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Entropy Binomial Tree Model for Option Pricing

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Abstract: We propose a new strategy to determine the parameters of the binomial tree model, which avoids the existing models' drawback of yielding a negative probability distribution p and avoids the restrictive conditions imposed on these models, such as $ud = 1$. Specifically, by regarding the price states of the underlying asset (stock) in the binomial tree model at the end of the period $t = n\Delta t$ as an information system, we establish an entropy optimization model based on the maximum-entropy principle, from which the probability density of the stock price distribution p, and consequently the up ratio, u, and down ratio, d, are derived. This model is not only easy to solve but also has clear economic and physical meaning. In particular, the solution yielded may be applied to various underlying asset price distribution types. Numerical comparisons with the classical binomial tree (CRR) model, the Black-Scholes (B-S) model, the Jarrow and Rudd (JR) model, and the Trigeorgis (TRG) model show that new model produces more reasonable values of p, u and d, and is easier to be used.

Keywords: Option pricing, maximum entropy principle, binomial tree model.

1. Introduction

Financial derivative instruments play an important role in the financial economy. Options, as representing one of the financial derivative instruments, encompass many other derivative securities though combination, and are used for both investing and hedging. The advent of the Black-Scholes [1] European option pricing formula, led to the emergence of active research on option pricing in finance. Noting that analytic solutions of American options and exotic options are difficult to be derived, and that option prices are usually not unique when the market is incomplete or when there is an arbitrage opportunity, some numerical methods have been proposed to compute option prices and have quickly drawn attention in theory and practice. At present, there are three popular numerical methods: the Monte Carlo method [2], the tree method [3], and the partial differential equation method (also known as the finite-difference method [4]).

The classical binomial tree model [3] (CRR) is the most fundamental of the tree models, but applying this model presents some difficulties, such as balancing the number of n-ary relationships, the convergence rate, the accuracy of solutions and the computational intensity, the determination of the model parameters p, u and d (with u, d representing the up/down ratios, respectively, and p representing the probability of going up), and so on. The structure of the model often leads it to yield a probability density $p < 0 \ \ (p > 1)$ or utilizes an imprecise condition, such as $ud = 1$, when determining u, d and p. Due to these deficiencies, this model may not effectively describe the process underlying asset price changes. Jarrow and Rudd [5] later adopted the strategy of letting $p = \frac{1}{2}$, which leads to the formula (JR):

$$p = \frac{1}{2},\ u = e^{((r-\frac{1}{2}\sigma^2)\Delta t+\sigma\sqrt{\Delta t})},\ d = e^{((r-\frac{1}{2}\sigma^2)\Delta t-\sigma\sqrt{\Delta t})}.$$

Then, in 1992, to maintain algorithm stability for binomial tree model in the short step time,Trigeorgis [6] proposed the following formula (TRG) for the parameters, under the condition that $ud = 1$:

$$p = \frac{1}{2} + \frac{1}{2}\frac{r\Delta t-\frac{1}{2}\sigma^2\Delta t}{\sqrt{\sigma^2\Delta t+(r\Delta t-\frac{1}{2}\sigma^2\Delta t)^2}},$$
$$u = e^{\sqrt{\sigma^2\Delta t+(r\Delta t-\frac{1}{2}\sigma^2\Delta t)^2}},\ \ d = e^{-\sqrt{\sigma^2\Delta t+(r\Delta t-\frac{1}{2}\sigma^2\Delta)^2}}.$$

Clearly, the conditions $p = \frac{1}{2}$ and $ud = 1$ imposed on the model are too stringent, as in practice, the product of and

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does not necessarily equal 1. Recently, Yisong [7], Johnson et al. [8], Diener et al. [9], and Benjamin et al. [10] set the parameters of the tree model using higher moments for return distribution of the underlying asset. However, this setting not only renders it difficult to calculate higher moments but also produces a model that is unsuitable for computing the option price, which follows a different distributive form.

In this paper, to overcome the traditional classical problems mentioned above, we propose a novel strategy to determine the parameters p, u and d based on the maximum-entropy principle [11]. Specifically, we set as the objective function the maximum information entropy, and the amount of different kinds of information, e.g., the expectation and variance of stock price change, as constraint conditions to establish an entropy optimization model about the probability density p, up ratio u and down ratio d . According to [11], this model may yield an unbiased and objective probability density p to uniformly approximate the actual probability distribution. In particular, this p is always non-negative, and the parameters p, u and d have a clear meaning. Numerical comparisons with those given by the B-S model, the CRR model, and the JR model show that the proposed method is effective.

This paper is organized as follows. In Section 2, we introduce the classical binomial tree model and its deficiencies. In Section 3, we propose a model based on the maximum-entropy principle. In Section 4, the performance of the new method is compared with that of existing methods. Finally, we conclude this paper.

Some words about our notations. Throughout this paper, S_0 denotes the initiative price of a share of stock, S represents the current price of a share of stock, K denotes the option strike price, μ is the expectation of stock return, and σ is the volatility of stock price. In addition, we use r and T to denote the riskless interest rate and option expiration, respectively.

We implement these models in Matlab 7 on a standard office PC, and perform a comparison among the four models.

2. CRR model

We consider stock option prices without dividend in a risk-neutral market and begin by setting the parameters p, u and d. In a binomial tree, every node of the tree has two possible future states at next time. That is, if the current stock price is S, then the future stock price will be uS with probability p or dS with probability $1-p$ after Δt time, where

$$u > 1,\ 0 < d \leq 1,\ d < 1 + r < u.$$

Because the market is risk-neutral, the expectation of the stock price change is $Se^{r\Delta t}$, for which we obtain that:

$$Se^{r\Delta t} = pSu + (1-p)Sd, e^{r\Delta t} = pu + (1-p)d. \quad (1)$$

Usually, one considers the change of the stock price S_t as governed by the Black-Scholes stochastic differential equation, i.e., $\Delta S = S\mu\Delta t + S\sigma\varepsilon\sqrt{\Delta t}$, with ε being stochastic number of a standard Brownian motion. Therefore, over Δt , the variance of the stock price change is:

$$E[(S+\Delta S)^2] - (E[(S+\Delta S)])^2 = S^2 e^{2r\Delta t}(e^{\sigma^2\Delta t} - 1)$$

where

$$\begin{aligned} S^2 e^{2r\Delta t}(e^{\sigma^2\Delta t} - 1) &= pS^2u^2 + (1-p)S^2d^2 \\ &\quad -S^2[pu + (1-p)d]^2, \\ e^{2r\Delta t + \sigma^2\Delta t} &= pu^2 + (1-p)d^2. \end{aligned} \quad (2)$$

For convenience of calculation, one usually imposes an additional condition $ud = 1$. Then, from Eq. (1), Eq. (2) and $ud = 1$, one may obtain the parameter formula of the CRR model:

$$p = \frac{e^{r\Delta t} - d}{u - d},\ \ u = e^{\sigma\sqrt{\Delta t}},\ \ d = e^{-\sigma\sqrt{\Delta t}}. \quad (3)$$

It is not difficult to see that the imposed condition $ud = 1$ lacks practically economic meaning, and in particular, when $\sigma < r\sqrt{\Delta t}$, the parameter p in (3) may be negative or p larger than 1. Therefore, it is necessary to study how to obtain well-founded and more precise parameter formula for the binomial tree model. Note that Eq. (1) and Eq. (2) both concern moments of stock price change and that parameter p represents the probability distribution of stock price change. Then, one naturally asks what the probability distribution is like? Motivated by the maximum-entropy principle, in the next section we propose a new strategy to determine the parameters p, u and d of the binomial tree model.

3. Entropy optimization model of parameters

3.1. Entropy optimization model

It is known that future stock price change correspond to a certain probability distribution. Then, how do we predict the probability distribution? The maximum-entropy principle [11] states that "in making inference on the basis of partial information one must use that probability distribution which has maximum entropy subject to whatever is known. This is the only unbiased assignment one can make; to use any other would amount to arbitrary assumption of information which by hypothesis one does not have." In light of this, we propose the following entropy optimization model:

$$\begin{aligned} \max \quad & -p\ln p - (1-p)\ln(1-p) \\ s.t. \quad & pu + (1-p)d = e^{r\Delta t} \\ & pu^2 + (1-p)d^2 = e^{2r\Delta t + \sigma^2\Delta t} \\ & u > 1 \\ & 0 < d \leq 1 \\ & 0 \leq p \leq 1\ . \end{aligned} \quad (4)$$

To determine the probability distribution of the stock price change in the binomial tree model at the end of the period $t = \Delta t$, provided that the expectation and variance are known. In model (4), p, u and d are unknown quantities, and r, σ and Δt are known quantities. The objective function of (4) is the probability distribution of the stock price change at the end of the period $t = \Delta t$, and the information (such as expectation, variance) is the constraints, with the first two constraints describing the first two-order moments of the stock price change by the no-arbitrage principle and the risk neutral principle and last three constraints reflecting the practical economic reality: p, which denotes the probability density of the stock price going up; u, which denotes the up ratio of the stock price in the next period; and d, which denotes the down ratio of the stock price in the next period.

3.2. Solving model (4)

First, u and d are computed by p :

$$\begin{cases} u = e^{r\Delta t} + \sqrt{\frac{1-p}{p}(e^{2r\Delta t+\sigma^2\Delta t} - e^{2r\Delta t})} \\ d = e^{r\Delta t} - \sqrt{\frac{p}{1-p}(e^{2r\Delta t+\sigma^2\Delta t} - e^{2r\Delta t})} \end{cases}$$

or

$$\begin{cases} u = e^{r\Delta t} - \sqrt{\frac{1-p}{p}(e^{2r\Delta t+\sigma^2\Delta t} - e^{2r\Delta t})} \\ d = e^{r\Delta t} + \sqrt{\frac{p}{1-p}(e^{2r\Delta t+\sigma^2\Delta t} - e^{2r\Delta t})} \end{cases},$$

if $d = e^{r\Delta t} + \sqrt{\frac{p}{1-p}(e^{2r\Delta t+\sigma^2\Delta t} - e^{2r\Delta t})}$ in the second group expression, it is trivially $d > 1$ and is not consistent with the constraint on d. Therefore,

$$\begin{cases} u = e^{r\Delta t} + \sqrt{\frac{1-p}{p}(e^{2r\Delta t+\sigma^2\Delta t} - e^{2r\Delta t})} \\ d = e^{r\Delta t} - \sqrt{\frac{p}{1-p}(e^{2r\Delta t+\sigma^2\Delta t} - e^{2r\Delta t})}\ . \end{cases} \quad (5)$$

We substitute Eq. (5) into Eq. (4):

$$\begin{aligned} \max \quad & -p\ln p - (1-p)\ln(1-p) \\ s.t. \quad & 0 \le p \le 1 \\ & e^{r\Delta t} + \sqrt{\tfrac{1-p}{p}(e^{2r\Delta t+\sigma^2\Delta t} - e^{2r\Delta t})} > 1 \\ & 0 < e^{r\Delta t} - \sqrt{\tfrac{p}{1-p}(e^{2r\Delta t+\sigma^2\Delta t} - e^{2r\Delta t})} \le 1\ . \end{aligned} \quad (6)$$

The second constraint in Eq. (6) always holds, and Eq. (6) become:

$$\begin{aligned} \max \quad & -p\ln p - (1-p)\ln(1-p) \\ s.t. \quad & \tfrac{(e^{r\Delta t}-1)^2}{e^{2r\Delta t+\sigma^2\Delta t}-2e^{r\Delta t}+1} \le p < e^{-\sigma^2\Delta t}\ . \end{aligned} \quad (7)$$

Second, we know that the figure of mathematical programming (PL):

$$\begin{aligned} \max \quad & -p\ln p - (1-p)\ln(1-p) \\ s.t. \quad & 0 \le p \le 1. \end{aligned} \quad (PL)$$

is as follows:

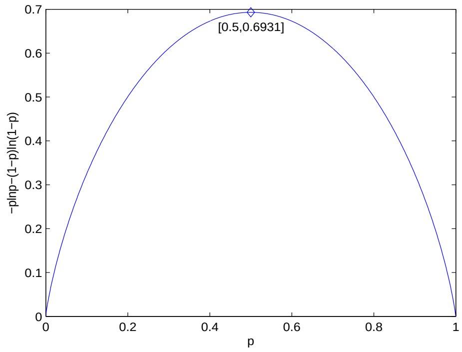

Figure 1 The mathematical programming (PL).

Figure 1 shows that the mathematical programming (PL) is a nonempty upper convex function and produces the maximum value at the point $[0.5,\ 0.6931]$. Because the feasible region of the mathematical programming (PL) contains that of Eq. (7), the figures produced by Eq. (7) and those produced by the mathematical programming (PL) are similar. Note that Eq. (7) has a nonempty and compact constraint set. Therefore, by Weierstrass' Theorem (see [12, Prop. A.8]), Eq. (7) has optimal solutions, and consequently the original entropy model (4) has optimal solutions. The optimal solution of Eq. (4) is as follows:

1) if $\frac{(e^{r\Delta t}-1)^2}{1-2e^{r\Delta t}+e^{2r\Delta t+\sigma^2\Delta t}} < e^{-\sigma^2\Delta t} < \frac{1}{2}$, the optimal solution is $p = e^{-\sigma^2\Delta t}$;

2) if $\frac{(e^{r\Delta t}-1)^2}{1-2e^{r\Delta t}+e^{2r\Delta t+\sigma^2\Delta t}} \le \frac{1}{2}, \quad e^{-\sigma^2\Delta t} \ge \frac{1}{2}$, the optimal solution is $p = \frac{1}{2}$;

3) if $\frac{1}{2} < \frac{(e^{r\Delta t}-1)^2}{1-2e^{r\Delta t}+e^{2r\Delta t+\sigma^2\Delta t}} < e^{-\sigma^2\Delta t}$, the optimal solution is $p = \frac{(e^{r\Delta t}-1)^2}{1-2e^{r\Delta t}+e^{2r\Delta t+\sigma^2\Delta t}}$.

Lastly, combined with Eq. (5), then, p, u, d are as follows:

$a)$ if $\frac{(e^{r\Delta t}-1)^2}{1-2e^{r\Delta t}+e^{2r\Delta t+\sigma^2\Delta t}} < e^{-\sigma^2\Delta t} < \frac{1}{2}$, then

$$\begin{cases} p = e^{-\sigma^2\Delta t} \\ u = e^{r\Delta t+\sigma^2\Delta t} \\ d = 0\ ; \end{cases}$$

$b)$ if $\frac{(e^{r\Delta t}-1)^2}{1-2e^{r\Delta t}+e^{2r\Delta t+\sigma^2\Delta t}} \le \frac{1}{2}, \quad e^{-\sigma^2\Delta t} \ge \frac{1}{2}$, then

$$\begin{cases} p = \frac{1}{2} \\ u = e^{r\Delta t} + \sqrt{e^{2r\Delta t+\sigma^2\Delta t} - e^{2r\Delta t}} \\ d = e^{r\Delta t} - \sqrt{e^{2r\Delta t+\sigma^2\Delta t} - e^{2r\Delta t}}\ ; \end{cases}$$

$c)$ if $\frac{1}{2} < \frac{(e^{r\Delta t}-1)^2}{1-2e^{r\Delta t}+e^{2r\Delta t+\sigma^2\Delta t}} < e^{-\sigma^2\Delta t}$, then

$$\begin{cases} p = \frac{(e^{r\Delta t}-1)^2}{1-2e^{r\Delta t}+e^{2r\Delta t+\sigma^2\Delta t}} \\ u = \frac{e^{2r\Delta t+\sigma^2\Delta t}-e^{r\Delta t}}{e^{r\Delta t}-1} \\ d = 1\ . \end{cases}$$

Therefore, the values of p, u, d of model (4) are computed by the three group expressions above a), b), c).

If we have more information, e.g., higher moments in the constraints of (4), the probability distribution p obtained will better approximate the real distribution. As a matter of fact, as soon as the expectation and variance of the stock price change are known, or if other higher moments of the stock price change that obey the probability distribution are known, the probability density value p can be obtained by solving the optimization problem (4). Therefore, the model does not require specific kind of underlying asset (stock) price change. Furthermore, this model always yields an unbiased nonnegative parameter p, and the constraints about u and d have practical clear economic meaning.

After obtaining the parameters p, u and d, the rest steps of calculating option price are the same as those of [3]. The prices of European options and American options without dividend can be calculated using model (4). In addition, after making suitable adjustments for Eq. (1), (2) and (4) in terms of specific situation of the options with dividend and exotic options [13], we may obtain the corresponding parameters p, u and d and then determine the option prices.

4. Numerical examples

Example 1. Consider European put option pricing without dividend at the expiration, given $S_0 = 8,\ K = 8.75,\ r = 0.12,\ \sigma = 0.01$ and the expirations of $1, 3, 6, 9$ and 12 months, with different time step $\Delta t = T/n$ at $n = 8, 16, 32,$ 64, Tables 1 and 2 report the parameters p, u, d and option prices under B-S model, CRR model, JR model and MEB model (the new model).

Table 1 clearly shows that parameter p in the CRR model is often greater than 1, which leads to $1 - p < 0$ and violates the meaning of p as a probability measure; compared with the other three models, the option price in the JR model approaches the option price of B-S, but the JR model sets $p = \frac{1}{2}$ and usually produces a value of d larger than 1, which violates the meaning of d in the binomial model. In addition, when $p = \frac{1}{2}$, the results of the MEB model coincide with those obtained by the JR model. The parameters p, u, d of the TRG model fit their practical meaning, but at long expirations, option prices calculated by the TRG model have low accuracy. The parameters p, u, d in the MEB model can have unbiased, definite meaning under the maximum-entropy principle, and the MEB model produces the segmented analytic solution and coincides with the JR model in the some cases.

Table 2, which describes European put option prices show that the MEB model has higher accuracy with n increase. For the same n, the calculation precision of the MEB model is not lower than the other methods'. When T is short, the MEB model is more effective. However, for the long T, the MEB model is slightly unstable when given the different steps. And it will be researched problem in the future, which is how the rest steps are found according to the MEB model parameters for option pricing.

To further analyze the performance of the MEB model, we consider the influence of volatility, expiration, out-of-the-money, at-the-money and in-the-money etc. on the MEB model. We thus compare the error of the three methods (JR, TRG, MEB) for American call (put) options. The error formula is $error_i = \frac{c_i - c_{CRR}}{c_{CRR}},\ \ i = \text{JR, TRG, MEB},$ where c_{CRR} is the option price of the CRR model when $n = 1000$.

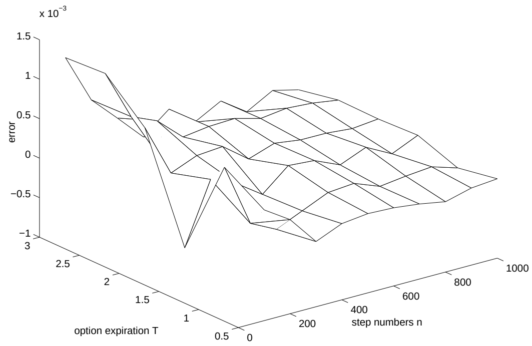

Figure 2 For American call options, influence of option expiration T on the MEB model (where $S_0 = 120,\ K = 100,\ r = 0.07,\ \sigma = 0.30,\ T = 0.5,\ 1,\ 1.5,\ 2,\ 2.5,\ 3,\ n = 100,\ 200,\ 300,\ 400,\ 500,\ 600,\ 700,\ 800,\ 900,\ 1000$).

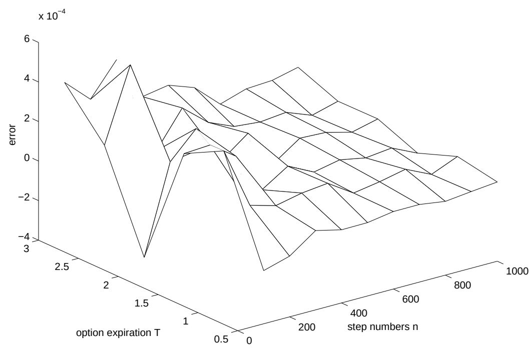

Figure 3 For American put options, influence of option expiration T on the MEB model (where $S_0 = 80,\ K = 100,\ r = 0.07,\ \sigma = 0.30,\ T = 0.5,\ 1,\ 1.5,\ 2,\ 2.5,\ 3,\ n = 100,\ 200,\ 300,\ 400,\ 500,\ 600,\ 700,\ 800,\ 900,\ 1000$).

Figure 2 and Figure 3 show that the MEB model produces slightly unstable option prices for the long option

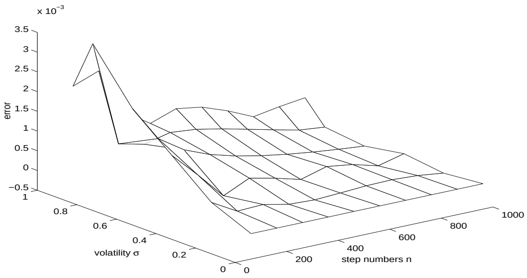

Figure 4 For American call options, influence of volatility σ on the MEB model (where $S_0 = 120,\ K = 100,\ r = 0.07,\ \sigma = 0.05,\ 0.25,\ 0.45,\ 0.65,\ 0.85,\ 0.95,\ T = 1,\ n = 100,\ 200,\ 300,\ 400,\ 500,\ 600,\ 700,\ 800,\ 900,\ 1000$).

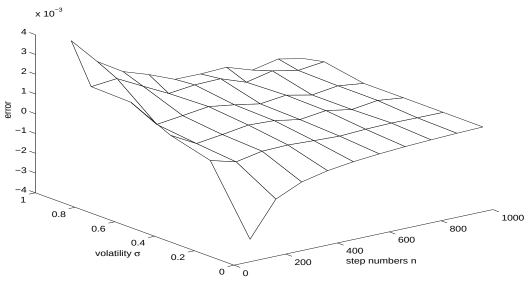

Figure 5 For American put options, influence of volatility σ on the MEB model (where $S_0 = 80,\ K = 100,\ r = 0.07,\ \sigma = 0.05,\ 0.25,\ 0.45,\ 0.65,\ 0.85,\ 0.95,\ T = 1,\ n = 100,\ 200,\ 300,\ 400,\ 500,\ 600,\ 700,\ 800,\ 900,\ 1000$).

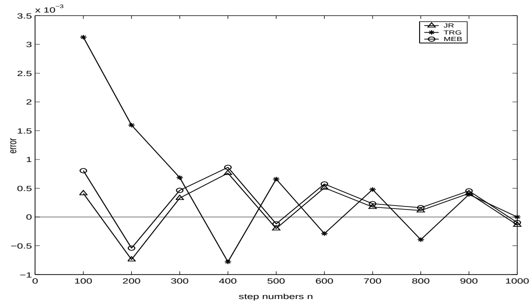

Figure 6 Out-of-the-money American call option prices under the JR, TRG and MEB models (where $S_0 = 90,\ K = 100,\ r = 0.07,\ \sigma = 0.30,\ T = 0.5,\ n = 100,\ 200,\ 300,\ 400,\ 500,\ 600,\ 700,\ 800,\ 900,\ 1000$).

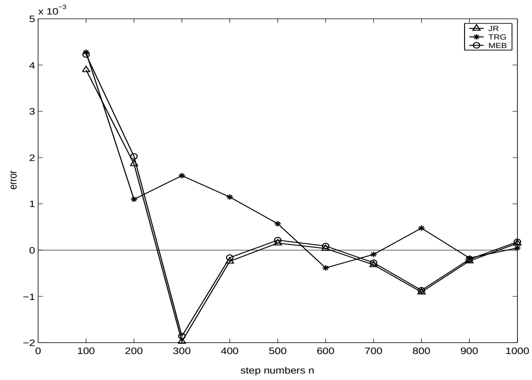

Figure 7 Out-of-the-money American put option prices under the JR, TRG and MEB models (where $S_0 = 100,\ K = 100,\ r = 0.07,\ \sigma = 0.30,\ T = 0.5,\ n = 100,\ 200,\ 300,\ 400,\ 500,\ 600,\ 700,\ 800,\ 900,\ 1000$).

expiration T, but produces stable American call option prices. Figure 4 and Figure 5 show that with the volatility σ increasing, the error in the MEB model becomes larger, especially for American put options. There are two explanations for the findings shown in figures 2-5: one is the deficiency of the CRR model in choosing parameters, which may result in negative option prices, and increases the error, and the other one is that the MEB model cannot effectively control the position of strike price in the final layer nodes of the tree, which increases the error.

For out-of-the-money options, Figure 6 and Figure 7 show that the MEB model is closer to the JR model and more accurate than the TRG model. The error of the MEB model declines slowly, with step numbers increasing, especially for at-the-money call options (e.g., Figure 8). The change in the error of the MEB model for in-the-money options is smaller than that for out-of-the-money, and at-the-money options (e.g. Figure 10 and Figure 11). When $n > 600$, the error under the MEB is approximately ± 0.5. As shown in figures 6 to 11, because the MEB model chooses unbiased, objective values for p, u and d, it produces the more reasonable option price. In particular, when $p = \frac{1}{2}$, the MEB model effectively produces the same results as the JR model.

5. Conclusion

In this paper, we propose a new strategy to determine the parameters p, u and d of the binomial tree for option pricing based on the maximum-entropy principle. This method yields an unbiased and objective probability density p via optimization problem (4), which both effectively overcomes the CRR model's shortcoming of yielding negative probability and avoids the imposed some restrictive conditions. Numerical examples demonstrate that the new model can yield a more accurate solution and is easier to operate. Further research into its stability is needed in the future.

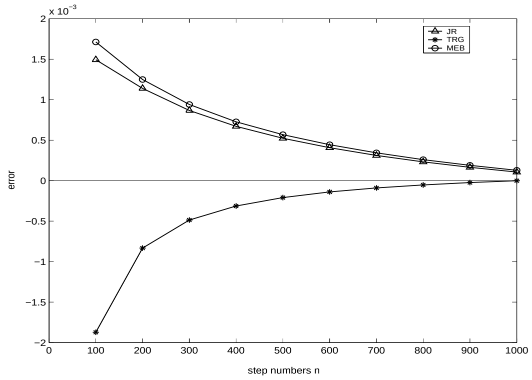

Figure 8 At-the-money American call option prices under the JR, TRG and MEB models (where $S_0 = 100,\ K = 100,\ r = 0.07,\ \sigma = 0.30,\ T = 0.5,\ n = 100,\ 200,\ 300,\ 400,\ 500,\ 600,\ 700,\ 800,\ 900,\ 1000$).

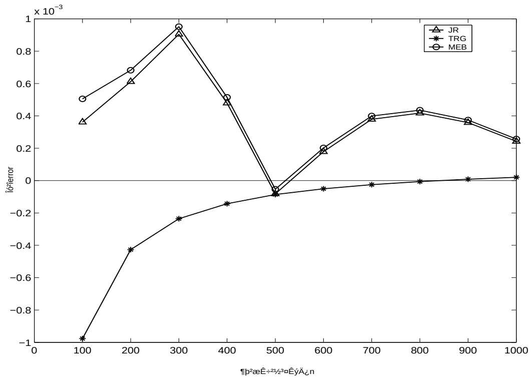

Figure 9 At-the-money American put option prices under the JR, TRG and MEB models (where $S_0 = 100,\ K = 100,\ r = 0.07,\ \sigma = 0.30,\ T = 0.5,\ n = 100,\ 200,\ 300,\ 400,\ 500,\ 600,\ 700,\ 800,\ 900,\ 1000$).

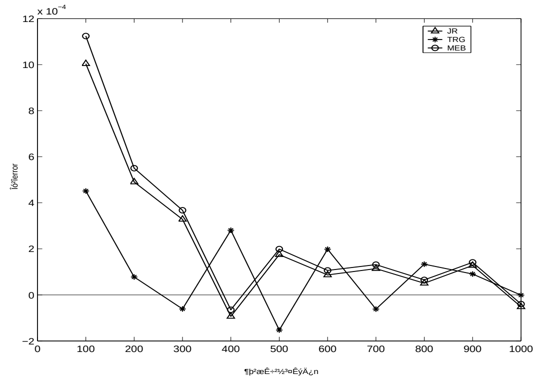

Figure 10 In-the-money American call option prices under the JR, TRG and MEB models (where $S_0 = 110,\ K = 100,\ r = 0.07,\ \sigma = 0.30,\ T = 0.5,\ n = 100,\ 200,\ 300,\ 400,\ 500,\ 600,\ 700,\ 800,\ 900,\ 1000$).

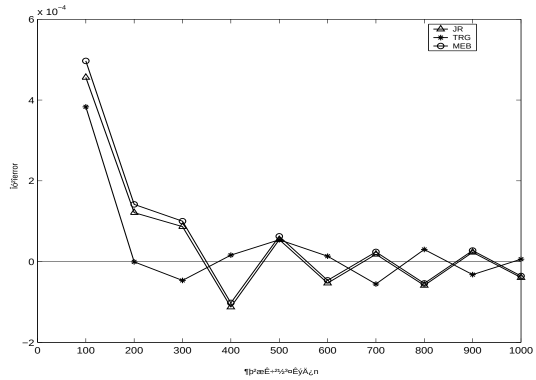

Figure 11 In-the-money American put option prices under the JR, TRG and MEB models (where $S_0 = 100,\ K = 110,\ r = 0.07,\ \sigma = 0.30,\ T = 0.5,\ n = 100,\ 200,\ 300,\ 400,\ 500,\ 600,\ 700,\ 800,\ 900,\ 1000$).

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Table 1 Comparisons of values of parameters p, u and d of binomial for European put options

Step	Model	Values of parameters p, u and d														
		Month1			Months3			Months6			Months9			Months12		
		p	u	d	p	u	d	p	u	d	p	u	d	p	u	d
n=8	CRR	1.1125	1.001	0.999	1.5622	1.0018	0.9982	2.005	1.0025	0.9975	2.3467	1.0031	0.9969	2.6364	1.0035	0.9965
	JR	0.5	1.0023	1.0002	0.5	1.0055	1.002	0.5	1.01	1.005	0.5	1.0144	1.0082	0.5	1.0187	1.0115
	TRG	0.8872	1.0016	0.9984	0.9522	1.0042	0.9959	0.9743	1.0079	0.9921	0.9824	1.0117	0.9884	0.9867	1.0155	0.9847
	MEB	0.5997	1.0021	1	0.8176	1.0046	1	0.8993	1.0084	1	0.9303	1.0122	1	0.9466	1.016	1
n=16	CRR	0.933	1.0007	0.9993	1.2504	1.0013	0.9988	1.5622	1.0018	0.9982	1.8022	1.0022	0.9978	2.005	1.0025	0.9975
	JR	0.5	1.0013	0.9999	0.5	1.0031	1.0006	0.5	1.0055	1.002	0.5	1.0078	1.0035	0.5	1.01	1.005
	TRG	0.8272	1.001	0.999	0.916	1.0023	0.9977	0.9522	1.0042	0.9959	0.9666	1.006	0.994	0.9743	1.0079	0.9921
	MEB	0.5	1.0013	0.9999	0.6919	1.0027	1	0.8176	1.0046	1	0.8703	1.0065	1	0.8993	1.0084	1
n=32	CRR	0.8061	1.0005	0.9995	1.0304	1.0009	0.9991	1.2504	1.0013	0.9988	1.4195	1.0015	0.9985	1.5622	1.0018	0.9982
	JR	0.5	1.0008	0.9998	0.5	1.0018	1.0001	0.5	1.0031	1.0006	0.5	1.0044	1.0013	0.5	1.0055	1.002
	TRG	0.761	1.0006	0.9994	0.8637	1.0013	0.9987	0.916	1.0023	0.9977	0.9391	1.0032	0.9968	0.9522	1.0042	0.9959
	MEB	0.5	1.0008	0.9998	0.5292	1.0018	1	0.6919	1.0027	1	0.7709	1.0037	1	0.8176	1.0046	1
n=64	CRR	0.7164	1.0004	0.9996	0.8749	1.0006	0.9994	1.0304	1.0009	0.9991	1.1497	1.0011	0.9989	1.2504	1.0013	0.9988
	JR	0.5	1.0005	0.9998	0.5	1.0011	0.9998	0.5	1.0018	1.0001	0.5	1.0025	1.0003	0.5	1.0031	1.0006
	TRG	0.6986	1.0004	0.9996	0.7999	1.0008	0.9992	0.8637	1.0013	0.9987	0.8961	1.0018	0.9982	0.916	1.0023	0.9977
	MEB	0.5	1.0005	0.9998	0.5	1.0011	0.9998	0.5292	1.0018	1	0.6276	1.0022	1	0.6919	1.0027	1

Table 2 Comparisons of European put option prices between five models

Step	Model	Option price c at the expiration T				
		Month1	Months3	Months6	Months9	Months12
n=8	B-S	0.662936	0.491398	0.2404398	0.026111	2.62E-05
	CRR	0.662936	0.491398	0.2404397	-0.003102	-0.239446
	JR	0.662936	0.491398	0.2404397	0.025623	0
	TRG	0.662936	0.491398	0.2404407	0.022106	0.001042
	MEB	0.662936	0.491398	0.2404397	0.028819	0.000619
n=16	B-S	0.662936	0.491398	0.2404398	0.026111	2.62E-05
	CRR	0.662936	0.491398	0.2404397	-0.003102	-0.239446
	JR	0.662936	0.491398	0.2404397	0.025935	0.00001
	TRG	0.662936	0.491398	0.2404402	0.028536	0.000682
	MEB	0.662936	0.491398	0.2404397	0.025947	0.000203
n=32	B-S	0.662936	0.491398	0.2404398	0.026111	2.62E-05
	CRR	0.662936	0.491398	0.2404397	-0.003102	-0.239455
	JR	0.662936	0.491398	0.2404397	0.026069	0.000021
	TRG	0.662936	0.491398	0.2404399	0.025089	0.00024
	MEB	0.662936	0.491398	0.2404397	0.026431	0.000087
n=64	B-S	0.662936	0.491398	0.2404398	0.026111	2.62E-05
	CRR	0.662936	0.491398	0.2404397	-0.003102	-0.239297
	JR	0.662936	0.491398	0.2404397	0.026119	0.000022
	TRG	0.662936	0.491398	0.2404397	0.026253	0.000107
	MEB	0.662936	0.491398	0.2404397	0.026018	0.00004