	0.0096
APARCH (δ)	2.0876	<0.0001	2.0771	<0.0001	1.9691	<0.0001
Q (10)	9.2291	0.4164	15.6247	0.0751	11.3782	0.1812
Q(15)	13.0386	0.5235	19.5533	0.1448	15.4569	0.2797
Q(20)	26.3605	0.1205	32.4107	0.0281	27.1830	0.0756
Q^2*(10)*	6.1708	0.6281	8.0896	0.4248	9.5857	0.2953
Q^2*(15)*	12.1931	0.5119	14.8650	0.3159	15.6586	0.2681
Q^2*(20)*	14.1883	0.7167	16.8046	0.5366	17.5727	0.4841
ARCH LM(5)	0.6485	0.6627	0.9423	0.4522	1.2714	0.2732
ARCH LM (10)	0.6191	0.7988	0.8055	0.6235	0.9633	0.4734
AIC	3.2131		3.1608		3.1601	

Table 12: ARFIMA-FIAPARCH parameter estimation with different error distributions and Diagnostic tests (Silver returns)

Parameters	*Normal*		*Student-t*		*Skewed Student-t*	
	Statistic	*p-value*	*Statistic*	*p-value*	*Statistic*	*p-value*
Cst(M)	-		-		-	
d-ARFIMA (d_m)	-		-		-	
AR(1)	-0.1037	<0.0001	-0.1181	<0.0001	-0.1195	<0.0001
MA(1)	-		-		-	
Cst(V)	-		-		-	
d-FIGARCH (d_v)	0.4047	<0.0001	0.4334	<0.0001	0.4336	<0.0001
ARCH(α_1)	0.3715	<0.0001	0.4241	<0.0001	0.4233	<0.0001
GARCH(β_1)	0.6731	<0.0001	0.7467	<0.0001	0.7468	<0.0001
APARCH(γ_1)	-0.1117	0.0569	-0.2169	0.0001	-0.2219	0.0001
APARCH (δ)	2.1424	<0.0001	2.0610	<0.0001	2.0596	<0.0001
Q (10)	14.2518	0.1136	17.2202	0.0454	17.6880	0.0390
Q(15)	20.4800	0.1157	23.6739	0.0501	24.1527	0.0439
Q(20)	26.2276	0.1240	29.3005	0.0614	29.7740	0.0547
$Q^2(10)$	4.3861	0.8207	4.7997	0.7788	4.8727	0.7711
$Q^2(15)$	6.1014	0.9423	6.8469	0.9099	6.9328	0.9056
$Q^2(20)$	10.1379	0.9273	10.5919	0.9109	10.6802	0.9074
ARCH LM(5)	0.5613	0.7298	0.6491	0.6622	0.6616	0.6527
ARCH LM (10)	0.4387	0.9281	0.4743	0.9076	0.4815	0.9030
AIC	3.9227		3.8330		3.8328	

4.2 ***Long memory-GARCH models with heavy-tailed distribution (VG, NIG, PIV)***

We now incorporate LM GARCH models where the innovation process follows VG, NIG and PIV. These distributions depict the tail distribution of the standardized residual series. The PIV distribution is fitted to the standardized residuals extracted from the ARFIMA-FIGARCH, ARFIMA-HYGARCH, ARFIMA-FIAPARCH with normal innovations. The parameters are estimated using the method of maximum likelihood. The maximum likelihood procedure is carried out using R package PearsonDS. Tables 13, 15 and 17 show the maximum likelihood estimates of PIV distribution fitted to the ARFIMA-FIGARCH, ARFIMA-HYGARCH, ARFIMA-FIAPARCH models with normal innovations respectively.

From table 13, 15 and 17, the values of $m > 0.5$ for all PIV models, thus satisfying the condition for PIV distribution. The AD statistics are significant, thus the PIV distribution is a good fit of the extracted standardized residuals from the ARFIMA-FIGARCH, ARFIMA-HYGARCH, ARFIMA-FIAPARCH models.

We fit NIG and VG to the standardized residuals extracted from the ARFIMA-FIGARCH, ARFIMA-HYGARCH, ARFIMA-FIAPARCH models. The MLE parameter estimates are shown in Tables 14, 16 and 18. The AD test results suggest that the extracted standardized residuals follow the fitted distributions. The high p-values indicate that we cannot reject the null hypothesis that the standardized residuals follow NIG and VG distributions. Table 13 shows that, for silver returns, VG is a good fit of the standardized residuals at 1% level of significance.

Table 13: ML estimates of the Pearson type-IV distribution for the ARFIMA-FIGARCH-PIV model

Returns	$\hat{m}$	$\hat{\upsilon}$	$\hat{\lambda}$	$\hat{a}$	*AD Test*	
					Statistic	*p-value*
Gold	1.8768	0.0118	0.0354	1.0610	2.1259	0.0784
Platinum	2.1325	0.1195	0.1156	1.6507	0.4068	0.842
Silver	2.0702	0.0131	0.0700	2.2458	0.9380	0.3915

Table 14: ML estimates of the GHDs for the ARFIMA-FIGARCH -GHDs model

Returns	*Distr*	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\delta}$	$\hat{\mu}$	$\hat{\lambda}$	*AD Test*	
							Statistic	*p-value*
Gold	*NIG*	*0.6146*	*-0.0052*	*0.6938*	*0.03161*	*-0.5*	*0.87056*	*0.4328*
	VG	*1.2749*	*0.0068*	*0*	*0.0185*	*0.8610*	*0.7535*	*0.5158*
Platinum	*NIG*	*0.5605*	*-0.0313*	*1.0731*	*0.0857*	*-0.5*	*0.4164*	*0.8324*
	VG	*1.0736*	*0.0133*	*0*	*0.0011*	*1.0655*	*3.3649*	*0.0179*
Silver	*NIG*	*0.3783*	*-0.0071*	*1.4601*	*0.0682*	*-0.5*	*1.071*	*0.3221*
	VG	*0.7452*	*0.0120*	*0*	*-0.0028*	*1.0218*	*2.6452*	*0.0416*

Table 15: ML estimates of the Pearson type-IV distribution for the ARFIRMA-HYGARCH-PIV model

Returns	$\hat{m}$	$\hat{\upsilon}$	$\hat{\lambda}$	$\hat{a}$	*AD Test*	
					Statistic	*p-value*
Gold	*1.8752*	*0.0224*	*0.0403*	*1.0593*	*2.12*	*0.0790*
Platinum	*2.1372*	*0.1042*	*0.1097*	*1.6480*	*0.2676*	*0.9603*
Silver	*2.0699*	*0.0639*	*0.1073*	*2.2416*	*0.7554*	*0.5143*

Table 16: ML estimates of the GHDs for the ARFIMA-HYGARCH -GHDs model

Returns	*Distr*	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\delta}$	$\hat{\mu}$	$\hat{\lambda}$	*AD Test*	
							Statistic	*p-value*
Gold	*NIG*	*0.6144*	*-0.0080*	*0.6933*	*0.0335*	*-0.5*	*0.8846*	*0.4239*
	VG	*1.2791*	*0.00142*	*0*	*0.0233*	*0.8650*	*0.7603*	*0.5105*
Platinum	*NIG*	*0.5648*	*-0.0283*	*1.0723*	*0.0843*	*-0.5*	*0.2933*	*0.9431*
	VG	*1.1191*	*-0.0197*	*0*	*0.0661*	*1.1329*	*1.5968*	*0.1550*
Silver	*NIG*	*0.3788*	*-0.0151*	*1.4567*	*0.0851*	*-0.5*	*0.8215*	*0.4658*
	VG	*0.7517*	*-0.0013*	*0*	*0.0319*	*1.0346*	*2.3783*	*0.0574*

Table 17: ML estimates of the Pearson type-IV distribution for the ARFIRMA-FIAPARCH-PIV model

Returns	$\hat{m}$	$\hat{\upsilon}$	$\hat{\lambda}$	$\hat{a}$	*AD Test*	
					Statistic	*p-value*
Gold	*1.8804*	*0.0224*	*0.0402*	*1.0634*	*2.0747*	*0.0836*
Platinum	*2.1452*	*0.1134*	*0.0840*	*1.6628*	*0.252*	*0.9693*
Silver	*2.0702*	*0.0653*	*0.1091*	*2.2506*	*0.8003*	*0.4808*

Table 18: ML estimates of the GHDs for the ARFIMA-FIAPARCH -GHDs model

Returns	*Distr*	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\delta}$	$\hat{\mu}$	$\hat{\lambda}$	*AD Test*	
							Statistic	*p-value*
Gold	*NIG*	*0.6170*	*-0.0078*	*0.6959*	*0.0334*	*-0.5000*	*0.8581*	*0.4410*
	VG	*1.2556*	*0.0232*	*<0.0001*	*<0.0001*	*0.8396*	*1.4082*	*0.2000*
Platinum	*NIG*	*0.5650*	*-0.0302*	*1.0809*	*0.0559*	*-0.5000*	*0.2864*	*0.9480*
	VG	*1.1183*	*-0.0279*	*<0.0001*	*0.0488*	*1.1406*	*1.503*	*0.1758*
Silver	*NIG*	*0.3774*	*-0.0154*	*1.4618*	*0.0867*	*-0.5000*	*0.8421*	*0.4516*
	VG	*0.7474*	*-0.0003*	*<0.0001*	*0.0280*	*1.0309*	*2.3916*	*0.0564*

4.3 VAR estimation and Backtesting of models

VaR is calculated at 1%, 2.5%, 5%, 95%, 97.5% and 99% for each model. The estimates are then backtested using the Kupiec LR test. The p-values of the Kupiec test for in-sample are summarized in Tables 19, 20 and 21.

ARFIMA-FIGARCH models

For gold returns, ARFIMA-FIGARCH-NIG gives the largest p -value at 1% VaR level and FIGARCH-VG at 2.5% level. ARFIMA-FIGARCH-STD is not a suitable model at 2.5%, 5%, 95% and 97.5% VaR levels. The VaR estimates from the ARFIMA-FIGARCH-N produced the lowest p-values at most VaR levels. However, the model is adequate at 5% and 95% VaR levels. It is interesting to note that ARFIMA-FIGARCH-PIV, ARFIMA-FIGARCH-NIG, ARFIMA-FIGARCH-VG models are all suitable at all VaR levels. For silver and platinum the three models are also suitable at all VaR levels.

Table 19: In-Sample VaR Backtesting: ARFIMA-FIGARCH models

Returns	*Model*	*p-values of Kupiec LR test*					
		Long positions			*Short positions*		
		1%	*2.5%*	*5%*	*95%*	*97.5%*	*99%*
Gold	*ARFMA-FIGARCH-N*	<0.0001	0.0002	0.1631	0.1828	<0.0001	<0.0001
	ARFIMA-FIGARCH-STD	*0.5783*	*0.0495*	*0.0437*	*0.0101*	*0.0023*	*0.8877*
	ARFIMA-FIGARCH-SSTD	*0.7797*	*0.0603*	*0.0878*	*0.0040*	*0.0005*	*0.4072*
	ARFIMA-FIGARCH-PIV	*0.7769*	*0.2696*	*0.2526*	*0.1285*	*0.8932*	*0.1451*
	ARFIMA-FIGARCH-NIG	*0.9998*	*0.7525*	*0.7473*	*0.9998*	*0.4970*	*0.1914*
	ARFIMA-FIGARCH-VG	*0.1722*	*0.8932*	*0.2166*	*0.1931*	*0.4408*	*0.6758*
Platinum	*ARFMA-FIGARCH-N*	*0.0001*	*0.2014*	*0.6988*	*0.5180*	*0.2337*	*0.0001*
	ARFIMA-FIGARCH-STD	*0.2179*	*0.3092*	*0.1136*	*0.9998*	*0.8231*	*0.4885*
	ARFIMA-FIGARCH-SSTD	*0.8884*	*0.6882*	*0.4073*	*0.4827*	*0.2014*	*0.2179*
	ARFIMA-FIGARCH-PIV	*0.5783*	*0.6882*	*0.7488*	*0.9998*	*0.9642*	*0.5684*
	ARFIMA-FIGARCH-NIG	*0.6758*	*0.6193*	*0.2421*	*0.2421*	*0.3883*	*0.5684*
	ARFIMA-FIGARCH-VG	*0.0434*	*0.6882*	*0.2421*	*0.0029*	*0.0726*	*0.5684*
Silver	*ARFMA-FIGARCH-N*	*0.0006*	*0.1316*	*0.0064*	*0.0546*	*0.7727*	*0.0006*
	ARFIMA-FIGARCH-STD	*0.3046*	*0.3220*	*0.7242*	*0.7260*	*0.8756*	*0.4489*
	ARFIMA-FIGARCH-SSTD	*0.0692*	*0.0355*	*0.4277*	*0.0096*	*0.05313*	*0.0276*
	ARFIMA-FIGARCH-PIV	*0.6278*	*0.0932*	*0.6331*	*0.9745*	*0.9466*	*0.3522*
	ARFIMA-FIGARCH-NIG	*0.7283*	*0.6697*	*0.2069*	*0.2579*	*0.4296*	*0.4328*
	ARFIMA-FIGARCH-VG	*0.0276*	*0.4661*	*0.1272*	*0.0014*	*0.2115*	*0.7241*

ARFIMA-HYGARCH models

For gold and platinum returns, ARFIMA-HYGARCH-STD, ARFIMA-HYGARCH-SSTD, ARFIMA-HYGARCH-PIV, ARFIMA-HYGARCH-NIG, ARFIMA-HYGARCH-VG are all suitable at all VaR levels. ARFIMA-HYGARCH-NIG is the overall best model for gold returns at it has highest p-values at all VaR levels, except at 99% VaR level. ARFIMA-HYGARCH-PIV, ARFIMA-HYGARCH-NIG compete favorably for the overall best model for platinum returns. For silver, ARFIMA-HYGARCH-STD, ARFIMA-HYGARCH-SSTD, ARFIMA-HYGARCH-PIV, ARFIMA-HYGARCH-NIG are suitable at all VAR levels. It is interesting to note that ARFIMA-HYGARCH-VG is not suitable at 1%, 5% and 95% VaR levels. i.e. ARFIMA-HYGARCH-VG did not perform well for silver returns.

Table 20: In-Sample VaR Backtesting: ARFIMA-HYGARCH models

Returns	***Model***	***p-values of Kupiec LR test***					
		Long positions			***Short positions***		
		1%	***2.5%***	***5%***	***95%***	***97.5%***	***99%***
Gold	*ARFMA-HYGARCH-N*	*0.0004*	*0.5051*	*0.1931*	*0.1931*	*0.0404*	*0.0024*
	ARFIMA-HYGARCH-STD	*0.3889*	*0.9442*	*0.3725*	*0.3396*	*0.1470*	*0.0554*
	ARFIMA-HYGARCH-SSTD	*0.2472*	*0.4970*	*0.7977*	*0.1450*	*0.0329*	*0.1914*
	ARFIMA-HYGARCH-PIV	*0.6698*	*0.3998*	*0.2042*	*0.1630*	*0.7546*	*0.1451*
	ARFIMA-HYGARCH-NIG	*0.9998*	*0.9642*	*0.8469*	*0.9487*	*0.4970*	*0.1451*
	ARFIMA-HYGARCH-VG	*0.2179*	*0.8231*	*0.3642*	*0.1715*	*0.4970*	*0.6758*
Platinum	*ARFMA-HYGARCH-N*	*0.0009*	*0.5213*	*0.3170*	*0.2069*	*0.3220*	*0.0004*
	ARFIMA-HYGARCH-STD	*0.9441*	*0.8413*	*0.7260*	*0.9264*	*0.9113*	*0.4489*
	ARFIMA-HYGARCH-SSTD	*0.1914*	*0.3397*	*0.5179*	*0.3725*	*0.2014*	*0.2720*
	ARFIMA-HYGARCH-PIV	*0.6278*	*0.5799*	*0.9745*	*0.8225*	*0.6052*	*0.8328*
	ARFIMA-HYGARCH-NIG	*0.7283*	*0.6697*	*0.0849*	*0.2069*	*0.2115*	*0.9441*
	ARFIMA-HYGARCH-VG	*0.0916*	*0.7727*	*0.0546*	*0.0635*	*0.2114*	*0.6278*
Silver	*ARFMA-HYGARCH-N*	*0.0002*	*0.2446*	*0.0202*	*0.0052*	*0.8413*	*0.0098*
	ARFIMA-HYGARCH-STD	*0.5343*	*0.4661*	*0.7242*	*0.5848*	*0.4848*	*0.9441*
	ARFIMA-HYGARCH-SSTD	*0.9440*	*0.8756*	*0.2580*	*0.7742*	*0.8413*	*0.5343*
	ARFIMA-HYGARCH-PIV	*0.7283*	*0.1550*	*0.7260*	*0.6331*	*0.9822*	*0.3522*
	ARFIMA-HYGARCH-NIG	*0.8344*	*0.8054*	*0.1635*	*0.7242*	*0.3306*	*0.4328*
	ARFIMA-HYGARCH-VG	*0.0380*	*0.5799*	*0.0468*	*0.0116*	*0.3306*	*0.8328*

ARFIMA-FIAPARCH models

For gold and platinum returns, ARFIMA-FIAPARCH-STD, ARFIMA-FIAPARCH-SSTD, ARFIMA-FIAPARCH-PIV, ARFIMA-FIAPARCH-NIG, ARFIMA-FIAPARCH-VG are all suitable at all VaR levels. For silver, ARFIMA-FIAPARCH-STD, ARFIMA-FIAPARCH-SSTD, ARFIMA-FIAPARCH-PIV, ARFIMA-FIAPARCH-NIG are suitable at all VAR levels. ARFIMA-HYGARCH-VG did not perform well for silver returns at 1% and 95% VaR levels. For gold, ARFIMA-FIAPARCH-NIG produced highest p-values at 1%, 2.5%, 5% and 95%. It is clearly the best model for gold returns. For platinum, ARFIMA-FIAPARCH-SSTD produced highest p-value at 1% VaR level and, ARFIMA-FIAPARCH-VG at 2.5% VaR level. ARFIMA-FIAPARCH-PIV has the largest p-values at 5%, 95% and 97.5% VaR levels. At 5% and 97.5% VaR levels, ARFIMA-FIAPARCH-PIV is the most robust model for estimating VaR for silver returns. While at 1% ARFIMA-FIAPARCH-NIG produce largest p-value. For silver returns, ARFIMA-FIAPARCH-SSTD produced highest p-values at 2.5% and 95% VaR levels.

5 Conclusion

In this study, we extended the work of Arouri *et al* [4], Diaz [9], Ranganai and Khubeka [8], Youssef *et al.* [10], Mabrouk and Saadi [11], Cochran *et al.* [5], McNeil and Frey [12], Bhattacharyya *et al.* [13] and Bhattacharyya and Madhav [14]. Our findings revealed that precious metal markets are characterized by asymmetry, heavy-tail and LM (long-range dependence). We examined the LM GARCH models under the GHDs, the SSTD and PIV assumptions. The conditional variance and LM in mean and volatility were modeled by ARFIMA-FIGARCH, ARFIMA-HYGARCH and ARFIMA-FIAPARCH models with pseudo-normal assumptions. The NIG, VG and PIV distributions are applied to capture the heavy-tail behavior for the extracted standardized residuals and VaR is calculated at different levels. Adequacy of the resulting VaR estimates were tested using the Kupiec LR test. Backtesting results have shown that ARFIMA-FIGARCH, ARFIMA-HYGARCH and ARFIMA-FIAPARCH models with PIV, NIG and VG governing the innovations are suitable for depicting VaR in gold, platinum and silver returns. However, for silver returns the ARFIMA-HYGARCH model with VG governing the innovations is not suitable at 1%, 5% and 95% VaR levels. The ARFIMA-HYGARCH with NIG governing the innovations is the overall best model for gold returns. For platinum, the ARFIMA-FIAPARCH with PIV governing the innovations is the best model. Our results are consistent with the results of Arouri et al, [4], however, our models incorporated heavy-tailedness exhibited by precious metals returns. With forecasted volatility, absolute magnitude of returns and quantiles may be predicted for risk management and portfolio management purposes. The volatility of a stock generally exhibits either a positive or negative shock which have different implications to the portfolio manager and risk management. For instance, the declining stock prices tend to increase the firm's leverage (debt-to-equity) ratio making the stock a riskier asset. This results in increased volatility of returns to stockholders and reduced demand for the stock due to investors' risk aversion. Thus, the results of this study

Table 21: In-Sample VaR Backtesting: ARFIMA-FIAPARCH models

Returns	*Model*	*p-values of Kupiec LR test*					
		Long positions			*Short positions*		
		1%	*2.5%*	*5%*	*95%*	*97.5%*	*99%*
Gold	*ARFMA-FIAPARCH-N*	*<0.0001*	*0.3526*	*0.1715*	*0.2696*	*0.0169*	*0.0007*
	ARFIMA- FIAPARCH -STD	*0.9998*	*0.5052*	*0.0878*	*0.1828*	*0.0603*	*0.8877*
	ARFIMA- FIAPARCH SSTD	*0.6698*	*0.9642.*	*0.1631*	*0.0437*	*0.0265*	*0.6758*
	ARFIMA-FIAPARCH-PIV	*0.6698*	*0.4507*	*0.1450*	*0.1136*	*0.8231*	*0.1451*
	ARFIMA-FIAPARCH-NIG	*0.9998*	*0.9642*	*0.8469*	*0.9998*	*0.6193*	*0.1913*
	ARFIMA-FIAPARCH -VG	*0.1722*	*0.6882*	*0.5179*	*0.0421*	*0.2950*	*0.8877*
Platinum	*ARFMA-FIAPARCH-N*	*<0.0001*	*0.0729*	*0.3725*	*0.1174*	*0.4507*	*0.0007*
	ARFIMA- FIAPARCH -STD	*0.4072*	*0.2696*	*0.1003*	*0.76667*	*0.7525*	*0.6758*
	ARFIMA- FIAPARCH SSTD	*0.7769*	*0.3883*	*0.8976*	*0.1003*	*0.2696*	*0.0784*
	ARFIMA-FIAPARCH-PIV	*0.6758*	*0.5631*	*0.9488*	*0.8469*	*0.6848*	*0.7769*
	ARFIMA-FIAPARCH-NIG	*0.6758*	*0.7525*	*0.0893*	*0.2166*	*0.2177*	*0.8877*
	ARFIMA-FIAPARCH -VG	*0.0434*	*0.8231*	*0.0214*	*0.0670*	*0.2177*	*0.5783*
Silver	*ARFMA-FIAPARCH-N*	*0.0001*	*0.0729*	*0.0068*	*0.0255*	*0.8231*	*0.0114*
	ARFIMA- FIAPARCH -STD	*0.4884*	*0.3998*	*0.6054*	*0.6515*	*0.6193*	*0.8884*
	ARFIMA- FIAPARCH SSTD	*0.7797*	*0.7546*	*0.2991*	*0.8979*	*0.8231*	*0.4885*
	ARFIMA-FIAPARCH-PIV	*0.6758*	*0.2337*	*0.6543*	*0.6090*	*0.9642*	*0.3031*
	ARFIMA-FIAPARCH-NIG	*0.9998*	*0.6193*	*0.1715*	*0.6515*	*0.3883*	*0.3889*
	ARFIMA-FIAPARCH -VG	*0.04344*	*0.5052*	*0.0576*	*0.0055*	*0.2177*	*0.8877*

provide an impetus for investors and the risk management community to consider a holistic, yet more accurate, measure of stock volatility on the gold, platinum and silver prices.

Further recommendations would be:

– Comparing the results of this study with the Generalised Autoregressive Score(GAS) models.
– Investigating the relative performance of the proposed models in VaR estimation of cryptocurrencies.

The main objective of the study was to propose a modelling framework which can be used in precious metal market for carrying out accurate risk management or assessment. The scope of the study was well defined and had no visible limitations in achieving the main objective of the study.

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Conflict of interest

The authors declare that they have no conflict of interest.

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