Numerical and Computational Methods in Sciences & Engineering
An International Journal

http://dx.doi.org/10.18576/ncmse/010205

On Breather and Cuspon waves solutions for the generalized higher-order nonlinear Schrödinger equation with light-wave promulgation in an optical fiber

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Received: 15 March 2019, Revised: 12 June 2019, Accepted: 22 June 2019
Published online: 1 July 2019

Abstract: This research paper investigates the unprecedented optical closed form of solutions for the generalized higher-order nonlinear Schrödinger (NLS) equation, which is considered as a fundamental model in the optical fiber by the implementation of the modified Khater method. The suggested equation describes the promulgation of the light-wave in an optical fiber. Some novel solutions are obtained by using the suggested method, which is considered as one of the recent methods developed in the last decades. The performance of the used method shows the power and effective of the method and its ability for applying to many different forms of nonlinear evolution equations. The obtained solutions verified with Maple 16 & Mathematica 12 by placing them back into the original equations.

Keywords: Light-wave promulgation in an optical fiber; Generalized higher-order nonlinear Schrödinger equation; Generalized Kudryashov method; Exact traveling wave solution.

1 Introduction

An ultra-short pulse phenomenon of the light is one of a basic phenomenon in optics physics [1]. This phenomenon is an electromagnetic pulse which the period of its equal pico-second (10^{12} second) or less. These pulses have a wide range of the optical spectrum and also can establish by mode-locked oscillators. These pulses are typically mentioned to as ultra-fast juveniles. The padding of ultra-short pulses approximately demands the technical of chirped pulse amplification. It is distinguished by a high peak intensity. This phenomenon was studied in the nonlinear optics field. The Egyptian scientist, Ahmed H. Zewail in 1999, was taken Nobel Prize in Chemistry for utilizing ultra-short pulses to notice the chemical reactions on the timescales. The studying of Ahmed H. Zewail has opened the window for an unprecedented branch of science which is femtochemistry [2]. Femtochemistry is considered as chemical reactions on extremely short timescales approximately (10^{15}) seconds or one femtosecond. Now, this branch is considered as one of the basic fields as it has many vital applications in the different field of science such as freezing atoms in motion, reactive intermediates, Proteomic and Metabolomic Analysis and so on. Femtochemistry has many areas that utilized such as a gas phase & molecular beam, condensed phase, mesoscopic phase, control, structures of UED & x-ray and femtobiology [3,4,5].

All these studies have contributed the greatest role in providing an opportunity to study the dynamic each of the next phenomena: The gas-to-liquid transition region, Small and large molecules in cyclodextrins, molecular (one-atom) caging, microscopic friction, liquid state and energy flow in polymers. The examples of the femto-second pulse are collinear programmable, transverse static, transverse programmable, and collinear static. According to these examples, we able to find the applications of ultra-short pulse in studying advanced material 3D micro-/nano-processing and Micro-machining.

According to all these studies, it was natural for the world's mathematicians to take its toll in this branch of modern

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science for the great benefit of the world. The great scientist Erwin Schrödinger derived Schrödinger equation [6] in the next formula:

$$i\hbar S_t - \widehat{H}\,S = 0, \tag{1}$$

where $S = S(x,t)$ is the wave function of the quantum system, $\hbar$ represents Planck constant, $\widehat{H}$ represents Hamiltonian operator and (x,t) represent the position vector and time respectively. This equation describes the dynamics of the light pulses and femtosecond pulses. This equation is also considered as one of the basic models in quantum mechanics.

The Schrödinger equation has taken a lot of forms and formulas from the first day of its appearance and that because of its properties and possibilities. Many scientists have tried to adapt this equation to decipher, dissolve, and characterize some mathematical and physical models. More than one equation has emerged to characterize the Schrödinger equation, and many sporting methods have been utilized to find the exact solutions to such an important model of equations and so to discover the physical properties that have not yet been revealed from these studies on the Schrödinger equation [6,7,8,9,10,11,12,13,14,15,16,17,18,19,20].

The remainder of this paper is governed as follows: In section 2, we utilize the modified Khater method [21,22,23,24,25] to get the exact and solitary traveling wave solutions of a light-wave promulgation in an optical fiber (generalized higher-order NLS equation) [26,27,28,29,30] , In section 3, conclusion is given.

2 Applications

This section applies the modified Khater method to the generalized higher-order NLS equation that is given as:

$$i\,u_t + c_1\,u_{xx} + \left(c_2\,|u|^{2m} + c_3\,|u|^{4m}\right)u + c_4\left(\frac{(|u|)_{xx}}{|u|}\right)u = 0, \tag{2}$$

Using the wave transformation $u = u(x,t) = \phi(\xi)\,e^{i\mu}$, $\xi = (k\,x - \omega\,t)$, $\mu = (\rho\,x + \delta\,t)$ on Eq. (2), leads to find a real and imaginary parts of a generalized higher-order NLS equation. Separating these parts, leads to

$$\begin{cases} \omega = 2\,c_1\,\rho\,k, \\ a\,\phi'' - b\,\phi + c\,\phi^{2m+1} + d\,\phi^{4m+1} = 0, \end{cases} \tag{3}$$

where $a = k\,(c_1 + c_4)$, $b = (\delta + c_1\,p^2)$, $c = c_3, d = c_4$. Balancing the highest order derivative and nonlinear terms in Eq. (3), obtains $n = \frac{1}{2m}$. Utilizing the following transformation $\phi = S^{\frac{1}{2m}}$ on Eq. (3), gives

$$\frac{a\,(1-2m)}{4m^2}\,S'^2 + \frac{a}{2m}\,S\,S'' - b\,S^2 + c\,S^3 + d\,S^4 = 0. \tag{4}$$

Balancing the highest order derivative and nonlinear terms in Eq.(4), leads to $n = 1$. According the general solutions that suggested by the modified Khater method, the general solution of Eq. (4) is given as

$$S(\xi) = \sum_{i=1}^{n} a_i K^{i f(\xi)} + \sum_{i=1}^{n} b_i K^{-i f(\xi)} + a_0 = a_0 + a_1\,K^{f(\xi)} + \frac{b_1}{K^{f(\xi)}}, \tag{5}$$

where a_0, a_1, b_1 are arbitrary constants and $f(\xi)$ is the solution of the next auxiliary equation

$$f'(\xi) = \frac{\beta + \alpha K^{-f(\xi)} + \sigma K^{f(\xi)}}{ln(k)}, \tag{6}$$

where α, β, σ are arbitrary constants. Exchanging Eq.(5) along (6) and its derivatives into Eq.(4). Combination all terms with the same power of $K^{i\,f(\xi)}$ where $(i = 4,3,...,1,0)$, leads to a system of algebraic system of equations. Solving this system by using any computer program (Maple, Mathematica, Matlab,...,etc.) to get the values of parameters that involved in Eq.(4), obtains:

Family I

$$a_1 \to -\frac{a_0\sqrt{\sigma}}{2\sqrt{\alpha}}, b_1 \to -\frac{\sqrt{\alpha}a_0}{2\sqrt{\sigma}}, b \to \frac{1}{4}\left(12a\sqrt{\alpha}\beta\sqrt{\sigma} + 20a\alpha\sigma + a\beta^2\right), c \to \frac{2\left(a\sqrt{\alpha}\beta\sqrt{\sigma} + 4a\alpha\sigma\right)}{a_0}, d \to -\frac{3a\alpha\sigma}{a_0^2}.$$

Thus, the solitary wave solutions the generalized higher-order NLS equation
When $\beta^2-4\alpha\sigma<0\,\&\,\sigma\neq 0$

$$u_1(x,t)=2^{\frac{-1}{m}}e^{i(\delta t+\rho x)}\left(\frac{a_0\left(2\sqrt{\alpha}\sqrt{\sigma}+\beta-\sqrt{4\alpha\sigma-\beta^2}\tan\left(\frac{1}{2}\sqrt{4\alpha\sigma-\beta^2}(kx-t\omega)\right)\right)^2}{\sqrt{\alpha}\sqrt{\sigma}\left(\beta-\sqrt{4\alpha\sigma-\beta^2}\tan\left(\frac{1}{2}\sqrt{4\alpha\sigma-\beta^2}(kx-t\omega)\right)\right)}\right)^{\frac{1}{2m}}, \tag{7}$$

$$u_2(x,t)=2^{\frac{-1}{m}}e^{i(\delta t+\rho x)}\left(\frac{a_0\left(2\sqrt{\alpha}\sqrt{\sigma}+\beta-\sqrt{4\alpha\sigma-\beta^2}\cot\left(\frac{1}{2}\sqrt{4\alpha\sigma-\beta^2}(kx-t\omega)\right)\right)^2}{\sqrt{\alpha}\sqrt{\sigma}\left(\beta-\sqrt{4\alpha\sigma-\beta^2}\cot\left(\frac{1}{2}\sqrt{4\alpha\sigma-\beta^2}(kx-t\omega)\right)\right)}\right)^{\frac{1}{2m}}. \tag{8}$$

When $\beta^2-4\alpha\sigma>0\,\&\,\sigma\neq 0$

$$u_3(x,t)=2^{\frac{-1}{m}}e^{i(\delta t+\rho x)}\left(\frac{a_0\left(2\sqrt{\alpha}\sqrt{\sigma}+\beta+\sqrt{\beta^2-4\alpha\sigma}\tanh\left(\frac{1}{2}\sqrt{\beta^2-4\alpha\sigma}(kx-t\omega)\right)\right)^2}{\sqrt{\alpha}\sqrt{\sigma}\left(\beta+\sqrt{\beta^2-4\alpha\sigma}\tanh\left(\frac{1}{2}\sqrt{\beta^2-4\alpha\sigma}(kx-t\omega)\right)\right)}\right)^{\frac{1}{2m}}, \tag{9}$$

$$u_4(x,t)=2^{\frac{-1}{m}}e^{i(\delta t+\rho x)}\left(\frac{a_0\left(2\sqrt{\alpha}\sqrt{\sigma}+\beta+\sqrt{\beta^2-4\alpha\sigma}\coth\left(\frac{1}{2}\sqrt{\beta^2-4\alpha\sigma}(kx-t\omega)\right)\right)^2}{\sqrt{\alpha}\sqrt{\sigma}\left(\beta+\sqrt{\beta^2-4\alpha\sigma}\coth\left(\frac{1}{2}\sqrt{\beta^2-4\alpha\sigma}(kx-t\omega)\right)\right)}\right)^{\frac{1}{2m}}. \tag{10}$$

When $\alpha\sigma>0\,\&\,\alpha\neq 0\,\&\,\sigma\neq 0\,\&\,\beta=0$

$$u_5(x,t)=e^{i(\delta t+\rho x)}\left(a_0\left(-\left(\csc\left(2\sqrt{\alpha}\sqrt{\sigma}(kx-t\omega)\right)-1\right)\right)\right)^{\frac{1}{2m}}, \tag{11}$$

$$u_6(x,t)=e^{i(\delta t+\rho x)}\left(a_0\left(\csc\left(2\sqrt{\alpha}\sqrt{\sigma}(kx-t\omega)\right)+1\right)\right)^{\frac{1}{2m}}. \tag{12}$$

When $\beta=0\,\&\,\alpha=-\sigma$

$$u_7(x,t)=e^{i(\delta t+\rho x)}\left(a_0\left(\frac{\sqrt{\alpha}\operatorname{csch}(2\alpha(kx-t\omega))}{\sqrt{-\alpha}}+1\right)\right)^{\frac{1}{2m}}. \tag{13}$$

When $\beta=0\,\&\,\alpha=\sigma$

$$u_8(x,t)=2^{\frac{-1}{2m}}e^{i(\delta t+\rho x)}\left(a_0\tan(C+\alpha kx+\alpha t\omega)\left(-(\cot(C+\alpha kx+\alpha t\omega)-1)^2\right)\right)^{\frac{1}{2m}}. \tag{14}$$

When $\beta^2-4\alpha\sigma=0$

$$u_9(x,t)=e^{i(\delta t+\rho x)}\left(\frac{a_0\beta^2(kx-t\omega)}{4\sqrt{\alpha}\sqrt{\sigma}(\beta(kx-t\omega)+2)}+\frac{\sqrt{\alpha}a_0\sqrt{\sigma}(\beta(kx-t\omega)+2)}{\beta^2(kx-t\omega)}+a_0\right)^{\frac{1}{2m}}. \tag{15}$$

Family II

$$a_1\to\frac{a_0\sigma}{\beta},b_1\to\frac{\alpha a_0}{\beta},b\to\frac{1}{4}\left(a\beta^2-4a\alpha\sigma\right),c\to\frac{a\beta^2}{a_0},d\to-\frac{3a\beta^2}{4a_0^2}.$$

Thus, the solitary wave solutions the generalized higher-order NLS equation
When $\beta^2-4\alpha\sigma<0\,\&\,\sigma\neq 0$

$$u_{10}(x,t)=2^{\frac{-1}{2m}}e^{i(\delta t+\rho x)}\left(\frac{a_0\left(\beta^2-4\alpha\sigma\right)\sec^2\left(\frac{1}{2}\sqrt{4\alpha\sigma-\beta^2}(kx-t\omega)\right)}{\beta\left(\beta-\sqrt{4\alpha\sigma-\beta^2}\tan\left(\frac{1}{2}\sqrt{4\alpha\sigma-\beta^2}(kx-t\omega)\right)\right)}\right)^{\frac{1}{2m}}, \tag{16}$$

$$u_{11}(x,t)=2^{\frac{-1}{2m}}e^{i(\delta t+\rho x)}\left(\frac{a_0\left(\beta^2-4\alpha\sigma\right)\csc^2\left(\frac{1}{2}\sqrt{4\alpha\sigma-\beta^2}(kx-t\omega)\right)}{\beta\left(\beta-\sqrt{4\alpha\sigma-\beta^2}\cot\left(\frac{1}{2}\sqrt{4\alpha\sigma-\beta^2}(kx-t\omega)\right)\right)}\right)^{\frac{1}{2m}}. \tag{17}$$

When $\beta^2-4\alpha\sigma>0\,\&\,\sigma\neq 0$

$$u_{12}(x,t)=2^{\frac{-1}{2m}}e^{i(\delta t+\rho x)}\left(\frac{a_0\left(\beta^2-4\alpha\sigma\right)\operatorname{sech}^2\left(\frac{1}{2}\sqrt{\beta^2-4\alpha\sigma}(kx-t\,\omega)\right)}{\beta\left(\beta+\sqrt{\beta^2-4\alpha\sigma}\tanh\left(\frac{1}{2}\sqrt{\beta^2-4\alpha\sigma}(kx-t\,\omega)\right)\right)}\right)^{\frac{1}{2m}}, \tag{18}$$

$$u_{13}(x,t)=2^{\frac{-1}{2m}}e^{i(\delta t+\rho x)}\left(-\frac{a_0\left(\beta^2-4\alpha\sigma\right)\operatorname{csch}^2\left(\frac{1}{2}\sqrt{\beta^2-4\alpha\sigma}(kx-t\,\omega)\right)}{\beta\left(\beta+\sqrt{\beta^2-4\alpha\sigma}\coth\left(\frac{1}{2}\sqrt{\beta^2-4\alpha\sigma}(kx-t\,\omega)\right)\right)}\right)^{\frac{1}{2m}}. \tag{19}$$

When $\beta=\frac{\alpha}{2}=\kappa\,\&\,\sigma=0$

$$u_{14}(x,t)=e^{i(\delta t+\rho x)}\left(a_0\left(\frac{2}{e^{\kappa(kx-t\,\omega)}-2}+1\right)\right)^{\frac{1}{2m}}. \tag{20}$$

When $\beta=\sigma=\kappa\,\&\,\alpha=0$

$$u_{15}(x,t)=e^{i(\delta t+\rho x)}\left(\frac{a_0}{1-e^{\kappa(kx-t\,\omega)}}\right)^{\frac{1}{2m}}. \tag{21}$$

When $\alpha=0\,\&\,\beta\neq 0\,\&\,\sigma\neq 0$

$$u_{16}(x,t)=2^{\frac{1}{2m}}e^{i(\delta t+\rho x)}\left(\frac{a_0}{2-\sigma e^{\beta(kx-t\,\omega)}}\right)^{\frac{1}{2m}}. \tag{22}$$

When $\sigma=0\,\&\,\beta\neq 0\,\&\,\alpha\neq 0$

$$u_{17}(x,t)=e^{i(\delta t+\rho x)}\left(a_0\left(\frac{\alpha}{\beta e^{\beta(kx-t\,\omega)}-\alpha}+1\right)\right)^{\frac{1}{2m}}. \tag{23}$$

When $\beta^2-4\alpha\sigma=0$

$$u_{18}(x,t)=2^{\frac{-1}{2m}}e^{i(\delta t+\rho x)}\left(a_0\left(\frac{4\alpha\sigma\left(-\beta-\frac{2}{kx-t\,\omega}\right)}{\beta^3}+\frac{2}{\beta kx-\beta t\omega+2}+1\right)\right)^{\frac{1}{2m}}. \tag{24}$$

Family III

$$a_1\to\frac{\sqrt{a_0^2\beta^2-4\alpha a_0^2\sigma}+a_0\beta}{2\alpha},b_1\to 0,b\to\frac{1}{4}\left(a\beta^2-4a\alpha\sigma\right),$$
$$c\to\frac{aa_0\left(\beta^2-4\alpha\sigma\right)-a\beta\sqrt{a_0^2\left(\beta^2-4\alpha\sigma\right)}}{2a_0^2},d\to\frac{3a\left(\beta\sqrt{a_0^2\left(\beta^2-4\alpha\sigma\right)}-a_0\left(\beta^2-2\alpha\sigma\right)\right)}{8a_0^3}.$$

Thus, the solitary wave solutions the generalized higher-order NLS equation
When $\beta^2-4\alpha\sigma<0\,\&\,\sigma\neq 0$

$$u_{19}(x,t)=e^{i(\delta t+\rho x)}\left(a_0-\frac{\left(\sqrt{a_0^2\left(\beta^2-4\alpha\sigma\right)}+a_0\beta\right)\left(\beta-\sqrt{4\alpha\sigma-\beta^2}\tan\left(\frac{1}{2}\sqrt{4\alpha\sigma-\beta^2}(kx-t\,\omega)\right)\right)}{4\alpha\sigma}\right)^{\frac{1}{2m}}, \tag{25}$$

$$u_{20}(x,t)=e^{i(\delta t+\rho x)}\left(a_0-\frac{\left(\sqrt{a_0^2\left(\beta^2-4\alpha\sigma\right)}+a_0\beta\right)\left(\beta-\sqrt{4\alpha\sigma-\beta^2}\cot\left(\frac{1}{2}\sqrt{4\alpha\sigma-\beta^2}(kx-t\,\omega)\right)\right)}{4\alpha\sigma}\right)^{\frac{1}{2m}}. \tag{26}$$

When $\beta^2-4\alpha\sigma>0\,\&\,\sigma\neq 0$

$$u_{21}(x,t)=e^{i(\delta t+\rho x)}\left(a_0-\frac{\left(\sqrt{a_0^2\left(\beta^2-4\alpha\sigma\right)}+a_0\beta\right)\left(\beta+\sqrt{\beta^2-4\alpha\sigma}\tanh\left(\frac{1}{2}\sqrt{\beta^2-4\alpha\sigma}(kx-t\,\omega)\right)\right)}{4\alpha\sigma}\right)^{\frac{1}{2m}}, \tag{27}$$

$$u_{22}(x,t)=e^{i(\delta t+\rho x)}\left(a_0-\frac{\left(\sqrt{a_0^2(\beta^2-4\alpha\sigma)}+a_0\beta\right)\left(\beta+\sqrt{\beta^2-4\alpha\sigma}\coth\left(\frac{1}{2}\sqrt{\beta^2-4\alpha\sigma}(kx-t\omega)\right)\right)}{4\alpha\sigma}\right)^{\frac{1}{2m}}. \quad (28)$$

When $\alpha\sigma>0\,\&\,\alpha\neq0\,\&\,\sigma\neq0\,\&\,\beta=0$

$$u_{23}(x,t)=e^{i(\delta t+\rho x)}\left(\frac{\sqrt{-\alpha a_0^2\sigma}\tan\left(\sqrt{\alpha\sigma}(kx-t\omega)\right)}{\sqrt{\alpha\sigma}}+a_0\right)^{\frac{1}{2m}}, \quad (29)$$

$$u_{24}(x,t)=e^{i(\delta t+\rho x)}\left(a_0-\frac{\sqrt{-\alpha a_0^2\sigma}\cot\left(\sqrt{\alpha\sigma}(kx-t\omega)\right)}{\sqrt{\alpha\sigma}}\right)^{\frac{1}{2m}}. \quad (30)$$

When $\alpha\sigma>0\,\&\,\alpha\neq0\,\&\,\sigma\neq0\,\&\,\beta=0$

$$u_{25}(x,t)=e^{i(\delta t+\rho x)}\left(\frac{\sqrt{-\alpha a_0^2\sigma}\tanh\left(\sqrt{-\alpha\sigma}(kx-t\omega)\right)}{\sqrt{-\alpha\sigma}}+a_0\right)^{\frac{1}{2m}}, \quad (31)$$

$$u_{26}(x,t)=e^{i(\delta t+\rho x)}\left(\frac{\sqrt{-\alpha a_0^2\sigma}\coth\left(\sqrt{-\alpha\sigma}(kx-t\omega)\right)}{\sqrt{-\alpha\sigma}}+a_0\right)^{\frac{1}{2m}}. \quad (32)$$

When $\beta=0\,\&\,\alpha=-\sigma$

$$u_{27}(x,t)=e^{i(\delta t+\rho x)}\left(\frac{\sqrt{\alpha^2a_0^2}\coth(\alpha(kx-t\omega))}{\alpha}+a_0\right)^{\frac{1}{2m}}. \quad (33)$$

When $\alpha=\sigma=0\,\&\,\beta\neq0$

$$u_{28}(x,t)=e^{i(\delta t+\rho x)}\left(a_1e^{\beta(kx-t\omega)}\right)^{\frac{1}{2m}}. \quad (34)$$

When $\beta=\sigma=\kappa\,\&\,\alpha=0$

$$u_{29}(x,t)=e^{i(\delta t+\rho x)}\left(a_1\left(\frac{1}{1-e^{\kappa(kx-t\omega)}}-1\right)\right)^{\frac{1}{2m}}. \quad (35)$$

When $\beta=0\,\&\,\alpha=\sigma$

$$u_{30}(x,t)=e^{i(\delta t+\rho x)}\left(\frac{\sqrt{-\alpha^2a_0^2}\tan(C+\alpha kx-\alpha t\omega)}{\alpha}+a_0\right)^{\frac{1}{2m}}. \quad (36)$$

When $\sigma=0\&\beta\neq0\,\&\,\alpha\neq0$

$$u_{31}(x,t)=e^{i(\delta t+\rho x)}\left(\frac{\left(\sqrt{a_0^2\beta^2}+a_0\beta\right)\left(e^{\beta(kx-t\omega)}-\frac{\alpha}{\beta}\right)}{2\alpha}+a_0\right)^{\frac{1}{2m}}. \quad (37)$$

When $\beta^2-4\alpha\sigma=0$

$$u_{32}(x,t)=2^{\frac{1}{2m}}e^{i(\delta t+\rho x)}\left(-\frac{a_0}{\beta kx-\beta t\omega}\right)^{\frac{1}{2m}}. \quad (38)$$

Family IV

$$a_1 \to 0, b_1 \to \frac{a_0\beta - \sqrt{a_0^2\beta^2 - 4\alpha a_0^2\sigma}}{2\sigma}, b \to \frac{1}{4}\left(a\beta^2 - 4a\alpha\sigma\right),$$

$$c \to \frac{a\beta\sqrt{a_0^2\left(\beta^2-4\alpha\sigma\right)} + aa_0\left(\beta^2-4\alpha\sigma\right)}{2a_0^2}, d \to -\frac{3a\left(\beta\sqrt{a_0^2\left(\beta^2-4\alpha\sigma\right)} + a_0\left(\beta^2-2\alpha\sigma\right)\right)}{8a_0^3}.$$

Thus, the solitary wave solutions the generalized higher-order NLS equation
When $\beta^2 - 4\alpha\sigma < 0 \,\&\, \sigma \neq 0$

$$u_{33}(x,t) = e^{i(\delta t+\rho x)}\left(\frac{\sqrt{a_0^2\left(\beta^2-4\alpha\sigma\right)} - a_0\beta}{\beta - \sqrt{4\alpha\sigma-\beta^2}\tan\left(\frac{1}{2}\sqrt{4\alpha\sigma-\beta^2}(kx-t\omega)\right)} + a_0\right)^{\frac{1}{2m}}, \tag{39}$$

$$u_{34}(x,t) = e^{i(\delta t+\rho x)}\left(\frac{\sqrt{a_0^2\left(\beta^2-4\alpha\sigma\right)} - a_0\beta}{\beta - \sqrt{4\alpha\sigma-\beta^2}\cot\left(\frac{1}{2}\sqrt{4\alpha\sigma-\beta^2}(kx-t\omega)\right)} + a_0\right)^{\frac{1}{2m}}. \tag{40}$$

When $\beta^2 - 4\alpha\sigma > 0 \,\&\, \sigma \neq 0$

$$u_{35}(x,t) = e^{i(\delta t+\rho x)}\left(\frac{\sqrt{a_0^2\left(\beta^2-4\alpha\sigma\right)} - a_0\beta}{\beta + \sqrt{\beta^2-4\alpha\sigma}\tanh\left(\frac{1}{2}\sqrt{\beta^2-4\alpha\sigma}(kx-t\omega)\right)} + a_0\right)^{\frac{1}{2m}}, \tag{41}$$

$$u_{36}(x,t) = e^{i(\delta t+\rho x)}\left(\frac{\sqrt{a_0^2\left(\beta^2-4\alpha\sigma\right)} - a_0\beta}{\beta + \sqrt{\beta^2-4\alpha\sigma}\coth\left(\frac{1}{2}\sqrt{\beta^2-4\alpha\sigma}(kx-t\omega)\right)} + a_0\right)^{\frac{1}{2m}}. \tag{42}$$

When $\alpha\sigma > 0 \,\&\, \alpha \neq 0 \,\&\, \sigma \neq 0 \,\&\, \beta = 0$

$$u_{37}(x,t) = e^{i(\delta t+\rho x)}\left(a_0 - \frac{\sqrt{-\alpha a_0^2\sigma}\cot\left(\sqrt{\alpha\sigma}(kx-t\omega)\right)}{\sqrt{\alpha\sigma}}\right)^{\frac{1}{2m}}, \tag{43}$$

$$u_{38}(x,t) = e^{i(\delta t+\rho x)}\left(\frac{\sqrt{-\alpha a_0^2\sigma}\tan\left(\sqrt{\alpha\sigma}(kx-t\omega)\right)}{\sqrt{\alpha\sigma}} + a_0\right)^{\frac{1}{2m}}. \tag{44}$$

When $\alpha\sigma > 0 \,\&\, \alpha \neq 0 \,\&\, \sigma \neq 0 \,\&\, \beta = 0$

$$u_{39}(x,t) = e^{i(\delta t+\rho x)}\left(\frac{\sqrt{-\alpha a_0^2\sigma}\coth\left(\sqrt{-\alpha\sigma}(kx-t\omega)\right)}{\sqrt{-\alpha\sigma}} + a_0\right)^{\frac{1}{2m}}, \tag{45}$$

$$u_{40}(x,t) = e^{i(\delta t+\rho x)}\left(\frac{\sqrt{-\alpha a_0^2\sigma}\tanh\left(\sqrt{-\alpha\sigma}(kx-t\omega)\right)}{\sqrt{-\alpha\sigma}} + a_0\right)^{\frac{1}{2m}}. \tag{46}$$

When $\beta = 0 \,\&\, \alpha = -\sigma$

$$u_{41}(x,t) = e^{i(\delta t+\rho x)}\left(\frac{\sqrt{\alpha^2 a_0^2}\tanh(\alpha(kx-t\omega))}{\alpha} + a_0\right)^{\frac{1}{2m}}. \tag{47}$$

When $\beta = \sigma = \kappa \,\&\, \alpha = 0$

$$u_{42}(x,t) = 6^{\frac{-1}{2m}} e^{i(\delta t+\rho x)} \left(a_1 \left(-\left(3\coth\left(\frac{1}{2}\kappa(kx-t\,\omega) \right) +2 \right) \right) \right)^{\frac{1}{2m}}. \tag{48}$$

When $\alpha = 0 \,\&\, \beta \neq 0 \,\&\, \sigma \neq 0$

$$u_{43}(x,t) = e^{i(\delta t+\rho x)} \left(\frac{\left(\sqrt{a_0^2\beta^2} - a_0\beta \right) e^{-\beta(kx-t\,\omega)} \left(\sigma e^{\beta(kx-t\,\omega)} - 2 \right)}{2\beta\sigma} + a_0 \right)^{\frac{1}{2m}}. \tag{49}$$

When $\beta = 0 \,\&\, \alpha = \sigma$

$$u_{44}(x,t) = e^{i(\delta t+\rho x)} \left(a_0 - \frac{\sqrt{-\alpha^2 a_0^2}\cot(C+\alpha kx - \alpha t\omega)}{\alpha} \right)^{\frac{1}{2m}}. \tag{50}$$

When $\beta^2 - 4\alpha\sigma = 0$

$$u_{45}(x,t) = e^{i(\delta t+\rho x)} \left(a_0 - \frac{a_0\beta^3(kx-t\,\omega)}{4\alpha\sigma(\beta kx - \beta t\omega + 2)} \right)^{\frac{1}{2m}}. \tag{51}$$

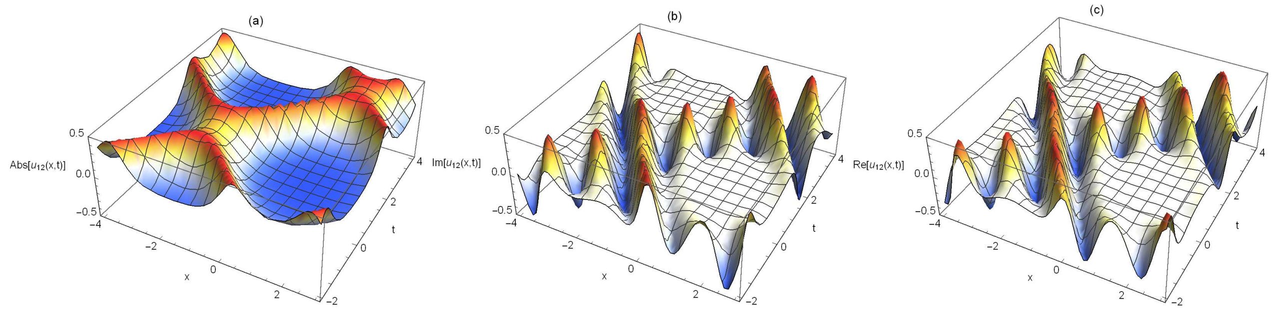

Fig. 1: Breather wave of the generalized higher-order NLS equation by using Eq. (18) in three-dimensional for absolute, imaginary, and real valued of the solution, when $\left[\alpha = 2,\, a_0 = 4,\, \beta = 3,\, \delta = 3,\, k = 5,\, m = 1,\, \rho = 4,\, \sigma = 1,\, \omega = 6\right]$

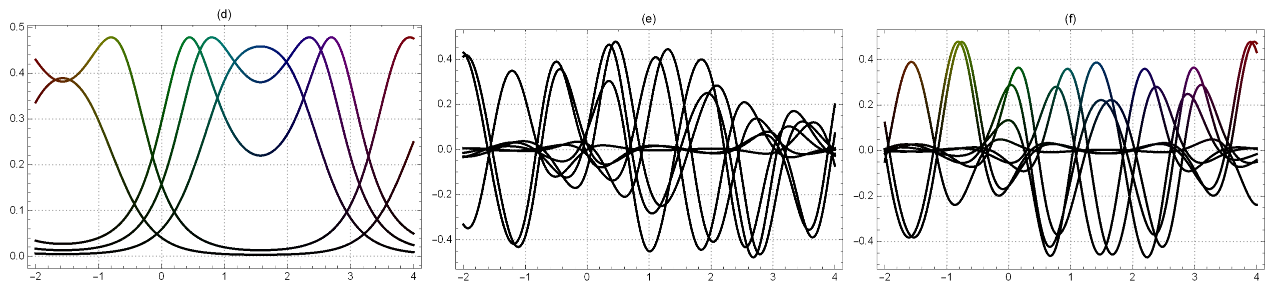

Fig. 2: Breather wave of the generalized higher-order NLS equation by using Eq. (18) in two-dimensional for absolute, imaginary, and real valued of the solution, when $\left[\alpha = 2,\, a_0 = 4,\, \beta = 3,\, \delta = 3,\, k = 5,\, m = 1,\, \rho = 4,\, \sigma = 1,\, \omega = 6\right]$

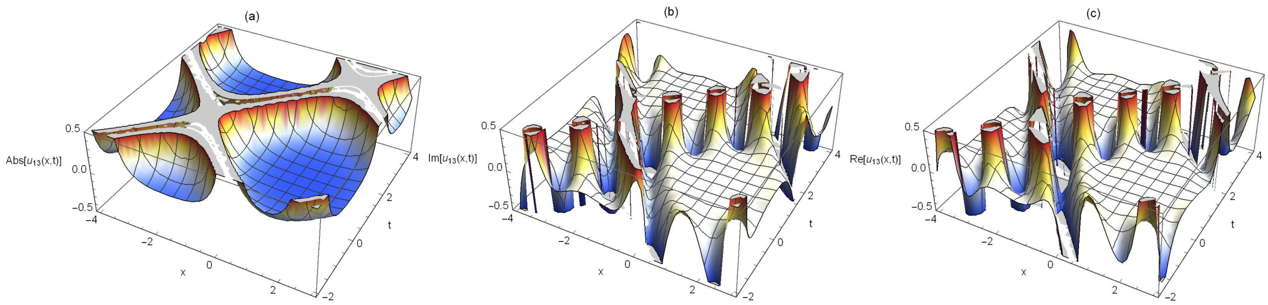

Fig. 3: Cuspon wave of the generalized higher-order NLS equation by using Eq. (19) in three-dimensional for absolute, imaginary, and real valued of the solution, when $\left[\alpha = 2,\, a_0 = 4,\, \beta = 3,\, \delta = 3,\, k = 5,\, m = 1,\, \rho = 4,\, \sigma = 1,\, \omega = 6\right]$

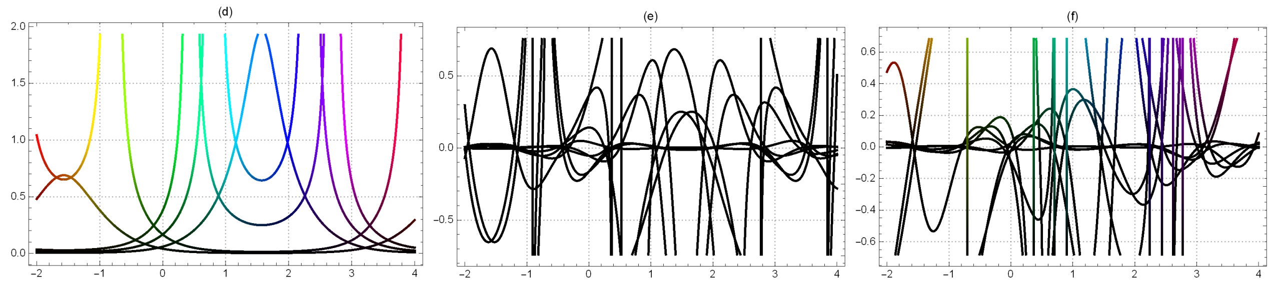

Fig. 4: Cuspon wave of the generalized higher-order NLS equation by using Eq. (19) in two-dimensional for absolute, imaginary, and real valued of the solution, when $\left[\alpha = 2,\, a_0 = 4,\, \beta = 3,\, \delta = 3,\, k = 5,\, m = 1,\, \rho = 4,\, \sigma = 1,\, \omega = 6\right]$

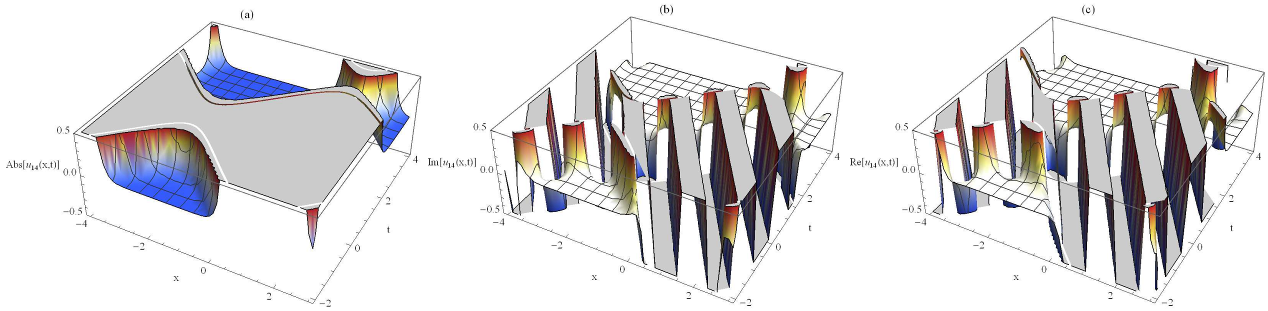

Fig. 5: Singular cuspon wave of the generalized higher-order NLS equation by using Eq. (20) in three-dimensional for absolute, imaginary, and real valued of the solution, when $\left[\alpha = 6,\, a_0 = 4,\, \beta = 3,\, \delta = 3,\, k = 5,\, m = 1,\, \rho = 4,\, \sigma = 0,\, \omega = 6,\, \kappa = 3\right]$

3 Conclusion

In this research paper, the modified Khater method is applied to the light-wave promulgation in an optical fiber (Generalized higher-order NLS equation). Some novel solutions are obtained, and some figures are also sketched to show more physical properties and dynamical behavior of the particles in the light-wave, specially in an optical fiber. These solutions show the power and effective of this method and also its ability to apply this method on many different formulae of nonlinear partial differential equations.

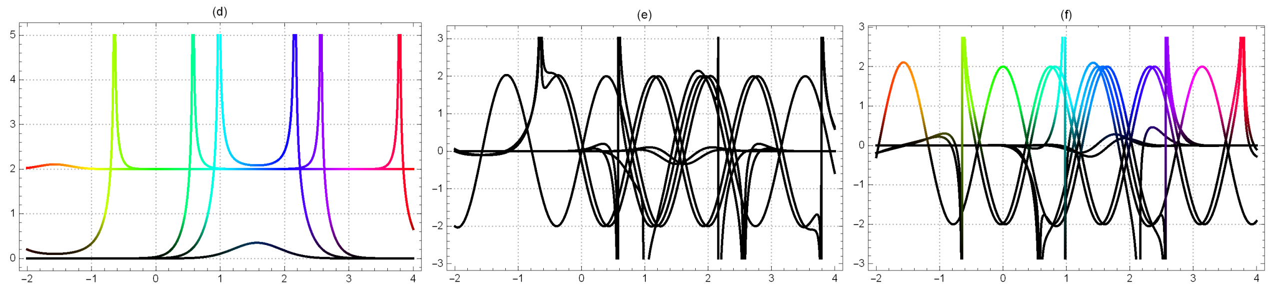

Fig. 6: Singular cuspon wave of the generalized higher-order NLS equation by using Eq. (20) in two-dimensional for absolute, imaginary, and real valued of the solution, when $\left[\alpha = 6, a_0 = 4, \beta = 3, \delta = 3, k = 5, m = 1, \rho = 4, \sigma = 0, \omega = 6, \kappa = 3\right]$

Acknowledgement

The authors greatly thank the referees for their comments, which improve the paper.

Conflict of Interests

There is no conflict of interests by authors regarding the publication of this manuscript.

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