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Constancy of Speed of Light in Inertial Frames

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Abstract: This article proves that speed of light in all uniformly moving inertial reference frames is absolute as postulated by Einstein. This is first done by considering light propagating with a speed c in all directions in an inertial frame of reference. If that frame is moving uniformly with a speed v relative to a second stationary inertial frame, we assume that light in the second frame is propagating in all directions with a different speed $c_\rho=\rho\, c$. Consequently, modified transformation Equations are formed. The established Poincaré ellipsoidal light waves are then used to find the Equation that governs the relation of ρ at any speed v. The analytical solution and numerical calculations to this equation yield a value ρ =1. This proves that speed of light propagates through empty space with speed c independent of the speed of the light source or the observer.

Keywords: Special Relativity, Lorentz transformations, Poincaré Ellipsoidal Light Waves

1 Introduction

For more than a century, the postulates of Special Relativity (SR) theory introduced by Albert Einstein [1] have been the fundamentals of all present physical phenomena and accepted theories that describe all interactions occurring in nature. Albert Einstein's second postulate declares that the speed of light c (about 3×10^8 ms^{-1}) in free space is the same in all inertial frames of reference. With today's continuous development towards achieving higher experimental precision (see [2]), there has been a stronger motivation to test this theory, especially since any deviation from Einstein's predictions would have overwhelming consequences for our understanding of nature. Furthermore, interest in testing the Lorentz Invariance (LI), which is a transformation that directly results from the second postulate, has been motivated by the need to unify quantum and general relativity theories. These unification attempts have only been possible through the violation of LI [3]. Although special and general relativity is the foundation of modern physics, scientists know that the present rules did not hold at the birth of the universe. Recently, in the study of critical geometry of a thermal big bang, scenarios for varying the speed of light (VSL) have showed that this may solve the cosmological problems tackled by inflation [4]. A review of VSL is also introduced [5] to discuss the physical meaning of a varying c, dismissing the fable that the constancy of c is a matter of logical consistency. In addition, evidence is reported that redshift depends on varying the dimensionless fine structure constant $a=e^2\hbar^{-1}c^{-1}$ indicating that e, $\hbar$, c, or a combination must vary [6]. However, some authors are keen to point out that it is meaningless to interpret the result of [6] for a varying c. For example, [7] indicates that the constant c is not only the speed of light in a vacuum, but it is the maximum velocity of any object in Nature, with photons being only one type of such objects, implying that c is Nature's fundamental unit of velocity. Moreover, [8] disfavors models of Planck scale physics in which the quantum nature of spacetime causes a linear variation of the speed of light with photon energy. If the VSL proposal is correct [5], that negates the predictions of inflation theory developed by Stephen Hawking [9], which have been tested well by many researchers such as in [10].

Finally, in the study of time dilation in relativistic two-particle interactions [11], the numerical simulations demonstrate that spacetime Lorentz Transformations (LT) do not hold for particle trajectories. Moreover, strong interaction potentials allow for particle velocities higher than the speed of light.

Many tests with different approaches have been performed to test the postulate of SR theory. For the second postulate

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and all its consequences on the SR Equations and results, one region of the achieved results can be classified for $0 < \beta \leq 0.85$, such as in [12-16]. The other region is classified for $0.99 \leq \beta < 1$, such as in references [17-18]. One of the tests of time dilation is related to observing muons at sea level. The proper lifetime of muons in a lab was reported to be $\Delta t' = 2.196980 \times 10^{-6}$s [19]. Muons are created when cosmic rays hit Earth's atmosphere at high altitudes of about 16 km and then reach the Earth's surface. The lifetime of muons moving at the speed v gets dilated in our Earth's frame of reference, and they reach us when $v = 0.9992c$. In addition, tests on time dilation predicted by SR were done using a network of optical clocks connected by optical fiber links to build a high accuracy atomic clock [20].The aforementioned time dilation tests agree completely with SR. MacArthur et al. [21] verified with good accuracy the SR Doppler formula for an atomic hydrogen H^0 beam moving at $\beta = 0.84$ and so light in SR with a speed *c*. This gives credibility to the postulate of the absoluteness of the speed of light suggested by Einstein. One of the experimental measurements of the SR energy relationship has been tested precisely for $0 < \beta < 1$ [22], but it remains to be verified for particles moving with a speed very close to the value *c*.

2 A Theoretical Approach

The preceding introduction indicates that the second postulate of the special theory of relativity has been well tested and verified experimentally. An attempt to find a proof of this postulate is accomplished by assuming quasi-form invariant spherical light waves with different values for the speed of light. When these waves transform into Poincaré elliptical light waves, they lead to light propagating through empty space with the same constant speed *c* independent of the speed of the source or observer.

In this context, the starting point is the fact that the speed of light *c* that is emitted by an observer o' at rest in the origin of an inertial frame S' is constant in empty space. Then assuming that S' is moving with a constant velocity $\vec{v}$ relative to an observer *o* at rest in inertial reference frame *S*. To simplify the steps, the coordinate axes in each frame, *S* and S', are chosen to be parallel (i.e., $x' \parallel x$, $y' \parallel y$, and $z' \parallel z$), and remain mutually perpendicular. In addition, the velocity of the relative motion $\vec{v}$ is chosen to the right and along the $o'x'$ and ox axes (i.e., $y = y'$ and $z = z'$), see the axes drawn in Figure 1. Also, the two frames are superimposed at $t' = t = 0$, i.e., the origins of both the Cartesian coordinate systems are at the same location, $(x', y', z') = (x, y, z) = (0, 0, 0)$. An event in S' can be completely specified by its four spacetime coordinates (x', y', z', t') and in *S* by (x, y, z, t). Suppose that at time $t' = t = 0$ a spherical wave is emitted from the observer o'. The Equation of this spherical wave with respect to o' at time t' will be:

$$x'^2 + y'^2 + z'^2 - c^2 t'^2 = 0. \tag{1}$$

At time $t > 0$, the origin o' acquires a horizontal position $x_{o'} = v t_{o'}$ with respect to the origin *o* of frame *S*. Einstein affirmed the form-invariance of the spherical light-wave given by Equation 1 in all other inertial frames of reference such as *S*. Since we are dealing with a coordinate transformation that has kinematic effects in deriving relativistic dynamical quantities, a quasi-form-invariant spherical light-wave can now be assumed with only a different value for the speed of light. Hence, all variables are kept form-invariant except *c*, and we assume that the speed of light is not absolute and will change to the value $c_\rho = \rho c$, where ρ is a multiplicative parameter which has not neen known yet. Therefore, the Equation of the propagation of the spherical light wave (SLW) with respect to *o* at time *t* will be given by:

$$x^2 + y^2 + z^2 - c_\rho^2 t^2 = 0. \tag{2}$$

Keeping in mind the selected choice of the Cartesian coordinates of S' and S, the relation between (x', y', z', t') and (x, y, z, t) could be written as:

$$t' = t'(x, y, z, t), \quad x' = x'(x, y, z, t), \quad y' = y, \quad z' = z'. \tag{3}$$

First, it is clear that the equations governing x' and t' must be linear due to the homogeneity of space and time. Thus:

$$\begin{aligned} t' &= \alpha x + \alpha' y + \alpha'' z + At, \\ x' &= \gamma_\rho x + \gamma'_\rho y + \gamma''_\rho z + Bt. \end{aligned} \tag{4}$$

Isotropy of space in all directions requires that $\alpha' = \alpha'' = 0$. Also, at point o', we have $x' = 0$, $y' = y = 0$, $z' = z = 0$, and $x = vt$. Therefore; $B = -\gamma_\rho v$ and Equation 4 become:

$$\begin{aligned} t' &= \alpha x + At, \\ x' &= \gamma_\rho (x - vt). \end{aligned} \tag{5}$$

Substituting Equation 5 into Equation 1 gives:

$$-(A^2c^2 - \gamma_\rho^2 v^2)t^2 - 2xt(\alpha A c^2 + v\gamma_\rho^2) + [\gamma_\rho^2 - \alpha^2 c^2]x^2 + y^2 + z^2 = 0 \tag{6}$$

Comparing Equation 6 with Equation 2 and performing some simple manipulations, the relation between the spacetime coordinates will lead to the following transformation Equations:

$$ct' = \gamma_\rho (c_\rho t - vx/c_\rho), \tag{7}$$

$$x' = \gamma_\rho (x - vt), \tag{8}$$

where, $$\gamma_\rho = 1/\sqrt{1 - v^2/c_\rho^2}, (\gamma_\rho \geq 1). \tag{9}$$

The corresponding inverse transformations of Equation 7 and Equation 8 are as follows:

$$ct=\gamma_\rho (c_\rho t'+v x'/c_\rho)\,, \tag{10}$$

$$x=\gamma_\rho (x'+vt')\,. \tag{11}$$

Equations 7 to 11 are different from those in SR (where in SR, we use $\gamma=(1-\beta^2)^{-1/2}$), because of their dependence on $c_\rho=\rho c$ and γ_ρ . If ρ equals 1 as postulated in SR, $\gamma_\rho=\gamma$ and all the previous Equations will be identical to the well-known Lorentz transformation in SR. A direct consequence of the formulated transformations with $\rho\neq 1$ might be questioned and misunderstood at this point because it will not lead to form-invariant versions of Maxwell's Equations. Such concerns must be set aside temporarily until we find the Equation that governs the parameter ρ.

3 Poincaré Ellipsoidal Light Waves

To simplify the steps, we consider light propagation in a two-dimensional plane, namely x and y. Then, Equation 1 and Equation 2 become:

$$x'^2+y'^2-c^2t'^2=0 \qquad (\text{In } S'), \tag{12}$$

$$x^2+y^2-c_\rho^2 t^2=0 \qquad (\text{In } S). \tag{13}$$

Based on Poincaré's relativity [23], the aim is to construct the location of each point on the circular wave produced by the source o' at time t' in S' when viewed by the observer o at time t in S at height $y=y'$ along the y-axis. Expressing x' in terms of x and t' can be obtained using Equation 11 as follows:

$$x'=\gamma_\rho^{-1}x-vt'\,. \tag{14}$$

Replacing x' in Equation 12 and setting $y=y'$, we reach the following elliptical Equation:

$$\frac{(x-h)^2}{a^2}+\frac{y^2}{b^2}=1\,, \tag{15}$$

where the ellipse has a semi-major axis $a=\gamma_\rho ct'=\gamma_\rho b$, a semi-minor axis $b=ct'$, and is centered at $(h=\gamma_\rho vt',0)$ from the origin o of S. This ellipse has a focal distance $f=(a^2-b^2)^{1/2}=h/\rho$ and eccentricity $e=f/a$. Since the origin o' has a coordinate $x'=0$ in frame S', then Equation 10 will give the time of o' in S in terms of t' in S' by:

$$t_{o'}=\rho\gamma_\rho t'\,. \tag{16}$$

A schematic diagram of the elliptical wave given by Equation 15 and all related parameters is displayed in Figure 1.

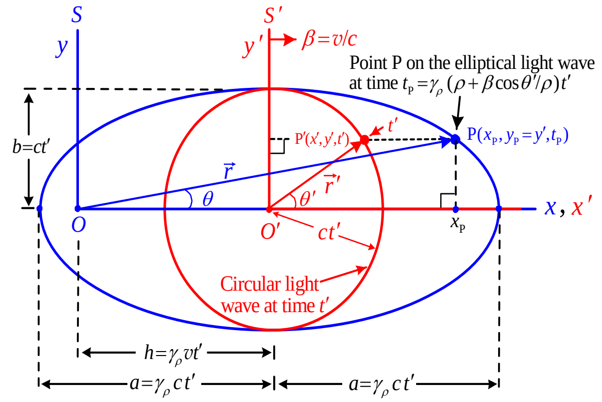

Fig. 1: Schematic diagram showing the circular light wave at time t' with respect to o' and the Poincaré elliptical light wave related to any general point on the circular wave.

According to Poincaré's remark of non-orthogonal induced deformations [23], light will travel in the S frame a distance r from o to point $\text{P}(x_\text{P},y_\text{P}=y',t_\text{P})$ on the ellipse with a speed $c_\rho=\rho c$, as shown in Figure 1. The time t_P at this point can be obtained from Equation 10 and is represented as:

$$ct_\text{P}=\gamma_\rho(\rho ct'+\beta x'/\rho)\,, \tag{17}$$

where the corresponding horizontal coordinate on the circular wave in S' has a coordinate $x'=ct'\cos\theta'$. Thus:

$$t_\text{P}=\gamma_\rho(\rho+\beta\cos\theta'/\rho)t'\,. \tag{18}$$

Therefore, r can be calculated as follows:

$$r=c_\rho t_\text{P}=\rho ct_\text{P}=\gamma_\rho(\rho^2+\beta\cos\theta')ct'\,. \tag{19}$$

Once we apply the general elliptical Equation 15 to point P, we find that the coordinate x_P takes the following form:

$$x_\text{P}=\gamma_\rho vt'+a\cos\theta'\,. \tag{20}$$

Applying $r^2=x_\text{P}^2+y_\text{P}^2$ to the right-angle triangle in Figure 1 and performing some algebra, we obtain the following cubic equation:

$$X^3+AX^2+BX+C=0\,, \tag{21}$$

where $\rho=\sqrt{X}$ and the coefficients A, B, and C are related to the variables β and θ' as follows:

$$\begin{aligned} A&=2\beta\cos\theta',\\ B&=-(\beta^2\sin^2\theta'+2\beta\cos\theta'+1),\\ C&=\beta^2\sin^2\theta'. \end{aligned} \tag{22}$$

The solution to the cubic Equation 21 was first published by Cardano in 1545 [24] and is given by:

$$X=P+\left\{Q+\sqrt{Q^2+(R-P^2)^3}\right\}^{1/3}+\left\{Q-\sqrt{Q^2+(R-P^2)^3}\right\}^{1/3}, \tag{23}$$

where

$$P = -A/3, \quad Q = P^3 + (AB - 3C)/6, \qquad R = B/3. \qquad (24)$$

The coefficients *A*, *B*, and *C* can take on a variety of values depending on β and θ'. Equation 23 might end up with a square root of a negative number. Hence, we would be calculating the cubic root of a complex number. This occurs as part of the intermediate calculation steps, even though these complex numbers do not appear in the problem or in its final answer. To avoid these cases when $Q^2 + (R - P^2)^3 < 0$ (which is always the case in calculating ρ), we write:

$$i\sqrt{|Q^2 + (R - P^2)^3|} = iS,$$

and rewrite the first curly bracket term in Equation 23 as:

$$Z = Q + iS = W\exp(i\varphi), \qquad (25)$$

with $W = \sqrt{Q^2 + S^2}$ and $\varphi = \tan^{-1}(S/Q)$.

After multiplying Z by $\exp(2i\pi k) = 1$, where k is an integer, the cubic root of *Z* will be:

$$Z^{1/3} = W^{1/3}\exp[i(\varphi/3 + 2\pi k/3)],$$

$$k = -1, 0, 1 \text{ (giving three roots for } Z) \qquad (26)$$

Since the third term of Equation 23 is the complex conjugate of the second term, $Z^{1/3}$, then the imaginary parts will be cancel out when adding the two terms together, and the solution of Equation 23 becomes:

$$X = P + 2W^{1/3}\cos(\varphi/3 + 2\pi k/3),$$

$$k = -1, 0, 1 \text{ (giving three roots for } X). \qquad (27)$$

Calculations related to this work generate real values for *X* only if $k = 0$. Thus, the other two complex roots are rejected. Hence the second term of Equation 27 can be written for $k = 0$ as:

$$V = 2W^{1/3}\cos(\varphi/3). \qquad (28)$$

Now the function ρ can be calculated from the relation:

$$\rho = \sqrt{P + V}. \qquad (29)$$

In all the calculations of the analytic solution of this work, we allow θ' to vary in steps of $1°$, and allow β to reach the value $\beta = 0.999991$ in steps of 10^{-5}. When considering any time such as when $t' = 1\text{s}$ and record the large resulting data points for φ, *P*, *V*, and ρ, the upper part of Figure 2 shows the variation of the angle φ, which has an important role in calculating *V* for the entire range of β and θ'. This angle starts from about $85°$ when both β and θ' are zero and increases to $+180°$ and then falls to $-180°$ when $\beta \to 1$ and $\theta' = 180°$. The lower part of Figure 2 shows the surface variation of both functions *P* and *V*, which are responsible for calculating ρ in Equation 29. The sum of the two surfaces is always 1 regardless the values of *P* and *V*. The same result is true for any value of t' (like Figure 2 but not displayed in this paper). Therefore, these calculations concluded that:

$$\rho = \sqrt{P + V} = 1 \quad \text{(for any value of } t'\text{)}. \qquad (30)$$

Accordingly, this proves that the speed of light propagates through empty space with speed *c* independent of the speed of the source or the observer. In other words, it is now a proven fact as well as a postulate as suggested by Einstein [1]. Also, it shows that Maxwell's Equations are form-invariant as expected.

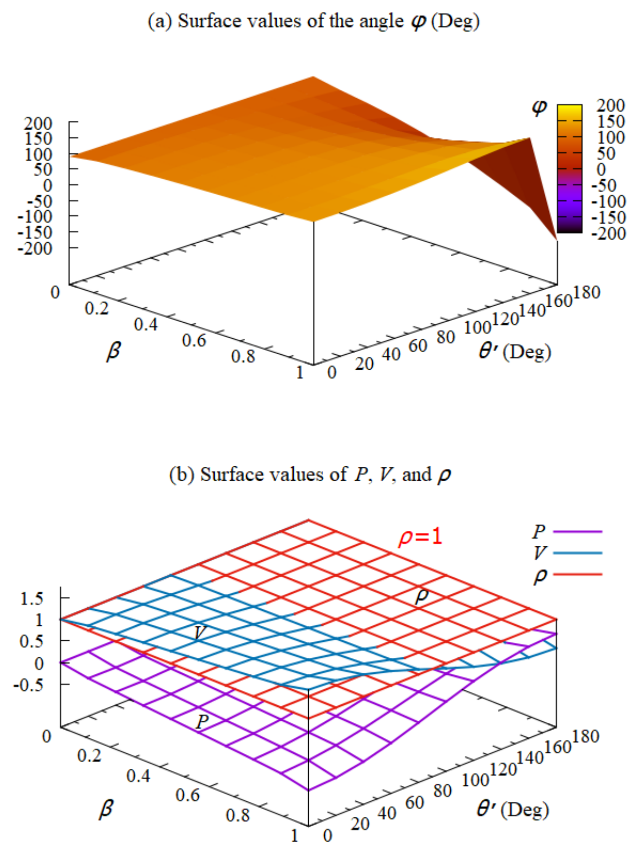

Fig. 2: Graphs for $t' = 1\text{s}$ and viewing angle (55°, 45°). Part (a) shows the surface change of the angle φ for all possible values of β and θ'. Part (b) indicates the surface variation of both *P* and *V*, where *P*+*V*=1. The function *P* is always less than 1 but can be less than zero. When *P* is negative, *V* is greater than 1 to compensate. When *P* is positive and less than 1, *V* is also positive and less than 1 with a value such that *P*+*V*=1.

4 Relativistic and Poincaré Relations

Since we proved that $\rho = 1$ and so $c_\rho = c$, then Equations 1 to 11 must be modified to cope with all the acceptable known SR relations based on the postulated speed of light, but now with a solid proof. The Equations that are directly related to the basic SR relations are:

[SLW]: $x'^2+y'^2+z'^2=c^2t'^2$ & $x^2+y^2+z^2=c^2t^2$, (31)

[LT]: $ct'=\gamma(ct-vx/c)$ & $x'=\gamma(x-vt)$, (32)

[ILT] : $ct=\gamma(ct'+vx'/c)$ & $x=\gamma(x'+vt')$, (33)

where $\gamma=1/\sqrt{1-v^2/c^2}$, $(\gamma\geq 1)$. (34)

Additionally, the Poincaré elliptical Cartesian Equation must be updated to take the following form:

$$\frac{(x-f)^2}{a^2}+\frac{y^2}{b^2}=1, \quad (35)$$

where the semi-major axis a, semi-minor axis b, focal distance f, eccentricity e, time $t_{o'}$ of o' in S, time t at any point $P(x, y=y', t)$ on the ellipse, $x_{o'}$, and the distance r from o to point P, are presented as follows:

$$a=\gamma ct'=\gamma b,$$
$$b=ct'=r',$$
$$f=(a^2-b^2)^{1/2}=\gamma vt',$$
$$e=f/a=\beta,$$
$$t_{o'}=\gamma t',$$
$$t=\gamma(1+\beta\cos\theta')t',$$
$$x_{o'}=vt_{o'}=\gamma vt',$$
$$r=\gamma(1+\beta\cos\theta')ct'. \quad (36)$$

Thus, Equation 35 represents an ellipse with an origin of the y-axis centered at the focal point $(f=\gamma vt',0)$. Moreover, this focal point is the location of the origin of the inertial frame S and $oo'=x_{o'}=vt_{o'}=\gamma vt'=f$. A schematic representation of the elliptical wave given by Equation 35 and its related parameters are given by Equation 36 and displayed in Figure 3.

The polar form of the Cartesian ellipse given by Equation 35 can be written in terms of θ or θ' as follows:

$$r=\frac{r'}{\gamma(1-\beta\cos\theta)}=\gamma r'(1+\beta\cos\theta'). \quad (37)$$

Finally, the angle θ can be related to the angle θ' using the following relation:

$$\theta=\cos^{-1}\left(\frac{\beta+\cos\theta'}{1+\beta\cos\theta'}\right), \qquad 0\leq\theta'\leq\pi. \quad (38)$$

For $\pi\leq\theta'\leq 2\pi$, we subtract the resulted value of Equation 38 from 2π. This relation does not have a time dependence feature, but the parameter β plays a significant role.

Figure 4 represents a two dimensional study of the evolution of the elliptical wave as a function of β, when the time t' is equal to 1s in frame S'. If β is much less than 0.6, the form of the elliptical waves is very close to spherical waves.

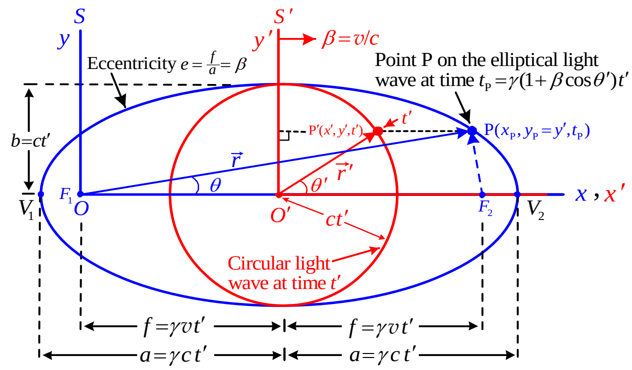

Fig. 3: A schematic diagram showing the circular light wave at time t' with respect to o' and the Poincaré elliptical light wave related to any general point on the circular wave. The independence of the speed of light c on either the speed of the source or the speed of the observer leads to an observer o always being at the left focal point, while the observe o' is at the center of the ellipse.

As β approaches unity, then γ, $t_{o'}$, and oo' acquire huge values. Consequently, the origin o (which coincides with the left focal point) approaches the left vertex of the ellipse, see the case when $\beta=0.9$.

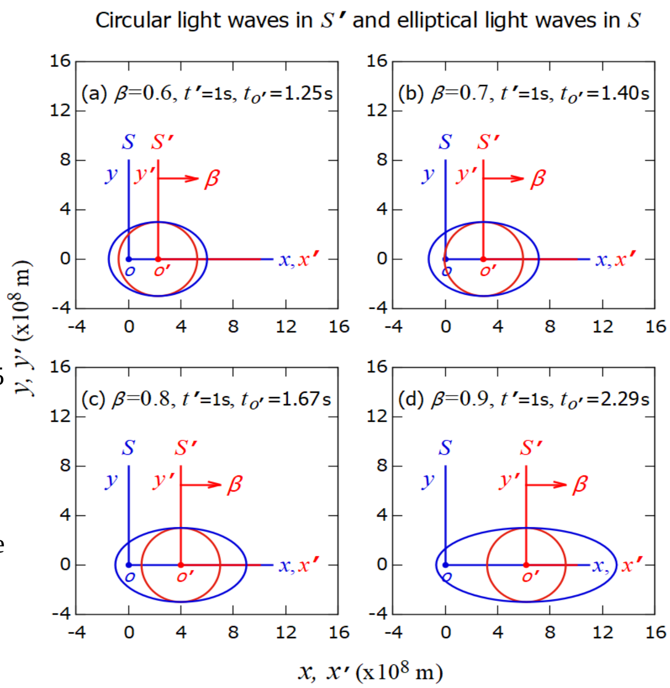

Fig.4: (a)-(d) Scaled geometrical shapes of the circular and elliptical light waves (representing the spherical and Poincaré ellipsoidal waves) at four different values of β to illustrate the evolution of the circular and elliptical waves when considering a certain instant of time (at $t'=1$s) for the propagation of circular waves in S'.

For the selected values $\beta = 0.8$ and $t' = 1\mathrm{s}$, Figure 5 shows the spherical waves (circular in the figure) observed by o' at time t' and by o at time $t = \gamma t'$. The figure also manifests a point P at time $t_P = \gamma(1 + \beta\cos\theta')t'$ on the elliptical wave that corresponds to the same height on the y'-axis of the point P′ at time t'. Only one-point Q lies on both circles as well as the elliptical wave. This point has a vertical distance ct' with respect to o' and a distance $\gamma ct'$ with respect to *o*.

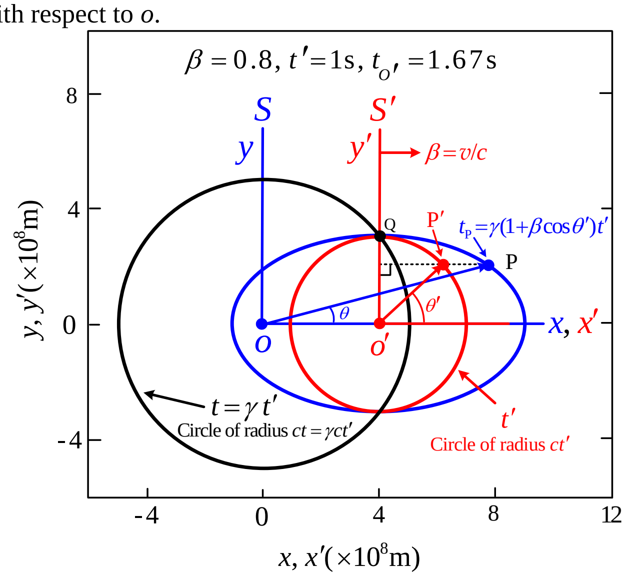

Fig. 5: Scaled geometrical circular waves generated by o' and observed by o (representing the two spherical waves given by Equation 31) and the elliptical wave generated by points on the circular wave emitted by o', Equation 35.

5 Discussion and Conclusion

We have exploited the conflicting insights between Einstein and Poincaré's theories to find a proof for the constancy of the speed of light in inertial reference frames. In both theories, the speed of light in different inertial reference frames was ***postulated*** to be the same with spherical propagation. Both led to the invariance of Maxwell–Lorentz Equations. On the other hand, the postulate of Einstein led to Lorentz's transformations, while Poincaré obtained these transformations only when leaving the Maxwell–Lorentz Equations as an invariant. In addition, Poincaré interpreted the Lorentz transformation as a geometric consideration representing an ellipsoidal wave which is a main gauge of Poincaré's kinematics of relativity.

In our approach, we started by assuming quasi-form-invariant spherical light-waves with a different value for the speed of light. Then, using the Poincaré ellipsoidal geometrical consideration, we ***proved*** analytically and by adopting numerical approach that light propagates through empty space with the same constant speed *c* independent of the speed of the source or the observer.

The preceding analytical and numerical proof is conceptually unique and not part of extended research frameworks such as: (1) very special relativity (VSR), in which space-time symmetries are certain proper subgroups of the Poincaré group [25], (2) the deformed special relativity (DSR), which imposes a maximal energy to SR by deforming Poincaré symmetry [26], and (3) the method that considers the right hand side in Equations 10 and 11 as parameters and use infinitesimal group transformation to seek what is called the excellent speed [27].

In conclusion, this article shows that the constancy of the speed of light in all inertial frames might be considered as a postulate as well as a proven fact. Also, this proof might reactivate all studies that call for a varying speed of light in SR.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there is no conflict of interest.

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