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Bound Lengths on Type-I Progressive Hybrid Rayleigh Data under SS-PALT

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Abstract: The present article discusses some Bayes prediction bound lengths in one-parameter Rayleigh distribution. For this, Type-I Progressive Hybrid censoring criterion (T-IPH Censoring) have been combined with Step-Stress Partially Accelerated Life Test (SS-PALT). No literature was found tackling the Bayes prediction bound lengths under T-IPH Censoring with SS-PALT. One-Sample & Two-Sample Bayes prediction bound lengths have been obtained in the above scenario. The analysis is done under simulated and real data set, by using Metropolis-Hastings (M-H) algorithm that has been discussed. The optimal stress change time also have been measured by the method of minimization of the asymptotic variance of ML Estimation. Approximate confidence lengths (ACL) also have been discussed along with Bayes prediction bound lengths.

Keywords: Step-Stress Partially Accelerated Life Test (SS-PALT); Type-I Progressive Hybrid censoring (T-IPH-censoring); Bayes Prediction Bound Length, Approximate Confidence Lengths (ACL).

1 Introduction

The One-Parameter Rayleigh distribution is recognized as an appropriate model by several researchers for several kinds of medical studies and life testing trials. The concerned distribution is also a vital model in applications such as noise theory, height of the sea waves and wave lengths. One-parameter Rayleigh distribution is considered as the underlying model for the present discussion. Probability density and the cumulative density functions for the underlying model are given as

$$f(y;\theta) = \frac{2y}{\theta} exp\left(-\frac{y^2}{\theta}\right) ; \theta > 0, y \geq 0 \tag{1}$$

and

$$F(y;\theta) = 1 - exp\left(-\frac{y^2}{\theta}\right) ; \theta > 0, y \geq 0. \tag{2}$$

The parameter θ represents the shape parameter of the Rayleigh model given in Eq. (1). A massive amount of literature for estimation regarding classical and Bayesian methodology are available, a few latest studies have discussed on Rayleigh distribution.

[1] discussed the properties of some Bayes estimators for 3-component mixture of Rayleigh distribution. [2] studied the properties of the Bayes estimators and one-sample Bayes prediction bound lengths, for the Rayleigh distribution under Type-II Progressive censoring criterion.

[3] discussed the properties of Bayes prediction bound lengths by pooling of two different progressive censored Rayleigh data. [4] has proposed some Bayes and shrinkage estimators for the unknown parameter of the Rayleigh distribution by using Al-Bayyati loss function. [5] has studied generalized Rayleigh distribution by using progressive Type-II censoring data with Binomial removals. They also developed optimum test plans to improve the quality of the statistical inference.

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[6] deals with E-Bayesian method under progressively Type-II censored Rayleigh sample for computing estimates of the parameter. They also discussed some Bayesian and E-Bayesian estimators obtained under squared error and LINEX loss functions. [7] discussed Bayes estimation for the Rayleigh distribution on simple random sample, ranked set sampling and maximum ranked set sampling procedure with unequal samples in one cycle and m-cycle cases. Recently, [8] discussed new aspects of compound Rayleigh distribution by using progressive first-failure censoring samples during step-stress partially accelerated life tests.

To decrease the duration of testing and to minimize test expenses, progressive censoring and the accelerated life testing are performed. So, the main objective of the present discussion is to combine SS-PALT with Type-I progressive hybrid censoring. One-Sample and Two-Sample Bayes prediction bound lengths have been obtained under the above scenario on one-parameter Rayleigh distribution. The optimal stress change time also was measured by the method of minimizing the asymptotic variance of ML estimation under study. Simulated samples were generated based on the M-H algorithm. An example based on a real data set also have been considered in the analysis of the proposed method.

2 Type-I Progressive Hybrid censoring

In Reliability analysis, the life-tests are performed to observe the life of units put to the test. In such trial, some surviving units are removed or lost due to time and cost constraints or due to the immediate needs of the units for other drives. The data obtained from such life tests are generally known as censored samples. The most common censoring scheme is Type-II censoring, in which the test will terminate after a pre-assumed number of failures. Another censoring scheme is a Type-I censoring, in which the test will stop at some pre-considered time. Both censoring schemes do not have the flexibility of elimination of units rather than the deadly point of life test. The progressive Type-II censoring scheme removes this lack of flexibility by providing removal of some units at each failure and at the final termination point there are no units available.

See [9] for extensive reviews on literature of the progressive censoring. A brief introduction of progressive Type-II censoring is, suppose from n test units are those placed on a life test with $T_1, T_2, \ldots, T_n$ corresponding lifetimes. All units are independent and identically distributed (Eq. (1)). Following [2], the trial stop at $m^{th}(m \leq n)$ failure and the progressively Type-II censored samples are $T_{1:m:n} \leq T_{2:m:n} \leq \ldots \leq T_{m:m:n}$ following the censoring pattern $R \equiv (R_1, R_2, \ldots, R_m)$.

In the present article, we are concerned about the study of the behavior of Bayes prediction bound length under T-IPH censoring in SS-PALT. In literature, no other studies have been found for the combination of these two under the Bayesian methodology. Few recent articles on T-IPH censoring have been discussed here. The properties of maximum likelihood estimators and Bayes estimators of unknown parameters for Exponential model were discussed by [10]. Classical estimation for two-parameter Weibull distribution have been discussed by [11] under Type-II progressive hybrid censored data. [12] discussed the Bayes estimates for Lindley distribution by using Type-II hybrid censored data. For more informative literature on hybrid censoring one may consult [13]. Inferences on Burr Type-XII distribution have been discussed recently by [14] by using Type-I progressive hybrid censoring.

The Type-I progressive hybrid censoring is now discussed as:

Let us assume, total n identical test units $x_1, x_2, \ldots, x_{m-1}, x_m, x_{m+1}, \ldots, x_n$ are subject to a life test trial. Under the progressive Type-II censoring scheme, $R \equiv (R_1, R_2, \ldots, R_m)$ be the prescribed censoring pattern which follows $n - m = R_1 + R_2 + \ldots + R_m$. Under T-1PH censoring, the test will terminate either at $m^{th}(m \leq n)$ failure or at time point t, which one occurred first and, both are pre-fixed at the time of commencement. In case the experiment stops at time t with a number of failures $X_{j:m:m}$, then they must satisfy the condition $X_{j:m:m} < t < X_{j+1:m:m}$ and the remaining life test units $n - R_1 - R_2 - \ldots - R_j - j = R^*$ (say) are removed from the test. So, the observed sample may be one of the following two types:

$$\begin{cases} I: & (X_{1:n:m}, X_{2:n:m}, \ldots, X_{\varepsilon:n:m}, X_{\varepsilon+1:n:m}, \ldots, X_{m:n:m},) \; ; if \, X_{m:n:m} < t \\ II: & (X_{1:n:m}, X_{2:n:m}, \ldots, X_{\varepsilon:n:m}, X_{\varepsilon+1:n:m}, \ldots, X_{j:n:m},) \; ; if \, X_{j:n:m} < t < X_{j+1:n:m}. \end{cases}$$

3 Step - Stress Partially Accelerated Life Test

It is much more difficult to obtain the failure evidence under the normal stress condition for the excellence and reliable products. It makes very costive and time consuming lifetime test on normal stress condition. For such motive, accelerated life test (ALT) is used to get information about the lifetime distribution of product or materials in a shorter time and less expensively. Accelerated life test is achieved by subjecting the test units to conditions that are more severe than the normal ones, such as higher levels of temperature, pressure, voltage, vibration, cycling rate, load, *etc*.

In step stress scheme, stress is put to test units in the way that, the stress will be altered at pre-specified time. Usually, a test starts at a specified low stress. If the unit does not fail at specified time, the stress is raised and held at specified times. Stress is repeatedly increased until the test unit fails or censoring scheme is reached. SS-PALT is used to get, rapidly, information for the lifetime of the product with high reliability; especially, when the mathematical model related to test conditions of mean lifetime of the product is unknown and cannot be assumed ([15]).

There is a great amount of literature available on SS-PALT, a few of them are [16], [17], [18] and [19]. [20] presents optimum SS-PALT concept for Rayleigh model that deals with the likelihood estimates of parameters and confidence intervals, based on the asymptotic normality of the MLE. Based on constant-stress partially accelerated life test on the truncated Logistic distribution, ML estimation and confidence limits have been obtained by [21] by using Type-I censoring. Under step-stress partially accelerated life test, some bound lengths and their properties are recently studied by [22] for Burr Type-II distribution.

Basically, in SS-PALT, all the test units are tested first at normal stress condition, if the unit does not fail for a pre-specified time, then it runs at accelerated condition till failure. In such case the switching to the higher stress level will shorten the life of the test item. An altered random variable model ([23]) for the lifetime of the units is considered under SS-PALT and is defined as

$$X = \begin{cases} Y & : 0 < Y \le \varepsilon \\ \varepsilon + \frac{Y-\varepsilon}{\beta} & : \quad Y > \varepsilon. \end{cases} \tag{3}$$

Here, the parameter ε and β are known respectively as stress change time and the acceleration factor. In SS-PALT, all of n units are tested first under normal stress condition, if units do not fail for a pre-specified time ε, then the test is switched to the higher level of stress and it continues until items fails. The effect of this shifting is to multiply the remaining lifetime of the item by the inverse of the acceleration factor β (Eq. (3)).

Hence, the total lifetime of a test item, denoted by X, passes through two stages, the first one is normal stress condition (denoted by I) and the second one is accelerated stress condition (denoted by II) and defined by an altered random variable model given in Eq. (3). Using Eq. (3), the probability density function of the considered model is rewritten as

$$f(x;\theta) = \begin{cases} I: f_1 = \frac{2x}{\theta} exp\left(-\frac{x^2}{\theta}\right) \quad ; & 0 < x \le \varepsilon \\ II: f_2 = \beta \frac{2\tilde{x}}{\theta} exp\left(-\frac{\tilde{x}^2}{\theta}\right) ; x > \varepsilon \, ; \tilde{x} = (x-\varepsilon)\beta + \varepsilon. \end{cases} \tag{4}$$

The joint probability density (likelihood) function on progressive Type-II censoring scheme under SS-PALT, is given as

$$L \propto \prod_{i=1}^{k} \left(f_1 (1-F_1)^{R_i}\right) \times \prod_{i=k+1}^{m} \left(f_2 (1-F_2)^{R_i}\right). \tag{5}$$

The Eq. (5) is rewritten on SS-PALT under T-IPH censoring scheme, as

$$\begin{cases} I: \qquad L \propto \prod_{i=1}^{k} \left(f_1 (1-F_1)^{R_i}\right) \prod_{i=k+1}^{m} \left(f_2 (1-F_2)^{R_i}\right) ; x_{(i)} \equiv X_{i:n:m} \\ II: L \propto \prod_{i=1}^{l} \left(f_1 (1-F_1)^{R_i} (1-F(t))^{R*}\right) \prod_{i=l+1}^{j} \left(f_2 (1-F_2)^{R_i} (1-F_2(t))^{R*}\right). \end{cases} \tag{6}$$

Using Eq. (4) in Eq. (6), we get

$$L \propto \frac{1}{\theta^d}\beta^{d-\delta} W_0 \, exp\left\{-\frac{1}{\theta}W_d\right\}; \tag{7}$$

where $W_0 = \prod_{i=\delta+1}^{d} \tilde{x}_{(i)}$, $W_1 = \sum_{i=1}^{k}(1+R_i)x_{(i)}^2$, $W_2 = \sum_{i=k+1}^{m}(1+R_i)\tilde{x}_{(i)}^2$, $W_3 = \sum_{i=1}^{l}(1+R_i)\tilde{x}_{(i)}^2 + R^* t^2$, $W_4 = \sum_{i=l+1}^{j}(1+R_i)\tilde{x}_{(i)}^2 + R^*((t-\varepsilon)\beta+\varepsilon)^2$, $d = \begin{cases} I : m \\ II : j \end{cases}$, $\delta = \begin{cases} I : k \\ II : l \end{cases}$ and $W_d = \begin{cases} I : W_1 + W_2 \\ II : W_3 + W_4 \end{cases}$.

4 The Bayes Prediction Bound Lengths

Prediction of the future observations based on an informative sample is an interesting topic and it has been used in various purposes. Several applications can be found in actuarial studies, rainfall extremes, guarantee data analysis and highest water levels. For example, in guarantee data analysis, setting up warranty period for a product, a manufacturer would use some of the known previous failure times to predict a suitable warranty period of the product.

In practice, the experimenter would like to know the failure times of the removed surviving units (censored units) based on the observed data. This problem leads to the prediction of the future sample based on censored sample. [24] has discussed several references on the applications of Bayes prediction in different areas of applied statistics. Our main focus is to study the bound lengths by using Bayesian approach under the current scenario.

Following [3], a natural family of the conjugate prior for the parameter θ is assumed here as the one-parameter Gamma distribution, having a probability density function

$$\pi_\theta \propto \theta^{-\alpha} e^{-1/\theta}; \alpha > 0, \theta > 0.$$

For the acceleration factor β, a vague prior is assumed here. The vague prior does not play any significant role in the analyses that follow. Thus, the joint prior probability density is given as

$$\pi_{(\theta,\beta)} \propto \frac{\theta^{-\alpha} e^{-1/\theta}}{\beta}; \theta > 0, \beta > 0, \alpha > 0. \tag{8}$$

Now, the joint and marginal posterior densities corresponding to the parameters θ and β are obtained respectively as

$$\pi^*_{(\theta,\beta)} = \frac{\frac{1}{\theta^{\alpha+d}}\beta^{d-\delta-1}W_0 \, exp\left(-\frac{W_d+1}{\theta}\right)}{\int_\beta \beta^{d-\delta-1}W_0 \int_\theta \frac{1}{\theta^{\alpha+d}} exp\left(-\frac{W_d+1}{\theta}\right) d\theta \, d\beta}$$

$$\Rightarrow \pi^*_{(\theta,\beta)} \propto \Omega \frac{1}{\theta^{\alpha+d}}\beta^{d-\delta-1}W_0 \, exp\left(-\frac{W_d+1}{\theta}\right); \; \Omega = \left\{\Gamma(\alpha+d-1)\int_\beta \frac{\beta^{d-\delta-1}W_0}{(W_d+1)^{\alpha+d-1}} d\beta\right\}^{-1} \tag{9}$$

and the marginal posteriors are

$$\pi^*_{(\theta)} \propto \Omega \frac{1}{\theta^{\alpha+d}} \int_\beta \beta^{d-\delta-1}W_0 \, exp\left(-\frac{W_d+1}{\theta}\right) d\beta \tag{10}$$

$$\pi^*_{(\beta)} \propto \Omega\Gamma(\alpha+d-1)\frac{\beta^{d-\delta-1}W_0}{(W_d+1)^{\alpha+d-1}}. \tag{11}$$

4.1 One-Sample Bayes Prediction Procedure

Let us assume that $\underline{x}\left(=x_{(1)},x_{(2)},...,x_{(d)}\right)$ be the first ordered observed items from the model given in Eq. (1) under the T-IPH censoring. If we assume that Z be the future random variable having independent ordered random sample $\underline{z}\left(=z_{(1)},z_{(2)},...,z_{(d)}\right)$ under similar censoring pattern. Then the Bayes predictive density function $h(z|\underline{x})$ (say) corresponding to the assumed future random variable Z, is defined as

$$h(z|\underline{x}) = \int_{\Theta} f(z;\theta)\,\pi_{\Theta}^{*} d\Theta\,; \Theta = \theta,\beta. \tag{12}$$

The Eq. (12) presents the Bayes predictive density function defined for the parameter θ and acceleration factor β separately. Since, the explicit solution of Eq. (12) does not exist for either parameter, so no mathematical solution exists. We continue with Eq. (12) numerically for both parameters respectively.

Now, if l_1 and l_2 are the lower and upper Bayes prediction bound limits in One-Sample criterion. Then $100(1-\tau)\%$ Bayes prediction bound length for the future observation corresponding to the parameter Θ, under the considered censoring scenario is obtained by solving the following equation

$$L_{One} = \left[P\left(Z \geq l_2 = \frac{\tau}{2}\right)\right] - \left[P\left(Z \leq l_1 = \frac{\tau}{2}\right)\right]. \tag{13}$$

Using Eq. (12) & Eq. (13), the Bayes prediction bound limits under one-sample technique for T-IPH censoring on SS-PALT is obtained by solving the following equalities for both parameters respectively

$$\left(1-\frac{\tau}{2}\right) = \int_{\Theta}\left(1-e^{-\frac{l_1^2}{\theta}}\right)\times\pi_{\Theta}^{*} d\Theta,$$

$$\left(\frac{\tau}{2}\right) = \int_{\Theta}\left(1-e^{-\frac{l_2^2}{\theta}}\right)\times\pi_{\Theta}^{*} d\Theta$$

and

$$L_{One} = l_2 - l_1. \tag{14}$$

4.2 Two-Sample Bayes Prediction Procedure

In the previous subsection, it was presumed that $\underline{x}\left(=x_{(1)},x_{(2)},...,x_{(d)}\right)$ be the first (observed) T-IPH censored samples with the progressive censoring scheme $R \equiv (R_1,R_2,...,R_d)$ of the considered model given in Eq. (1). If $\underline{Z}\left(=z_{(1)},z_{(2)},...,z_{(d)}\right)$ be another T-IPH censored samples, drawn from same model independently, then the first sample is referred to as the informative sample, while the second is mentioned as the future sample. Based on an informative sample, the k^{th} order statistic from the future sample will be predicted. For this, the cumulative predictive density $G(z|\underline{x})$ is obtained as

$$G(z|\underline{x}) = Pr(Z \leq z) = \int_{\Theta}\left(1-e^{\frac{z^2}{\theta}}\right)\times\pi_{\Theta}^{*} d\Theta; \Theta = \theta,\beta. \tag{15}$$

If we assume Z_k be the k^{th} order statistic from the future sample of size $m(1 \leq k \leq m)$. Then the probability density function of k^{th} ordered future observation is defined as

$$\phi(z_k) = k({}^{m}C_k)\left(G(z|\underline{x})\right)^{k-1}\left(1-G(z|\underline{x})\right)^{m-k} h(z|\underline{x}). \tag{16}$$

One may obtain the solution of Eq. (16) by substituting Eq. (12) & Eq. (15), for both parameters respectively. However, the closed form of the Eq. (16) for either parameter does not exist. If l_{2k} & l_{1k} be the upper and lower Bayes prediction limits in Two-Sample approach for the k^{th} future observation, then the Bayes prediction bound length under Two-Sample approach can be obtained by solving following equality

$$L_{Two} = \left[P\left(Y \geq l_{2k} = \frac{\tau}{2}\right)\right] - \left[P\left(Z \leq l_{1k} = \frac{\tau}{2}\right)\right] \tag{17}$$

Here, again the simplified forms of the Bayes prediction bound length for the smallest future observation $k(=1)$ and the largest future observation $k(=m)$ are not possible to obtain from the Eq. (17). Numerical technique is applied herewith for the numerical illustration.

5 The Approximate Confidence Lengths (ACL) & ML Estimation

The logarithm joint probability function based on concerned scenario is obtained, by using Eq. (7) as

$$Log\,L = -d\,log\theta + (d-\delta)\,log\,\beta + log\,W_0 - \frac{W_d}{\theta}. \tag{18}$$

Differentiating Eq. (18) with respect to both parameters separately and equating to zero, we have

$$\frac{\partial}{\partial\theta}LogL = -\frac{d}{\theta} + \frac{W_d}{\theta^2}$$

and

$$\frac{\partial}{\partial\beta}LogL = \frac{d-\delta}{\beta} + \sum_{i=\delta+1}^{d}(x_{(i)}-\varepsilon)\left(\frac{1}{\tilde{x}_{(i)}} - \frac{2(1+R_i)}{\theta}\tilde{x}_{(i)}\right) - \begin{cases} I:0 \\ II:\ \frac{2(t-\varepsilon)R*}{\theta((t-\varepsilon)\beta+\varepsilon)^{-1}} \end{cases}.$$

The maximum likelihood estimators corresponding to the parameters θ and β are denoted respectively by $\hat{\theta}_{Ml}$ and $\hat{\beta}_{Ml}$, and obtained as

$$\hat{\theta}_{Ml} = \frac{W_d}{d} \tag{19}$$

and

$$\lambda = h(\beta)\,say = \frac{d-\delta}{\sum_{i=\delta+1}^{d}(x_{(i)}+\varepsilon)\left(\frac{2(1+R_i)}{\hat{\theta}_{Ml}}\tilde{x}_{(i)} - \frac{1}{\tilde{x}_{(i)}}\right) + W_{d0}}. \tag{20}$$

[25] proposed a simple iterative scheme for a mathematical solution of Eq. (19-20). The procedures is: start with an initial guess value of λ, say $\lambda_{(0)}$, then obtain $\lambda_{(1)} = h\left(\lambda_{(0)}\right)$ and continue in this way iteratively to obtain $\lambda_{(n+1)} = h\left(\lambda_{(n)}\right)$. Stop this procedure until $|\lambda_{(n+1)} - \lambda_{(n)} < \upsilon,|$ for some pre-assigned tolerance limit and obtained $\hat{\beta}_{Ml}$ for the parameter β and then $\hat{\theta}_{Ml}$ is obtained easily.

The precise mathematical expression for the expectation is also hard to find. So it can be approximated numerically inverting the asymptotic Fisher's information matrix and written as

$$F = -\begin{bmatrix} \frac{\partial^2}{\partial\theta^2}LogL & \frac{\partial^2}{\partial\theta\,\partial\beta}LogL \\ \frac{\partial^2}{\partial\beta\partial\theta}LogL & \frac{\partial^2}{\partial\beta^2}LogL \end{bmatrix}.$$

where

$$\frac{\partial^2}{\partial\theta^2}LogL = \frac{d}{\theta^2} - 2\frac{W_d}{\theta^3},$$

$$\frac{\partial^2}{\partial\theta\,\partial\beta}LogL = \frac{\partial^2}{\partial\beta\partial\theta}LogL = \frac{2}{\theta^2}\sum_{i=\delta+1}^{d}(1+R_i)\tilde{x}_i\left(x_{(i)}-\varepsilon\right) + \frac{W_{d0}}{\theta}$$

and

$$\frac{\partial^2}{\partial\beta^2}LogL = \frac{d-\delta}{\beta^2} - \sum_{i=\delta+1}^{d}\left(x_{(i)}-\varepsilon\right)^2\left(\frac{1}{\tilde{x}_{(i)}^2} + \frac{2(1+R_i)}{\theta}\right) - \begin{cases} I:0 \\ II:\ \frac{2(t-\varepsilon)^2}{\theta}R* \end{cases}.$$

In practice, we usually estimate F^{-1} under the ML estimates. [26] stated the most common method to set confidence bounds for the parameters which is to use asymptotic normal distribution of maximum likelihood estimators. Asymptotically, the ML estimators, under suitable regularity conditions, are consistent and normally distributed. Thus, $100(1-\tau)\%$ normal Approximate Confidence Intervals for both the parameters θ and β can be obtained as

$$\hat{\theta}_{Ml} \mp Z_{\tau/2}\sqrt{v_{11}}$$

and

$$\hat{\beta}_{Ml} \mp Z_{\tau/2}\sqrt{v_{22}}$$

where v_{11} and v_{22} are the elements on the main diagonal of the variance covariance matrix and $Z_{\tau/2}$ is the percentile of the standard normal distribution with right-tail probability $\frac{\tau}{2}$.

6 The Optimization Criterion in SS-PALT

The major issue in SS-PALT procedures is, how to measure optimum stress change time, the time in which stress level changed from normal to higher. The optimization criterion cracked this issue, and provided the duration of lower stress level (normal stress). [27] discussed the optimum test plan criterion, which is based on the determinant of the Fisher's information matrix. Maximizing that determinant is equivalent to minimizing the generalized asymptotic variance of the ML estimation of the model parameters θ and the acceleration factor β.

Following [28], the generalized asymptotic variance is the reciprocal of the determinant of the Fisher's information matrix. Hence, the optimal value of ε is chosen such that the determinant of the Fisher's information matrix is maximized and then the generalized asymptotic variance is minimized and is obtained by using Wolfram Mathematica software 10.0. This criterion is further known as D-optimality criterion.

7 Simulation Study

In this section, a simulation study has been discussed for the numerical illustration. Random numbers have been generated from this distribution, by using Metropolis-Hastings method with the normal proposal distribution. [29] and [30] have deliberated an algorithm for simulating samples from a given posterior distribution by use of an arbitrary proposal distribution and this algorithm is broadly used to provide an alternative way for numerical computation. Ensuing [14], the random sample generated from the posterior distribution. One may explore recent articles [31] and [32] for more details on M-H algorithm.

The objective of the present discussion is based on the Bayes predictive bound lengths on One-Sample and Two-Sample scenario. For this, we considered here One-Parameter Rayleigh distribution under the SS-PALT setup by combining T-IPH censoring. The ML estimate and ACL are also obtained. Now for evaluating the behavior, the values are computed by using Monte Carlo simulations on the basis of $10,000$ replications based on Metropolis-Hastings (M-H) algorithm following [14].

The total number of test units is fixed first at $n(=30)$. Three different progressive censoring stages have been assumed with pre-supposed censoring pattern to study the behavior of censored sample size. These censoring stages are $(10,15,20)$, the test will terminate at these stages. The pre-supposed progressive censoring patterns for these stages are $(2,4,1,2,3,0,2,3,2,1)$, $(0,1,1,0,1,2,3,0,2,0,2,1,0,1,1)$ and $(0,0,0,1,0,1,0,1,2,0,1,0,2,0,0,0,1,0,1,0)$. For censored sample size $m(=15)$, we considered one more progressive censoring pattern $(1,0,0,0,2,1,2,2,0,4,0,1,0,0,2)$ for perceiving the effect of censoring pattern when other parametric values are fixed.

The optimal stress change time ε is calculated by the method of minimizing the asymptotic variance of ML Estimation of the parameter θ and the acceleration factor β as discussed in the previous section. For this we assumed the values of the underlying parameter θ and accelerated factor β are $\theta = 0.50(0.10)2.50 = \beta$. We observed the effects at each value of these parameters from 0.50 up to 2.50 with an increment of 0.10. The selected hyper parametric values were assumed as $\alpha = 0.40(0.10)2.50$. [33] studied the properties of the Bayes estimation in the generalized Exponential distribution under the optimum SS-PALT scenario and found out the value of the acceleration factor for which the magnitude of Bayes risks is minimized numerically when other parametric fixed values. So, in the present discussion, we also catch out the values β,θ and α for which the bound length touches maximum. The pre-determined failure time was assumed here as $t(=04,07)$ for the numerical analysis.

The numerical findings based on simulated data are presented in Tables 1-2 in terms of Bayes prediction bound lengths under One-Sample scenario for the parameter θ and acceleration parameter β respectively for selected parametric values at $\tau(=99\%,95\%,90\%)$. It is observed that, the boost in the confidence level increases the bound lengths. The opposite trend has been seen with the censored sample size m. Similar properties also have been seen when the censoring time t

Table 1: Bound Length under One-Sample Criterion for Parameter θ

$n=30, \theta=1.80$			$t=04$			$t=07$		
β	α	$m \downarrow \tau \rightarrow$	99%	95%	90%	99%	95%	90%
0.50	0.50	10	1.1786	1.1617	1.1348	1.1529	1.1225	1.0985
		15 (I)	1.1179	1.1109	1.0811	1.0854	1.0706	1.0457
		15 (II)	1.1174	1.0974	1.0634	1.0829	1.0718	1.0416
		20	1.0915	1.0327	1.0138	1.0055	0.9958	0.9828
	1.30	10	1.2512	1.2168	1.1881	1.2238	1.1756	1.1508
		15 (I)	1.1796	1.1627	1.1326	1.1519	1.1215	1.0949
		15 (II)	1.1646	1.1503	1.1142	1.1497	1.1221	1.0906
		20	1.0939	1.0821	1.0624	1.0681	1.0435	1.0299
	2.50	10	1.1098	1.0987	1.0826	1.0865	1.0716	1.0485
		15 (I)	1.0664	1.0595	1.0317	1.0522	1.0219	0.9973
		15 (II)	1.0637	1.0476	1.0148	1.0309	1.0226	0.9936
		20	1.0206	0.9855	0.9672	0.9669	0.9511	0.9379
1.60	0.50	10	1.2838	1.2648	1.2352	1.2548	1.2225	1.1961
		15 (I)	1.2178	1.2088	1.1776	1.1812	1.1666	1.1384
		15 (II)	1.2165	1.1955	1.1585	1.1789	1.1665	1.1338
		20	1.1891	1.1253	1.1049	1.0954	1.0851	1.0709
	1.30	10	1.3616	1.3245	1.2934	1.3316	1.2795	1.2525
		15 (I)	1.2847	1.2658	1.2333	1.2538	1.2217	1.1923
		15 (II)	1.2675	1.2514	1.2135	1.2513	1.2215	1.1874
		20	1.1912	1.1786	1.1575	1.1632	1.1367	1.1219
	2.50	10	1.2086	1.1967	1.1794	1.1822	1.1676	1.1425
		15 (I)	1.1615	1.1541	1.1241	1.1459	1.1133	1.0868
		15 (II)	1.1583	1.1416	1.1059	1.1221	1.1138	1.0824
		20	1.1118	1.0741	1.0545	1.0536	1.0357	1.0222
2.50	0.50	10	1.1238	1.1077	1.0813	1.0994	1.0703	1.0473
		15 (I)	1.0693	1.0582	1.0305	1.0342	1.0205	0.9961
		15 (II)	1.0458	1.0365	1.0136	1.0326	1.0214	0.9924
		20	1.0021	0.9842	0.9667	0.9592	0.9489	0.9366
	1.30	10	1.1934	1.1604	1.1328	1.1673	1.1211	1.0985
		15 (I)	1.1248	1.1087	1.0802	1.0984	1.0693	1.0438
		15 (II)	1.1066	1.0935	1.0621	1.0945	1.0686	1.0378
		20	1.0428	1.0329	1.0146	1.0182	0.9956	0.9806
	2.50	10	1.1442	1.0602	1.0326	1.1409	1.0235	1.0024
		15 (I)	1.0996	1.0089	0.9812	1.0865	0.9734	0.9515
		15 (II)	1.0422	0.9955	0.9674	1.0425	0.9728	0.9459
		20	1.0253	0.9394	0.9234	0.9335	0.9073	0.8935

increases. It is observed for the pre-assumed values of θ that, the bound lengths increase first when the value of θ rises up to $\theta(=1.80)$ and then bound lengths decrease with the further increase of θ. That shows the widest bound length at $\theta(=1.80)$. Hence, all the numerical findings are presented here only for $\theta=1.80$. Similar, behavior also has been noted for the parameter $\beta(=1.60)$ and $\alpha(=1.30)$. Therefore, all the tables presents the numerical findings only for selected values of $\alpha(=0.50,1.30,2.50)$ and $\beta(=0.50,1.60,2.50)$.

Further, the censored sample size $m=15$ has two different censoring patterns (I) & (II). The first censoring pattern shows the wide bound length as compared to second one. However, the difference in magnitude is nominal. This shows that, different censoring patterns show different magnitudes of bound lengths. However, the difference in magnitude is nominal with similar behavior.

Table 2 presents the bound lengths for the parameter β. All the properties have been seen similar as discussed above, but the magnitude of bound length have been seen narrower when compared with the bound length for the parameter θ under similar parametric values.

The Tables 3-6 present the Bayes prediction bound lengths under Two-Sample approach for the parameter θ and β

Table 2: Bound Length under One-Sample Criterion for Parameter β

$n=30, \theta=1.80$			$t=04$			$t=07$		
β	α	$m \downarrow \tau \rightarrow$	99%	95%	90%	99%	95%	90%
0.50	0.50	10	1.1186	1.1019	1.0763	1.0937	1.0647	1.0418
		15 (I)	1.0603	1.0527	1.0251	1.0294	1.0153	0.9909
		15 (II)	1.0609	1.0408	1.0083	1.0272	1.0166	0.9872
		20	1.0351	0.9791	0.9614	0.9534	0.9444	0.9317
	1.30	10	1.1872	1.1544	1.1275	1.1613	1.1153	1.0916
		15 (I)	1.1196	1.1029	1.0741	1.0927	1.0637	1.0384
		15 (II)	1.1049	1.0911	1.0566	1.0908	1.0645	1.0344
		20	1.0374	1.0261	1.0073	1.0137	0.9894	0.9765
	2.50	10	1.0525	1.0419	1.0266	1.0305	1.0163	0.9942
		15 (I)	1.0113	1.0047	0.9781	0.9978	0.9689	0.9455
		15 (II)	1.0088	0.9934	0.9622	0.9768	0.9697	0.9425
		20	0.9676	0.9341	0.9166	0.9166	0.9014	0.8889
1.60	0.50	10	1.2175	1.2001	1.1718	1.1908	1.1595	1.1348
		15 (I)	1.1546	1.1468	1.1176	1.1206	1.1067	1.0798
		15 (II)	1.1543	1.1337	1.0988	1.1186	1.1068	1.0755
		20	1.1281	1.0672	1.0477	1.0395	1.0291	1.0156
	1.30	10	1.2923	1.2569	1.2273	1.2639	1.2143	1.1885
		15 (I)	1.2185	1.2011	1.1701	1.1898	1.1585	1.1312
		15 (II)	1.2029	1.1875	1.1512	1.1875	1.1591	1.1266
		20	1.1301	1.1185	1.0978	1.1035	1.0782	1.0641
	2.50	10	1.1466	1.1352	1.1187	1.1216	1.1077	1.0833
		15 (I)	1.1018	1.0947	1.0661	1.0871	1.0559	1.0306
		15 (II)	1.0989	1.0823	1.0487	1.0645	1.0566	1.0266
		20	1.0545	1.0185	0.9997	0.9992	0.9823	0.9692
2.50	0.50	10	1.0658	1.0505	1.0253	1.0428	1.0158	0.9931
		15 (I)	1.0144	1.0034	0.9773	0.9807	0.9676	0.9443
		15 (II)	0.9917	0.9823	0.9608	0.9793	0.9686	0.9409
		20	0.9589	0.9329	0.9154	0.9093	0.8994	0.8877
	1.30	10	1.1321	1.1007	1.0743	1.1074	1.0635	1.0414
		15 (I)	1.0668	1.0515	1.0243	1.0418	1.0145	0.9897
		15 (II)	1.0495	1.0372	1.0074	1.0381	1.0129	0.9841
		20	0.9887	0.9793	0.9618	0.9655	0.9438	0.9295
	2.50	10	1.0855	1.0054	0.9783	1.0815	0.9706	0.9504
		15 (I)	1.0429	0.9564	0.9299	1.0305	0.9223	0.9019
		15 (II)	0.9881	0.9437	0.9164	0.9881	0.9222	0.8966
		20	0.9721	0.8898	0.8749	0.8849	0.8599	0.8466

respectively for the smallest future observation $(k=1)$ and the largest future observation $(k=m)$. All the properties that have been discussed above are seen similar. On different censoring patterns (I) & (II) for censored sample size $m=15$, no clear trend have been seen to be discussed for a better censoring pattern. However, as the censored sample size gets wider the bound lengths become narrower.

Table 7 presents the ML estimation based on simulated data for all pre-assumed values of the parameters. A similar trend has been noted in terms of estimate. It was observed that, the ML estimate first increases when θ increases for $(=1.80)$ and then ML estimate decreases otherwise. Hence, numerical findings are presented here only for $\theta=1.80$. ML estimate increases first as β increases up to $(=1.60)$ and then decreases for other values of β. Further, a decreasing trend also have been seen if the censored sample size m increases or censored time t increases. It is noted further that, the magnitude of ML estimate is larger for the parameter β as compared with θ when other parametric values are fixed.

The approximate confidence lengths (ACL) for both parameters are measured for all pre-assumed parametric values. The numerical finding are presented here only for $\theta(=1.80)$, for both concerned parameters in Tables 8-9 respectively. All the properties have been seen similar as discussed above. One remarkable point is that, the magnitude of ACL for parameter θ was noted wider when compared to ACL of the parameter β.

Table 3: Bound Length under Two-Sample Criterion for Parameter θ

$n=30, k=1, \theta=1.80$			$t=04$			$t=07$		
β	α	$m\downarrow \tau\rightarrow$	99%	95%	90%	99%	95%	90%
0.50	0.50	10	0.7327	0.7171	0.6935	0.7214	0.6951	0.6738
		15 (I)	0.6868	0.6791	0.6532	0.6655	0.6516	0.6296
		15 (II)	0.6932	0.6728	0.6367	0.6705	0.6594	0.6331
		20	0.6658	0.6147	0.5953	0.6022	0.5989	0.5819
	1.30	10	0.7904	0.7608	0.7357	0.7778	0.7372	0.7153
		15 (I)	0.7358	0.7201	0.6994	0.7185	0.6919	0.6686
		15 (II)	0.7309	0.7149	0.6771	0.7234	0.6993	0.6729
		20	0.6678	0.6539	0.6338	0.6582	0.6278	0.6193
	2.50	10	0.6782	0.6671	0.6592	0.6687	0.6546	0.6342
		15 (I)	0.6496	0.6382	0.6139	0.6392	0.6131	0.5911
		15 (II)	0.6507	0.6334	0.5982	0.6292	0.6203	0.5952
		20	0.6097	0.5771	0.5583	0.5716	0.5544	0.5462
1.60	0.50	10	0.8162	0.7989	0.7731	0.8023	0.7745	0.7513
		15 (I)	0.7662	0.7568	0.7297	0.7416	0.7278	0.7031
		15 (II)	0.7719	0.7508	0.7122	0.7468	0.7345	0.7064
		20	0.7434	0.6881	0.6674	0.6736	0.6608	0.6519
	1.30	10	0.8788	0.8464	0.8194	0.8633	0.8196	0.7959
		15 (I)	0.8193	0.8019	0.7738	0.7992	0.7715	0.7459
		15 (II)	0.8124	0.7951	0.7558	0.8041	0.7781	0.7489
		20	0.7451	0.7305	0.7092	0.7274	0.7017	0.6924
	2.50	10	0.7566	0.7448	0.7289	0.7446	0.7381	0.7087
		15 (I)	0.7214	0.7133	0.6872	0.7136	0.6856	0.6621
		15 (II)	0.7258	0.7078	0.6705	0.7016	0.6928	0.6655
		20	0.6822	0.6475	0.6275	0.6404	0.6217	0.6132
2.50	0.50	10	0.6894	0.6743	0.6591	0.6791	0.6536	0.6331
		15 (I)	0.6483	0.6372	0.6131	0.6295	0.6119	0.5903
		15 (II)	0.6364	0.6245	0.5973	0.6306	0.6194	0.5941
		20	0.5949	0.5763	0.5578	0.5654	0.5527	0.5453
	1.30	10	0.7444	0.7161	0.6919	0.7329	0.6994	0.6738
		15 (I)	0.6923	0.6773	0.6523	0.6769	0.6506	0.6298
		15 (II)	0.6757	0.6629	0.6357	0.6751	0.6523	0.6255
		20	0.6272	0.6172	0.6003	0.6122	0.5921	0.5778
	2.50	10	0.7146	0.6433	0.6122	0.7166	0.6211	0.6021
		15 (I)	0.6723	0.5959	0.5692	0.6665	0.5722	0.5571
		15 (II)	0.6245	0.5851	0.5606	0.6338	0.5763	0.5526
		20	0.6111	0.5407	0.5257	0.5473	0.5243	0.5191

8 Numerical Illustration

A numerical illustration is presented in this section by using a real data set. The concerned real data set is taken from the data set of wind speed in Taiz, located southwest of Yemen, which was used by [34] and recently by [3]. The monthly wind speed for the year 2002 has been used for the investigation. [3] discussed the fitting of the data by using Kolmogorov-Smirnov (KS) test (test statistic value = 0.3436 with p-value 0.0671) and Chi-Square goodness of fit test (test statistic value = 0.7821 with p-value = 0.3236). Based on results, one-parameter Rayleigh distribution affords a satisfactory fit to this data set.

For numerical illustration, we considered here a set of $30(=n)$ wind speed data. Using all above pre-assumed parametric values, the Bayes prediction bound lengths under the One-Sample and Two-Sample criterion for both parameters in Tables 10-12 are presented for $\theta = 1.80\,\&\,\beta = 1.60$ only. All the properties have been seen similar as discussed above. The remarkable point is that, the wider bound lengths have been noticed for real data set when compared with corresponding simulated data. Further, for censored sample size $m = 15$, the second censoring pattern (II) shows wider bound length as compared with the first censoring pattern (I), when other parametric values are assumed to be fixed.

Table 4: Bound Length under Two-Sample Criterion for Parameter θ

$n = 30, k = m, \theta = 1.80$			$t = 04$			$t = 07$		
β	α	$m \downarrow \tau \rightarrow$	99%	95%	90%	99%	95%	90%
0.50	0.50	10	1.4488	1.4117	1.3564	1.4321	1.3710	1.3212
		15 (I)	1.3488	1.3286	1.2684	1.3046	1.2712	1.2197
		15 (II)	1.3689	1.3188	1.2292	1.3224	1.2953	1.2342
		20	1.3008	1.1821	1.1341	1.1627	1.1391	1.1175
	1.30	10	1.5782	1.5095	1.4511	1.5586	1.4654	1.4141
		15 (I)	1.4588	1.4205	1.3598	1.4235	1.3616	1.3072
		15 (II)	1.4535	1.4131	1.3198	1.4411	1.3848	1.3214
		20	1.3054	1.2698	1.2204	1.2743	1.2157	1.1989
	2.50	10	1.3265	1.2994	1.2634	1.3184	1.2802	1.2324
		15 (I)	1.2565	1.2369	1.1804	1.2455	1.1885	1.1334
		15 (II)	1.2737	1.2305	1.1428	1.2298	1.2077	1.1493
		20	1.1753	1.0977	1.0513	1.0984	1.0511	1.0375
1.60	0.50	10	1.6359	1.5985	1.5349	1.6136	1.5479	1.4947
		15 (I)	1.5276	1.5028	1.4397	1.4753	1.4421	1.3847
		15 (II)	1.5453	1.4937	1.3984	1.4934	1.4638	1.3985
		20	1.4749	1.3466	1.2958	1.3228	1.2896	1.2772
	1.30	10	1.7746	1.7016	1.6387	1.7503	1.6501	1.5947
		15 (I)	1.6451	1.6039	1.5388	1.6044	1.5401	1.4805
		15 (II)	1.6362	1.5929	1.4962	1.6218	1.5615	1.4938
		20	1.4788	1.4417	1.3895	1.4435	1.3814	1.3628
	2.50	10	1.5023	1.4738	1.4361	1.4841	1.4514	1.3993
		15 (I)	1.4256	1.4052	1.3445	1.4124	1.3475	1.2926
		15 (II)	1.4462	1.3977	1.3049	1.3921	1.3703	1.3068
		20	1.3376	1.2556	1.2063	1.2484	1.2092	1.1852
2.50	0.50	10	1.3517	1.3157	1.2612	1.3373	1.2798	1.2298
		15 (I)	1.2617	1.2346	1.1785	1.2138	1.1823	1.1317
		15 (II)	1.2416	1.2106	1.1409	1.2393	1.2056	1.1468
		20	1.1492	1.0959	1.0501	1.0803	1.0475	1.0393
	1.30	10	1.4749	1.4093	1.3529	1.4579	1.3685	1.3212
		15 (I)	1.3604	1.3246	1.2663	1.3282	1.2691	1.2162
		15 (II)	1.3291	1.2998	1.2269	1.3284	1.2795	1.2128
		20	1.2144	1.1898	1.1497	1.1851	1.1378	1.1037
	2.50	10	1.4369	1.2527	1.1742	1.4257	1.2095	1.1647
		15 (I)	1.3156	1.1398	1.0757	1.3069	1.0912	1.0593
		15 (II)	1.2562	1.1156	1.0586	1.2357	1.1047	1.0493
		20	1.1762	1.0161	0.9802	1.0418	0.9881	0.9562

The ML estimate and ACL for both parameters under real data set have been presented in Tables 7-9. All the properties that have been discussed above are seen similar. The remarkable point is that, the magnitude of ML estimate was noted larger when compared with simulated data respectively. Similarly, the wider ACL was obtained from a real data set.

9 Conclusion

In the present article, we discussed Bayes prediction bound lengths on One-Parameter Rayleigh distribution. We have combined here Type-I Progressive Hybrid censoring with Step-Stress Partially Accelerated Life Test (SS-PALT). One-Sample & Two-Sample Bayes prediction bound lengths have been obtained for t analysis by using Metropolis-Hastings algorithm under simulated data and real data set. The optimal stress change time also have been measured by the method of minimization of asymptotic variance of ML Estimation.

For numerical analysis, three censoring size with four different censoring patterns have been selected along with several pre-assumed parametric values and studied their properties in different aspects. All findings are discussed in previous section along with analysis on real data set. The present discussion, also shows the parametric values for which the

Table 5: Bound Length under Two-Sample Criterion for Parameter β

$n=30, k=1, \theta=1.80$			$t=04$			$t=07$		
β	α	$m\downarrow\tau\rightarrow$	99%	95%	90%	99%	95%	90%
0.50	0.50	10	0.6695	0.6496	0.6203	0.6644	0.6321	0.6055
		15 (I)	0.6185	0.6077	0.5758	0.5979	0.5799	0.5526
		15 (II)	0.6319	0.6044	0.5548	0.6096	0.5948	0.5625
		20	0.5944	0.5314	0.5051	0.5249	0.5068	0.4995
	1.30	10	0.7362	0.7091	0.6691	0.7295	0.6806	0.6534
		15 (I)	0.6751	0.6585	0.6229	0.6591	0.6264	0.5976
		15 (II)	0.6754	0.6583	0.6014	0.6707	0.6409	0.6074
		20	0.5966	0.5766	0.5494	0.5824	0.5505	0.5426
	2.50	10	0.6067	0.5919	0.5725	0.6036	0.5854	0.5598
		15 (I)	0.5714	0.5605	0.5305	0.5676	0.5354	0.5081
		15 (II)	0.5829	0.5589	0.5104	0.5682	0.5497	0.5187
		20	0.5295	0.4888	0.4623	0.4895	0.4657	0.4583
1.60	0.50	10	0.7659	0.7484	0.7123	0.7578	0.7237	0.6985
		15 (I)	0.7102	0.6974	0.6641	0.6857	0.6679	0.6373
		15 (II)	0.7228	0.6944	0.6482	0.6977	0.6816	0.6471
		20	0.6839	0.6161	0.5883	0.6073	0.5886	0.5803
	1.30	10	0.8373	0.7988	0.7657	0.8282	0.7758	0.7465
		15 (I)	0.7715	0.7494	0.7195	0.7522	0.7183	0.6868
		15 (II)	0.7695	0.7455	0.6923	0.7638	0.7319	0.6961
		20	0.6859	0.6651	0.6365	0.6694	0.6358	0.6279
	2.50	10	0.6971	0.6815	0.6614	0.6912	0.6735	0.6458
		15 (I)	0.6585	0.6472	0.6151	0.6534	0.6191	0.5909
		15 (II)	0.6695	0.6475	0.5938	0.6456	0.6334	0.5999
		20	0.6132	0.5692	0.5422	0.5689	0.5434	0.5356
2.50	0.50	10	0.6196	0.6002	0.5713	0.6157	0.5842	0.5585
		15 (I)	0.5741	0.5594	0.5295	0.5512	0.5341	0.5072
		15 (II)	0.5663	0.5487	0.5093	0.5636	0.5477	0.5175
		20	0.5125	0.4871	0.4617	0.4824	0.4638	0.4572
	1.30	10	0.6873	0.6484	0.6185	0.6777	0.6308	0.6055
		15 (I)	0.6249	0.6056	0.5748	0.6101	0.5787	0.5507
		15 (II)	0.6237	0.5871	0.5537	0.6119	0.5827	0.5498
		20	0.5498	0.5363	0.5147	0.5364	0.5113	0.4927
	2.50	10	0.6666	0.5703	0.5265	0.6529	0.5507	0.5268
		15 (I)	0.6018	0.5097	0.4749	0.5991	0.4863	0.4708
		15 (II)	0.5747	0.4992	0.4669	0.5633	0.4965	0.4656
		20	0.5292	0.4469	0.4267	0.4635	0.4349	0.4177

bound lengths show wider tendency. Further, for a fixed censored sample size the different censoring patterns do not play any significant role.

Table 6: Bound Length under Two-Sample Criterion for Parameter β

$n=30, k=m, \theta=1.80$			$t=04$			$t=07$		
β	α	$m\downarrow \tau\rightarrow$	99%	95%	90%	99%	95%	90%
0.50	0.50	10	1.2167	1.1715	1.1061	1.2158	1.1438	1.0841
		15 (I)	1.1096	1.0884	1.0129	1.0702	1.0292	0.9688
		15 (II)	1.1486	1.0818	0.9652	1.1029	1.0686	0.9966
		20	1.0577	0.9175	0.8559	0.9134	0.8695	0.8564
	1.30	10	1.3601	1.2993	1.2108	1.3557	1.2488	1.1871
		15 (I)	1.2312	1.1931	1.1142	1.2017	1.1291	1.0648
		15 (II)	1.2394	1.1977	1.0654	1.2343	1.1676	1.0932
		20	1.0625	1.0146	0.9581	1.0371	0.9634	0.9489
	2.50	10	1.0898	1.0474	1.0032	1.0852	1.0434	0.9859
		15 (I)	1.0085	0.9824	0.9155	1.0052	0.9335	0.8725
		15 (II)	1.0407	0.9894	0.8698	1.0139	0.9717	0.9027
		20	0.9183	0.8258	0.7639	0.8373	0.7812	0.7677
1.60	0.50	10	1.4284	1.3838	1.3038	1.4166	1.3407	1.2894
		15 (I)	1.3067	1.2766	1.2025	1.2591	1.2183	1.1598
		15 (II)	1.3411	1.2752	1.1696	1.2924	1.2553	1.1786
		20	1.2501	1.0993	1.0347	1.0906	1.0453	1.0309
	1.30	10	1.5774	1.4922	1.4184	1.5678	1.4526	1.3872
		15 (I)	1.4385	1.3885	1.3218	1.4092	1.3267	1.2564
		15 (II)	1.4417	1.3895	1.2607	1.4345	1.3633	1.2839
		20	1.2544	1.2048	1.1384	1.2241	1.1467	1.1324
	2.50	10	1.2762	1.2401	1.1943	1.2734	1.2329	1.1708
		15 (I)	1.1955	1.1688	1.0972	1.1896	1.1134	1.0503
		15 (II)	1.2267	1.1744	1.0491	1.1805	1.1515	1.0771
		20	1.0983	0.9985	0.9356	1.0081	0.9481	0.9384
2.50	0.50	10	1.1295	1.0653	1.0007	1.1111	1.0409	0.9831
		15 (I)	1.0142	0.9801	0.9133	0.9997	0.9307	0.8704
		15 (II)	1.0095	0.9721	0.8674	1.0094	0.9674	0.9509
		20	0.8817	0.8222	0.7627	0.8222	0.7772	0.7655
	1.30	10	1.2549	1.1688	1.1021	1.2444	1.1409	1.0841
		15 (I)	1.1234	1.0794	1.0108	1.0964	1.0265	0.9684
		15 (II)	1.1183	1.0382	0.9629	1.1093	1.0376	0.9644
		20	0.9619	0.9303	0.8815	0.9383	0.8817	0.8392
	2.50	10	1.2204	1.0086	0.9044	1.1961	0.9794	0.9201
		15 (I)	1.0738	0.8708	0.7991	1.0729	0.8254	0.7947
		15 (II)	1.0129	0.8681	0.7763	0.9984	0.8524	0.7834
		20	0.9195	0.7356	0.6897	0.7839	0.7299	0.6805

Table 7: Maximum Likelihood Estimates for Both Parameter

$n=30$		$\hat{\theta}_{Ml}$				$\hat{\beta}_{Ml}$			
$\theta=1.80$		Simulated Data		Real Data		Simulated Data		Real Data	
β	$m\downarrow t\rightarrow$	04	07	04	07	04	07	04	07
0.50	10	1.0678	1.0421	1.1225	1.1092	1.1003	1.0778	1.1726	1.1434
	15 (I)	1.0134	0.9869	1.0611	1.0297	1.0369	1.0091	1.0965	1.0565
	15 (II)	1.0177	0.9856	1.0635	1.0301	1.0483	1.0139	1.1057	1.0633
	20	0.9356	0.9232	1.0339	0.9475	0.9431	0.9324	1.0616	0.9595
1.60	10	1.1224	1.0959	1.1971	1.1729	1.1662	1.1426	1.2625	1.2312
	15 (I)	1.0658	1.0374	1.1245	1.0988	1.1098	1.0798	1.1729	1.1389
	15 (II)	1.0694	1.0369	1.1121	1.0988	1.1107	1.0746	1.1642	1.1462
	20	0.9846	0.9716	1.0364	1.0119	1.0022	0.9907	1.0646	1.0371

2.50	10	1.0155	0.9907	1.0517	1.0318	1.0373	1.0158	1.0872	1.0612
	15 (I)	0.9634	0.9371	1.0081	0.9955	0.9766	0.9491	1.0326	1.0153
	15 (II)	0.9671	0.9363	1.0084	0.9766	0.9874	0.9544	1.0392	0.9988
	20	0.8896	0.8779	0.9611	0.9079	0.8877	0.8766	0.9739	0.9118

Table 8: ACL for Parameter θ

$n = 30, \theta = 1.80$		$t = 04$			$t = 07$		
β	$m \downarrow \tau \rightarrow$	99%	95%	90%	99%	95%	90%
ACL On Simulated Data							
0.50	10	1.2402	1.2148	1.1763	1.2324	1.1769	1.1423
	15 (I)	1.1643	1.1519	1.1095	1.1287	1.1059	1.0701
	15 (II)	1.1736	1.1418	1.0832	1.1353	1.1175	1.0747
	20	1.1298	1.0464	1.0151	1.0242	1.0057	0.9913
1.60	10	1.3349	1.2867	1.2458	1.3125	1.2463	1.2105
	15 (I)	1.2448	1.2193	1.1768	1.2154	1.1724	1.1342
	15 (II)	1.2353	1.2099	1.1494	1.2226	1.1832	1.1398
	20	1.1339	1.1109	1.0627	1.1067	1.0672	1.0527
2.50	10	1.1503	1.1325	1.1082	1.1333	1.1106	1.0776
	15 (I)	1.0979	1.0548	1.0451	1.0852	1.0424	1.0069
	15 (II)	1.1037	1.0759	1.0198	1.0674	1.0533	1.0121
	20	1.0374	0.9858	0.9543	0.9746	0.9465	0.9331
ACL On Real Data							
0.50	10	1.2648	1.2298	1.1783	1.2685	1.1953	1.1487
	15 (I)	1.1733	1.1542	1.0978	1.1348	1.1031	1.0554
	15 (II)	1.1943	1.1487	1.0617	1.1528	1.1273	1.0705
	20	1.1298	1.0188	0.9732	1.0041	0.9744	0.9595
1.60	10	1.3834	1.3198	1.2653	1.3683	1.2821	1.2346
	15 (I)	1.2737	1.2386	1.1821	1.2435	1.1863	1.1353
	15 (II)	1.2716	1.2333	1.1446	1.2622	1.2096	1.1521
	20	1.1349	1.0995	1.0328	1.1073	1.0513	1.0365
2.50	10	1.1522	1.1267	1.0929	1.1439	1.1122	1.0676
	15 (I)	1.0898	1.0325	1.0172	1.0804	1.0235	0.9759
	15 (II)	1.1068	1.0655	0.9822	1.0677	1.0469	0.9927
	20	1.0149	0.9429	0.8976	0.9419	0.9002	0.8868

Table 9: ACL for Parameter β

$n = 30, \theta = 1.80$		$t = 04$			$t = 07$		
β	$m \downarrow \tau \rightarrow$	99%	95%	90%	99%	95%	90%
ACL On Simulated Data							
0.50	10	1.1225	1.1041	1.0754	1.1098	1.0678	1.0421
	15 (I)	1.0611	1.0529	1.0212	1.0297	1.0134	0.9869
	15 (II)	1.0635	1.0411	1.0021	1.0301	1.0177	0.9856
	20	1.0339	0.9715	0.9501	0.9475	0.9356	0.9232
1.60	10	1.1971	1.1607	1.1302	1.1729	1.1224	1.0959
	15 (I)	1.1245	1.1061	1.0742	1.0981	1.0658	1.0374
	15 (II)	1.1121	1.0954	1.0543	1.0988	1.0694	1.0369
	20	1.0364	1.0223	0.9876	1.0119	0.9846	0.9716
2.50	10	1.0517	1.0393	1.0218	1.0318	1.0155	0.9907
	15 (I)	1.0081	0.9765	0.9705	0.9955	0.9634	0.9371
	15 (II)	1.0084	0.9898	0.9521	0.9766	0.9671	0.9363
	20	0.9611	0.9231	0.9022	0.9079	0.8896	0.8773

ACL On Real Data							
0.50	10	1.2362	1.2118	1.1728	1.2283	1.1733	1.1388
	15 (I)	1.1608	1.1484	1.1063	1.1251	1.1026	1.0671
	15 (II)	1.1698	1.1375	1.0803	1.1317	1.1141	1.0716
	20	1.1264	1.0436	1.0126	1.0214	1.0024	0.9888
1.60	10	1.3304	1.2824	1.2418	1.3087	1.2422	1.2067
	15 (I)	1.2407	1.2156	1.1733	1.2115	1.1688	1.1309
	15 (II)	1.2312	1.2061	1.1462	1.2185	1.1794	1.1363
	20	1.1296	1.1078	1.0599	1.1027	1.0643	1.0498
2.50	10	1.1469	1.1292	1.1059	1.1298	1.1073	1.0747
	15 (I)	1.0939	1.0519	1.0424	1.0821	1.0395	1.0042
	15 (II)	1.1003	1.0729	1.0172	1.0642	1.0503	1.0094
	20	1.0345	0.9826	0.9521	0.9715	0.9443	0.9308

Table 10: Bound Length under One-Sample Criterion for Real Data

$n = 30, \beta = 1.60$		$t = 04$			$t = 07$		
α	$m \downarrow \tau \rightarrow$	99%	95%	90%	99%	95%	90%
For Parameter θ							
0.50	10	1.3904	1.3684	1.3331	1.362	1.3239	1.2929
	15 (I)	1.3152	1.3046	1.2675	1.2759	1.2579	1.2243
	15 (II)	1.3177	1.2908	1.2444	1.2764	1.2618	1.2229
	20	1.2829	1.2076	1.1813	1.1766	1.1626	1.1472
1.30	10	1.4813	1.4374	1.4004	1.4509	1.3896	1.3572
	15 (I)	1.3927	1.3706	1.3319	1.3599	1.3209	1.2866
	15 (II)	1.3768	1.3561	1.3086	1.3602	1.3246	1.2841
	20	1.2853	1.2687	1.2422	1.2551	1.2224	1.2062
2.50	10	1.3043	1.2896	1.2685	1.2788	1.2601	1.2294
	15 (I)	1.2514	1.2414	1.2056	1.2351	1.1963	1.1646
	15 (II)	1.2504	1.2283	1.1835	1.2106	1.2008	1.1626
	20	1.1935	1.1478	1.1231	1.1282	1.1055	1.0909
For Acceleration Factor β							
0.50	10	1.2175	1.2001	1.1718	1.1908	1.1595	1.1348
	15 (I)	1.1546	1.1468	1.1176	1.1206	1.1067	1.0798
	15 (II)	1.1543	1.1337	1.0988	1.1186	1.1068	1.0755
	20	1.1281	1.0672	1.0477	1.0395	1.0291	1.0156
1.30	10	1.2923	1.2569	1.2273	1.2639	1.2143	1.1885
	15 (I)	1.2185	1.2011	1.1701	1.1898	1.1585	1.1312
	15 (II)	1.2029	1.1875	1.1512	1.1875	1.1591	1.1266
	20	1.1301	1.1185	1.0978	1.1035	1.0782	1.0641
2.50	10	1.1466	1.1352	1.1187	1.1216	1.1077	1.0833
	15 (I)	1.1018	1.0947	1.0661	1.0871	1.0559	1.0306
	15 (II)	1.0989	1.0823	1.0487	1.0645	1.0566	1.0266
	20	1.0545	1.0185	0.9997	0.9992	0.9823	0.9692

Table 11: Bound Length for θ under Two-Sample Criterion for Real Data

$n=30, \beta=1.60$		$t=04$			$t=07$		
α	$m \downarrow \tau \rightarrow$	99%	95%	90%	99%	95%	90%
				$k=1$			
0.50	10	0.8878	0.8658	0.8333	0.8757	0.8408	0.8115
	15 (I)	0.8285	0.8158	0.7819	0.8008	0.7883	0.7518
	15 (II)	0.8387	0.8109	0.7593	0.8106	0.7945	0.7592
	20	0.8007	0.7313	0.7038	0.7184	0.7005	0.6908
1.30	10	0.9641	0.9234	0.8895	0.9497	0.8956	0.8656
	15 (I)	0.8938	0.8706	0.8353	0.8708	0.8386	0.8037
	15 (II)	0.8879	0.8646	0.8122	0.8802	0.8475	0.8109
	20	0.8028	0.7827	0.7545	0.7836	0.7501	0.7487
2.50	10	0.8156	0.8002	0.7797	0.8056	0.7966	0.7598
	15 (I)	0.7739	0.7683	0.7302	0.7669	0.7317	0.7021
	15 (II)	0.7829	0.7587	0.7087	0.7559	0.7439	0.7097
	20	0.7265	0.6821	0.6555	0.6788	0.6529	0.6439
				$k=m$			
0.50	10	1.7006	1.6551	1.5795	1.6834	1.6052	1.5416
	15 (I)	1.5775	1.5466	1.4714	1.5213	1.4807	1.4121
	15 (II)	1.6044	1.5404	1.4216	1.5487	1.5123	1.4346
	20	1.5165	1.3637	1.3008	1.3448	1.3022	1.2945
1.30	10	1.8691	1.7744	1.6994	1.8416	1.7235	1.6571
	15 (I)	1.7133	1.6635	1.5861	1.6706	1.5984	1.5229
	15 (II)	1.7095	1.6585	1.5346	1.6971	1.6253	1.5448
	20	1.5281	1.4737	1.4091	1.4845	1.4084	1.3889
2.50	10	1.5496	1.5109	1.4651	1.5336	1.4936	1.4311
	15 (I)	1.4595	1.4338	1.3613	1.4486	1.3712	1.3056
	15 (II)	1.4897	1.4293	1.3134	1.4316	1.4042	1.3285
	20	1.3578	1.2586	1.1972	1.2588	1.2092	1.1836

Table 12: Bound Length for β under Two-Sample Criterion for Real Data

$n=30, \beta=1.60$		$t=04$			$t=07$		
α	$m \downarrow \tau \rightarrow$	99%	95%	90%	99%	95%	90%
				$k=1$			
0.50	10	0.8098	0.7824	0.7315	0.8097	0.7599	0.7228
	15 (I)	0.7361	0.7164	0.6697	0.7082	0.6819	0.6386
	15 (II)	0.7603	0.7174	0.6458	0.7316	0.7076	0.6589
	20	0.7009	0.6253	0.5933	0.6234	0.5932	0.5847
1.30	10	0.9036	0.8497	0.8073	0.9011	0.8287	0.7897
	15 (I)	0.8178	0.7861	0.7435	0.7971	0.7494	0.7048
	15 (II)	0.8226	0.7858	0.7065	0.8199	0.7749	0.7246
	20	0.7036	0.6707	0.6277	0.6864	0.6365	0.6283
2.50	10	0.7161	0.6929	0.6636	0.7179	0.6919	0.6524
	15 (I)	0.6667	0.6494	0.6074	0.6651	0.6168	0.5766
	15 (II)	0.6879	0.6546	0.5773	0.6617	0.6431	0.5959
	20	0.6163	0.5727	0.5615	0.5782	0.5453	0.5395

$k = m$							
0.50	10	1.4805	1.4302	1.3411	1.4716	1.3869	1.3294
	15 (I)	1.3474	1.3131	1.2304	1.2971	1.2581	1.1855
	15 (II)	1.3884	1.3138	1.1932	1.3369	1.2985	1.2094
	20	1.2885	1.1168	1.0435	1.1114	1.0593	1.0445
1.30	10	1.6446	1.5496	1.4673	1.6382	1.5103	1.4371
	15 (I)	1.4926	1.4365	1.3628	1.4624	1.3705	1.2919
	15 (II)	1.4993	1.4396	1.2935	1.4935	1.4139	1.3254
	20	1.2897	1.2331	1.1578	1.2585	1.1711	1.1564
2.50	10	1.3127	1.2719	1.2204	1.3139	1.2681	1.1987
	15 (I)	1.2249	1.1944	1.1144	1.2205	1.1354	1.0649
	15 (II)	1.2623	1.2026	1.0604	1.2135	1.1806	1.0975
	20	1.1178	1.0057	0.9343	1.0205	0.9523	0.9426

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