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Bayesian Estimation for Logarithmic Transformed Exponential Distribution under Different Loss Functions

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Abstract: The aim of the present article is to find the better estimator for the parameter of logarithmic transformed exponential distribution using informative and non-informative prior under squared error, linex and general entropy loss functions. The performances of these proposed estimators have been compared based on their simulated risk.

Keywords: Logarithmic transformed exponential distribution, General entropy loss function, Bayes estimator, Metropolis-Hastings algorithm, Simulated risk.

1 Introduction

Lifetime distributions are used to describe the real life circumstances in various fields of life like; medical, engineering and so on. There are lots of lifetime distributions available in the statistical literature to describe such types of situations. Weibull and exponential distributions are very useful lifetime distributions having some interesting properties. In the context of increasing flexibility in distributions, many generalization or transformation methods are available in the literature based on some baseline distribution. Recently, [1] proposed one parameter lifetime distribution (named as logarithmic transformed exponential (LT exponential) distribution) having increasing failure rate based on exponential distribution and discussed its statistical properties along with its suitability over other models.
A random variable X is said to have LT exponential distribution, if its distribution function is given by

$$F(x) = 1 - \frac{\log(1+e^{-\theta x})}{\log 2}; \qquad x > 0, \theta > 0 \tag{1}$$

and its associated probability density function (pdf) is:

$$f(x) = \frac{\theta e^{-\theta x}}{(1+e^{-\theta x})\log 2}; \qquad x > 0, \theta > 0 \tag{2}$$

and hazard rate is:

$$h(x) = \frac{\theta e^{-\theta x}}{(1+e^{-\theta x})\log(1+e^{-\theta x})} \tag{3}$$

where, θ is the scale parameter of the LT exponential model.

In Bayesian estimation, it requires a probability distribution for observed data and a prior probability for the unknown parameter, then all inferences are based on the combination of these two probability distributions. Many authors have been discussing Bayesian approach of estimation in both parametric and non-parametric case. [2] used Bayesian approach to infer the unknown failure time distribution under piece-wise exponential distribution. [3] and [4] introduced the Bayes methods for the estimation of parameters under compound Rayleigh distribution and exponential distribution respectively. [5] obtained Bayes estimates under different loss functions using Gamma prior in case of exponential distribution. The

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reason behind using gamma prior is its flexibility (see [6], [7] and [8] for more details about prior choice). One may also follow [9], [10], [11], [12], [13], [14], [15] and [16]). An important element in the Bayes point estimation problem, is the specification of the loss function. The most popular loss function used in the estimation problem is the squared error loss function (SELF), which can be easily justified on the grounds of minimum variance-unbiased estimation. However, the weakness of this loss function is that it is symmetric and gives an equal weight to the overestimation and underestimation of the same magnitude. A number of asymmetric loss functions are also available in the statistical literature, and one of the most widely used asymmetric loss functions is the linex loss function (LLF), originally proposed by [17] and popularized by [18]. One more asymmetric loss function has been introduced by [19], which is nowadays, very popular among the young researchers.

Motivated by these literatures, here, we are trying to find better estimator for the parameter of logarithmic transformed exponential distribution using informative and non-informative prior under squared error, linex and general entropy loss functions. The rest of the article is organized as follows: Section 2, describes the technique of Bayes estimation under squared error, linex and general entropy loss function using gamma prior. The techniques of MCMC simulation by Metropolis-Hastings algorithm is discussed in Section 3. In Section 4, Bayes interval i.e. highest posterior density interval has been discussed. Section 5, deals with the simulation study which is done on 10,000 samples from pdf given in equation (2). Conclusion is presented in the last section of the article i.e. Section 6.

2 Bayes Estimation

A statistical estimation problem is a special kind of statistical decision problems in which the decision made by the investigator is the estimate of the value of an unknown parameter θ. For the estimation of the parameter θ, an informative gamma prior is used, which can be defined as

$$g(\theta) = \frac{a^b}{\Gamma b} e^{-a\theta} \theta^{b-1}; \qquad a, b, \theta > 0 \tag{4}$$

The formula for the evaluation of posterior distribution of θ is given by

$$g(\theta \mid x) = \frac{L(\theta \mid x) g(\theta)}{\int\limits_{\forall\theta} L(\theta \mid x) g(\theta) d\theta} \tag{5}$$

where $L(\theta \mid x)$ is the likelihood function of the LT exponential distribution, given as follows:

$$L(\theta \mid x) = \left(\frac{\theta}{\log 2}\right)^n \frac{e^{-\theta \sum\limits_{i=1}^{n} x_i}}{\prod\limits_{i=1}^{n} (1 + e^{-\theta x_i})}; \qquad x, \theta > 0. \tag{6}$$

Therefore, the posterior density can be written with the help of equation (4), (5) and (6) as:

$$g(\theta \mid x) = \frac{a^b}{n \log 2 \Gamma b} \frac{\theta^{n+b-1} e^{-\theta\left(\sum\limits_{i=1}^{n} x_i + a\right)}}{\prod\limits_{i=1}^{n} (1 + e^{-\theta x_i})}. \tag{7}$$

For the Bayes estimation techniques, loss function plays very important role. Here, in this article, we are using three different type of loss functions, as mentioned above.

2.1 Bayes estimation under squared error loss function

The squared error loss function (SELF) is a very famous symmetric type of loss functions, which gives same weight to under and over estimation problem. SELF is defined as

$$L(\hat{\theta}, \theta) = (\hat{\theta} - \theta)^2 \tag{8}$$

where $\hat{\theta}$ is the Bayes estimator of θ and under the SELF, the Bayes estimator is nothing but the posterior mean. The squared error loss function is widely used by [9], [20], [10] and [21]. Afterward some time researchers [22], [23], [17], [24] and [25] pointed out that the use of symmetric loss function is inappropriate in some situations. There may be a situation where a negative error may be more serious than positive error or vice-versa. For eg. in dam construction, underestimation of peak of water level is more serious than overestimation. In the same fashion, in reliability context, the overestimation of blast of bomb is more serious than underestimation. Therefore, asymmetric loss function came in picture.

2.2 Bayes estimation under linex loss function

The Linex loss function (see [17]) is asymmetric loss function and defined as

$$L(\hat{\theta},\theta) = e^{\delta(\hat{\theta}-\theta)} - \delta(\hat{\theta}-\theta) - 1 \tag{9}$$

where $\delta \neq 0$. Here, constant δ determines the shape of the loss function. The sign of the constant shows the direction of asymmetry and magnitude of the shape parameter shows the degree of asymmetry. If $\delta > 0$, the overestimation is more serious than underestimation and vice-versa. For δ close to 0, the linex loss is approximately squared error loss and therefore almost symmetric. Under the linex loss, Bayes estimator of θ is that which minimizes the expected loss. Therefore, the Bayes estimator under linex loss is given by

$$\hat{\theta}_L = \frac{-1}{\delta}\log\left[E_\theta(e^{-\delta\theta} \mid X)\right] \tag{10}$$

[23] used linex loss for estimating the mean of normal distribution and [11] used linex loss in estimation of reliability under exponential distribution.

2.3 Bayes estimation under general entropy loss function

Among the asymmetric loss functions available in statistical literature, general entropy loss function (GELF) is the frequently used loss function. It was introduced by [19] and given as:

$$L(\hat{\theta},\theta) = \left(\frac{\hat{\theta}}{\theta}\right)^{\delta_1} - \delta_1 \log\left(\frac{\hat{\theta}}{\theta}\right) - 1 \tag{11}$$

where, δ_1 is the shape parameter of general entropy loss function (GELF). The sign and magnitude of the shape parameter δ_1, defines the direction and degree of asymmetry respectively. Also, when $\delta_1 > 0$, a positive error cause more serious consequence than negative error. This loss function is the generalization of entropy loss function, as put $\delta_1 = 1$. The Bayes estimator under GELF is given by

$$\hat{\theta}_G = \left[E_\theta\left(\theta^{-\delta_1} \mid X\right)\right]^{-1/\delta_1} \tag{12}$$

It is easy to see that, when put δ_1=1, it gives Bayes estimator under weighted SELF and when put δ_1 = -1, it gives Bayes estimator under SELF.

Since, it is not possible to solve equation (7) analytically, we perform Monte Carlo Markov Chain (MCMC) simulation technique.

3 Metropolis-Hastings Algorithm

MCMC is a powerful technique for solving integration by using simulation. This technique used in Bayesian inference nowadays at regular basis. [26] used MCMC first time in statistics to study the equation of a state of a two dimensional rigid sphere system. Metropolis-Hastings algorithm is a generalization of a basic algorithm originated by [27]. [28] generalized the original method by allowing arbitrary proposal distribution and this method became very famous method, Metropolis-Hastings algorithm. One can also see [29] and [30] for review literature. A general way of constructing a MCMC sample was given by Metropolis-Hastings (M-H) algorithm. Suppose, we wish to simulate a sample of size n from a posterior density $g(\theta \mid x)$. The Metropolis-Hastings algorithm can be started with the following iterative steps

1.Set an initial guess value, say θ^0
2.Simulate a candidate value θ^* from a proposal density $p(\theta^* \mid \theta)$.
3.Calculate the probability α that the candidate value θ^* will be accepted as the next value in the sequence

$$\alpha = min\left(1, \frac{g(\theta^* \mid x)p(\theta \mid \theta^*)}{g(\theta \mid x)p(\theta^* \mid \theta)}\right).$$

4.Sample a value θ^t such that $\theta^t = \theta^*$ with probability α; t = 1, 2, $\cdots$, N.

Contour plot technique can be used for guess value in MCMC process. Also, the choice of variance is an important point; one can follow [31] for more details. Another choice for variance would be the Fisher information criterion (see [29], [32], [33] and [30] for more details).

4 Highest Posterior Density Interval

Recently in Bayesian inference, researchers summarized the marginal posterior distribution by $100(1-\alpha)\%$ posterior credible intervals for the parameter of interest. These credible intervals can be easily obtained by analytical solution or MCMC simulation technique (see [34]).

The HPD credible interval (see [35]) of the parameter θ is obtained, say for example on the basis of ordered MCMC samples of θ as $\theta_{(1)}, \theta_{(2)}, \cdots, \theta_{(N)}$. After that $100(1-\alpha)\%$ credible interval for the parameter θ is obtained as $\left((\theta_{(1)}, \theta_{[(1-\alpha)N]+1}), \cdots, (\theta_{[N\alpha]}, \theta_N)\right)$. Where $[Y]$ denotes the largest integer less than or equal to Y. The shortest interval among all of the Bayesian credible intervals is called highest posterior density (HPD) interval. [9] has discussed that the HPD interval has two main properties as:

1. The density of any point within interval is either equal to or greater than the density of any point outside the interval.
2. For a given probability say $(1-\alpha)$, the interval is of shortest length.

For more literature reader may refer [36], [37], [38], [39], [40] and [41] etc. for technical details on HPD interval.

5 Simulation Study

In order to compare the different Bayes estimators under different loss functions with gamma prior, we use the Monte Carlo simulation technique in the following way:

In the simulation study, we have used gamma prior $Gamma(a,b)$, where we have taken the values of hyper parameters a and b in two ways: firstly, we considered gamma prior as non-informative or flat prior by taking $a = 0.1$, $b = 0.1$; secondly, we have taken $a = 1$, $b = 4$, that means we consider gamma prior as informative prior. For the purpose of Bayesian estimation, three different loss functions: symmetric and asymmetric have been used. In symmetric loss function, squared error loss function (SELF) and in asymmetric loss function, linex loss function (LLF) and general entropy loss functions (GELF) have been used. The sign of shape parameter δ involved in LLF is considered positive and negative values both for the purpose of direction of asymmetry. Thus, the values of δ are taken as ($\delta = 0.5, 2.0$ & 3.5) and ($\delta = -0.5, -2.0$ & -3.5). In the similar fashion, in case of GELF, we have taken three different values for positive ($\delta_1 = 0.5, 2.0$ & 3.5) and three different values for negative ($\delta_1 = -0.5, -2.0$ & -3.5) case for the shape parameter of GELF.

In this section, Monte Carlo simulation technique is performed to compare the method of estimation by using Bayes risk. For this purpose, we have taken sample size variations as $n = 10, 30, 50$ and 80 to represent small, moderate and large sample sizes from LT exponential distribution with the scale parameter θ = 0.2, 0.5 & 1.2 with replication number 10,000. In the following resulting tables, $\hat{\theta}_S$,$\hat{\theta}_L$, $\hat{\theta}_G$ stands for the Bayes estimates under SELF, LLF and GELF respectively, see Table 1 and 2. BR_s, BR_{Li} and BR_{Gi} stands for the Bayes risk of the estimates of parameter θ under SELF, LLF and GELF respectively. Here, $i = 1, 2, \cdots, 6$ as δ and δ_1 taking six different values (as δ = 0.5, 2.0, 3.5, -0.5, -2.0, -3.5 and δ_1 = 0.5, 2.0, 3.5, -0.5, -2.0, -3.5) for LLF and GELF.

Table 1: Bayes estimators under different loss functions for various values of θ and n, when prior is $Gamma(1,4)$

			δ=0.5	δ=2	δ=3.5	δ=-0.5	δ=-2	δ=-3.5	δ_1=0.5	δ_1=2	δ_1=3.5	δ_1=-0.5	δ_1=-2	δ_1=-3.5
n	θ	$\hat{\theta}_S$	$\hat{\theta}_{L1}$	$\hat{\theta}_{L2}$	$\hat{\theta}_{L3}$	$\hat{\theta}_{L4}$	$\hat{\theta}_{L5}$	$\hat{\theta}_{L6}$	$\hat{\theta}_{G1}$	$\hat{\theta}_{G2}$	$\hat{\theta}_{G3}$	$\hat{\theta}_{G4}$	$\hat{\theta}_{G5}$	$\hat{\theta}_{G6}$
	0.2	0.2190	0.2180	0.2151	0.2122	0.2200	0.2232	0.2264	0.2062	0.1930	0.1793	0.2148	0.2274	0.2397
10	0.5	0.4955	0.4904	0.4760	0.4627	0.5007	0.5173	0.5355	0.4662	0.4360	0.4046	0.4858	0.5145	0.5426
	1.2	0.9608	0.9420	0.8909	0.8465	0.9806	1.0473	1.1272	0.9029	0.8429	0.7806	0.9417	0.9985	1.0539
	0.2	0.2065	0.2062	0.2054	0.2045	0.2068	0.2077	0.2087	0.2023	0.1980	0.1937	0.2051	0.2093	0.2135
30	0.5	0.4986	0.4968	0.4917	0.4867	0.5003	0.5057	0.5112	0.4883	0.4780	0.4675	0.4952	0.5054	0.5155
	1.2	1.1130	1.1044	1.0794	1.0559	1.1219	1.1495	1.1790	1.0900	1.0667	1.0431	1.1054	1.1283	1.1510
	0.2	0.2038	0.2036	0.2031	0.2026	0.2040	0.2045	0.2050	0.2013	0.1987	0.1962	0.2030	0.2055	0.2080
50	0.5	0.4987	0.4977	0.4946	0.4915	0.4998	0.5030	0.5062	0.4925	0.4863	0.4800	0.4967	0.5028	0.5090
	1.2	1.1464	1.1408	1.1247	1.1091	1.1520	1.1692	1.1871	1.1320	1.1176	1.1031	1.1416	1.1559	1.1701
	0.2	0.2026	0.2025	0.2022	0.2019	0.2027	0.2031	0.2034	0.2010	0.1995	0.1979	0.2021	0.2037	0.2053
80	0.5	0.5002	0.4995	0.4975	0.4956	0.5008	0.5028	0.5048	0.4962	0.4923	0.4884	0.4989	0.5028	0.5066
	1.2	1.1673	1.1637	1.1532	1.1428	1.1709	1.1820	1.1932	1.1582	1.1490	1.1397	1.1643	1.1734	1.1825

Table 2: Bayes estimators under different loss functions for various values of θ and n, when prior is $Gamma(0.1, 0.1)$

			δ=0.5	δ=2	δ=3.5	δ=-0.5	δ=-2	δ=-3.5	δ_1=0.5	δ_1=2	δ_1=3.5	δ_1=-0.5	δ_1=-2	δ_1=-3.5
n	θ	$\hat{\theta}_S$	$\hat{\theta}_{L1}$	$\hat{\theta}_{L2}$	$\hat{\theta}_{L3}$	$\hat{\theta}_{L4}$	$\hat{\theta}_{L5}$	$\hat{\theta}_{L6}$	$\hat{\theta}_{G1}$	$\hat{\theta}_{G2}$	$\hat{\theta}_{G3}$	$\hat{\theta}_{G4}$	$\hat{\theta}_{G5}$	$\hat{\theta}_{G6}$
	0.2	0.2205	0.2194	0.2161	0.2130	0.2216	0.2251	0.2287	0.2066	0.1922	0.1772	0.2159	0.2295	0.2428
10	0.5	0.5464	0.5397	0.5207	0.5035	0.5534	0.5761	0.6018	0.5120	0.4764	0.4392	0.5351	0.5688	0.6018
	1.2	1.3102	1.2723	1.1746	1.0946	1.3515	1.5014	1.6884	1.2278	1.1423	1.0531	1.2830	1.3639	1.4427
	0.2	0.2060	0.2057	0.2048	0.2039	0.2063	0.2072	0.2081	0.2016	0.1973	0.1928	0.2045	0.2088	0.2131
30	0.5	0.5148	0.5129	0.5073	0.5018	0.5167	0.5226	0.5286	0.5040	0.4930	0.4820	0.5112	0.5219	0.5326
	1.2	1.2381	1.2272	1.1959	1.1665	1.2493	1.2845	1.3225	1.2121	1.1858	1.1592	1.2295	1.2553	1.2810
	0.2	0.2039	0.2037	0.2032	0.2027	0.2041	0.2046	0.2051	0.2013	0.1987	0.1961	0.2030	0.2056	0.2082
50	0.5	0.5104	0.5093	0.5060	0.5028	0.5116	0.5149	0.5184	0.5040	0.4975	0.4910	0.5083	0.5147	0.5211
	1.2	1.2192	1.2129	1.1944	1.1766	1.2256	1.2453	1.2659	1.2038	1.1883	1.1727	1.2141	1.2294	1.2446
	0.2	0.2021	0.2020	0.2017	0.2014	0.2022	0.2026	0.2029	0.2005	0.1989	0.1973	0.2016	0.2032	0.2048
80	0.5	0.5057	0.5051	0.5030	0.5010	0.5064	0.5085	0.5106	0.5017	0.4977	0.4937	0.5044	0.5084	0.5124
	1.2	1.2124	1.2085	1.1970	1.1858	1.2163	1.2283	1.2406	1.2028	1.1932	1.1835	1.2092	1.2187	1.2283

Table 3: Bayes risk of estimators under different loss functions for variations in θ at different sample size n, when prior is $Gamma(1, 4)$

			δ=0.5	δ=2	δ=3.5	δ=-0.5	δ=-2	δ=-3.5	δ_1=0.5	δ_1=2	δ_1=3.5	δ_1=-0.5	δ_1=-2	δ_1=-3.5
n	θ	BR_S	BR_{L1}	BR_{L2}	BR_{L3}	BR_{L4}	BR_{L5}	BR_{L6}	BR_{G1}	BR_{G2}	BR_{G3}	BR_{G4}	BR_{G5}	BR_{G6}
	0.2	0.0045	0.0006	0.0087	0.0261	0.0006	0.0092	0.0287	0.0097	0.1606	0.5060	0.0095	0.1475	0.4365
10	0.5	0.0164	0.0020	0.0299	0.0856	0.0021	0.0361	0.1188	0.0087	0.1477	0.4847	0.0085	0.1304	0.3864
	1.2	0.0959	0.0120	0.1905	0.5755	0.0120	0.1947	0.6156	0.0154	0.2605	0.8399	0.0148	0.2237	0.6447
	0.2	0.0013	0.0002	0.0025	0.0076	0.0002	0.0025	0.0078	0.0034	0.0554	0.1727	0.0034	0.0531	0.1604
30	0.5	0.0066	0.0008	0.0129	0.0388	0.0008	0.0135	0.0420	0.0033	0.0539	0.1676	0.0033	0.0517	0.1561
	1.2	0.0356	0.0044	0.0695	0.2094	0.0045	0.0733	0.2303	0.0040	0.0656	0.2051	0.0040	0.0623	0.1875
	0.2	0.0007	0.0001	0.0014	0.0044	0.0001	0.0015	0.0045	0.0021	0.0337	0.1039	0.0021	0.0330	0.1002
50	0.5	0.0042	0.0005	0.0083	0.0251	0.0005	0.0084	0.0259	0.0021	0.0337	0.1044	0.0021	0.0326	0.0986
	1.2	0.0224	0.0028	0.0438	0.1319	0.0028	0.0458	0.1430	0.0023	0.0372	0.1149	0.0023	0.0362	0.1099
	0.2	0.0004	0.0001	0.0009	0.0027	0.0001	0.0009	0.0027	0.0013	0.0209	0.0645	0.0013	0.0206	0.0627
80	0.5	0.0026	0.0003	0.0051	0.0155	0.0003	0.0052	0.0159	0.0013	0.0206	0.0633	0.0013	0.0203	0.0617
	1.2	0.0146	0.0018	0.0287	0.0871	0.0018	0.0296	0.0916	0.0014	0.0226	0.0695	0.0014	0.0222	0.0676

Table 4: Bayes risk of estimators under different loss functions for variations in θ at different sample size n, when prior is $Gamma(0.1, 0.1)$

			δ=0.5	δ=2	δ=3.5	δ=-0.5	δ=-2	δ=-3.5	δ_1=0.5	δ_1=2	δ_1=3.5	δ_1=-0.5	δ_1=-2	δ_1=-3.5
n	θ	BR_S	BR_{L1}	BR_{L2}	BR_{L3}	BR_{L4}	BR_{L5}	BR_{L6}	BR_{G1}	BR_{G2}	BR_{G3}	BR_{G4}	BR_{G5}	BR_{G6}
	0.2	0.0055	0.0007	0.0108	0.0328	0.0007	0.0111	0.0344	0.0112	0.1904	0.6225	0.0109	0.1658	0.4880
10	0.5	0.0324	0.0040	0.0621	0.1844	0.0041	0.0681	0.2182	0.0110	0.1862	0.6036	0.0107	0.1624	0.4755
	1.2	0.1839	0.0223	0.3278	0.8740	0.0237	0.4253	0.9401	0.0110	0.1847	0.5951	0.0107	0.1635	0.4804
	0.2	0.0013	0.0002	0.0026	0.0079	0.0002	0.0026	0.0081	0.0036	0.0583	0.1808	0.0036	0.0564	0.1703
30	0.5	0.0081	0.0010	0.0160	0.0484	0.0010	0.0165	0.0510	0.0036	0.0580	0.1809	0.0035	0.0555	0.1674
	1.2	0.0468	0.0058	0.0888	0.2608	0.0059	0.0982	0.3115	0.0036	0.0578	0.1790	0.0035	0.0559	0.1686
	0.2	0.0007	0.0001	0.0015	0.0045	0.0001	0.0015	0.0046	0.0021	0.0344	0.1063	0.0021	0.0337	0.1024
50	0.5	0.0046	0.0006	0.0090	0.0274	0.0006	0.0092	0.0286	0.0021	0.0338	0.1043	0.0021	0.0331	0.1005
	1.2	0.0268	0.0033	0.0520	0.1559	0.0034	0.0551	0.1722	0.0022	0.0351	0.1081	0.0022	0.0344	0.1048
	0.2	0.0004	0.0001	0.0009	0.0027	0.0001	0.0009	0.0027	0.0013	0.0209	0.0645	0.0013	0.0206	0.0628
80	0.5	0.0027	0.0003	0.0054	0.0166	0.0003	0.0055	0.0170	0.0013	0.0210	0.0647	0.0013	0.0208	0.0633
	1.2	0.0158	0.0020	0.0311	0.0939	0.0020	0.0323	0.1002	0.0013	0.0212	0.0651	0.0013	0.0209	0.0637

Form Table 1-5, we can conclude that

1. Risk is minimum in case of linex loss function for each variation of θ and n under both priors (see Table 3 and 4).
2. Risk increases as the value of the shape parameter increases for LLF and GELF both in case of positive shape parameter of loss functions (see Table 3 and 4).
3. Risk decreases as the value of the shape parameter increases for LLF and GELF both in case of negative shape parameter of loss functions (see Table 3 and 4). This pattern is the same as discussed by [17] for LLF.

Table 5: HPD interval for variations in n and θ along with coverage probabilities

		$a=1, b=4$				$a=0.1, b=0.1$			
n	θ	LL	UL	length	cp	LL	UL	length	cp
	0.2	0.1044	0.3301	0.2257	0.958	0.1008	0.3343	0.2336	0.949
10	0.5	0.2372	0.7541	0.5169	0.948	0.2497	0.8288	0.5792	0.950
	1.2	0.4595	1.4799	1.0204	0.837	0.6002	1.9916	1.3913	0.942
	0.2	0.1406	0.2710	0.1304	0.952	0.1395	0.2711	0.1316	0.946
30	0.5	0.3397	0.6554	0.3157	0.948	0.3486	0.6775	0.3289	0.949
	1.2	0.7583	1.4664	0.7080	0.910	0.8386	1.6304	0.7919	0.949
	0.2	0.1530	0.2540	0.1010	0.949	0.1526	0.2544	0.1018	0.949
50	0.5	0.3740	0.6211	0.2471	0.946	0.3822	0.6369	0.2548	0.949
	1.2	0.8602	1.4306	0.5704	0.926	0.9128	1.5215	0.6087	0.949
	0.2	0.1624	0.2423	0.0799	0.950	0.1618	0.2419	0.0801	0.951
80	0.5	0.4010	0.5984	0.1974	0.941	0.4050	0.6054	0.2004	0.947
	1.2	0.9353	1.3969	0.4616	0.933	0.9707	1.4515	0.4808	0.950

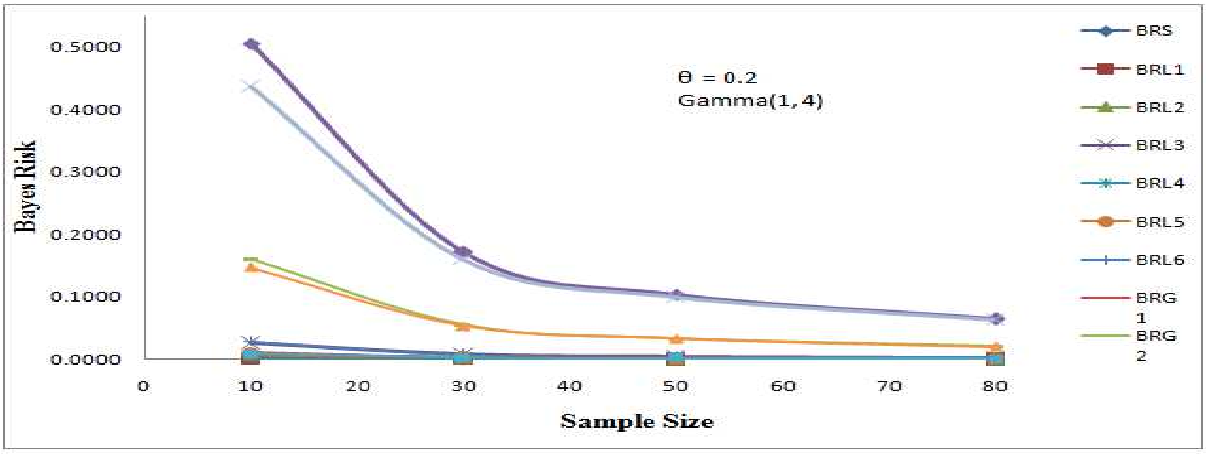

Fig. 1: Bayes risk of estimators under different loss functions for θ = 0.2 at different sample size n, when prior is Gamma (1, 4)

4.The length of the HPD interval is increases with the increase in the value of the parameter θ, for all variations of sample size n, i.e. n = 10, 30, 50 & 80 (see Table 5).
5.Coverage probability of HPD interval decreases as the increase in the value of the parameter θ of the model (see Table 5).
6.Also, the length of the HPD interval decreases as the increase the sample size for fixed parameter θ, in all variations of θ, taken in this study with both type of priors i.e. $Gamma(0.1,0.1)$ and $Gamma(1,4)$ (see Table 5).
7.Form Table 3 and 4, we can conclude that the risk is minimum under $Gamma(1,4)$ prior as compared to $Gamma(0.1,0.1)$ prior for all variations of θ and n, that are taken in this study.

These findings can also be verified graphically by the Figures 1-6. Where Fig. 1-3 represent Bayes risk for different considered loss functions with prior Gamma (1, 4), whereas Fig. 4-6 represent Bayes risk for different considered loss functions with prior Gamma (0.1, 0.1).

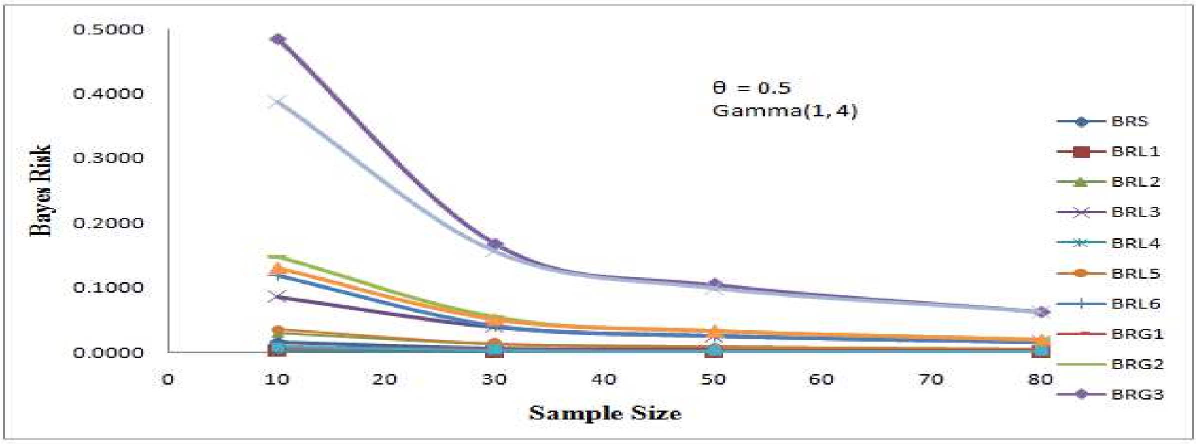

Fig. 2: Bayes risk of estimators under different loss functions for θ = 0.5 at different sample size n, when prior is Gamma (1, 4)

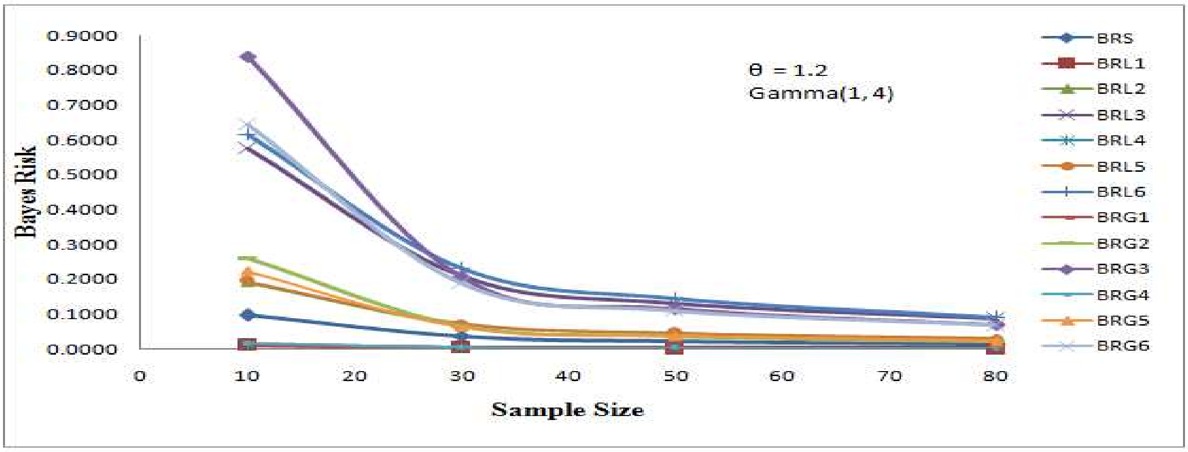

Fig. 3: Bayes risk of estimators under different loss functions for θ = 1.2 at different sample size n, when prior is Gamma (1, 4)

6 Conclusion

In this article, Bayes estimators are calculated for three different values of θ and four different values of sample size n as n= 10, 30, 50 & 80 by using gamma as a prior distribution with two sets of hyper parameters as $a = 1$, $b = 4$ and $a = 0.1$, $b = 0.1$ as informative and non-informative prior. All estimators are calculated under three different loss functions named as squared error, linex loss and general entropy loss function for LT exponential distribution. After this study, we can conclude that, Bayes estimators under linex loss are better among the all other estimators with lowest value of loss function shape parameter δ. In other words, as we increase the value δ, the risk of Bayes estimator will increase for LT exponential distribution.

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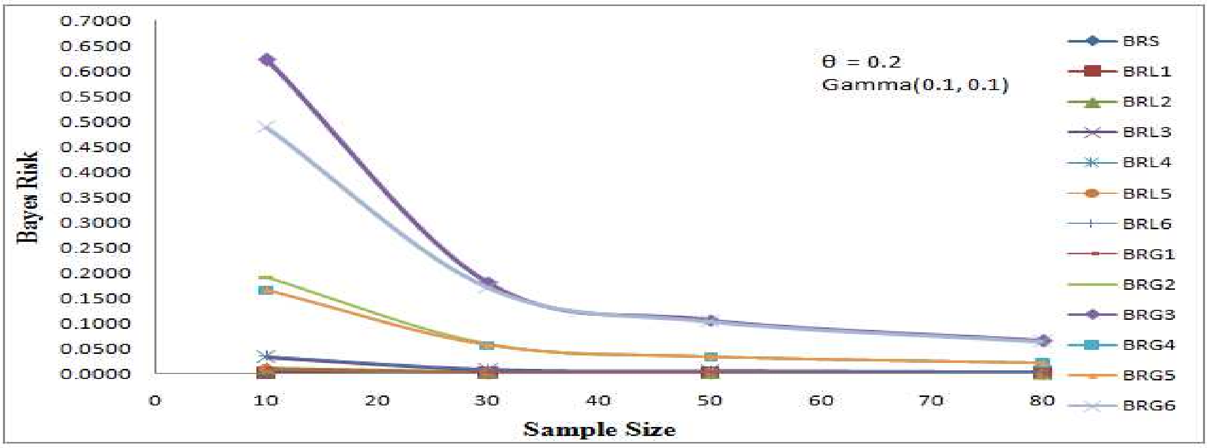

Fig. 4: Bayes risk of estimators under different loss functions for $\theta = 0.2$ at different sample size n, when prior is Gamma (0.1, 0.1)

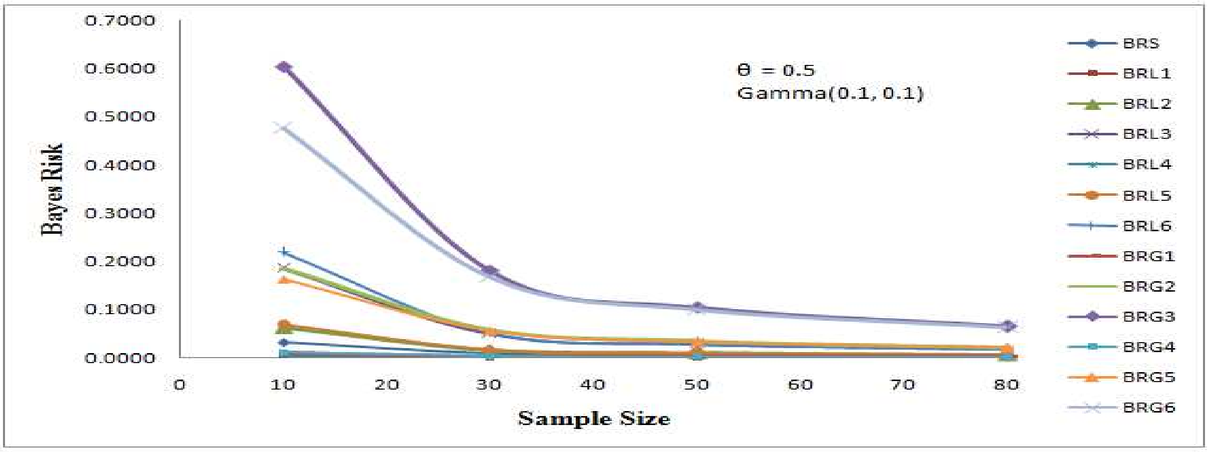

Fig. 5: Bayes risk of estimators under different loss functions for $\theta = 0.5$ at different sample size n, when prior is Gamma (0.1, 0.1)

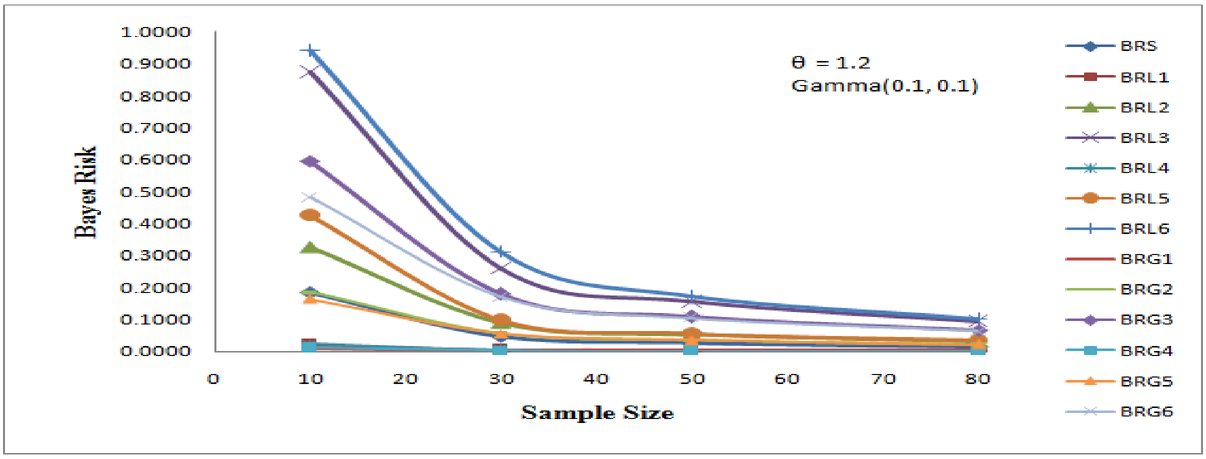

Fig. 6: Bayes risk of estimators under different loss functions for $\theta = 1.2$ at different sample size n, when prior is Gamma (0.1, 0.1)

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