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Influence of Viscous Dissipation and Radiation on Unsteady MHD Mixed Convection Flow of Micropolar Fluids

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In this study, we investigate the influence of viscous dissipation and radiation on the problem of unsteady MHD mixed convection two-dimensional laminar flow of an incompressible electrically conducting micropolar fluid past a semi-infinite vertical moving porous plate embedded in a porous medium. By taking the radiative heat flux in the differential form and imposing an oscillatory time-dependent perturbation, the coupled nonlinear equations are solved for the temperature and velocity distributions. The effects of the material parameters on the temperature and velocity profiles are discussed quantitatively. Especially, the effects of non-zero values of micro-gyration vector on the velocity, angular velocity and temperature fields across the boundary layer are studied. Numerical evaluation of the analytical results is performed and some graphical results for the velocity, angular velocity and temperature profiles within the boundary layer have been illustrated.

Keywords: Analytical solution, micropolar fluid, mixed convection, unsteady flow, viscous dissipation, radiation, porous media.

NOMENCLATURE

A	–	Small positive parameter;
B	–	Planck's function;
C_P	–	Specific heat capacity;
Ec	–	Eckert number;
g	–	gravitational acceleration;
Gr	–	Grashof number (free-convection parameter);

H'_o – Constant transverse magnetic field;
K – Dimensional porosity parameter;
k – thermal conductivity;
M^2 – Non-dimensional magnetic parameter;
Pr – Prandtl number;
$q^*_{z^*}$ – Radiative heat flux;
R^2 – Radiation parameter;
t^* – Dimensional time;
T_w – Wall temperature;
T_∞ – Free stream temperature;
u^*, v^*, w^* – Dimensional velocity components;
U_o – Mean velocity of $U^*(t^*)$;
U^* – Dimensional free stream velocity;
w^*_o – Dimensional suction velocity;
x^*, y^*, z^* – Dimensional Cartesian coordinates;

Greek Symbols

α^2 – Absorption coefficient;
δ – Radiation absorption coefficient along the surface;
ε – Small positive parameter;
χ^2 – Darcy number;
λ – Frequency;
γ – Spin gradient viscosity;
μ – Dynamic viscosity of the fluid;
ν – Kinematic viscosity;
ρ – Fluid density;
σ_o – Electrical conductivity;
ω^* – Dimensional free stream frequency of oscillation;

Subscripts

w – Surface conditions;
∞ – Conditions far a way form the surface.

1 Introduction

Flow through porous medium past infinite vertical plate is common in nature and has many applications in engineering and science. A number of workers have investigated such flows and excellent literature on the properties and phenomenon may be found in literature ([1]- [3]). For example, Soundalgekar [3] investigated the effects of free-convection currents on the oscillatory type boundary layer flow past an infinite vertical plate with constant suction where the plate temperature differs from the free stream temperature. Kim [4] has considered the case of a semi-infinite moving porous plate in a porous medium with the presence of pressure gradient and constant velocity in the flow direction when the magnetic field is imposed transverse to the plate. He also considered the free stream to consist of a mean velocity and temperature over which are superimposed an exponentially varying with time.

On the other hand, heat transfer by simultaneous free or mixed convection and thermal radiation in the case of a micropolar fluid has not received as much attention. This is unfortunate because thermal radiation plays an important role in determining the overall surface heat transfer in situations where convective heat transfer coefficients are small. Such situations are common in space technology [5]. Raptis [6] studied numerically the case of a steady two-dimensional flow of a micropolar fluid past a continuously moving plate with a constant velocity in the presence of thermal radiation. Gorla and Tornabene [7] investigated the effects of thermal radiation on mixed convection flow over a vertical plate with nonuniform heat flux boundary conditions.

Micropolar fluids are fluids with microstructure belonging to a class of fluids with asymmetrical stress tensor. Physically, they represent fluids consisting of randomly oriented particles suspended in a viscous medium ([8]– [11]). The micropolar fluid considered here is a gray, absorbing-emitting but non-scattering optically thick medium. The problem of micropolar fluids past through a porous media has many applications, such as, porous rocks, foams and foamed solids, aerogels, alloys, polymer blends and microemulsions. The simultaneous effects of a fluid inertia force and boundary viscous resistance upon flow and heat transfer in a constant porosity porous medium were analyzed by Vafai and Tien [12]. Raptis [13] studied boundary layer flow of a micropolar fluid through a porous medium. Kim [14] has considered the case of a semi-infinite moving porous plate with a constant velocity in the longitudinal direction when the magnetic field is imposed transversely to the plate. Kim [15] has considered the effect of nonzero values of microgyration vector on the semi-infinite moving porous plate with a constant velocity in the longitudinal direction when the magnetic field is imposed transversely to the plate.

Most of the previous studies of the same problem neglected viscous dissipation and radiation. In the present work we consider the case of mixed convection flow of a micropolar fluid past a semi-infinite, steadily moving porous plate with varying suction velocity normal to the plate in the presence of thermal radiation and viscous dissipation.

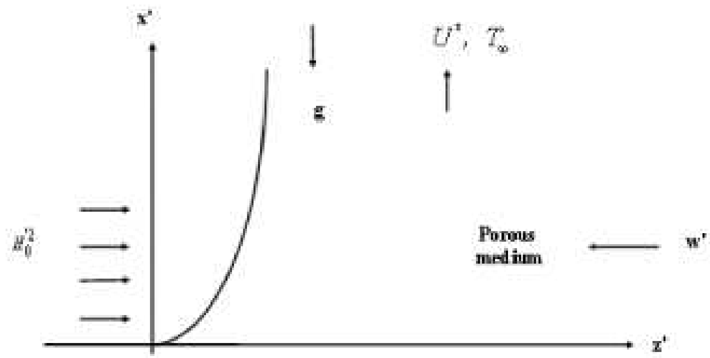

Figure 1.1: The physical model and coordinate system of the problem.

2 Mathematical Analysis

The problem that we will study is the two-dimensional, unsteady, mixed convection flow of a laminar, incompressible, micropolar fluid over a semi-infinite vertical porous moving plate in the presence of a magnetic field. It is also assumed here that the hole size of the porous plate is significantly larger than a characteristic microscopic length scale of a micropolar fluid. The x^* -axis is taken along the planar surface in the upward direction and the z^* axis is taken to be normal to the plate. At time $t^* = 0$, the plate is maintained at a temperature T_w, which is high enough to initiate radiative heat transfer. A constant magnetic field is maintained in the z^* direction and the plate moves uniformly along the positive x^* direction with velocity U_0. Under Boussinesq approximation the flow is governed by the following equations:

$$\frac{\partial w^*}{\partial z^*} = 0, \tag{2.1}$$

$$\begin{aligned}\frac{\partial u^*}{\partial t^*} + w^* \frac{\partial u^*}{\partial z^*} &= (\nu + \nu_r)\frac{\partial^2 u^*}{\partial z^{*2}} + \frac{\partial U^*}{\partial t^*} - (\frac{\mu^2 \sigma_o H_o^2}{\rho} + \frac{\nu}{K^*})(u^* - U^*) \\ &+ g\beta(T^* - T_\infty) + 2\nu_r \frac{\partial N^*}{\partial z^*},\end{aligned} \tag{2.2}$$

$$\frac{\partial N^*}{\partial t^*} + w^* \frac{\partial N^*}{\partial z^*} = \frac{\gamma}{\rho j^*}\frac{\partial^2 N^*}{\partial z^{*2}}, \tag{2.3}$$

$$\frac{\partial T^*}{\partial t^*} + w^* \frac{\partial T^*}{\partial z^*} = \frac{k}{\rho c_p}(\frac{\partial^2 T^*}{\partial z^{*2}} - \nabla q^*_{z^*}) + \frac{\mu}{\rho c_p}(\frac{\partial u^*}{\partial z^*})^2, \tag{2.4}$$

$$\frac{\partial^2 q^*_{z^*}}{\partial z^{*2}} - 3\alpha^2 q^*_{z^*} - 16\alpha\sigma T^3_\infty \frac{\partial T^*}{\partial z^*} = 0, \tag{2.5}$$

the boundary conditions are

$$\begin{aligned} &u^* = u^*_p, \qquad N^* = -n\frac{\partial u^*}{\partial z^*}, \qquad T^* = T_w \qquad z^* = 0, \\ &u^* = U^*(t^*) = w^*_o(1 + \varepsilon e^{i\omega^* t^*}),\ N^* \longrightarrow 0,\ T^* \longrightarrow T_\infty \qquad z^* \longrightarrow \infty, \end{aligned} \tag{2.6}$$

where the medium is optically thin with relativity low density and $\alpha \ll 1$ the radiative heat flux given by Eq. (2.5) in the spirit of Cogley *et al.* [16] becomes

$$\frac{\partial q^*_{z^*}}{\partial z^*} = 4\alpha^2(T^* - T_\infty). \tag{2.7}$$

Since

$$\alpha^2 = \int_0^\infty \delta\lambda \frac{\partial B}{\partial T^*}, \tag{2.8}$$

it is clear from Eq. (2.1) that the suction velocity at the plate surface is a function of time only. Assuming that it takes the following exponential form

$$w^* = -w_o(1 + \varepsilon A e^{i\omega^* t^*}), \tag{2.9}$$

where A is a real positive constant, ε and εA are small and less than unity, and w_o is a scale of suction velocity which has non-zero positive constant. The second equation in (2.6) is the boundary condition for microrotation variable N^* that describes its relationship with the surface stress. In this equation, the parameter n is a number between 0 and 1 that relates the micro-gyration vector to the shear stress. The value $n = 0$ corresponds to the case where the particle density is sufficiently large so that microelements close to the wall are unable to rotate. The value $n = 0.5$ is indicative of weak concentrations, and when $n = 1$ flows are believed to represent turbulent boundary layers in Rees and Bassom [17].

We shall use the dimensionless variables

$$\begin{aligned} &u = \frac{u^*}{U_o},\ w = \frac{w^*}{w_0},\ N = \frac{\nu}{U_0 w_o}N^*,\ z = \frac{w_o z^*}{\nu},\ t = \frac{w_o^2 t^*}{4\nu},\ U = \frac{U^*}{U_0},\ U_p = \frac{u^*_p}{U_o}, \\ &j = \frac{w_0^2}{\nu^2}j^*,\ \theta = \frac{T^* - T_\infty}{T_w - T_\infty},\ \omega = \frac{4\nu\omega^*}{w_o^2},\ \chi^2 = \frac{\nu^2}{K^* w_0^2},\ Ec = \frac{U_o^2}{c_{p(T_w - T_\infty)}}, \\ &Gr = \frac{\nu g\beta(T_w - T_\infty)}{U_o w_o^2},\ R^2 = \frac{4\alpha^2(T_w - T_\infty)}{\rho c_p k w_o^2},\ Pr = \frac{\mu c_p}{k},\ M^2 = \frac{\mu^2\sigma_o H_o^2}{\rho w_o^2}, \end{aligned} \tag{2.10}$$

where Pr is the Prandtl number , Gr is the Grashof number, Ec is the Eckert number, R is the radiation parameter, M is the magnetic field parameter and χ^2 is the Darcy number.

Furthermore, the spin-gradient viscosity γ which gives some relationship between the coefficients of viscosity and micro-inertia, is defined as

$$\gamma = (\mu + \frac{k}{2})j^* = \mu j^*(1 + \frac{1}{2}\beta); \quad \beta = \frac{k}{\mu}. \tag{2.11}$$

Here β denotes the dimensionless viscosity ratio in which k is the coefficient of gyro-viscosity (or vortex viscosity). In view of Eqs.(2.5)-(2.11), Eqs. (2.2)-(2.4) become

$$\frac{1}{4}\frac{\partial u}{\partial t} - (1+\varepsilon A e^{i\omega t})\frac{\partial u}{\partial z} = (1+\beta)\frac{\partial^2 u}{\partial z^2} + \frac{1}{4}\frac{\partial U}{\partial t} - (M^2+\chi^2)(u-U) + Gr\theta + 2\beta\frac{\partial N}{\partial z}, \tag{2.12}$$

$$\frac{1}{4}\frac{\partial N}{\partial t} - (1+\varepsilon A e^{i\omega t})\frac{\partial N}{\partial z} = \frac{1}{\eta}\frac{\partial^2 N}{\partial z^2}, \tag{2.13}$$

$$\frac{Pr}{4}\frac{\partial \theta}{\partial t} - Pr(1+\varepsilon A e^{i\omega t})\frac{\partial \theta}{\partial z} = (\frac{\partial^2}{\partial z^2} - R^2)\theta + PrEc(\frac{\partial u}{\partial z})^2. \tag{2.14}$$

where

$$\eta = \frac{\mu j^*}{\gamma} = \frac{2}{2+\beta}.$$

The boundary conditions are

$$\begin{aligned} &u = U_p,\ \theta = 1,\ N = -n\frac{\partial u}{\partial z}, && on \quad z = 0, \\ &u \to (1+\varepsilon e^{i\omega t}),\ \theta \longrightarrow 0,\ N \longrightarrow 0, && as \quad z \longrightarrow \infty. \end{aligned} \tag{2.15}$$

The mathematical statement of the problem is now complete and embodies the solution of Eqs.(2.12)-(2.14) subject to boundary conditions (2.15)

3 Method of Solution

In order to reduce the above system of partial differential equations to a system of ordinary differential equations in dimensionless form, we may represent the linear velocity, microrotation and temperature as

$$\begin{aligned} u(z,t) &= u_o(z) + \varepsilon e^{i\omega t} u_1(z) + O(\varepsilon^2), \\ N(z,t) &= N_o(z) + \varepsilon e^{i\omega t} N_1(z) + O(\varepsilon^2), \\ \theta(z,t) &= \theta_o(z) + \varepsilon e^{i\omega t} \theta_1(z) + O(\varepsilon^2). \end{aligned} \tag{3.1}$$

Substituting Eqs.(3.1) into Eqs.(2.12-2.15), equating the harmonic and non-harmonic terms, and neglecting the higher order of $O(\varepsilon^2)$,and simplifying we obtains the following pairs of equations u_o, N_o, θ_o and u_1, N_1, θ_1

$$(1+\beta)u_o'' + u_o' - (M^2+\chi^2)u_o = -(M^2+\chi^2) - Gr\theta_o - 2\beta N_o', \tag{3.2}$$

$$N_o'' + \eta N_o' = 0, \tag{3.3}$$

$$\theta_o'' + Pr\theta_o' - R^2\theta_o = -PrEcu_o'^2, \tag{3.4}$$

subject to the boundary conditions

$$\begin{aligned} &u_o = U_p,\ \theta_0 = 1,\ N_o = -nu_o', \quad on \quad z = 0, \\ &u_o = 1,\ \theta_0 = 0,\ N_o = 0, \qquad as \quad z \longrightarrow \infty, \end{aligned} \tag{3.5}$$

for $O(1)$ equations, and

$$(1+\beta)u_1'' + u_1' - (M^2+\chi^2+\frac{i\omega}{4})u_1 = -(M^2+\chi^2+\frac{i\omega}{4}) - Gr\theta_1 - 2\beta N_1 - Au_o', \tag{3.6}$$

$$N_1'' + \eta N_1' - (\frac{i\omega}{4})\eta N_1 = -AN_o', \tag{3.7}$$

$$\theta_1'' + Pr\theta_1' - (R^2 + \frac{i\omega}{4}Pr)\theta_1 = -PrA\theta_0' - 2PrEcu_o'u_1', \tag{3.8}$$

where a prime denotes ordinary differentiation with respect to z. The corresponding boundary conditions can be written as

$$\begin{aligned} &u_1 = 0,\ \theta_1 = 0,\ N_1 = -nu_1', \quad on \quad z = 0, \\ &u_1 = 1,\ \theta_1 = 0,\ N_1 = 0, \qquad as \quad z \longrightarrow \infty, \end{aligned} \tag{3.9}$$

for $O(\varepsilon)$ equations.

To solve the nonlinear-coupled Eqs.(3.2-3.8) and (3.9), we further assume that the viscous dissipation parameter (Eckert number Ec) is small, and therefore, advance an asymptotic expansion for the liner velocity, microrotation and temperature as follows:

$$\begin{aligned} u_o(z) &= u_{o1}(z) + Ecu_{o2}(z) + O(Ec^2), \\ N_o(z) &= N_{o1}(z) + EcN_{02}(z) + O(Ec^2), \\ \theta_o(z) &= \theta_{o1}(z) + Ec\theta_{02}(z) + O(Ec^2), \\ u_1(z) &= u_{11}(z) + Ecu_{12}(z) + O(Ec^2), \\ N_1(z) &= N_{11}(z) + EcN_{12}(z) + O(Ec^2), \\ \theta_1(z) &= \theta_{11}(z) + Ec\theta_{12}(z) + O(Ec^2). \end{aligned} \tag{3.10}$$

Substituting Eqs.(3.10) into Eqs.(3.2-3.9), we obtain the following sequence of approximations:

$$(1+\beta)u_{o1}'' + u_{o1}' - (M^2+\chi^2)u_{o1} = -(M^2+\chi^2) - Gr\theta_{o1} - 2\beta N_{o1}', \tag{3.11}$$

$$N_{o1}'' + \eta N_{o1}' = 0, \tag{3.12}$$

$$\theta''_{o1} + Pr\theta'_{o1} - R^2\theta_{o1} = 0, \tag{3.13}$$

$$(1+\beta)u''_{o2} + u'_{o2} - (M^2+\chi^2)u_{o2} = -Gr\theta_{o2} - 2\beta N'_{o2}, \tag{3.14}$$

$$N''_{o2} + \eta N'_{o2} = 0, \tag{3.15}$$

$$\theta''_{o2} + Pr\theta'_{o2} - R^2\theta_{o2} = -Pru'^2_{o1}, \tag{3.16}$$

subject to

$$\begin{aligned} u_{o1} &= U_p,\ \theta_{o1} = 1,\ N_{o1} = -nu'_{o1},\ u_{02} = \theta_{o2} = 0,\ N_{o2} = -nu'_{o2},\ \text{on} \quad z = 0 \\ u_{o1} &= u_{o2} = \theta_{o1} = \theta_{o2} = N_{o1} = N_{o2} = 0, \qquad \text{as} \quad z \longrightarrow \infty, \end{aligned} \tag{3.17}$$

for O(1) equations, and

$$\begin{aligned} (1+\beta)u''_{11} + u'_{11} - (M^2+\chi^2+\frac{i\omega}{4})u_{11} &= -(M^2+\chi^2+\frac{i\omega}{4}) - Gr\theta_{11} \\ &\quad -2\beta N'_{11} - Au'_{o1}, \end{aligned} \tag{3.18}$$

$$N''_{11} + \eta N'_{11} - \frac{i\omega}{4}\eta N_{11} = -AN'_{o1}, \tag{3.19}$$

$$\theta''_{11} + Pr\theta'_{11} - (R^2 + \frac{i\omega}{4}Pr)\theta_{11} = -PrA\theta'_{01}, \tag{3.20}$$

$$(1+\beta)u''_{12} + u'_{12} - (M^2+\chi^2+\frac{i\omega}{4})u_{12} = -Gr\theta_{12} - 2\beta N'_{12} - Au'_{o2}, \tag{3.21}$$

$$N''_{12} + \eta N'_{12} - \frac{i\omega}{4}\eta N_{12} = -AN'_{o2}, \tag{3.22}$$

$$\theta''_{12} + Pr\theta'_{12} - (R^2 + \frac{i\omega}{4}Pr)\theta_{12} = -PrA\theta'_{o2} - 2Pru'_{o1}u'_{11}, \tag{3.23}$$

subject to

$$\begin{aligned} u_{11} &= \theta_{11} = 0,\ N_{11} = -nu'_{11},\ u_{12} = \theta_{12} = 0,\ N_{12} = -nu'_{12},\ \text{on} \quad z = 0 \\ u_{11} &= 1,\ u_{12} = \theta_{11} = \theta_{12} = N_{11} = N_{12} = 0, \qquad \text{as} \quad z \longrightarrow \infty, \end{aligned} \tag{3.24}$$

for $O(Ec)$ equations.

Solving Eqs.(3.11-3.16) under the boundary conditions (3.17) and Eqs. (3.18-3.23) under the boundary conditions (3.24) and substituting into Eqs. (3.10) and (3.1) we obtain the temperature, angular velocity and velocity profiles of the flow respectively as

$$
\begin{aligned}
&\theta(z,t)\\
&= e^{-m_1 z} + Ec[\alpha_6 e^{-2m_2 z} + \alpha_7 e^{-2m_1 z} + \alpha_8 e^{-(m_1+m_2)z} + \alpha_9 e^{-2\eta z} + \alpha_{10} e^{-(m_1+\eta)}\\
&\quad +\alpha_{11} e^{-(m_2+\eta)z} + \alpha_{12} e^{-m_3 z}] + \varepsilon e^{i\omega t}\{\alpha_{23}[e^{-m_1 z} - e^{-m_5 z}] + Ec[\alpha_{48} e^{-(m_1+m_2)z}\\
&\quad +\alpha_{49} e^{-(m_1+m_5)z} + \alpha_{50} e^{-(m_1+m_6)z} + \alpha_{51} e^{-(m_2+m_5)z} + \alpha_{52} e^{-(m_2+m_5)z}\\
&\quad +\alpha_{53} e^{-2m_1 z} + \alpha_{54} e^{-2m_2 z} + \alpha_{55} e^{-m_3 z} + \alpha_{56} e^{-(m_1+\eta)z} + \alpha_{57} e^{-(m_2+\eta)z}\\
&\quad +\alpha_{58} e^{-2\eta z} + \alpha_{59} e^{-(m_1+m_9)z} + \alpha_{60} e^{-(m_2+m_9)z} + \alpha_{61} e^{-(m_5+\eta)z}\\
&\quad +\alpha_{62} e^{-(m_6+\eta)z} + \alpha_{63} e^{-(m_9+\eta)z} + \alpha_{64} e^{-m_7 z}]\},
\end{aligned}
\tag{3.25}
$$

$$
\begin{aligned}
&u(z,t)\\
&= 1+\alpha_1 e^{-m_1 z}+\alpha_4 e^{-\eta z} + \alpha_5 e^{-m_2 z}+Ec[\alpha_{13} e^{-2m_2 z}+\alpha_{14} e^{-2m_1 z}+\alpha_{15} e^{-(m_1+m_2)z}\\
&\quad +\alpha_{16} e^{-m_3 z}+\alpha_{17} e^{-2\eta z}+\alpha_{18} e^{-(m_1+\eta)}+\alpha_{19} e^{-(m_2+\eta)}+\alpha_{20} C_2 e^{-\eta z}+\alpha_{22} e^{-m_4 z})]\\
&\quad +\varepsilon e^{i\omega t}\{1 + \alpha_{24} e^{-m_2 z} + \alpha_{25} e^{-m_1 z} + \alpha_{26} e^{-m_5 z} + \alpha_{27} e^{-\eta z} + \alpha_{28} C_3 e^{-m_9 z}\\
&\quad +\alpha_{31} e^{-m_6 z} + Ec[\alpha_{65} e^{-m_7 z} + \alpha_{66} e^{-(m_1+m_2)z} + \alpha_{67} e^{-2m_1 z} + \alpha_{68} e^{-2m_2 z}\\
&\quad +\alpha_{69} e^{-m_3 z} + \alpha_{70} e^{-m_4 z} + \alpha_{71} e^{-(m_1+m_5)z} + \alpha_{72} e^{-(m_1+m_6)z}\\
&\quad +\alpha_{73} e^{-(m_2+m_5)z} + \alpha_{74} e^{-(m_2+m_6)z} + \alpha_{75} e^{-2\eta z} + \alpha_{76} e^{-(m_1+\eta)}\\
&\quad +\alpha_{77} e^{-(m_2+\eta)} + \alpha_{78} e^{-\eta z} + \alpha_{79} e^{-(m_1+m_9)z} + \alpha_{80} e^{-(m_2+m_9)z}\\
&\quad +\alpha_{81} e^{-(m_5+\eta)z} + \alpha_{82} e^{-(m_6+\eta)z} + \alpha_{83} e^{-(m_9+\eta)z} + \alpha_{84} C_4 e^{-m_9 z} + \alpha_{86} e^{-m_8 z}]\},
\end{aligned}
\tag{3.26}
$$

$$
\begin{aligned}
N(z,t) \;=\; &\alpha_3 e^{-\eta z} + Ec\alpha_{21} e^{-\eta z} + \varepsilon e^{i\omega t}\{\alpha_{29} e^{-m_9 z} + \frac{4i\alpha_3 A}{\omega} e^{-\eta z}\\
&+Ec[\alpha_{85} e^{-m_9 z} + \frac{4i\alpha_{21} A}{\omega} e^{-\eta z}]\}.
\end{aligned}
\tag{3.27}
$$

The physical quantities of interest are the wall shear stress τ_w and the local surface heat transfer rate q_w. These are defined by

$$
\begin{aligned}
\tau_w &= \rho[(\nu+\nu_r)\frac{\partial u^*}{\partial z_*}]_{z^*=0} + \nu_r[\frac{\partial N^*}{\partial z_*}]_{z^*=0} = \rho U_o^2[(1+\beta)u'(0) - n\beta u'(0)]\\
&= \rho U_o^2[1+\beta(1-n)]u'(0).
\end{aligned}
\tag{3.28}
$$

Therefore, the local friction factor C_f is given by

$$
C_f = \frac{2\tau_w}{\rho U_o^2} = 2[(1+\beta)u'(0) - n\beta u'(0)].
$$

The couple stress at the wall is defined from the definition of

$$
M_w = \gamma\frac{\partial N^*}{\partial z^*}|_{z^*=0} = \frac{\gamma U_o^3}{\nu^2}N'(0),
\tag{3.29}
$$

the local surface heat flux defined by

$$q_w = -k\frac{\partial T^*}{\partial z^*}, \tag{3.30}$$

(where k is the effective thermal conductivity), together with the definition of the local Nusselt number

$$Nu = \frac{q_w}{T_w - T_\infty}\frac{x}{k}, \tag{3.31}$$

one can write

$$\frac{Nu}{Re} = -\theta'(0), \tag{3.32}$$

where the constants are given in Appendix, $Re = U_o x/\nu$.

4 Results and Discussion

The formulation of the problem that accounts for the effect of radiation field on the flow and heat transfer of an incompressible micropolar fluid along a semi-infinite, moving vertical porous plate was accomplished out in the preceding sections. This enables us to carry out the numerical computations for the velocity, microrotation and temperature fields for various values of the flow conditions and fluid properties. Figures 4.1-4.7 show representative plots of the streamwise velocity and angular velocity as well as temperature profiles for a micropolar fluid with the fixed conditions $\omega = 2, A = 0.01, t = 2, \varepsilon = 0.01$, and $E = 0.01$ while $\beta, Gr, \chi, M, n, Pr, Up$ and R are varied over a range, which are listed in the figures legend.

Figure $4.1a$ shows the streamwise velocity distribution within the boundary layer. As the material parameter β increases, we observe that the magnitude of the stream wise velocity decreases and the inflection point for the velocity distribution moves further away from the surface. The numerical results show that the velocity distribution is lower for a Newtonian fluid $\beta = 0$ with the fixed flow and material parameters. Figure $4.1b$ presents buoyancy-assisted results for angular velocity profile as a function of spanwise coordinate z within the boundary layer. It can be shown that as the viscosity ratio increases the amplitude of the angular velocity profiles decreases.

The velocity and angular velocity profiles against spanwise coordinate z for different values of Grashof number Gr are described in Figures. $4.2a$ and $4.2b$. It is observed that an increase in Gr leads to arise in the values of velocity, but decreases due to angular velocity. Here the positive value of Gr corresponds to a cooling of the surface by natural convection. In addition, the curves show that the peak value of velocity increases rapidly near the wall of the porous plate as Grashof number increases, and then decays to the free stream velocity.

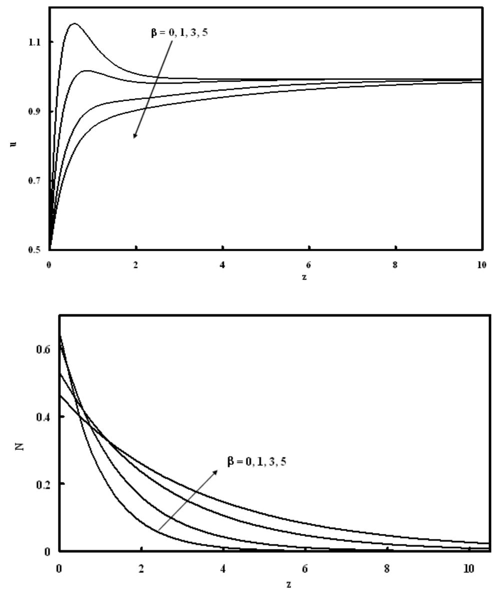

Figure 4.1: Effects of (the micropolar parameter) on the velocity and angular velocity distribution for $R = 2, n = 0.5, M = 2, Up = 0.5, Gr = 10, \chi = 2$ and $Pr = 0.71$.

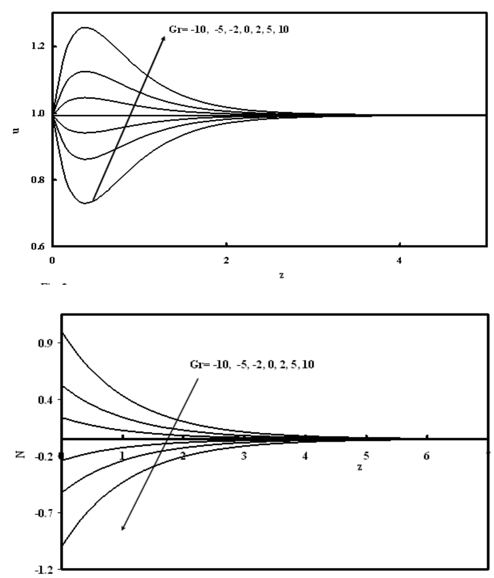

Figure 4.2: Effects of (Grashof number) on the velocity, and angular velocity distribution for $R = 2, n = 0.5, M = 2, Up = 1, \beta = 0.2, \chi = 2$ and $Pr = 0.71$.

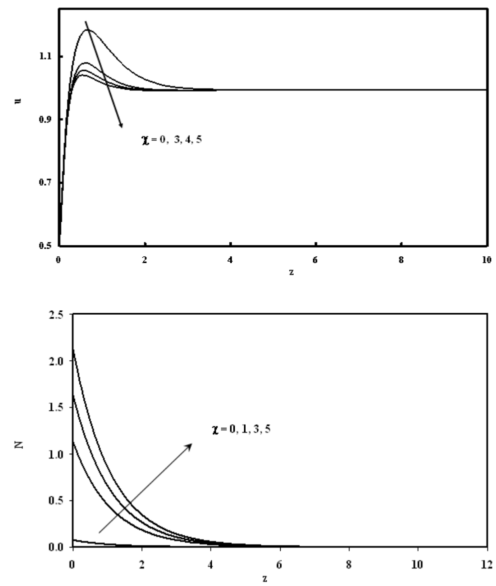

Figure 4.3: Effects of (χ parameter) on velocity and angular velocity distribution for$R = 2, n = 0.5, M = 2, Up = 0.5, \beta = 0.2, Gr = 10$ and $Pr = 0.71$.

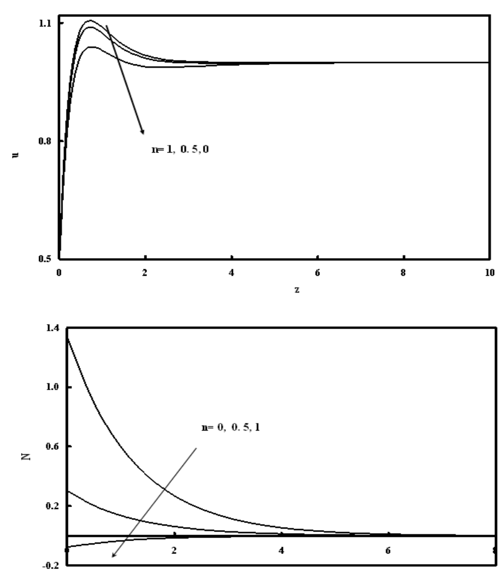

Figure 4.4: Effects of (n parameter) on velocity and angular velocity distribution for$R = 2, M = 2, \chi = 2, Up = 0.5, \beta = 0.5, Gr = 10$ and $Pr = 0.71$.

Figure $4.3a$ shows the velocity profiles for different values of the Darcy number χ. Clearly as χ increases the peak value of velocity tends to decrease. Figure $4.3b$ illustrates variation of the angular velocity across the boundary layer for various values of the Darcy number χ in the direction of fluid flow. The values of angular velocity on the porous plate are increased as the permeability increases.

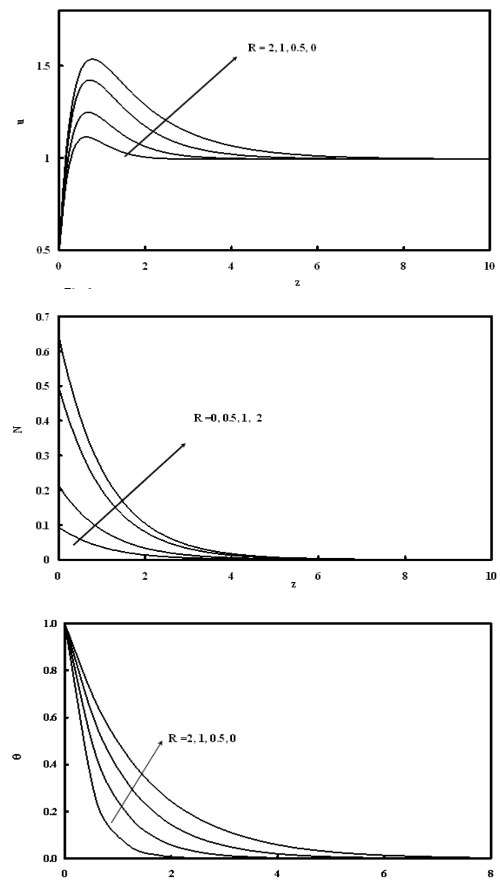

Figure 4.5: Effects of (R rotational parameter) on velocity and angular velocity distribution for$Pr = 0.71, M = 2, \chi = 2, Up = 0.5, \beta = 0.2, Gr = 10$ and $n = 0.5$.

For the case of a micropolar fluid $\beta = 0.2$, the profiles of streamwise velocity and micro-rotation against the spanwise coordinate z for the variations of the parameter n in the boundary condition for micro-gyration vector are shown in Figures $4.4a, 4.4b$. The results show that increasing values of n-parameter results in an increasing velocity within the boundary layer, which eventually approaches to the relevant free stream velocity at the edge of boundary layer. However, the micro-rotation profiles decreases as the n-parameter increases.

For different values of the radiation parameter R, the velocity and temperature profiles are plotted in Figures $4.5a, 4.5b$ and $4.5c$. It is obvious that an increase in the radiation parameter R results in decreasing velocity and temperature within the boundary layer and increasing the angular velocity, as well as a decreased thickness of the velocity, and temperature boundary layers. This is because the large R-values correspond to an increased dominance of conduction over radiation thereby decreasing buoyancy force (thus, vertical velocity) and thickness of the thermal and momentum boundary layers.

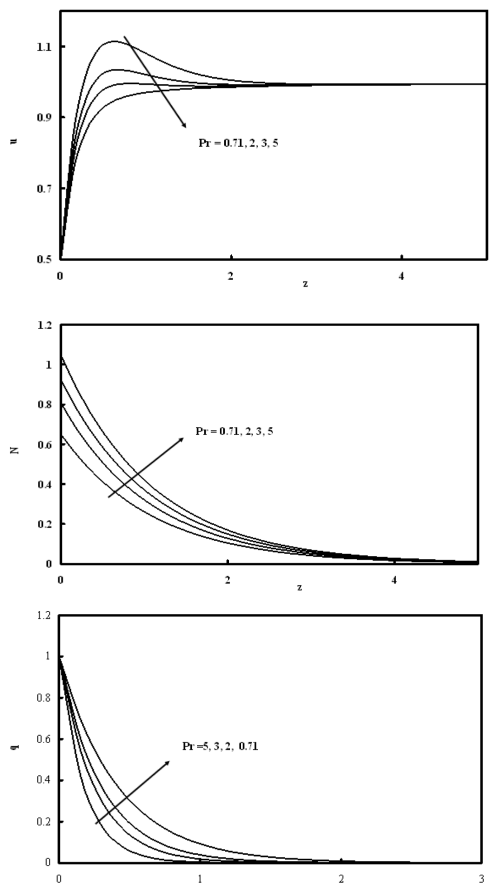

Figure 4.6: Effects of (Pr prandtl number) on velocity and angular velocity distribution for$R = 2, M = 2, \chi = 2, Up = 0.5, \beta = 0.2, Gr = 10$ and $n = 0.5$.

Figure $4.6a$ shows the velocity profiles against spanwise coordinate z for different values of Prandtl number Pr. The numerical results show that the effect of increasing values of Prandtl number results in a decreasing velocity. The results also reveal that the peak value of velocity decreases as Pr increases. For the case of a micropolar fluid $\beta = 0.2$, the profiles of micro-rotation against the spanwise coordinate z for the variations of the parameter Pr in the boundary condition for micro-gyration vector are shown in Figure $4.6b$ The results show that increasing values of Pr parameter results in a decreasing the micro-rotation profiles.

Typical variations in the temperature profiles along the spanwise coordinate are shown in Figure $4.6c$ for different values of the Prandtl number Pr. As expected, the numerical results show that an increase in the Prandtl number results in a decrease of the thermal boundary layer thickness and in general lower average temperature within the boundary layer. The reason is that smaller values of Pr are equivalent to increasing the thermal conductivity of the fluid, and therefore heat is able to diffuse away from the heated surface more rapidly than for higher values of Pr. Hence in the case of smaller Prandtl numbers the thermal boundary layer is thicker and the rate of heat transfer is reduced.

For different values of the magnetic field parameter M, the velocity and angular velocity profiles are plotted in Figure $4.7a$, and $4.7b$. It is obvious that the effect of increasing values of magnetic field parameter results in a decreasing velocity distribution across the boundary layer. Furthermore, the results show that the values of angular velocity on the porous plate are increased as M increases.

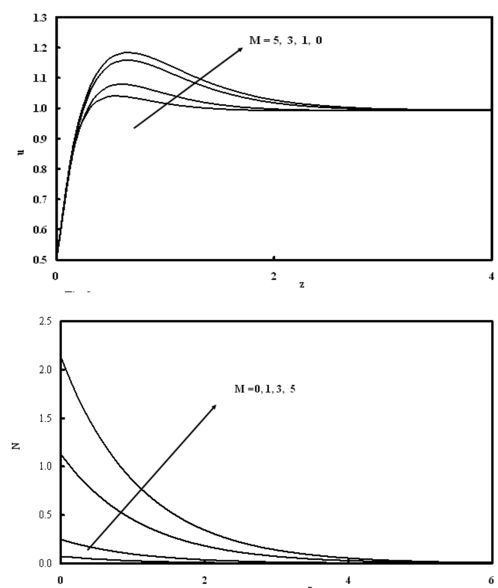

Figure 4.7: Effects of (magnetic parameter) on velocity and angular velocity distribution for$R = 2, n = 0.5, \chi = 2, Up = 0.5, \beta = 0.2, Gr = 10$ and $Pr = 0.71$.

Concluding Remarks

In this paper, the flow of an unsteady MHD mixed-convection flow of micropolar fluid past an infinite vertical plate with time-dependent suction under the simultaneous effects of viscous dissipation and radiation is affected by the material parameters. The governing boundary layer equations for the velocity, microrotation component and temperature have been solved analytically. The resulting partial differential equations were transformed into a set of ordinary differential equations using two-term series and solved in closed-form. Numerical evaluations of the closed-form results were performed and some graphical results were obtained to illustrate the details of the flow and heat and mass transfer characteristics and their dependence on some of the physical parameters. In addition, an increase temperature profile is a function of an increase in viscous dissipation. Whereas an increase in radiation and magnetic field parameters led to a decrease in the temperature profile on cooling. Equally, cooling of the plate by convection currents with increases in the radiation, magnetic field and Darcy parameters led to a decrease in the velocity profile. Finally, increased cooling of the plate and viscous dissipation resulted in an increase in the velocity profile.

Appendix

$$m_1 = m_3 = \tfrac{1}{2}(Pr + \sqrt{Pr^2 + 4R^2})$$
$$m_2 = m_4 = \tfrac{1}{2(1+\beta)}(1 + \sqrt{1 + 4(M^2 + \chi^2)})$$
$$m_5 = m_7 = \tfrac{1}{2}(Pr + \sqrt{Pr^2 + 4N_2})$$
$$m_6 = m_8 = \tfrac{1}{2(1+\beta)}(1 + \sqrt{1 + 4N_1})$$
$$m_9 = m_{10} = \tfrac{1}{2}(\eta + \sqrt{\eta^2 + i\omega\eta})$$
$$\alpha_1 = \frac{-Gr}{(1+\beta)m_1^2 - m_1 - M^2 - \chi^2}$$
$$\alpha_2 = \frac{2\beta\eta}{(1+\beta)\eta^2 - \eta - M^2 - \chi^2}$$
$$\alpha_3 = \frac{n((m_1 - m_2)\alpha_1 + (1 - U_p)m_2)}{1 - n\alpha_2(\eta - m_2)}$$
$$\alpha_4 = \alpha_2\alpha_3\alpha_5 = U_p - (1 + \alpha_1 + \alpha_4)$$
$$\alpha_6 = \frac{-Pr\alpha_5^2 m_2^2}{4m_2^2 - 2Prm_2 - R^2}$$
$$\alpha_7 = \frac{-Pr\alpha_1^2 m_1^2}{4m_1^2 - 2Prm_1 - R^2}$$
$$\alpha_8 = \frac{-2Pr\alpha_5\alpha_1 m_1 m_2}{(m_1 + m_2)^2 - Pr(m_1 + m_2) - R^2}$$
$$\alpha_9 = \frac{-Pr\alpha_4^2\eta^2}{4\eta^2 - 2Pr\eta - R^2}$$
$$\alpha_{10} = \frac{-2Pr\alpha_1\alpha_4 m_1\eta}{(m_1 + \eta)^2 - Pr(m_1 + \eta) - R^2}$$

$$\alpha_{11} = \frac{-2Pr\alpha_5\alpha_4 m_2\eta}{(m_2+\eta)^2 - Pr(m_2+\eta) - R^2}$$
$$\alpha_{12} = -(\alpha_6+\alpha_7+\alpha_8+\alpha_9+\alpha_{10}+\alpha_{11})$$
$$\alpha_{13} = \frac{-Gr\alpha_6}{4(1+\beta)m_2^2 - 2m_2 - M^2 - \chi^2}$$
$$\alpha_{14} = \frac{-Gr\alpha_7}{4(1+\beta)m_1^2 - 2m_1 - M^2 - \chi^2}$$
$$\alpha_{15} = \frac{-Gr\alpha_8}{(1+\beta)(m_1+m_2)^2 - (m_1+m_2) - M^2 - \chi^2}$$
$$\alpha_{16} = \frac{-Gr\alpha_{12}}{(1+\beta)m_3^2 - m_3 - M^2 - \chi^2}$$
$$\alpha_{17} = \frac{-Gr\alpha_9}{4(1+\beta)\eta^2 - 2\eta - M^2 - \chi^2}$$
$$\alpha_{18} = \frac{-Gr\alpha_{10}}{(1+\beta)(m_1+\eta)^2 - (m_1+\eta) - M^2 - \chi^2}$$
$$\alpha_{19} = \frac{-Gr\alpha_{11}}{(1+\beta)(m_2+\eta)^2 - (m_2+\eta) - M^2 - \chi^2}$$
$$\alpha_{20} = \frac{2\beta\eta}{(1+\beta)\eta^2 - \eta - M^2 - \chi^2}$$
$$\alpha_{21} = \{n[\alpha_{13}(2m_2-m_4)+\alpha_{14}(2m_1-m_4)+\alpha_{15}(m_1+m_2-m_4)+\alpha_{16}(m_3-m_4) \\ +\alpha_{17}(2\eta-m_4)+\alpha_{18}(m_1+\eta-m_4)+\alpha_{19}(m_2+\eta-m_4)]\}/[1-n\alpha_{20}(\eta-m_4)]$$
$$\alpha_{22} = (\alpha_{13}+\alpha_{14}+\alpha_{15}+\alpha_{16}+\alpha_{17}+\alpha_{18}+\alpha_{19}+\alpha_{20}\alpha_{21}$$
$$N_1 = M^2 + K^2 + \frac{iw}{4} \qquad N_2 = R^2 + \frac{iw}{4}Pr$$
$$\alpha_{23} = \frac{Pr}{m_1^2 - Prm_1 - N_1} \qquad \alpha_{24} = \frac{A\alpha_5 m_2}{(1+\beta)m_2^2 - m_2 - N_1}$$
$$\alpha_{25} = \frac{A\alpha_1 m_1}{(1+\beta)m_1^2 - m_1 - N_1} \qquad \alpha_{26} = \frac{-Gr}{(1+\beta)m_5^2 - m_5 - N_1}$$
$$\alpha_{27} = \frac{A\alpha_4\eta + (4i\alpha_3 A/\omega)}{(1+\beta)\eta^2 - \eta - N_1}$$
$$\alpha_{29} = \frac{n[\alpha_{24}(m_2-m_6)+\alpha_{25}(m_1-m_6)+\alpha_{26}(m_5-m_6)+\alpha_{27}(\eta-m_6)]}{1-n\alpha_{28}(m_9-m_6)} - \frac{4i\alpha_3 A}{\omega}$$
$$\alpha_{28} = \frac{2\beta m_9}{(1+\beta)m_9^2 - m_9 - N_1} \qquad \alpha_{30} = \alpha_{29}\alpha_{28}$$
$$\alpha_{31} = -(\alpha_{24}+\alpha_{25}+\alpha_{26}+\alpha_{27}+\alpha_{28}\alpha_{29})$$
$$\alpha_{32} = APr(m_1+m_2)\alpha_8 - 2Pr\alpha_1 m_1 m_2\alpha_{24} - 2Pr\alpha_5 m_1 m_2 \alpha_{25}$$
$$\alpha_{33} = -2Pr\alpha_1 m_1 m_5 \alpha_{26}$$
$$\alpha_{34} = -2Pr\alpha_1\alpha_{31} m_1 m_6$$
$$\alpha_{35} = -2Pr\alpha_5\alpha_{26} m_2 m_5$$
$$\alpha_{36} = -2Pr\alpha_5\alpha_{31} m_2 m_6$$
$$\alpha_{43} = -2Pr\alpha_1\alpha_{30} m_1 m_9$$
$$\alpha_{37} = -2PrAm_1\alpha_7 - 2Pr\alpha_1 m_1^2\alpha_{25}$$
$$\alpha_{38} = -2PrAm_2\alpha_6 - 2Pr\alpha_5 m_2^2\alpha_{24}$$
$$\alpha_{39} = PrA\alpha_{12} m_3$$

$$\alpha_{40} = APr(m_1+\eta)\alpha_{10} - 2Pr\alpha_4\alpha_{25}\eta m_1 - 2Pr\alpha_1\alpha_{27}\eta m_1$$
$$\alpha_{41} = APr(m_2+\eta)\alpha_{11} - 2Pr\alpha_5\alpha_{27}\eta m_2 - 2Pr\alpha_4\eta m_2\alpha_{24}$$
$$\alpha_{42} = -2PrA\alpha_9\eta - 2Pr\alpha_4\alpha_{27}\eta^2$$
$$\alpha_{44} = -2Pr\alpha_5\alpha_{30}m_2m_9$$
$$\alpha_{45} = -2Pr\alpha_4 m_5\eta\alpha_{26}$$
$$\alpha_{46} = -2Pr\alpha_{31}\alpha_4\eta m_6 \qquad \alpha_{47} = -2Pr\alpha_{30}\alpha_4\eta m_9$$
$$\alpha_{48} = \frac{\alpha_{32}}{(m_1+m_2)^2 - Pr(m_1+m_2) - N_2}$$
$$\alpha_{49} = \frac{\alpha_{33}}{(m_1+m_5)^2 - Pr(m_1+m_5) - N_2}$$
$$\alpha_{50} = \frac{\alpha_{34}}{(m_1+m_6)^2 - Pr(m_1+m_6) - N_2}$$
$$\alpha_{51} = \frac{d_{35}}{(m_5+m_2)^2 - Pr(m_5+m_2) - N_2}$$
$$\alpha_{52} = \frac{d_{36}}{(m_6+m_2)^2 - Pr(m_6+m_2) - N_2}$$
$$\alpha_{53} = \frac{d_{37}}{m_1^2 - 2Prm_1 - N_2} \qquad \alpha_{54} = \frac{d_{38}}{4m_2^2 - 2Prm_2 - N_2}$$
$$\alpha_{55} = \frac{d_{39}}{m_3^2 - Prm_3 - N_2}$$
$$\alpha_{56} = \frac{\alpha_{40}}{(m_1+\eta)^2 - Pr(m_1+\eta) - N_2}$$
$$\alpha_{57} = \frac{\alpha_{41}}{(m_2+\eta)^2 - Pr(m_2+\eta) - N_2}$$
$$\alpha_{58} = \frac{h_{42}}{4\eta^2 - 2Pr\eta - N_2}$$
$$\alpha_{59} = \frac{h_{43}}{(m_1+m_9 - Pr(m_1+m_9) - N_2}$$
$$\alpha_{60} = \frac{\alpha_{44}}{(m_2+m_9)^2 - Pr(m_2+m_9) - N_2}$$
$$\alpha_{61} = \frac{\alpha_{45}}{(m_5+\eta)^2 - Pr(m_5+\eta) - N_2}$$
$$\alpha_{62} = \frac{\alpha_{46}}{(m_6+\eta)^2 - Pr(m_6+\eta) - N_2}$$
$$\alpha_{63} = \frac{\alpha_{47}}{(m_9+\eta)^2 - Pr(m_9+\eta) - N_2}$$
$$\alpha_{64} = -(\alpha_{48}+\alpha_{49}+\cdots+\alpha_{63})$$
$$\alpha_{65} = \frac{-Gr\alpha_{64}}{(1+\beta)m_7^2 - m_7 - N_1}$$
$$\alpha_{66} = \frac{A\alpha_{15}(m_1+m_2) - Gr\alpha_{48}}{(1+\beta)(m_1+m_2)^2 - (m_1+m_2 - N_1}$$
$$\alpha_{67} = \frac{2Am_1\alpha_{14} - Gr\alpha_{53}}{4(1+\beta)m_1^2 - 2m_1 - N_1}$$
$$\alpha_{68} = \frac{2Am_2\alpha_{13} - Gr\alpha_{54}}{4(1+\beta)m_2^2 - 2m_2 - N_1}$$
$$\alpha_{69} = \frac{Am_3\alpha_{16} - Gr\alpha_{55}}{(1+\beta)m_3^2 - m_3 - N_1}$$
$$\alpha_{70} = \frac{A\alpha_{22}m_4}{(1+\beta)m_4^2 - m_4 - N_1}$$

$$\alpha_{71} = \frac{-Gr\alpha_{48}}{(1+\beta)(m_5+m_1)^2 - (m_5+m_1) - N_1}$$
$$\alpha_{72} = \frac{-Gr\alpha_{50}}{(1+\beta)(m_1+m_6)^2 - (m_1+m_6) - N_1}$$
$$\alpha_{73} = \frac{-Gr\alpha_{51}}{(1+\beta)(m_5+m_2)^2 - (m_5+m_2) - N_1}$$
$$\alpha_{74} = \frac{-Gr\alpha_{52}}{(1+\beta)(m_6+m_2)^2 - (m_6+m_2) - N_1}$$
$$\alpha_{75} = \frac{2A\alpha_{17}\eta - Gr\alpha_{58}}{4(1+\beta)\eta^2 - 2\eta - N_1}$$
$$\alpha_{76} = \frac{A\alpha_{18}(m_1+\eta) - Gr\alpha_{56}}{(1+\beta)(m_1+\eta)^2 - (m_1+\eta) - N_1}$$
$$\alpha_{77} = \frac{A\alpha_{19}(m_2+\eta) - Gr\alpha_{57}}{(1+\beta)(m_2+\eta)^2 - (m_2+\eta) - N_1}$$
$$\alpha_{78} = \frac{A\alpha_{20}\alpha_{21}\eta + 8i\beta\alpha_{21}\eta/\omega}{(1+\beta)\eta^2 - \eta - N_1}$$
$$\alpha_{79} = \frac{-Gr\alpha_{59}}{(1+\beta)(m_1+m_9)^2 - (m_1+m_9) - N_1}$$
$$\alpha_{80} = \frac{-Grh_{60}}{(1+\beta)(m_2+m_9)^2 - (m_2+m_9) - N_1}$$
$$\alpha_{81} = \frac{-Gr\alpha_{61}}{(1+\beta)(m_5+\eta)^2 - (m_5+\eta) - N_1}$$
$$\alpha_{82} = \frac{-Gr\alpha_{62}}{(1+\beta)(m_6+\eta)^2 - (m_6+\eta) - N_1}$$
$$\alpha_{83} = \frac{-Grh_{63}}{(1+\beta)(m_9+\eta)^2 - (m_9+\eta) - N_1}$$
$$\alpha_{84} = \frac{2\beta m_9}{(1+\beta)m_9^2 - m_9 - N_1}$$
$$\begin{aligned}\alpha_{85} = \{&n[\alpha_{65}(m_7-m_8) + \alpha_{66}(m_1+m_2-m_8) + \alpha_{67}(2m_1-m_8) + \alpha_{68}(2m_2-m_8) \\ &+\alpha_{69}(m_3-m_8) + \alpha_{70}(m_4-m_8) + \alpha_{71}(m_1+m_5-m_8) \\ &+\alpha_{72}(m_1+m_6-m_8) + \alpha_{73}(m_5+m_2-m_8) + \alpha_{74}(m_6+m_2-m_8) \\ &+\alpha_{75}(2\eta-m_8) + \alpha_{76}(m_1+\eta-m_8) + \alpha_{77}(m_2+\eta-m_8) + \alpha_{78}(\eta-m_8) \\ &+\alpha_{79}(m_1+m_9-m_8) + \alpha_{80}(m_2+m_9-m_8) + \alpha_{81}(m_5+\eta-m_8) \\ &+\alpha_{82}(m_6+\eta-m_8) + \alpha_{83}(m_9+\eta-m_8)] - 4iA\alpha_{21}/\omega\}/[1-n\alpha_{84}(m_9-m_8)].\end{aligned}$$

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