Progress in Fractional Differentiation and Applications
An International Journal

http://dx.doi.org/10.18576/pfda/080202

Analysis and Modelling of HIV/AIDS Model with Fractional Order Parameter Estimation

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Received: 2 May 2020, Revised: 18 Jun. 2020, Accepted: 26 Jul. 2020
Published online: 1 Apr. 2022

Abstract: In this paper, nonlinear fractional order HIV/AIDS mathematical model is discussed epidemic problems for the complex transmission of the disease. It is accepted that susceptible wind up contaminated by means of sexual contacts with infective eventually create AIDS. The point of this task was to amend transmission models recently created to represent HIV transmission and AIDS related mortality. The Caputo-Fabrizio fractional derivative operator of order $\beta \in (0,1)$ is used to obtain fractional differential equations structure. The stability fractional order model was developed and the unique non-negative solution was tested. The numerical simulations are performed using an iterative technique. Some new results are being viewed with the help of Sumudo transform. Nonetheless, according to Banach, the related findings are given nonlinear functional analysis and fixed point theory. However, mathematical simulations are also acknowledged to evaluate the impact of the model's parameter by decreasing the fractional values and showing the effect of the β fractional parameter on our obtained solutions. The impact of various parameters is represented graphically.

Keywords: Caputo-Fabrizio derivative, fractional-order HIV/AIDS, Sumudu transform, existence and uniqueness.

1 Introduction

Human immunodeficiency virus is a worldwide lentivirus which causes HIV infection. It is one of the world's most well-researched contagious disease. If an infected person does not use HIV medications, this person can only survived for $9-11$ years [1]. HIV-1 and HIV-2 are two different types of HIV. The originally discovered HIV was HIV-1 virus and it infects most people as compared to HIV-2 virus [2]. In the late 1980s, soon after the virus itself was discovered, the first model for HIV infection among individual persons was created. Such models were based upon the bread-and-butter models developed by Kendrick and McCormack in the early 1900s in statistical epidemiologies such as SIS and SIR models, which are used widely for describing the spread of infections in a single population [3]. In comparison, model "viral dynamics" were created to explain the spread of viruses through infected cells in a specific person's body [4,5]. It is clinically relevant to establish methods which can be used either between successive consultations or to forecast evolution or disease progression for a more detailed explanation of individual times [6].

Several researchers and mathematicians have shown that mathematical integration models reflect the very formal representation of natural realities by fractional extensions [7–14]. Caputo and Fabrizio recently gave a non-integer-order idea detail given in [15]. According to different mathematical modeling for HIV/AIDS spread elements were arranged in pervious researches. Global stability of equilibrium point for scientific models of HIV/AIDS spread elements have been pondered by various creators. The plague model thinks an inactive stage and inoculation of new conceived and vulnerable. It is normal that the HIV encourge feasts both through horizontal and vertical transmission [16]. The new scheme just requires solving a sequence of linear fractional-order BVPs in [17] and mainly study the chaos control and synchronization of a hyperchaotic model in both the frameworks of classical and fractional calculus, respectively [18]. In [19] fractionalize the classical Lagrangian of the system, and then we obtain the corresponding fractional

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Euler–Lagrange equations (FELEs) and Caputo and the Atangana-Baleanu-Caputo (ABC) fractional derivatives are given to verify the theoretical analysis. The new fractional model with Mittag-Leffler kernel fits the real data better than the other fractional and integer-order models for HRSV with nonsingular derivative operator is proposed in [20]. Recently, Toufik and Atangana [21] developed a numerical scheme to solve a nonlinear fractional differential equation considering the Atangana–Baleanu fractional derivative. This method is a combination of the fundamental fractional calculus theorem with two-step Lagrange polynomial which is successfully used to solve many real-world problems [22].

In this paper, we develop the system of complex nonlinear differential equations for HIV/AIDS epidemic model by using Caputo Fabrizio derivatives method for purposed system. In section 2 and 3 define some basic definition of fractional calculus and develop the fractional order model with Caputo Fabrizio derivative. Section 4 and 5 is the stability fractional order model was developed and the unique non-negative solution was tested. According to Banach, the related findings are given nonlinear functional analysis and fixed point theory and numerical simulations are performed using an iterative technique respectively.

2 Preliminaries

Definition 1. Let $\Theta \in H^1(c,d)$, $d > c$, $\Omega \in (0,1)$, then the new fractional order in Caputo derivative sense is as follows [15,23,24].

$$^{C}D_t^{\Omega}(\Theta(t)) = \frac{N(\Omega)}{1-\Omega}\int_c^t \Theta'(y)exp\left[-\Omega\frac{t-y}{1-\Omega}\right]dy,$$

where $N(\Omega)$ represents the normalization function with $N(0) = N(1) = 1$. But, if the function does not belong to $H^1(c,d)$, then the derivative can be write as

$$^{C}D_t^{\Omega}(\Theta(t)) = \frac{\Omega N(\Omega)}{1-\Omega}\int_c^t (\Theta(t)-\Theta(y))exp\left[-\Omega\frac{t-y}{1-\Omega}\right]dy.$$

Remark: If we take $\sigma = \frac{1-\Omega}{\Omega} \in [0,\infty]$, then the new Caputo derivative having fractional order as

$$^{C}D_t^{\Omega}(\Theta(t)) = \frac{N(\sigma)}{\sigma}\int_c^t \Theta'(y)exp\left[-\frac{t-y}{1-\sigma}\right]dy, N(0) = N(\infty) = 1$$

Further,

$$lim_{\sigma\to 0}\frac{1}{\sigma}exp[-\frac{t-y}{\sigma}] = \delta(y-t).$$

Definition 2. Let $\Omega \in (0,1)$, then the fractional integral of order Ω of a function $\Theta(t)$ is given by [23,24].

$$^{CF}I_t^{\Omega}(\Theta(t)) = \frac{2(1-\Omega)}{(2-\Omega)N(\Omega)} + \frac{2\Omega}{(2-\Omega)N(\Omega)}\int_0^t \Theta(s)ds, t \geq 0.$$

Remark: This definition indicates that the fractional integral of the Caputo form of function of order $0 < \Omega \leq 1$ is an average of Θ function and its integral of order one. Hence we get

$$\frac{2(1-\Omega)}{(2-\Omega)N(\Omega)} + \frac{2\Omega}{(2-\Omega)N(\Omega)} = 1.$$

This equation gives an explicit formula for

$$N(\Omega) = \frac{2}{2-\Omega}, 0 \leq \Omega \leq 1.$$

Assuming this derivative, the new Caputo derivative of order $0 < \Omega < 1$ has been reformulated as

$$^{C}D_t^{\Omega}(\Theta(t)) = \frac{1}{1-\Omega}\int_c^t \Theta'(y)exp\left[-\Omega\frac{t-y}{1-\Omega}\right]dy.$$

Definition 3. Let $\Theta(t)$ be a function for which the Caputo-Fabrizio exists, then the Sumudu transform of the Caputo-Fabrizio fractional derivative of $\Theta(t)$ is given as [23].

$$\mathscr{S}(^{CF}_{0}D_t^{\Omega})(\Theta(t)) = M(\Omega)\frac{\mathscr{S}(\Theta(t)) - \Theta(0)}{1-\Omega+\Omega\Delta}$$

where the normalization function is denoted by $M(\Omega)$ with $M(0) = M(1) = 1$.

3 Fractional order HIV/ AIDS model

In this section, HIV/AIDS spread through variable population estimate in a reliably merger populace are presented with fractional order. Human population is divided into four different parts in an given system [4] susceptible people (S); HIV patients but with mild or no symptoms and they can transfer HIV to others (I); HIV-contaminated patients (C); and HIV patients having signs and symptoms (A). The aggregate populace at time t, indicated by $G(t)$, is specified by

$$G(t) = S(t) + I(t) + C(t) + A(t),$$

The viable contact with individuals tainted with HIV is at a rate λ, given by $\lambda = \kappa(I + \eta_C C + \eta_A A)$. Here, the HIV assembly rate of diffusion is represented by κ. The parameter $\eta_A \geq 1$ tells about virtual indefatigable of people with AIDS side effect. Viral load and irresistibleness are inversely proportional to each other and people who are suffering from AIDS with side effects have more irresistible as compared to pre-AIDS patients thus have more viral load is present in people have AIDS with side effects [25]. On the other hand, $\eta_C \geq 1$ tells about HIV disease that is treated with ART [26, 27]. ART treatment can used to treat patients suffering from AIDS with side effects. HIV-contaminated people without any symptoms of AIDS I, development to the session of people with HIV disease with ART cure, C at rate ϕ, and people who have symptoms of HIV with ART cure at rate α. Patients of HIV with symptoms are denoted by A, that begin action, moving period for tainted people I. HIV- patients without any symptoms of AIDS, I , that don't continue with cure of ART, development to the AIDS period A at rate ρ. Accept just tainted people through side effects, A, anguish as of an AIDS influenced demise, at a rate d. The classical model in [26], will modified as fractional order HIV/AID model with Caputo-Fabrizio derivative and fractional order β such as $\beta \in (0,1)$ is given as

$$ {}_0^{CF}D_t^{\beta} S(t) = \wedge - \kappa(\eta_c C + \eta_A A)S(t) - \mu S \tag{1}$$

$$ {}_0^{CF}D_t^{\beta} I(t) = \kappa(I + \eta_c C + \eta_A A)S - (\rho + \phi + \mu)I + \omega C + \alpha A \tag{2}$$

$$ {}_0^{CF}D_t^{\beta} C(t) = \phi I - (\omega + \mu)C \tag{3}$$

$$ {}_0^{CF}D_t^{\beta} A(t) = \rho I - (\alpha + \mu + d)A \tag{4}$$

with given initial condition $S(t) = S_0 \geq, I(t) = I_0 \geq, C(t) = C_0 \geq, A(t) = A_0 \geq$, and $\beta \in (0,1]$ is used as fractional order parameters in the system.

3.1 Qualitative analysis

If the population has not been diagnosed with HIV/AIDS, the population will then remain free of the disease. It means that the whole population is susceptible to the infection. There are no infectious people in this way. So, the model (1)-(4) has a disease-free equilibrium, provided to it

$$\Gamma = (S, I, C, A) = (\frac{\wedge}{\mu}, 0, 0, 0) = (140, 0, 0)$$

and endemic point

$$\Gamma^* = (S^*, I^*, C^*, A^*) = (\frac{M}{N}, \frac{p_1 p_2(\wedge N - \mu M)}{MN}, \frac{p_1 \phi(\wedge N - \mu M)}{MN}, \frac{p_2 \rho(\wedge N - \mu M)}{MN})$$

$$= (76.582, 3.99593, 38.3171, 0.297253).$$

Where $M = \mu(p_2(\rho + p_1) + \phi p_1 + \rho d) + \rho \omega d, p_1 = \alpha + \mu + d, p_2 = \omega + \mu$, and $N = \kappa(p_2(\rho \eta_A + p_1) + \eta_c \phi p_1$. If there is a positive solution to $R_0 > 1$, however, that a disease-persistent equilibrium will occur only when $R_0 < 1$ and endemic point when $R_0 > 1$has been reached according to the model parameter used in the system. Hence the reproductive number of the system is

$$R_0 = \frac{S(\kappa(p_2(\rho \eta_A + p_1) + \eta_c \phi p_1)}{\mu(p_2(\rho + p_1) + \phi p_1 + \rho d) + \rho \omega d}$$

Theorem 3.1: The solution to the initial value problem given by $(1-4)$ remains unique while $R_4^+, w \geq 0$ still remains the solution.

Proof: The uniqueness and existence for the solution of $(1-4)$, in $(0,a)$. Our aim is to show the domain $R^4, w \geq 0$ is positively invariant. Since

$${}_0^{CF}D_t^{\beta} S|_{S=0} = \wedge \geq 0$$

$$D_0^{CF}D_t^{\beta} I|_{I=0} = \kappa(\eta_c C + \eta_A A)S + \omega C + \alpha A \geq 0$$

$${}_0^{CF}D_t^{\beta} C|_{C=0} = \phi I \geq 0$$

$${}_0^{CF}D_t^{\beta} A|_{A=0} = \rho I \geq 0$$

The vector field points were satisfied by the non-negative solution in R_4^+.

3.2 Caputo Fabrizio technique with Sumudu transform

Apply Caputo-Fabrizo derivative and fractional order β such that $\beta \in [0,1]$

$$M(\beta)\frac{S(S(t)-S(0))}{1-\beta+\beta s} = S[\wedge - \kappa + \eta_c C + \eta_A A)S(t) - \mu S]$$

$$M(\beta)\frac{S(I(t)-I(0))}{1-\beta+\beta s} = S[\kappa(I + \eta_c C + \eta_A A)S - (\rho + \phi + \mu)I + \omega C + \alpha A]$$

$$M(\beta)\frac{S(C(t)-C(0))}{1-\beta+\beta s} = S[\phi I - (\omega + \mu)C]$$

$$M(\beta)\frac{S(A(t)-A(0))}{1-\beta+\beta s} = S[\rho I - (\alpha + \mu + d)A]$$

after simplification, we get

$$S(S(t)) = S(0) + \frac{1-\beta+\beta s}{M(\beta)}S[\wedge - \kappa + \eta_c C + \eta_A A)S(t) - \mu S]$$

$$S(I(t)) = I(0) + \frac{1-\beta+\beta s}{M(\beta)}S[\kappa(I + \eta_c C + \eta_A A)S - (\rho + \phi + \mu)I + \omega C + \alpha A]$$

$$S(C(t)) = C(0) + \frac{1-\beta+\beta s}{M(\beta)}S[\phi I - (\omega + \mu)C]$$

$$S(A(t)) = A(0) + \frac{1-\beta+\beta s}{M(\beta)}S[\rho I - (\alpha + \mu + d)A]$$

The following recursive formula is given

$$S\{S_{k+1}\} = \frac{1-\beta+\beta s}{M(\beta)}[-\kappa(S\{D_k\} + \eta_c\{E_k\} + \eta_A S\{F_k\}) - \mu S\{S_k\}] \tag{5}$$

$$S\{I_{k+1}(t)\} = \frac{1-\beta+\beta s}{M(\beta)}[-\kappa(S\{D_k\} + \eta_c\{E_k\} + \eta_A S\{F_k\}) - (\rho + \phi + \mu)S\{I_k\} + \omega S\{C_k\} + \alpha S\{A_k\}] \tag{6}$$

$$S\{C_{k+1}\} = \frac{1-\beta+\beta s}{M(\beta)}[\phi S\{I_k\} - (\omega + \mu)S\{C_k\})] \tag{7}$$

$$S\{A_{k+1}\} = \frac{1-\beta+\beta s}{M(\beta)}[\rho S\{I_k\} - (\alpha + \mu + d)S\{A_k\})] \tag{8}$$

and the solution of system is given by

$$S(t) = lim_{n\to\infty}S_n(t), I(t) = lim_{n\to\infty}I_n(t)$$

$$C(t) = lim_{n\to\infty}C_n(t), A(t) = lim_{n\to\infty}A_n(t)$$

4 Stability and fixed point theorem analysis as application

Theorem 5.1: Let $(Z_1, \|.\|)$ be a Banach space and P be a self-map of Z_1 satisfying $\|P_z - P_y\| \leq C\|z - P_z\| + c\|z - y\|$ for all z, y$\in Z_1$ where $0 \leq C, 0 \leq c < 1$. Suppose that P is picard P-stable. Let the following recursive formula is

$$S_{(n+1)}(t) = S_n(0) + S^{-1}[\frac{1-\beta+\beta t}{G(\nu)} S[(\wedge - \kappa I + \eta_c C + \eta_A A)S - \mu S]]$$

$$I_{(n+1)}(t) = I_n(0) + S^{-1}[\frac{1-\beta+\beta t}{G(\nu)} S[\kappa(I + \eta_c C + \eta_A A)S - (\rho + \phi + \mu)I + \omega C + \alpha A]]$$

$$C_{(n+1)}(t) = C_n(0) + S^{-1}[\frac{1-\beta+\beta t}{G(\nu)} S[\phi I - (\omega + \mu)C]]$$

$$A_{(n+1)}(t) = A_n(0) + S^{-1}[\frac{1-\beta+\beta t}{G(\nu)} S[\phi I - (\rho I - (\alpha + \mu + d)A]]$$

Where $\frac{1-\beta+\beta t}{G(\nu)}$ is the fractional Lagrange multiplier. We defined a self-map as $'p'$ and is P-stable in $L^1(a,b)$ if

$$\{1 + f(q) - ag(q) + Mh(q) + LI(q)\} < 1$$

$$\{1 - bf(q) - (M + M_1)g_1(q)\} < 1 \tag{9}$$

$$\{1 - f_2(q) - cg_2(q)\} < 1$$

Proof:
We'll demonstrate that P has fixed point in the first phase, for which we will assess the following

$$P(S_n(t)) - P(S_m(t)) = S_n(t) - S_m(t) + S^{-1}[\frac{1-\beta+\beta t}{G(\nu)} S[(\wedge - \kappa I_n + \eta_c C_n + \eta_A A_n)S_n - \mu S_n]]$$

$$-S^{-1}[\frac{1-\beta+\beta t}{G(\nu)} S[(\wedge - \kappa I_m + \eta_c C_m + \eta_A A_m)S_m - \mu S_m]] \tag{10}$$

$$P(I_n(t)) - P(I_m(t)) = I_n(t) - I_m(t) + S^{-1}[\frac{1-\beta+\beta t}{G(\nu)} S[\kappa I_n + \eta_c C_n + \eta_A A_n)S_n - (\rho + \phi + \mu)I_n$$

$$+\omega C_n + \alpha A_n]] - S^{-1}[\frac{1-\beta+\beta t}{G(\nu)} S[\kappa(I_m + \eta_c C_m + \eta_A A_m)S_m - (\rho + \phi + \mu)I_m + \omega C_m + \alpha A_m]] \tag{11}$$

$$P(C_n(t)) - P(C_m(t)) = C_n(t) - C_m(t) + S^{-1}[\frac{1-\beta+\beta t}{G(\nu)} S[\phi I_n - (\omega + \mu)C_n]]$$

$$-S^{-1}[\frac{1-\beta+\beta t}{G(\nu)} S[\phi I_m - (\omega + \mu)C_m]]$$

$$P(A_n(t)) - P(A_m(t)) = A_n(t) - A_m(t) + S^{-1}[\frac{1-\beta+\beta t}{G(\nu)} S[\rho I_n - (\alpha + \mu + d)A_n]]$$

$$-S^{-1}[\frac{1-\beta+\beta t}{G(\nu)} S[\rho I_m - (\alpha + \mu + d)A_m]]$$

Taking norm on both sides of equation (10), we get,

$$||P(S_n(t)) - P(S_m(t))|| = ||S_n(t) - S_m(t)|| + ||S^{-1}[\frac{1-\beta+\beta t}{G(\nu)} S[(\wedge - \kappa I_n + \eta_c C_n + \eta_A A_n)S_n - \mu S_n]]$$

$$-S^{-1}[\frac{1-\beta+\beta t}{G(\nu)} S[(\wedge - \kappa I_m + \eta_c C_m + \eta_A A_m)S_m - \mu S_m]]|| \tag{12}$$

by making use of triangular inequality ,

$$||P(S_n(t))-P(S_m(t))|| \leq ||S_n(t)-S_m(t)|| + ||S^{-1}[\frac{1-\beta+\beta t}{G(\nu)}S[(\wedge-\kappa I_n+\eta_c C_n+\eta_A A_n)S_n-\mu S_n]]$$

$$-S^{-1}[\frac{1-\beta+\beta t}{G(\nu)}S[(\wedge-\kappa I_m+\eta_c C_m+\eta_A A_m)S_m-\mu S_m]]|| \quad (13)$$

Further simplification,

$$||P(S_n(t))-P(S_m(t))|| \leq ||S_n(t)-S_m(t)|| + S^{-1}[\frac{1-\beta+\beta t}{G(\nu)}[\wedge-||\beta S_n(I_n-I_m)||+||\beta I_m(S_n-S_m)||$$

$$+||\eta_c S_n(C_n-C_m)||+||\eta_c C_m(S_n-S_m)||+||\eta_A S_n(A_n-A_m)||+||\eta_A A_m(S_n-S_m)||-\mu||S_n-S_m||]] \quad (14)$$

Since solutions plays the same role, in this situation we'll suppose that

$$||S_n(t)-S_m(t)|| \cong ||I_n(t)-I_m(t)||$$

$$||S_n(t)-S_m(t)|| \cong ||C_n(t)-C_m(t)||$$

$$||S_n(t)-S_m(t)|| \cong ||A_n(t)-A_m(t)||$$

By putting this in equation (14),we have

$$||P(S_n(t))-P(S_m(t))|| \leq ||S_n(t)-S_m(t)|| + S^{-1}[\frac{1-\beta+\beta t}{G(\nu)}[\wedge-||\beta S_n(S_n-S_m)||+||\beta I_m(S_n-S_m)||$$

$$+||\eta_c S_n(S_n-S_m)||+||\eta_c C_m(S_n-S_m)||+||\eta_A S_n(S_n-S_m)||+||\eta_A A_m(S_n-S_m)||-\mu||S_n-S_m||]] \quad (15)$$

Since S_n, S_m, I_m, C_m, A_m are bounded and they are convergent, so we get M,M_1 L, N and Q for all t like

$$||S_n|| < M, ||S_m|| < M_1, ||I_m|| < L, ||C_m|| < N, ||A_m|| < Q \quad (16)$$

we have

$$||P(S_n(t))-P(S_m(t))|| \leq 1-\beta M f(q)-\beta L g(q)-\beta\eta_c M h(q)-\beta\eta_c N j(q)-\beta\eta_A M k(q)$$

$$-\beta\eta_A Q r(q)-\mu v(q)||S_n-S_m|| \quad (17)$$

Where f,g,h,j,k,r and u are functions from $S^{-1}[\frac{1-\beta+\beta t}{G(\nu)}]$. In the same path, we obtain

$$||P(I_n(t))-P(I_m(t))|| \leq 1+\beta M f_1(q)+\beta L g_1(q)+\beta\eta_c M h_1(q)+\beta\eta_c N j_1(q)+\beta\eta_A M k_1(q)$$

$$+\beta\eta_A Q r_1(q)-(\rho+\phi+\nu)v_1(q)+\omega x(q)+\alpha y(q)||I_n-I_m|| \quad (18)$$

$$||P(C_n(t))-P(C_m(t))|| \leq 1+\phi f_2(q)-(\omega+\nu)g_2(q)||C_n-C_m|| \quad (19)$$

$$||P(A_n(t))-P(A_m(t))|| \leq 1+\rho f_3(q)-(\alpha+\nu+d)g_3(q)||A_n-A_m|| \quad (20)$$

where

$$1-\beta M f(q)-\beta L g(q)-\beta\eta_c M h(q)-\beta\eta_c N j(q)-\beta\eta_A M k(q)-\beta\eta_A Q r(q)-\mu v(q) \leq 1 \quad (21)$$

$$1+\beta M f_1(q)+\beta L g_1(q)+\beta\eta_c M h_1(q)+\beta\eta_c N j_1(q)+\beta\eta_A M k_1(q)$$

$$-(\rho+\phi+\nu)v_1(q)+\omega y(q)+\alpha x(q) \leq 1 \quad (22)$$

$$1+\phi f_2(q)-(\omega+\nu)g_2(q) \leq 1 \quad (23)$$

$$1+\rho f_3(q)-(\alpha+\nu+d)g_3(q) \leq 1 \quad (24)$$

Thus the self-mapping of non-linear p has a fixed point and stable.

4.1 Uniqueness of the special solution

Theorem 5.2: Here, by using iteration method we will show that the uniqueness of special solution of system (1). we suppose that system (1) has an exact solution by which, the special solution converges for a large number m.Hilbert space $H = L(a,b)\times(0,T)$ defined by $y:(a,b)\times(0,T)\longrightarrow$ R, such that $\int\int uydudy < \infty$. We have to consider the operator below:

$$P(x,y,z) = \begin{array}{r}(\wedge-\kappa+\eta_c C+\eta_A A)S(t)-\mu S\\ \kappa(I+\eta_c C+\eta_A A)S-(\rho+\phi+\mu)I+\omega C+\alpha A\\ \phi I-(\omega+\mu)C\ \rho I-(\alpha+\mu+d)A\end{array}$$

Proof: The purpose of this portion is to demonstrate the inner product of

$P((Z_{11}-Z_{12},Z_{21}-Z_{22},Z_{31}-Z_{32},Z_{41}-Z_{42})(w_1,w_2,w_3,w_4))$

where $(Z_{11}-Z_{12})$, $(Z_{21}-Z_{22})$,$(Z_{31}-Z_{32})$,$Z_{41}-Z_{42}$ are special solution of system.

$P((Z_{11}-Z_{12},Z_{21}-Z_{22},Z_{31}-Z_{32},Z_{41}-Z_{42})(w_1,w_2,w_3,w_4))$

$$=[-\beta[(Z_{21}-Z_{22})+\eta_c(Z_{31}-Z_{32})+\eta_A(Z_{41}-Z_{42})](Z_{12}-Z_{22})-\nu(Z_{11}-Z_{12}),w_1] \tag{25}$$

$$[\beta[(Z_{21}-Z_{22})+\eta_c(Z_{31}-Z_{32})+\eta_A(Z_{41}-Z_{42})](Z_{12}-Z_{22})-(\rho+\phi+\mu)(Z_{21}-Z_{22})$$

$$+\omega(Z_{31}-Z_{32})+\alpha(Z_{41}-Z_{42}),w_2]$$

$$[\phi+(Z_{21}-Z_{22})-(\omega+\nu)(Z_{31}-Z_{32}),w_3],[\rho(Z_{21}-Z_{22})-(\alpha+\nu+d)(Z_{41}-Z_{42}),w_4] \tag{26}$$

We shall evaluate the first equation in the system without loss of generality

$$[-\beta[(Z_{21}-Z_{22})+\eta_c(Z_{31}-Z_{32})+\eta_A(Z_{41}-Z_{42})](Z_{12}-Z_{22})-\nu(Z_{11}-Z_{12}),w_1]$$

$$\cong[-\beta(Z_{11}-Z_{12})(Z_{12}-Z_{22}),w_1]+[-\beta\eta_c(Z_{11}-Z_{12})(Z_{31}-Z_{32}),w_1]+$$

$$[-\beta\eta_A(Z_{11}-Z_{12})(Z_{41}-Z_{42}),w_1]+[-\nu(Z_{11}-Z_{12}),w_1]$$

assume that

$$(Z_{11}-Z_{12})\cong(Z_{12}-Z_{22})\cong(Z_{31}-Z_{32})\cong(Z_{41}-Z_{42})$$

We have

$$\cong[-\beta(Z_{11}-Z_{12})^2,w_1]+[-\kappa\eta_c(Z_{11}-Z_{12})^2,w_1]+-\beta\eta_A(Z_{11}-Z_{12})^2,w_1]$$

$$+[-\mu(Z_{11}-Z_{12}),w_1] \tag{27}$$

We obtain the following based on the connection between norm and inner product and repeating the same fusion, we have

$$\beta(Z_{21}-Z_{22})^2+\beta\eta_c(Z_{21}-Z_{22})^2+\kappa\eta_A(Z_{21}-Z_{22})^2-(\rho+\phi+\mu)(Z_{21}-Z_{22})$$

$$+\omega(Z_{21}-Z_{22})+\alpha(Z_{21}-Z_{22}),w_2$$

$$\leq(\beta\bar{w_2}+\kappa\eta_c\bar{w_2}+\kappa\eta\bar{w_2}-(\rho+\phi+\mu)+\omega+\alpha)||Z_{21}-Z_{22}||||w_2|| \tag{28}$$

$$[\phi(Z_{21}-Z_{22})-(\omega+\mu)(Z_{31}-Z_{32}),w_3]\leq\phi-(\omega+\mu)||Z_{31}-Z_{32}||||w_3|| \tag{29}$$

$$[\rho(Z_{21}-Z_{22})-(\alpha+\mu+d)(Z_{41}-Z_{42}),w_4]\leq\rho-(\alpha+\mu+d)||Z_{41}-Z_{42}||||w_4|| \tag{30}$$

hence we have

$$\leq(1-\kappa\bar{w_2}-\kappa\eta_c\kappa w_2-\kappa\eta\bar{w_2}-\mu)||Z_{11}-Z_{12}||||w_1||$$

$$\leq(\tag{31}$$

$$kappa\bar{w_2}+\kappa\eta_c\bar{w_2}+\kappa\eta\bar{w_2}-(\rho+\phi+\mu)+\omega+\alpha)||Z_{21}-Z_{22}||||w_2|| \tag{32}$$

$$\leq\phi-(\omega+\mu)||Z_{31}-Z_{32}||||w_3||$$

$\rho-(\alpha+\mu+d)||Z_{41}-Z_{42}||||w_4||$

But for sufficiently large values of M with $i=1,2,3$. Both solutions converge to the exact solution using the topological concept, three tiny non-negative parameters lm_1, lm_2, lm_3, lm_4 exist, such that

$$||S-Z_{11}||,||S-Z_{12}|| < \frac{lm_1}{(1-\kappa\bar{w_2}-\kappa\eta_c\bar{w_2}-\kappa\eta\bar{w_2}-\mu)||Z_{11}-Z_{12}||||w_1||}$$

$$||I-Z_{21}||,||I-Z_{22}|| < \frac{lm_2}{(\kappa\bar{w_2}+\kappa\eta_c\bar{w_2}+\kappa\eta\bar{w_2}-(\rho+\phi+\mu)+\omega+\alpha)||Z_{21}-Z_{22}||||w_2||}$$

$$||C-Z_{31}||,||C-Z_{32}|| < \frac{lm_3}{\phi-(\omega+\mu)||Z_{31}-Z_{32}||||w_3||}$$

$$||A-Z_{41}||,||A-Z_{42}|| < \frac{lm_4}{\rho-(\alpha+\mu+d)||Z_{41}-Z_{42}||||w_4||}$$

Connecting the exact solution and implementing the triangular inequality by taking

$$M = max(m_1, m_2, m_3, m_4)$$
$$l = max(lm_1, lm_2, lm_3, lm_4)$$

$$(1-\kappa\bar{w_2}-\kappa\eta_c\bar{w_2}-\kappa\eta\bar{w_2}-\mu)||Z_{11}-Z_{12}||||w_1|| < 1$$

$$(\kappa\bar{w_2}+\kappa\eta_c\bar{w_2}+\kappa\eta\bar{w_2}-(\rho+\phi+\mu)+\omega+\alpha)||Z_{21}-Z_{22}||||w_2|| < 1 \quad (33)$$

$$\phi-(\omega+\mu)||Z_{31}-Z_{32}||||w_3|| < 1$$

$$\rho-(\alpha+\mu+d)||Z_{41}-Z_{42}||||w_4|| < 1$$

As we know l is small positive parameter, we obtain

$$(1-\kappa\bar{w_2}-\kappa\eta_c\bar{w_2}-\kappa\eta\bar{w_2}-\mu)||Z_{11}-Z_{12}||||w_1|| < 0$$

$$(\kappa\bar{w_2}+\kappa\eta_c\kappa w_2+\kappa\eta\bar{w_2}-(\rho+\phi+\mu)+\omega+\alpha)||Z_{21}-Z_{22}||||w_2|| < 0$$

$$\phi-(\omega+\mu)||Z_{31}-Z_{32}||||w_3|| < 0$$

$$\rho-(\alpha+\mu+d)||Z_{41}-Z_{42}||||w_4|| < 0$$

But it is obvious that

$$(1-\kappa\bar{w_2}-\kappa\eta_c\kappa w_2-\kappa\eta\bar{w_2}-\mu)||Z_{11}-Z_{12}||||w_1|| \neq 0$$

$$(\kappa\bar{w_2}+\kappa\eta_c\bar{w_2}+\kappa\eta\bar{w_2}-(\rho+\phi+\mu)+\omega+\alpha)||Z_{21}-Z_{22}||||w_2|| \neq 0$$

$$\phi-(\omega+\mu)||Z_{31}-Z_{32}||||w_3|| \neq 0$$

$$\rho-(\alpha+\mu+d)||Z_{41}-Z_{42}||||w_4|| \neq 0$$

So we've got,

$$||Z_{11}-Z_{12}||=0,\ \ ||Z_{21}-Z_{22}||=0,\ \ ||Z_{31}-Z_{32}||=0,\ \ ||Z_{41}-Z_{42}||=0$$

Which accept that

$$Z_{11}=Z_{12},\ \ Z_{21}=Z_{22},\ \ Z_{31}=Z_{32},\ \ Z_{41}=Z_{42}$$

We have finished the proof in this manner.

5 Result and discussion

Non-linear occurrence with mathematical analysis of outbreak of HIV/AIDS has been presented. HIV/AIDS model effected parameters can measured by changing the value of parameters for disease free and endemic equilibrium point. Model consists of four subcomponent which are S(t), I(t), C(t) and A(t)and parameters values in [17, 18]. By using Caputo Fabrizio fractional derivative, the numerical outcomes of the model for different fractional values of β are obtained according to equilibrium point. Figures (1-4) represents the graphical solution of Caputo Fabrizio derivative of HIV/AIDS transmission for disease free equilibrium point. Figures (5-8) represents the numerical solution for endemic equilibrium at different fractional value of β. We observed in the Figures that susceptible start to increase by decreasing the fractional value while the infected start to decrease. It has also been observed that HIV-touched people with no restorative signs of AIDS, transfer HIV to different people, chronic stage with short-lived epidemiological contents and HIV-touched people with AIDS medicinal signs have increased. From figures by taking non-integer fractional parameter values, remarkable responses are acquired from the compartments of the developed model. Different fractional parameter values are estimated for endemic and disease free point. we can observe impact of estimated fractional parameter value in all figures with respect to time where the solution lay in feasible reason. We can control and check dynamical behaviour of disease through estimated parameter values. These results also shown better conversion with respect to time and can develop control strategy to overcome risk of HIV/AIDS in the society. The endemic of HIV/AIDS is increases day by day. we can control spread of HIV/AIDS in society with the help of our results. Treatment strategy on obtained results are applied to make HIV/AIDS free society. Numerical results indicate that system keeps the chaotic motion for β. Following is the table of parameter values which are used in model

Table 1: Parameters and it's values for system

Parameter	Value	Parameter	Value
μ	1/70	α	0.33
$\wedge$	2	ϕ	1
β	0.001	ρ	0.1
η_C	0.04	ω	0.09
η_A	1.3	d	1

6 Conclusion

On the fractional order HIV/AIDS transmission system, according to the steady state the Summudo transformation technique was used to obtain the necessary solution. The efficiency of the proposed scheme is provided by performing convergence analysis. The fractional parameter is effected by our alternatives that are displayed in graphs. It is essential to note that significant modifications and memory impacts in fractional derivatives can be seen in comparison with ordinary derivatives. Stability of the fractional HIV/AIDS is tested. It is proved that the HIV/AIDs transmission should meet the quadratic form that can be interpreted as the preservation of HIV instruments. In addition, a system of HIV/AIDS state feedback is intended. To demonstrate the effectiveness of the used technique, we use the fixed-point theorem to explore the method's stability evaluation to solve new equation. This model will assist the public health planar in framing a HIV/AIDS control policy. In addition, we will expand the model incorporating determinist and stochastic model comparisons with fractional technique, as well as using optimal control theory for new outcomes. We can measure estimated paramet4er value in graphs with respect to time. These results are also helpful to overcome effects of HIV/AIDS in our society.

Conflicts of Interests
The authors declare that they have no conflicts of interests.

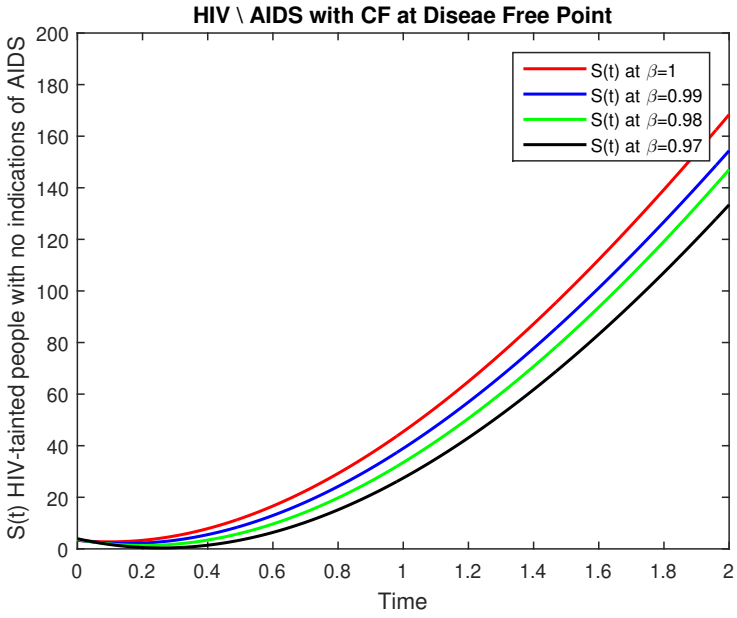

Figure 1. $S(t)$ **in time (t) with Caputo Fabrizio Derivatives at different values of** β.

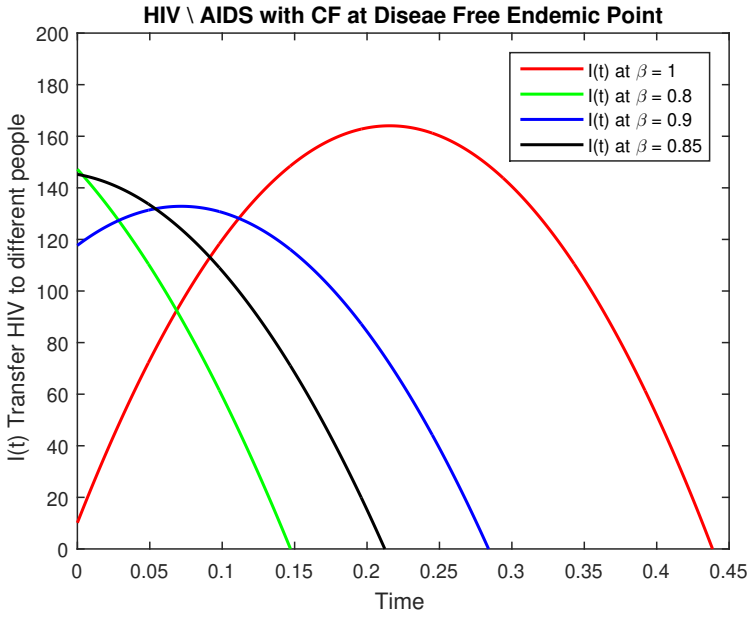

Figure 2.$I(t)$ **in time (t) with Caputo Fabrizio Derivatives at different values of** β.

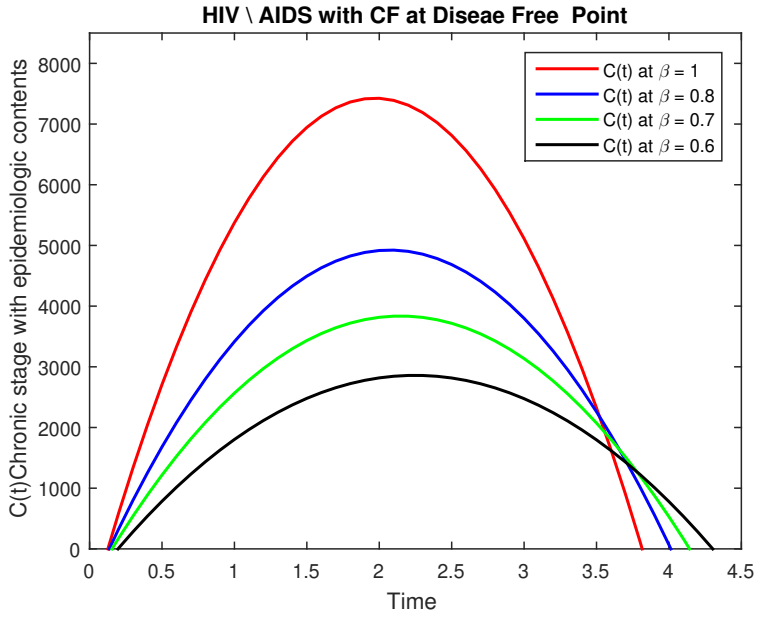

Figure 3.$C(t)$ **in time (t) with Caputo Fabrizio Derivatives at different values of** β.

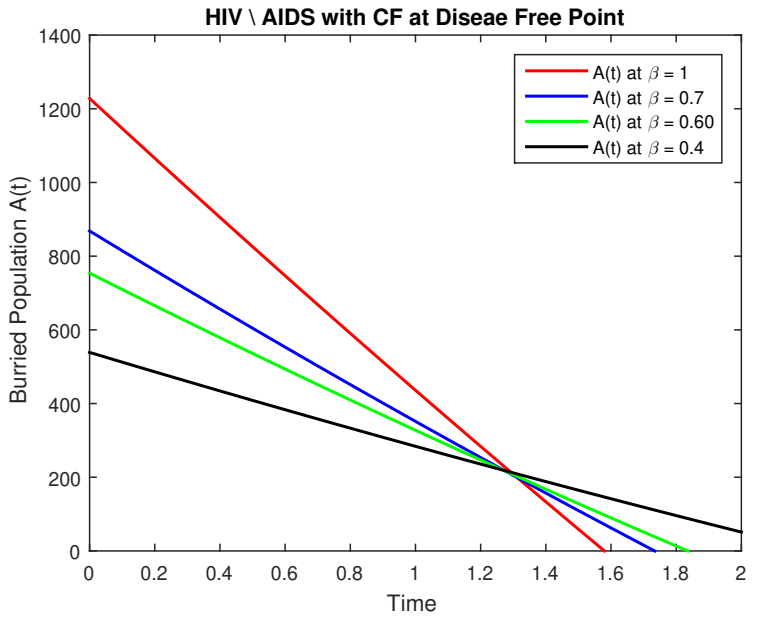

Figure 4.$A(t)$ **in time (t) with Caputo Fabrizio Derivatives at different values of** β.

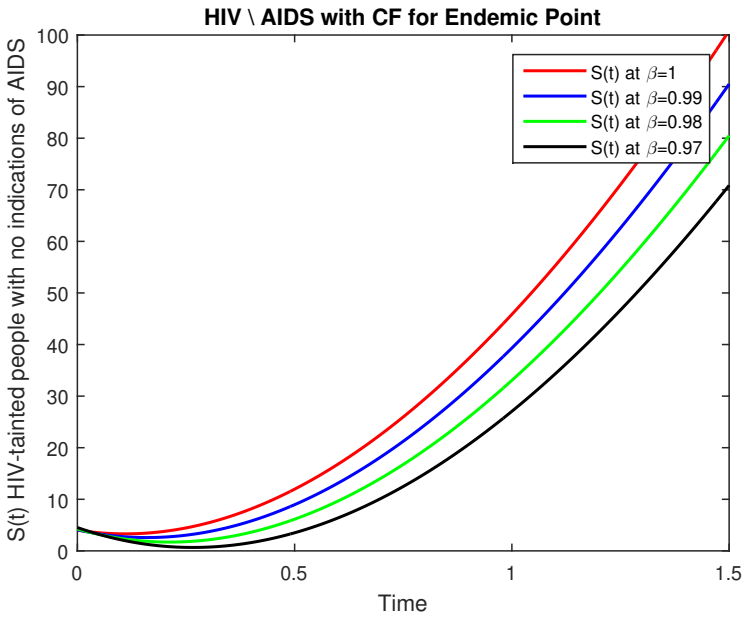

Figure 5. $S(t)$ **in time (t) with Caputo Fabrizio Derivatives at different values of** β.

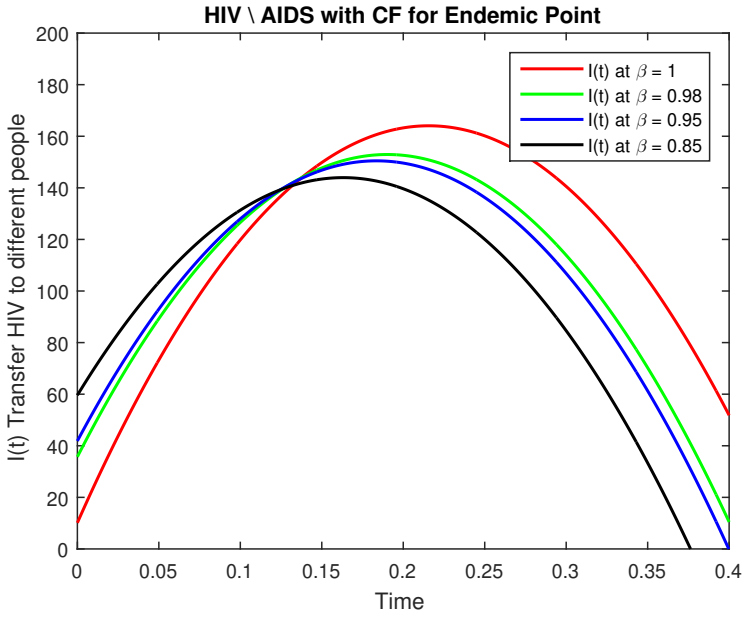

Figure 6. $I(t)$ **in time (t) with Caputo Fabrizio Derivatives at different values of** β.

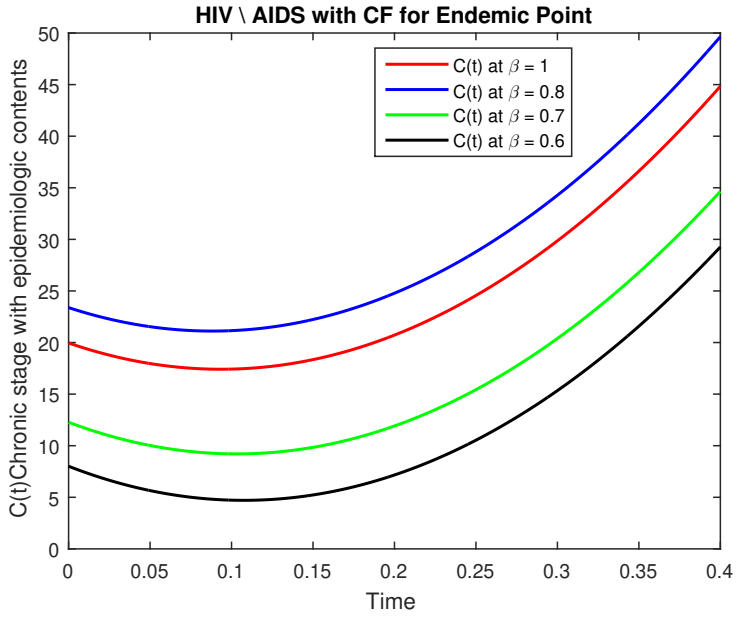

Figure 7. $C(t)$ **in time (t) with Caputo Fabrizio Derivatives at different values of** β.

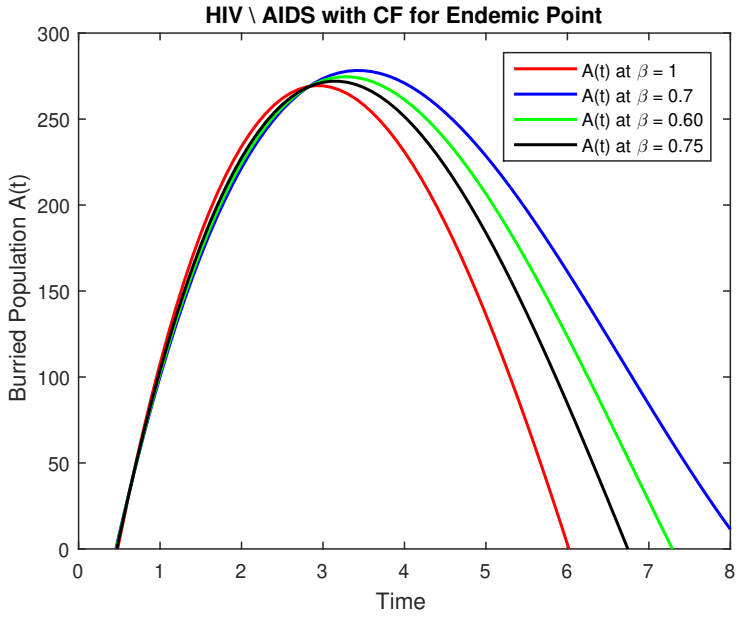

Figure 8. $A(t)$ **in time (t) with Caputo Fabrizio Derivatives at different values of** β.

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