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Analysis and Modeling of HIV Dynamical Transmission

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Abstract: In this article, HIV fractional order model is analyzed to reduce its effect on community and for control strategy. Verify the unique solutions of the proposed system as well as proved the stability analysis. Fractional order system is solved by using the Caputo fractional derivative operator $\beta \in (0,1]$ to check the effect of fractional parameter. Simulations are made to check the actual behavior of the HIV disease in the society. Such kind of analysis help to understand the outbreak of HIV and for future control strategy.

Keywords: Dynamical transmission, Caputo fractional derivative, HIV, stability analysis.

1 Introduction

Human immunodeficiency virus is a worldwide lentivirus-related health problem which causes HIV infection. It is one of the world's most well-researched contagious diseases. If an infected person does not use HIV medications, this person is $9-11$ years old in total survival time [1]. Two type HIV-1 and HIV-2 are separated. The originally discovered HIV-1 virus is thought to mostly infect HIV worldwide [2]. In the late 1980s, soon after the virus itself was discovered, the first model for HIV infection among individual persons was created. Such models were based upon the bread-and-butter models developed by Kendrick and McCormack in the early 1900s in statistical epidemiologies such as SIS and SIR models, which are used widely for describing the spread of infections in a single population [3]. In comparison, model "viral dynamics" was created to explain the spread of viruses through infected cells in a specific person's body [4]. In comparison, model "viral dynamics" were created to explain the spread of viruses through infected cells in a specific person's body [5]. In these types of cases it is clinically applicable to inaugurate methods which can be used between successive consultations as well as prognostication evolution for individuals [6]. Many researchers and mathematicians have proven that the different mathematical integration models reproduce the proper representation of normal realities by fractional additions [7, 8, 9, 10, 11, 12, 13]. Caputo and Fabrizio provide idea about non-integer-order [14]. Non integral order derivative used in diabetes patients [15]. The combination of Laplace and adomian decomposition method leads strong process and is helpful for both deterministic and stochastic differential equations as well as linear and non-linear and also for integer order and non-integer order differential equations. In addition, it does not need predefined steps such as $RK4$. This approach doesn't also depend on a parameter such as (HPM) and (HAM) homotopy analysis technique. While the solutions found for this strategy are the same as ADM's [16, 17]. A new fractional derivative with a non-singular kernel using exponential and trigonometric functions is studied in [18] and some related new new work for epidemic models are illustrated here [19, 20].

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In this paper, fractional order HIV model is analyzed using the Caputo fractional derivative. Laplace with Adomian Decomposition Method is used to solve the fractional order HIV model. Also numerical simulations are carried out to check the actual behavior of HIV outbreak in the community as well as show results in tabular form.

2 Preliminaries

Definition 1. The order of Riemann-Liouville integral operator with $\kappa \geq 0$ $f \in C_\kappa$ is written as follows

$$D_t^\kappa g(y) = \frac{1}{\Gamma(\kappa)} \int_t^y (y-t)^{\kappa-1} g(t)dt, \quad \text{where} \quad y > \kappa,$$

$$D_t^0 g(y) = g(y),$$

The fractional derivative of Caputo [13-15] is presented as

$${}_{t_0}^{C}D_t^\kappa\{g(t)\} = \frac{1}{\Gamma(1-\kappa)} \int_{t_0}^t \frac{d}{d\psi} g(\psi)(t-\psi)^{-\kappa} d\psi,$$

where $\Gamma(.)$ refers to the function of Gamma. Laplace transform of the above derivative is obtained as

$$\mathscr{L}\{{}_0^{C}D_t^\kappa\{g(t)\}\}(s) = S^\kappa G(s) - \sum_{k=0}^{m-1} S^{\kappa-k-1} g^{(k)}(0).$$

3 Mathematical model of HIV

The model is made up by susceptible class A and infection classes B, and is constructed in four separate sections, infected with lower plasma viral class $C(t)$, infected by upper plasma viral $D(t)$, infected individual treatment class 1 $E(t)$ and infected individual therapy Class 2 $F(t)$, respectively. The natural rate of removal μ is the death rate, while AIDS caused death rates in classes D and F are defined by δ_1 and δ_2. $\alpha_1$and α_2 on a diagnosis are respectively the rate of progression from B to C and the rate from B to D. The C to D progression rate shall be η_1. The diagnosis rates between C and E are $\gamma_1$and γ_2, respectively, from D to F. The progression between these immunological classes is indicated by η_2 and η_3 respectively [21].
We consider a $N(t)$ size population at time t is written as

$$N(t) = A(t) + B(t) + C(t) + D(t) + E(t) + F(t)$$

The rate of infection among susceptible persons is determined

$$\lambda = \frac{\beta_1 B + \beta_2 C + \beta_3 D + \beta_4 E + \beta_5 F}{N}$$

(i) The first $0 \leq \phi_1 \leq 1$intervention tests the efficacy of the efforts to protect people who are susceptible to infection. This modifies the terms of the infection given in above is following

$$\lambda_c = (1-\phi_1)\lambda$$

(ii) The $0 \leq \phi_2 \leq 1$ second interference reflects consulting and research activities. Hence we modify α_i to $a_i^c = \phi_2 \alpha_i$, for $i = 1,2$.
(iii) The 3rd treatment procedure of $0 \leq \phi_3 \leq 1$is efficient. The use of ARV drugs decreases primarily the risk of death. [22,23]. Thus, we are therefore modifying γ_i to $\gamma_i^c = \phi_3 \gamma_i$, for $i = 1,2$.

$$\frac{dA}{dt} = \pi - (\lambda_c + \mu)A, \tag{1}$$

$$\frac{dB}{dt} = \lambda_c A - ((\alpha_1 + \alpha_2)\phi_2 + \mu)B, \tag{2}$$

$$\frac{dC}{dt} = \phi_2\alpha_1 B - (\eta_1 + \phi_3\gamma_1 + \mu)C, \tag{3}$$

$$\frac{dD}{dt} = \phi_2\alpha_2 B + \eta_1 C - (\phi_3\gamma_2 + \mu + \delta_1)D, \tag{4}$$

$$\frac{dE}{dt} = \phi_3\gamma_1 C + \eta_2 F - (\eta_3 + \mu)E, \tag{5}$$

$$\frac{dF}{dt} = \phi_3\gamma_2 D + \eta_3 E - (\eta_2 + \mu + \delta_2)F, \tag{6}$$

with initial conditions

$$A(0) = 800,000, B(0) = 50,000, C(0) = 10,000, D(0) = 25,000, E(0) = 5000, F(0) = 2000$$

The classical model of HIV is given in [21], we purposed the fractional order differential equation model for HIV is given as

$$D^\beta A(t) = \mathscr{L}\{\Pi\} - (\lambda_c + \mu)\mathscr{L}\{A\}, \tag{7}$$

$$D^\beta B(t) = \lambda_c\mathscr{L}\{A\} - ((\alpha_1 + \alpha_2)\phi_2 + \mu)\mathscr{L}\{B\}, \tag{8}$$

$$D^\beta C(t) = \phi_2\alpha_1\mathscr{L}\{B\} - (\eta_1 + \phi_3\gamma_1 + \mu)\mathscr{L}\{C\}, \tag{9}$$

$$D^\beta D(t) = \phi_2\alpha_2\mathscr{L}\{B\} + \eta_1\mathscr{L}\{C\} - (\phi_3\gamma_2 + \mu + \delta_1)\mathscr{L}\{D\}, \tag{10}$$

$$D^\beta E(t) = \phi_3\gamma_1\mathscr{L}\{C\} + \eta_2\mathscr{L}\{F\} - (\eta_3 + \mu)\mathscr{L}\{E\}, \tag{11}$$

$$D^\beta F(t) = \phi_3\gamma_2\mathscr{L}\{D\} + \eta_3\mathscr{L}\{E\} - (\eta_2 + \mu + \delta_2)\mathscr{L}\{F\} \tag{12}$$

3.1 Qualitative analysis

If the population has not been diagnosed with HIV, the population will then remain free of the disease. It means that the whole population is susceptible to the infection. There are no infectious people in this way. So, the model (7)-(12) has a disease-free equilibrium, provided to it

$$\Gamma = [\frac{\Pi}{\mu}, 0, 0, 0, 0, 0]$$

For the purposes of calculating the system's endemic equilibrium $(7)-(12)$, the RHS is equal to 0, and the infection force mentioned in $(1)-(6)$ is specified in [21]. $B_0 < 0$ If there is a positive solution to $R_0 > 1$, however, that a disease-persistent equilibrium will occur only when $R_0 > 1$ has been reached according to the model parameter in [21].

Theorem 1. The solution to the initial value problem given by $(7)-(12)$ remains unique while $R^6, w \geq 0$ still remains the solution.

Proof: The uniqueness and existence for the solution of $(7)-(12)$, in $(0,a)$. Our aim is to show the domain $R^5, w \geq 0$ is positively invariant. Since

$$D^\beta A(t)|_{A=0} = \pi \geq 0, \tag{13}$$

$$D^\beta B(t)|_{B=0} = \lambda_c A \geq 0, \tag{14}$$

$$D^\beta C(t)|_{C=0} = \phi_2\alpha_1 B \geq 0, \tag{15}$$

$$D^\beta D(t)|_{D=0} = \phi_2\alpha_2 B + \eta_1 C \geq 0, \tag{16}$$

$$D^\beta E(t)|_{E=0} = \phi_3\gamma_1 C + \eta_2 F \geq 0, \tag{17}$$

$$D^\beta F(t)|_{F=0} = \phi_3\gamma_2 D + \eta_3 E \geq 0, \tag{18}$$

The vector field points were satisfied by the non-negative solution in R^6+. The solution lie in feasible region and hold the condition for positive solution.

Theorem 2. The disease-free equilibrium ε_o of the system $(7-12)$ is globally asymptotically stable if $R_o < 1$ and unstable otherwise. The disease-free equilibrium ε_o is the only equilibrium when $R_o \leq 1$

Proof: Let

$$V = \xi_1 B + \xi_2 C + \xi_3 D + \xi_4 E + \xi_4 F$$

be the Lyapunov function which involves individuals who contribute to infection in the population. The constants $\xi_1, \xi_2, \xi_3, \xi_4 and \xi_5$ are all non negative and we ought to find them. The time derivative of the Lyapunov function (7) is given by

$$D^\beta V(t) = \xi_1 D^\beta B(t) + \xi_2 D^\beta C(t) + \xi_3 D^\beta D(t) + \xi_4 D^\beta E(t) + \xi_5 D^\beta F(t)$$

$$\leq \xi_1[(1-\phi_1)(\beta_1 B + \beta_2 C + \beta_3 D + \beta_4 E + \beta_5 F) - P_1 B] + \xi_2[P_2 B - P_3 C]$$

$$+\xi_3[P_4 B + \eta_1 C - P_5 D] + \xi_4[P_6 C + \eta_2 F - P_7 E] + \xi_5[P_8 D + \eta_3 E - P_9 F]$$

$$= [\xi_1(1-\phi_1)\beta_1 + \xi_2 P_2 + \xi_3 P_4 - \xi_1 P_1]B + [\xi_1(1-\phi_1)\beta_2 + \xi_3\eta_1 + \xi_4 P_6 - \xi_2 P_3]C$$

$$+[\xi_1(1-\phi_1)\beta_3 + \xi_5 P_8 - \xi_3 P_5]D + [\xi_1(1-\phi_1)\beta_4 + \xi_5\eta_3 - \xi_4 P_7]E$$

$$+[\xi_1(1-\phi_1)\beta_5 + \xi_4\eta_2 + \xi_5 P_9]F$$

Where $P_1 = (\alpha_1+\alpha_2)\phi_2 + \mu$, $P_2 = \alpha_1\phi_2$, $P_3 = \eta_1 + \phi_3\gamma_1 + \mu$, $P_4 = \alpha_2\phi_2$, $P_5 = \alpha_3\gamma_2 + \mu + \delta_1$, $P_6 = \alpha_3\gamma_1$, $P_7 = \mu + \eta_3$, $P_8 = \phi_3\gamma_2$, $P_9 = \eta_2 + \mu + \delta_2$ We equate the coefficients of the components C, D, E, F to zero and solve for the coefficients of the Lyopunov function obtaining

$$\xi_1 = \frac{P_5 P_7 P_9(1-\phi_4)}{(1-\phi_1)}$$

$$\xi_2 = \frac{P_7 P_9(1-\phi_1)P_5\beta_2 + \eta_1\beta_3 + (P_5 P_6 P_9 + P_8\eta_1\eta_2)\beta_4 + (P_7 P_8\eta_1 + P_5 P_6\eta_3)\beta_5}{P_3}$$

$$\xi_3 = P_7 P_9\beta_3(1-\phi_1) + P_8(\eta_2\beta_4 + P_7\beta_5)$$

$$\xi_4 = P_5(\beta_5\eta_3 + \beta_4 P_9)$$

$$\xi_5 = P_5(\beta_4\eta_2 + \beta_5 P_7)$$

Substituting the constants $\xi_1, \xi_2, \xi_3, \xi_4$ and ξ_5 into (8), we obtain

$$D^\beta V(t) \leq [\frac{P_1 P_5 P_7 P_9(1-\phi_1)}{(1-\phi_1)}][R_o - 1]$$

Thus , $D^\beta V(t) \leq 0$ when $R_o \leq 1$. Furthermore, $D^\beta V(t)$ if and only if either $B = C = D = E = F = 0$ or $R_o = 1$ are satisfied. In either case, the largest compact invariant subset of the set

$$\varepsilon = \{A(t), B(t), C(t), D(t), E(t), F(t)\varepsilon\tau : D^\beta V(t) = 0\} \tag{19}$$

is the singleton ε_o. By LaSalle's Invariance Principle, this implies that ε_o is globally stable in τ provided $R_o \leq 1$.

Theorem 3 The disease persistent equilibrium ε_1 is globally asympotically stable whenever R_o is greater than unity.

ProofWe know that $M = \frac{\Pi}{\mu}$ as $t \to \infty$. Therefore, using the relation $A = \frac{\Pi}{\mu} - B - C - D - E - F$ and substituting in (4) gives the following limiting system

$$D^\beta B(t) = \lambda_c[\frac{\Pi}{\mu} - B - C - D - E - F] - P_1 B$$

$$D^\beta C(t) = P_2 B - P_3 C$$

$$D^\beta D(t) = P_4 B + \eta_1 C - P_5 D$$

$$D^{\beta}E(t) = P_6C + \eta_2F - P_7E$$

$$D^{\beta}F(t) = P_8D + \eta_3E - P_9F$$

It follows that

$$\partial\phi_2 = \frac{\partial}{\partial B}[(\frac{\mu(1-\phi_1)(\beta_1B+\beta_2C+\beta_3D+\beta_4E+\beta_5F)}{BCDEF\Pi})$$

$$(\frac{\Pi}{\mu} - B - C - D - E - F) - \frac{P_1}{CDEF}] + \frac{\partial}{\partial D}[\frac{P_4}{CDEF} + \frac{\eta_1}{BCEF} - P_5BCEF]$$

$$= -[\frac{\Pi P_4B^2 + \eta_1\Pi BC + \phi_3}{\Pi B^2CD^2EF}]$$

where

$$\phi_3 = B^2D\beta_1\mu(1-\phi_1) + D(1-\phi_1)(\Pi - \mu(C+D+E+F))[\beta_2C+\beta_3D+\beta_4E+\beta_5F]$$

So, there are no periodic orbits in τ, according to Dulacs criterion, hence the system is stable. Fractional order system has stable according to condition of stability. This is also similar results which prove for classical system of equation given in [21]. Fractional order system also satisfied the criteria for stability and positive solution.

4 Numerical solution of fractional order model

By applying the Laplace with Adomian decomposition method on system (7-12), we get the following equations

$$\mathscr{L}\{D^{\beta}A(t)\} = \mathscr{L}\{\Pi\} - (\lambda_c+\mu)\mathscr{L}\{A\}, \tag{20}$$

$$\mathscr{L}\{D^{\beta}B(t)\} = \lambda_c\mathscr{L}\{A\} - ((\alpha_1+\alpha_2)\phi_2+\mu)\mathscr{L}\{B\}, \tag{21}$$

$$\mathscr{L}\{D^{\beta}C(t)\} = \phi_2\alpha_1\mathscr{L}\{B\} - (\eta_1+\phi_3\gamma_1+\mu)\mathscr{L}\{C\}, \tag{22}$$

$$\mathscr{L}\{D^{\beta}D(t)\} = \phi_2\alpha_2\mathscr{L}\{B\} + \eta_1\mathscr{L}\{C\} - (\phi_3\gamma_2+\mu+\delta_1)\mathscr{L}\{D\}, \tag{23}$$

$$\mathscr{L}\{D^{\beta}E(t)\} = \phi_3\gamma_1\mathscr{L}\{C\} + \eta_2\mathscr{L}\{F\} - (\eta_3+\mu)\mathscr{L}\{E\}, \tag{24}$$

$$\mathscr{L}\{D^{\beta}F(t)\} = \phi_3\gamma_2\mathscr{L}\{D\} + \eta_3\mathscr{L}\{E\} - (\eta_2+\mu+\delta_2)\mathscr{L}\{F\}, \tag{25}$$

Using property of Laplace transform, we get

$$S^{\beta}\mathscr{L}\{A(t)\} - \{S^{\beta-1}A(0)\} = \mathscr{L}\{\Pi\} - (\lambda_c-\mu)\mathscr{L}\{A\}, \tag{26}$$

$$S^{\beta}\mathscr{L}\{B(t)\} - \{S^{\beta-1}B(0)\} = \lambda_c\mathscr{L}\{A\} - ((\alpha_1+\alpha_2)\phi_2+\mu)\mathscr{L}\{B\}, \tag{27}$$

$$S^{\beta}\mathscr{L}\{C(t)\} - \{S^{\beta-1}C(0)\} = \phi_2\alpha_1\mathscr{L}\{A\} - (\eta_1+\phi_3\gamma_1+\mu)\mathscr{L}\{C\}, \tag{28}$$

$$S^{\beta}\mathscr{L}\{D(t)\} - \{S^{\beta-1}D(0)\} = \phi_2\alpha_2\mathscr{L}\{B\} + \eta_1\mathscr{L}\{C\} - (\phi_3\gamma_2+\mu+\delta_1)\mathscr{L}\{D\}, \tag{29}$$

$$S^{\beta}\mathscr{L}\{E(t)\} - \{S^{\beta-1}E(0)\} = \phi_3\gamma_1\mathscr{L}\{C\} + \eta_2\mathscr{L}\{F\} - (\eta_3+\mu)\mathscr{L}\{E\}, \tag{30}$$

$$S^{\beta}\mathscr{L}\{F(t)\} - \{S^{\beta-1}F(0)\} = \phi_3\gamma_2\mathscr{L}\{D\} + \eta_3\mathscr{L}\{E\} - (\eta_2+\mu+\delta_2)\mathscr{L}\{F\}, \tag{31}$$

$$\mathscr{L}\{A(t)\} = \frac{1}{S}A(0) + \frac{\mathscr{L}\{\Pi\}}{S^{\beta}} - \frac{(\lambda_c+\mu)}{s^{\beta}}\{A\}, \tag{32}$$

$$\mathscr{L}\{B(t)\} = \frac{1}{S}B(0) + \frac{\lambda_c}{S^{\beta}}\mathscr{L}\{A\} - \frac{1}{S^{\beta}}((\alpha_1+\alpha_2)\phi_2+\mu)\mathscr{L}\{B\}, \tag{33}$$

$$\mathscr{L}\{C(t)\} = \frac{1}{S}C(0) + \frac{\phi_2\alpha_1}{S^\beta}\mathscr{L}\{B\} - \frac{1}{S^\beta}(\eta_1 + \phi_3\gamma_1 - \mu)\mathscr{L}\{C\}, \tag{34}$$

$$\mathscr{L}\{D(t)\} = \frac{1}{S}D(0) + \frac{1}{S^\beta}\phi_2\alpha_2\mathscr{L}\{B\} + \frac{1}{S^\beta}\eta_1\mathscr{L}\{C\} - \frac{1}{S^\beta}(\phi_3\gamma_2 + \mu + \delta_1)\mathscr{L}\{D\}, \tag{35}$$

$$\mathscr{L}\{E(t)\} = \frac{1}{S}E(0) + \frac{1}{S^\beta}\phi_3\gamma_1\mathscr{L}\{C\} + \frac{1}{S^\beta}\eta_2\mathscr{L}\{F\} - \frac{1}{S^\beta}(\eta_3 + \mu)\mathscr{L}\{E\}, \tag{36}$$

$$\mathscr{L}\{F(t)\} = \frac{1}{S}F(0) + \frac{1}{S^\beta}\phi_3\gamma_2\mathscr{L}\{D\} + \frac{1}{S^\beta}\eta_3\mathscr{L}\{E\} - \frac{1}{S^\beta}(\eta_2 + \mu + \delta_2)\mathscr{L}\{F\} \tag{37}$$

The method assumes the solution as an infinite series

$$A = \sum_{k=0}^{\infty} A_k, B = \sum_{k=0}^{\infty} B_k, C = \sum_{k=0}^{\infty} C_k,$$

$$D = \sum_{k=0}^{\infty} D_k, D = \sum_{k=0}^{\infty} D_k, E = \sum_{k=0}^{\infty} E_k, F = \sum_{k=0}^{\infty} F_k$$

The generalized form is

$$\mathscr{L}\{A_{k+1}(t)\} = \frac{1}{S}A(0) + \frac{\mathscr{L}\{\Pi\}}{S^\beta} - \frac{(\lambda_c + \mu)}{s^\beta}\{A_k\}, \tag{38}$$

$$\mathscr{L}\{B_{k+1}(t)\} = \frac{1}{S}B(0) + \frac{\lambda_c}{S^\beta}\mathscr{L}\{A_k\} - \frac{1}{S^\beta}((\alpha_1 + \alpha_2)\phi_2 + \mu)\mathscr{L}\{B_k\}, \tag{39}$$

$$\mathscr{L}\{C_{k+1}(t)\} = \frac{1}{S}C(0) + \frac{\phi_2\alpha_1}{S^\beta}\mathscr{L}\{B_k\} - \frac{1}{S^\beta}(\eta_1 + \phi_3\gamma_1 - \mu)\mathscr{L}\{C_k\}, \tag{40}$$

$$\mathscr{L}\{D_{k+1}(t)\} = \frac{1}{S}D(0) + \frac{1}{S^\beta}\phi_2\alpha_2\mathscr{L}\{B_k\} + \frac{1}{S^\beta}\eta_1\mathscr{L}\{C_k\} - \frac{1}{S^\beta}(\phi_3\gamma_2 + \mu + \delta_1)\mathscr{L}\{D_k\}, \tag{41}$$

$$\mathscr{L}\{E_{k+1}(t)\} = \frac{1}{S}E(0) + \frac{1}{S^\beta}\phi_3\gamma_1\mathscr{L}\{C_k\} + \frac{1}{S^\beta}\eta_2\mathscr{L}\{F_k\} - \frac{1}{S^\beta}(\eta_3 + \mu)\mathscr{L}\{E_k\}, \tag{42}$$

$$\mathscr{L}\{F_{k+1}(t)\} = \frac{1}{S}F(0) + \frac{1}{S^\beta}\phi_3\gamma_2\mathscr{L}\{D_k\} + \frac{1}{S^\beta}\eta_3\mathscr{L}\{E_k\} - \frac{1}{S^\beta}(\eta_2 + \mu + \delta_2)\mathscr{L}\{F_k\}, \tag{43}$$

The series solution is

$$A = 800{,}000 - 45724.64\frac{t^\beta}{\beta!} + 3633.087851\frac{t^2\beta}{2\beta!} - 288.6699017\frac{t^3\beta}{3\beta!} - 8.948941399\frac{t^4\beta}{4\beta!}, \tag{44}$$

$$B = 50{,}000 - 12150.36\frac{t^\beta}{\beta!} + 11792.5799\frac{t^2\beta}{2\beta!} - 13867.87003\frac{t^3\beta}{3\beta!} + 1620.269627\frac{t^4\beta}{4\beta!}, \tag{45}$$

$$C = 10{,}000 + 28545\frac{t^\beta}{\beta!} - 54018.74996\frac{t^2\beta}{2\beta!} + 92425.8281\frac{t^3\beta}{3\beta!} - 228.514364\frac{t^4\beta}{4\beta!}, \tag{46}$$

$$D = 25{,}000 - 1537.5\frac{t^\beta}{\beta!} + 23802.90067\frac{t^2\beta}{2\beta!} - 68656.01163\frac{t^3\beta}{3\beta!} + 748.2313337\frac{t^4\beta}{4\beta!}, \tag{47}$$

$$E = 5000 - 7045.1\frac{t^\beta}{\beta!}29271.42433\frac{t^2\beta}{2\beta!} + 23668.69114\frac{t^3\beta}{3\beta!} + 556.7993813\frac{t^4\beta}{4\beta!}, \tag{48}$$

$$F = 2000 + 20610.4\frac{t^\beta}{\beta!} - 13067.80898\frac{t^2\beta}{2\beta!} + 32320.38265\frac{t^3\beta}{3\beta!} + 260.3497978\frac{t^4\beta}{4\beta!}, \tag{49}$$

5 Results and discussions

The numerical evaluation of the HIV epidemic model was presented with the fractional order differential equation. The numerical solutions of different fractional values of β were explained in Tables 1-6 and Figures 1-6 for the accurate investigation. Figure 1 therefore provides a steady state solution, but we analyzed the results of the fractional values compare with the classical derivative. The rate of susceptible humans will start to decrease if we increase the fractional value. From Figure 2 we note that the solution provides better convergence to steady state for exposed by increasing fractional values, while in Figures 3-6 the fractional values converge in according to steady state. In terms of infinite power series, Laplace Adomian decomposing approach is an analytical solution. We can therefore easily observe a good, consistent and more efficient state solution with a fractional order derivative than ordinary. Results discussed in [21] for integer order model, but we proved our results with non-integer order derivative model and compare with integer order derivative.

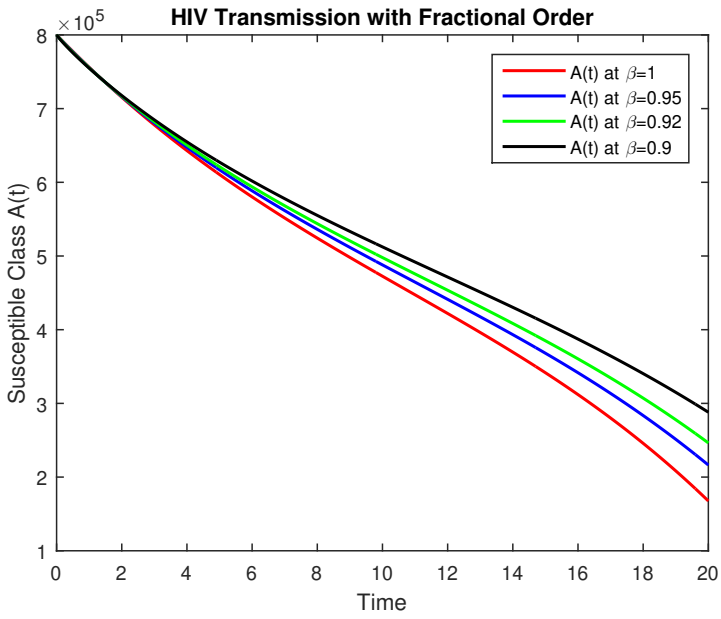

Fig. 1: Susceptible $A(t)$ population in time t in different β values.

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Table 1: Table of susceptible class $A(t)$ with different β values

t	$\beta = 1$	$\beta = 0.97$	$\beta = 0.95$	$\beta = 0.92$
0.1	795446	794970	794624	794241
4.08	636527	637793	638548	639543
8.06	489644	499835	505997	514414
12.04	297849	332742	352620	378324
14.02	164393	223156	255794	296957

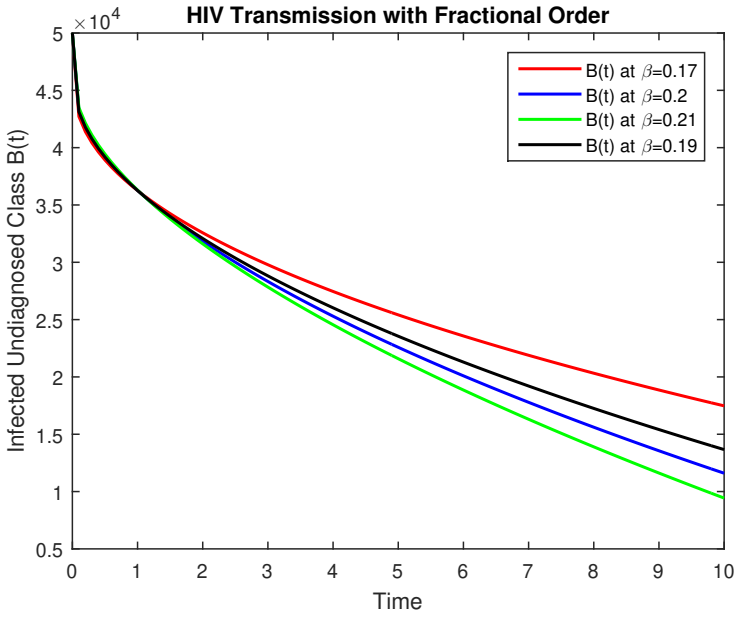

Fig. 2: Infected Undiagnosed population $B(t)$ on time at t with different β values.

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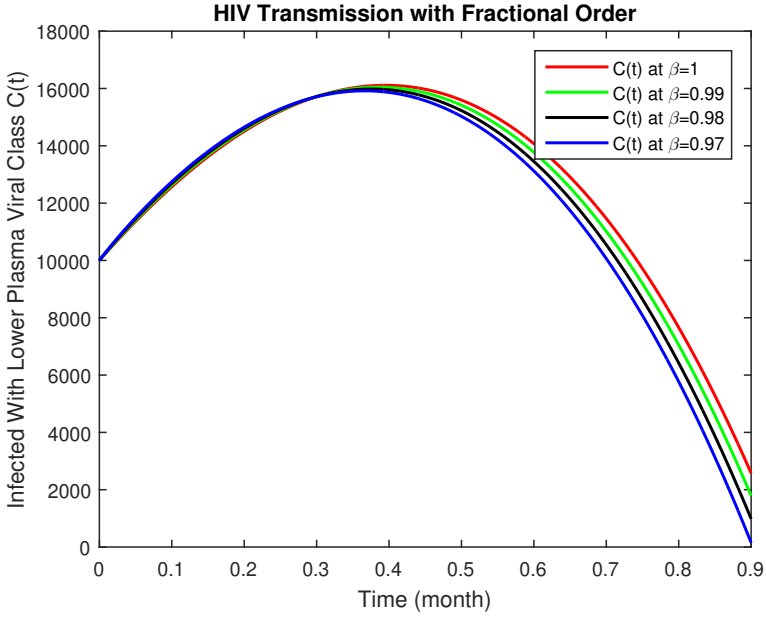

Fig. 3: Infected With Lower Plasma Viral population $C(t)$ on time at t with different β values.

Table 2: Table of infected undiagnosed class $B(t)$ with different values of β

t	$\beta = 0.21$	$\beta = 0.2$	$\beta = 0.19$	$\beta = 0.17$
0.1	43146.3	43012.5	42875.7	42593.7
2.08	36092.2	36267.2	36440.6	36784.1
4.06	33019.5	33418	33805.1	34548.2
6.04	30674.1	31272.3	31846.7	32929.9
8.02	28692.7	29476.3	30222.3	31611.6
10	26940.2	27898.7	28805.3	30477.2

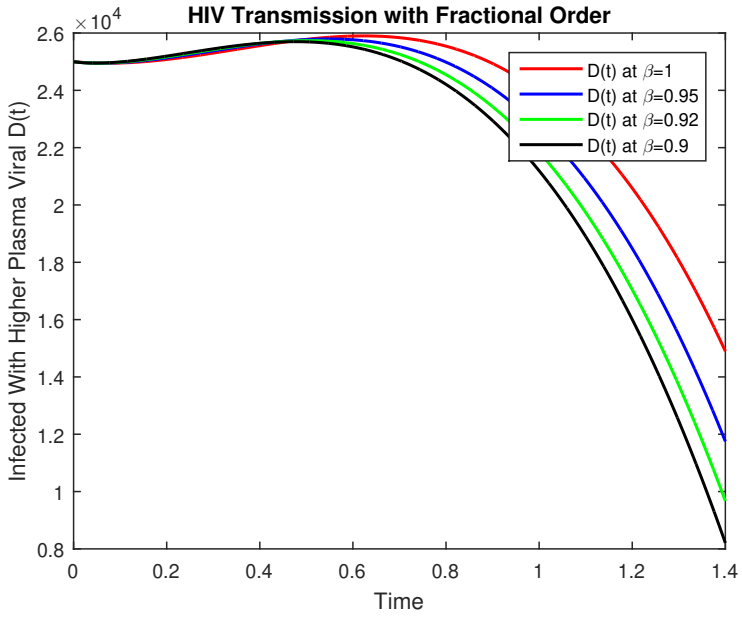

Fig. 4: Infected With Higher Plasma Viral population $D(t)$ on time t with different β values.

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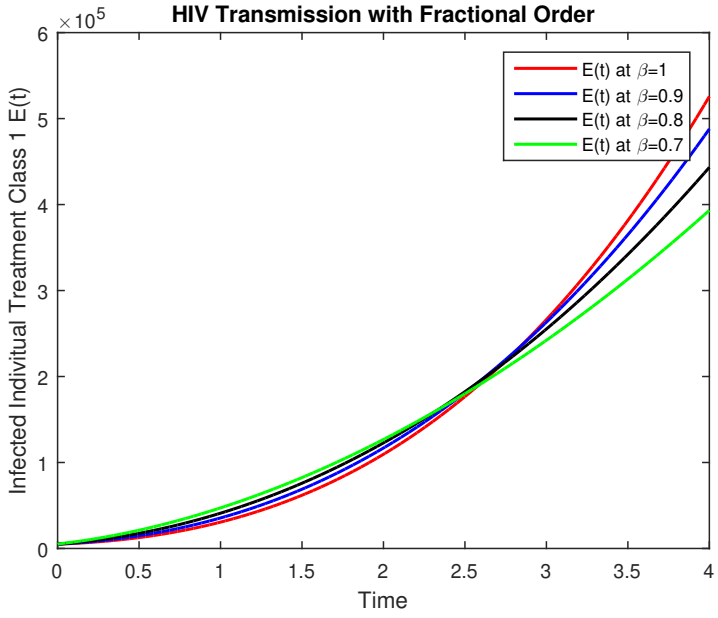

Fig. 5: Infected Individual Treatment population 1 $E(t)$ in time t in different β values.

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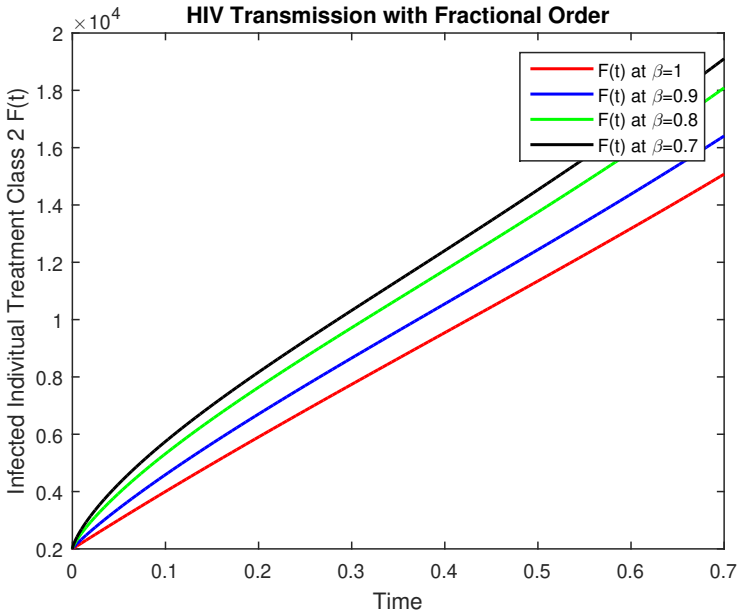

Fig. 6: Infected Individual Treatment population 2 $F(t)$ in time t (year) with different β values.

Table 3: Table of infected with lower plasma viral $C(t)$ with different values of β

t	$\beta = 1$	$\beta = 0.9$	$\beta = 0.8$	$\beta = 0.7$
0.001	10028.5	10057.1	10114.2	10227.9
0.8008	31336.1	32244.3	33208.9	34435.2
1.6006	112452	100383	89933.8	81414
2.4004	347108	269575	210568	166611
3.2002	828473	583046	412119	294632
4	1649160	1078850	707547	468214

Table 4: Table of infected with higher plasma viral $D(t)$ with different values of β

t	$\beta = 1$	$\beta = 0.95$	$\beta = 0.92$	$\beta = 0.90$
0.00001	25000	25000	25000	25000
0.280008	25001.3	24994.2	24986.6	24979.8
0.560006	23870.6	23678.6	23551	23460.5
0.840004	18634.9	18280.2	18063.7	17918.1
1.120002	6349.15	6351.17	6367.33	6384.6

Table 5: Table of Infected Individual Treatment Class 1 $E(t)$ with different values of β

t	$\beta = 0.9$	$\beta = 0.8$	$\beta = 0.7$	$\beta = 0.1$
0.001	5014.68	5030.36	5062.61	13209.3
0.8008	25767.1	27389.3	28968	33398.3
0.6006	81667	76239.4	70786.1	37717.1
2.4004	185331	155768	130584	40584.1
3.2002	348541	269566	208470	42789.3
4	582666	420925	304666	44605.3

Table 6: Table of Infected Individual Treatment Class 2 $F(t)$ with different values of β

t	$\beta = 1$	$\beta = 0.9$	$\beta = 0.8$	$\beta = 0.75$
0.0001	2002.06	2005.38	2013.96	2022.42
0.14008	4788.53	5511.84	6393.89	6902.79
0.28006	7496.72	8489.82	9624.77	10249.8
0.42004	10304.8	11470.8	12751.4	13435
0.56002	13391.7	14674	16029.3	16730.3
0.7	16936.6	18264.8	19607.2	20275.7

6 Conclusion

Dynamical system of HIV is analysed qualitatively to verify the steady state as well as uniqueness has been found for better results. Caputo fractional derivative is used to develop the fractional order non linear system. We develop a scheme for the solution of HIV using Laplace Adomian Decomposition Process with and without demographic impacts. It is worthy to note that the fractional order derivative shows significant improvements and memory effect as compared with ordinary derivatives. The influence of the fractional parameter on our obtained solutions are shown in graphs and tables which provide us the actual behavior of HIV transmission. Developed techniques provide better results which are useful to understand the HIV outbreak in the community. The techniques developed to provide good results which are useful for understanding the HIV disease. Also the comparison has been made with classical order derivative and observe that non-integer order give more suitable results according to steady state.

References

[1] S. D. Lawn and A. I. Zumla, Tuberculosis, *Lancet* **378**(9785), 57-72 (2011).
[2] M. Tahir, A. S. Inayat and G. Zaman, Prevention strategy for superinfection mathematical model tuberculosis and HIV associated with AIDS, *Cogent Math. Stat.* **6**, (2019).
[3] R. M. Anderson and R. M. May, *Infectious diseases of humans: dynamics and control*, Oxford University Press, USA, 1991.
[4] M. A. Nowak and R. M. May, *Virus dynamics: mathematical principles of immunology and virology*, Oxford University Press, USA, 2000.
[5] L. H. Alison, Mathematical models of HIV latency, *Curr. Top. Microbio. Immunol.* **417**, 131-156 (2018).
[6] Y. Huang, H. Wu and E. P Acosta, Hierarchical Bayesian inference for HIV dynamic differential equation models incorporating multiple treatment factors, *Biomet. J.* **52**, 470-486 (2010).
[7] M. Caputo, *Elasticita e dissipazione*, Zani-Chelli, Bologna, 1969.
[8] I. Podlubny, *Fractional differential equations*, Academic Press, New York, 1999.
[9] K. S. Miller and B. Ross, *An introduction to the fractional calculus and fractional differential equations*, Wiley, New-York, 1993.
[10] D. Baleanu, Z. B. Guvenc and J. A. Tenreiro, *New trends in nanotechnology and fractional calculus applications*, Springer, Dordrecht 2010.
[11] J. Singh, D. Kumar, Z. Hammouch and A. Atangana, A fractional epidemiological model for computer viruses pertaining to a new fractional derivative, *Appl. Math. Comput.* **316**, 504-515 (2018).

[12] J. Singh, D. Kumar and D. Baleanu, On the analysis of chemical kinetics system pertaining to a fractional derivative with Mittag–Leffler type kernel, *Chaos Solit. Fract.* **27**, 103-113 (2017).
[13] D. Kumar, J. Singh, D. Baleanu and Sushila, Analysis of regularized long-wave equation associated with a new fractional operator with Mittag Leffler type kernel, *Physica A* **492**, 155-167 (2018).
[14] M. Caputo and M. Fabrizio, A new definition of fractional derivative without singular kernel, *Prog. Fract. Differ. Appl.* **1**, 73-85 (2015).
[15] J. Losada and J. J. Nieto, Properties of a new fractional derivative without singular kernel, *Prog. Fract. Differ. App.* **1**, 8792 (2015).
[16] F. Haq, K. Shah, G. Rahman and M. Shahzad, Numerical solution of fractional order smoking model via laplace Adomian decomposition method, *Alexandria Eng. J.* **57**(2), 1061-1069 (2018).
[17] M. Farman, M. U. Saleem, A. Ahmad and M. O. Ahmad, Analysis and numerical solution of SEIR epidemic model of measles with non-integer time fractional derivatives by using Laplace Adomian decomposition method, *Ain Shams Engin. J.* **9**(4), 3391-3397 (2018).
[18] S. Kumar, S. Ghosh, M. S. M. Lotayif and B. Samet, A model for describing the velocity of a particle in Brownian motion by Robotnov function based fractional operator, *Alexandria Eng. J.* **59**(3), 1435-1449 (2020).
[19] M. Farman, A. Ahmad, A. Akgul and S. Imtiaz, Analysis and dynamical behavior of fractional order cancer model with vaccine strategy, *Math. Meth. App. Sci.*,https://doi.org/10.1002/mma.6240 (2020).
[20] M. Farman, M. U. Saleem, A. Ahmad, S. Imtiaz, M. F. Tabassm, S. Akram and M. O. Ahmad, A control of glucose level in insulin therapies for the development of artificial pancreas by Atangana-Baleanu fractional derivative, *Alexandria Eng. J.* **59**, 2639-2648 (2020).
[21] E. O. Omondi, R. W. Mbogo and L. S. Luboobi, Mathematical modelling of the impact of testing, treatment and control of HIV transmission in Kenya, *Cogent Math. Stat.* **5**, (2018).
[22] B. G. Williams, *Optimizing control of HIV in Kenya*, arXiv preprint arXiv:1407.7801, (2014).
[23] B. G. Williams, J. W. Hargrove and J. H. Humphrey, The benefits of early treatment for HIV, *AIDS* **24**(11), 1790-1791 (2010).