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Stress-Strength Reliability Of Power Function Distribution Based On Records

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Abstract: This paper deals with the estimation of Stress Strength reliability, R=$P(Y < X)$ when X and Y are two independent Power function distributions with different shape parameters but having the same scale parameter and the data on strength are record values. The maximum likelihood estimators and the Bayes estimators under squared error loss function and linex loss function of the reliability under stress- strength model for the power function distribution are obtained. Effectiveness of these estimators are evaluated using Monte Carlo simulation study.

Keywords: Maximum likelihood estimation, Bayesian estimation, Stress- strength model, Power function distribution and Records.

1 Introduction

Now a days we come across new records being created in events such as stock market prices, rainfall, temperature, flood level, sales of goods, sports events etc. [1]introduced and studied the properties of record values and [2],[3] [4], [5] ,[6] provided a detailed account of theory of records and the inference problems associated with records. Let $X_1, X_2,$ be an infinite sequence of iid random variables. An observation Xj is called a record if its value is greater than all previous observations, that is $Xj > Xi$ for every $i < j$. In the context of reliability, the stress-strength model describes the life of a component which has a random strength X which is subjected to random stress Y. The equipment fails at the instant the stress applied to it exceeds the strength and the equipment will function satisfactorily whenever $X > Y$. Thus $R = P(X > Y)$ is a measure of component reliability.

In the present paper we focus on the estimation of $R = P(X > Y)$ for the power function distribution, when the data on strength is record values. Power-law distributions occur in many situations of scientific interest and have significant consequences for our understanding of natural and man-made phenomena. The sizes of solar flares, the populations of cities, and the intensities of earthquakes, for example, are all quantities whose distributions are thought to follow power laws. [**?**] has studied the inference on the stress strength in the two-Parameter Weibull model based on Records. [8]discussed the inferences on the stress strength in power function distribution.

A power law implies that small occurrence are extremely common, whereas large instance are extremely rare. The density of the power function, P(,) is given by

$$f(x,\beta,\alpha) = \frac{\alpha}{\beta^{\alpha}} x^{(\alpha-1)}, 0 < x < \beta, \alpha, \beta > 0. \tag{1}$$

where β and α are the scale and shape parameters. In Section 2 we derive the maximum likelihood estimator and the associated large sample intervals. Bayesian procedures based on records are discussed in Section 3. Finally in Section 4 we assess the performance of the estimates using Monte Carlo simulation.

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2 Maximum Likelihood Estimation of R

Here we consider the case when the strength X follows power function distribution $P(\alpha_1,\beta)$ and the stress Y follows $P(\alpha_2,\beta)$ .The reliability function R can be defined as

$$R = P(Y < X) = \frac{\alpha_1}{\alpha_1+\alpha_2}, 0 < R < 1. \tag{2}$$

Let $\underline{r} = (r_0, r_1, r_2, ...r_n)$ be the sequence of the first 'n+1' record values with $r_0 = x_1$ defined over a random sample from $P(\alpha_1,\beta)$ and $\underline{y} = (y_1, y_2, ...y_m)$ be a complete data from $P(\alpha_2,\beta)$ Then the likelihood function is given [cf.Arnold et al. (1998)] as follows

$$\ell(\underline{r} \mid \alpha_1,\beta) = \frac{\alpha_1^{(n+1)} r_n^{(\alpha_1-1)}}{\beta^{\alpha_1}} \prod_{i=0}^{n} \frac{r_i^{(\alpha_1-1)}}{(\beta^{\alpha_1} - r_i^{\alpha_1})}. \tag{3}$$

and

$$\ell(\underline{y} \mid \alpha_2,\beta) = \frac{\alpha_2^m}{\beta^{m\alpha_2}} \prod_{j=1}^{m} y_j^{(\alpha_2-1)}. \tag{4}$$

The joint likelihood function can be obtained as

$$\begin{aligned} \log L = n\log\alpha_1 + m\log\alpha_2 - (n\alpha_1 + m\alpha_2)\log\beta + (\alpha_1-1)\sum_{i=1}^{n}\log x_i \\ +(\alpha_2-1)\sum_{j=1}^{m}\log y_j \end{aligned} \tag{5}$$

$$\ell(\underline{x},\underline{y} \mid \alpha_1,\alpha_2) = \frac{\alpha_1^n \alpha_2^m}{\beta^{(n\alpha_1+m\alpha_2)}} \prod_{i=1}^{n} x_i^{(\alpha_1-1)} \prod_{j=1}^{m} y_j^{(\alpha_2-1)}. \tag{6}$$

The log likelihood function of the sample is

$$\begin{aligned} \log\ell(\underline{r},\underline{y} \mid \alpha_1,\alpha_2,\beta) = (n+1)\log\alpha_1 + m\log\alpha_2 + (\alpha_1-1)\log r_n - (\alpha_1 + m\alpha_2)\log\beta \\ + (\alpha_1-1)\sum_{i=0}^{n}\log r_i - \sum_{i=0}^{n}\log(\beta^{\alpha_1} - r_i^{\alpha_1}) + (\alpha_2-1)\sum_{j=1}^{m}\log y_j. \end{aligned} \tag{7}$$

The MLEs of α_1,α_2,β say$\hat{\alpha}_1,\hat{\alpha}_2,\hat{\beta}$ can be obtained as the solutions of the following equations.

$$\frac{\partial \log\ell}{\partial\alpha_1} = 0 \Rightarrow \frac{n+1}{\alpha_1} - \log\beta + \log r_n + \sum_{i=0}^{n}\log r_i - \sum_{i=0}^{n}\left(\frac{(\beta^{\alpha_1}\log\beta - r_i^{\alpha_1}\log r_i}{\beta^{\alpha_1} - r_i^{\alpha_1}}\right) = 0 \tag{8}$$

and

$$\frac{\partial \log\ell}{\partial\alpha_2} = 0 \Rightarrow \frac{m}{\alpha_2} - m\log\beta + \sum_{j=1}^{m}\log y_j = 0. \tag{9}$$

The likelihood function can be maximized by taking β as the largest sample observation. So we define the MLE of β as

$$\hat{\beta} = max(r_{(n)}, y_{(m)}). \tag{10}$$

From (9), we obtain the MLE of α_2 as

$$\hat{\alpha}_2 = \frac{m}{\sum_{j=1}^{m} \log(\frac{\hat{\beta}}{y_j})}. \quad (11)$$

$\hat{\alpha}_1$ can be attained as a solution of the equation of the form

$$g(\alpha_1) = \alpha_1, \quad (12)$$

where $g(\alpha_1) = \frac{n+1}{\log\hat{\beta} + \log r_n + \sum_{i=0}^{n} \log r_i - \sum_{i=0}^{n} \left(\frac{(\hat{\beta}^{\alpha_1} \log\hat{\beta} - r_i^{\alpha_1} \log r_i}{\hat{\beta}^{\alpha_1} - r_i^{\alpha_1}} \right)}$.

Since $\hat{\alpha}_1$ is a fixed point solution of the equation (12), it can be obtained by using an iterative procedure.

Hence by substituting $\hat{\alpha}_1$ and $\hat{\alpha}_2$ in (2), we get

$$\hat{R} = \frac{\hat{\alpha}_1}{\hat{\alpha}_1 + \hat{\alpha}_2}. \quad (13)$$

As $n \longrightarrow \infty, m \longrightarrow \infty$ and $\frac{n}{m} \longrightarrow p$, then $\sqrt{n}(\hat{R} - R) \longrightarrow N(0, B)$

where

$$B = \frac{\alpha_2^2(U_{22}U_{33} - U_{23}^2) - 2\alpha_1\alpha_2\sqrt{p}U_{23}U_{31} + \alpha_1^2 p(U_{11}U_{33} - U_{13}^2)}{K(\alpha_1 + \alpha_2)^4}$$

and

$$K = U_{11}U_{22}U_{33} - U_{12}^2 U_{33} - U_{13}U_{31}U_{22}.$$

The Fisher Information matrix of θ=

$$I(\theta) = \begin{bmatrix} I_{11} & I_{12} & I_{13} \\ I_{21} & I_{22} & I_{23} \\ I_{31} & I_{32} & I_{33} \end{bmatrix} (say)$$

where

$$I_{11} = E(-\frac{\partial^2 \log L}{\partial \alpha_1^2}) = E(\frac{n}{\alpha_1^2}) = \frac{n}{\alpha_1^2},$$

$$I_{22} = E(-\frac{\partial^2 \log L}{\partial \alpha_2^2}) = E(\frac{m}{\alpha_2^2}) = \frac{m}{\alpha_2^2},$$

$$I_{33} = E(-\frac{\partial^2 \log L}{\partial \beta^2}) = E(\frac{-(n\alpha_1 + m\alpha_2)}{\beta^2}) = \frac{-(n\alpha_1 + m\alpha_2)}{\beta^2},$$

$$I_{12} = I_{21} = E(-\frac{\partial^2 \log L}{\partial \alpha_1 \partial \alpha_2}) = 0,$$

$$I_{13} = I_{31} = E(-\frac{\partial^2 \log L}{\partial \alpha_1 \partial \beta}) = E(\frac{n}{\beta}) = \frac{n}{\beta},$$

$$I_{23} = I_{32} = E(-\frac{\partial^2 \log L}{\partial \alpha_2 \partial \beta}) = E(\frac{m}{\beta}) = \frac{m}{\beta}.$$

Theorem 1 :

As $n \longrightarrow \infty, m \longrightarrow \infty$ and $\frac{n}{m} \longrightarrow p$, then

$$[\sqrt{n}(\hat{\alpha_1} - \alpha_1), \sqrt{m}(\hat{\alpha_2} - \alpha_2), \sqrt{n}(\hat{\beta_1} - \beta)] \longrightarrow N(0.A^{-1}(\alpha_1, \alpha_2, \beta))$$

where

$A(\alpha_1, \alpha_2, \beta) = \begin{bmatrix} U_{11} & 0 & U_{13} \\ 0 & U_{22} & U_{23} \\ U_{31} & U_{32} & U_{33} \end{bmatrix}$,

$U_{11} = \frac{1}{n} I_{11} = \frac{1}{\alpha_1^2}$,

$U_{22} = \frac{1}{m} I_{22} = \frac{1}{\alpha_2^2}$,

$U_{33} = \frac{1}{n} I_{33} = -\frac{(n\alpha_1 + m\alpha_2)}{n\beta^2}$,

$U_{12} = U_{21} = \frac{I_{12}}{\sqrt{(nm)}} = 0$,

$U_{13} = U_{31} = \frac{I_{13}}{n} = \frac{1}{\beta}$,

$U_{23} = U_{32} = \frac{1}{\sqrt{nm}} I_{23} = \frac{1}{\sqrt{p}\beta}$.

Proof of the above theorem follows from the asymptotic property of MLE.

3 Bayesian Estimation

3.1 Estimation when β is Known

Let $\underline{r} = (r_0, r_1, r_2, ..., r_n)$ be the first (n+1) records from (1). Using the likelihood function as given in (3) and a gamma prior for α_1 as given by,

$$g(\alpha_1) \propto \alpha_1^{(p-1)} exp - (\alpha_1 \tau), p, \tau, \alpha_1 > 0, \tag{14}$$

the posterior density for α_1 is

$$f(\alpha_1 \mid \underline{r}) \propto \frac{\alpha_1^{(n+p)}}{\beta^{\alpha_1}} exp(-\alpha_1 \tau) r_n^{(\alpha_1 - 1)} \prod_{i=0}^{n} \frac{r_i^{(\alpha_1 - 1)}}{(\beta^{\alpha_1} - r_i^{\alpha_1})} \tag{15}$$

and similarly the posterior density of α_2 is

$$f(\alpha_2 \mid \underline{y}) \propto \alpha_2^{(M-1)} exp - (\alpha_2 Q), \alpha_2 > 0, \tag{16}$$

where $M = m + q$ and $Q = \varphi + m \log \beta - \sum_{j=1}^{m} log y_j$.

Assuming α_1 and α_2 to be independently distributed, the joint posterior density of (α_1, α_2) can be written as

$$f(\alpha_1, \alpha_2 \mid \underline{r}, \underline{y}) \propto \frac{\alpha_1^N \alpha_2^{(M-1)}}{\beta^{\alpha_1}} exp(-(\alpha_1 \tau + \alpha_2 Q)) r_n^{(\alpha_1 - 1)} \prod_{i=0}^{n} \frac{r_i^{(\alpha_1 - 1)}}{(\beta^{\alpha_1} - r_i^{\alpha_1})}, \alpha_1, \alpha_2 > 0, \tag{17}$$

where N = (n+p) and M = (m+q).

Applying the transformations $R = \frac{\alpha_1}{\alpha_1+\alpha_2}$ and $U = \alpha_1 + \alpha_2$,

$$f(R,U \mid \underline{r},\underline{y}) \propto \frac{U^{(M+N-1)}}{\beta^{UR}} R^N (1-R)^{(M-1)} r_n^{(UR-1)}$$
$$exp(-U(R\tau + (1-R)Q) + (UR-1)\sum_{i=0}^{n} \log r_i - \sum_{i=0}^{n} \log(\beta^{UR} - r_i^{UR})), \tag{18}$$
$$U > 0, 0 < R < 1.$$

Under the squared error loss function, the Bayes estimate of the parameter R,

$$\hat{R_{SER}} = E(R \mid \underline{r},\underline{y}) = \frac{[C_1(1)]}{[C_1(0)]} \tag{19}$$

and the Bayes risk,

$$V(R \mid \underline{r},\underline{y}) = \frac{[C_1(2)]}{[C_1(0)]} - \hat{R_{SER}}^2, \tag{20}$$

where

$C_1(d) = \int_0^1 \int_0^\infty \frac{U^{(M+N-1)}}{\beta^{UR}} R^{N+d} (1-R)^{(M-1)} r_n^{(UR-1)}$
$exp(-U(R\tau + (1-R)Q) + (UR-1)\sum_{i=0}^{n} \log r_i - \sum_{i=0}^{n} \log(\beta^{UR} - r_i^{UR}))dUdR$

The Bayes estimator under linex loss function linex loss function of R is given by

$$\hat{R}_{LR} = \frac{1}{a} \log G_1 \tag{21}$$

where

$G_1 = [C_1(0)]^{-1} \int_0^1 \int_0^\infty \frac{U^{(M+N-1)}}{\beta^{UR}} R^N (1-R)^{(M-1)} r_n^{(UR-1)}$
$exp(-U(R\tau + (1-R)Q) + (UR-1)\sum_{i=0}^{n} \log r_i - \sum_{i=0}^{n} \log(\beta^{UR} - r_i^{UR}))dUdR.$

A numerical integration procedure is required to obtain the above estimates.

3.2 Bayesian estimation when α and β are unknown

In this section we obtain the Bayes estimate of R when both α and β are unknown. Here we take the joint prior distribution of α_1 and β as

$$g(\alpha_1, \beta) \propto \frac{exp - (\alpha_1 \tau)\alpha_1^{(p-1)}}{\beta}, p, \tau, \alpha_1 > 0. \tag{22}$$

where

$$g_1(\beta \mid \alpha_1) \propto \beta^{-1} \tag{23}$$

the posterior density for (α_1, β) is

$$\begin{aligned} f(\alpha_1, \beta \mid \underline{r}) &\propto \frac{\alpha_1^{(n+p)}}{\beta^{(\alpha_1+1)}} exp(-\alpha_1 \tau) r_n^{(\alpha_1-1)} \prod_{i=0}^{n} \frac{r_i^{(\alpha_1-1)}}{(\beta^{\alpha_1} - r_i^{\alpha_1})} \\ &\propto \frac{\alpha_1^{(N)}}{\beta^{(\alpha_1+1)}} exp(-\alpha_1 \tau) r_n^{(\alpha_1-1)} \prod_{i=0}^{n} \frac{r_i^{(\alpha_1-1)}}{(\beta^{\alpha_1} - r_i^{\alpha_1})}, \alpha_1 > 0, \beta > \sigma = r_{(n)}, \end{aligned} \tag{24}$$

where N=n+p

Using the posterior density function of (α_2, β) as defined and the likelihood,the joint posterior density function of $(\alpha_1, \alpha_2, \beta)$ is defined as

$$\begin{aligned} f(\alpha_1, \alpha_2, \beta \mid \underline{r}, \underline{y}) \propto \frac{\alpha_1^N \alpha_2^{(M-1)}}{\beta^{(\alpha_1} + m\alpha_2 + 2)} exp(-(\alpha_1 \tau + \alpha_2 Q)) r_n^{(\alpha_1-1)} \prod_{i=0}^{n} \frac{r_i^{(\alpha_1-1)}}{(\beta^{\alpha_1} - r_i^{\alpha_1})}, \\ \alpha_1, \alpha_2 > 0, \beta > \sigma = max(r_{(n)}, y_{(m)}). \end{aligned} \tag{25}$$

Applying the transformations $R = \frac{\alpha_1}{\alpha_1+\alpha_2}$ and $U = \alpha_1 + \alpha_2$,

$$\begin{aligned} f(R, U, \beta \mid \underline{r}, \underline{y}) \propto & \frac{U^{(M+N-1)}}{\beta^{(U+2)}} R^N (1-R)^{(M-1)} r_n^{(UR-1)} \\ & exp(-U(R\tau + (1-R)Q) + (UR-1) \sum_{i=0}^{n} \log r_i - \sum_{i=0}^{n} \log(\beta^{UR} - r_i^{UR})), \\ & U > 0, 0 < R < 1, \beta > \sigma = max(r_{(n)}, y_{(m)}). \end{aligned} \tag{26}$$

Integrating out U and β we get the density function of R.

The Bayes estimate of R under squared error loss function is given by,

$$R_{1\hat{SER}} = E(R \mid \underline{r}, \underline{y}) = \frac{[C_2(1)]}{[C_2(0)]} \tag{27}$$

and the variance,

$$V(R \mid \underline{r}.\underline{y}) = \frac{[C_2(2)]}{[C_2(0)]} - R_{1\hat{SER}}^{\;2}, \tag{28}$$

where $C_2(d) = \int_0^1 \int_\sigma^\infty \int_0^\infty \frac{U^{(M+N-1)}}{\beta^{(U+2)}} R^{N+d} (1-R)^{(M-1)} r_n^{(UR-1)}$

$exp(-U(R\tau + (1-R)Q) + (UR-1) \sum_{i=0}^n \log r_i - \sum_{i=0}^n \log(\beta^{UR} - r_i^{UR})) dU d\beta dR.$

The Bayes estimate of R under linex loss is

$$\hat{R}_{LR1} = \frac{1}{a} \log G_2, \tag{29}$$

where

$G_2 = [C_2(0)]^{-1} = \int_0^1 \int_\sigma^\infty \int_0^\infty \frac{U^{(M+N-1)}}{\beta^{(U+2)}} R^N (1-R)^{(M-1)} r_n^{(UR-1)}$

$$exp(-U(R\tau+(1-R)Q)+(UR-1)\sum_{i=0}^{n}\log r_i-\sum_{i=0}^{n}\log(\beta^{UR}-r_i^{UR}))dUd\beta dR.$$

$$\frac{dk(t)}{dt}=\frac{\varepsilon}{v}[k(t)]^a-\frac{\mu}{v}k(t). \tag{30}$$

3.3 Simulation study and discussion

In the absence of real data, we study the performance of the estimators obtained so far using simulated data. With different values of the parameters of the model, we compare the bias and the mean square errors of R. An adaptive multidimensional integration over hyper cubes is carried out using Cubature-package of R for the evaluation of estimates. A Monte Carlo simulation has been carried out for establishing the performance of the estimates. For this purpose random samples of different sizes have been generated and the above measures are calculated empirically using 1,000 Monte Carlo runs for different choices of the parameters. The bias and MSEs of the different estimators are given in table. We also computed the 95% C.I.s for R based on the asymptotic distributions of the MLE. We also observe the HPD intervals to provide the smallest average confidence credible lengths for different censoring schemes, and for different parameter values. We also compute the $\left[\frac{Min}{Max}\right]$ indices of the estimates for different values of the hyper parameters by keeping the others fixed for different values of the population parameters, for samples of sizes 25,50 and 75. It is found that the Bayes estimates are robust with respect to the respective hyper parameters, in all the cases, because all the $\left[\frac{Min}{Max}\right]$ indices computed are high.

4 Concluding Remarks

1.The Bayes estimator is robust with respect to the respective hyper parameters, in all the cases,because all the $\left[\frac{Min}{Max}\right]$ indices computed are fairly high.

2.For smaller values of the parameters, the Bayesian estimates are found to be better than the maximum likelihood estimates.

3.The bias and MSEs are higher for higher values of the parameters.

Table 1: (Min/Max) index(R) for different values of the hyper parameters using record values when the scale parameter β is known with $n=25, m=25, \alpha_1=0.5, \alpha_2=1.5, \beta=10$

p	q	τ/φ	1	2	3	4	Min/Max
1	1	1	.1695	.1771	.1844	.1915	.8851
		2	.1610	.1679	.1647	.1712	.9404
		3	.1560	.1584	.1607	.1648	.9466
		4	.1436	.1496	.1555	.1612	.8908
		Min/Max	.8472	.8447	.8433	.8418	
	2	1	.1547	.1621	.1612	.1762	.8780
		2	.1465	.1533	.1599	.1664	.8804
		3	.1318	.1381	.1443	.1502	.8775
		4	.1298	.1306	.1313	.1469	.8836
		Min/Max	.8390	.8057	.8145	.8337	
2	1	1	.2110	.2207	.2300	.2390	.8828
		2	.2080	.2171	.2258	.2343	.8878
		3	.1890	.1975	.2057	.2137	.8848
		4	.1731	.1811	.1888	.1964	.8814
		Min/Max	.8204	.8206	.8209	.8218	
	2	1	.2048	.2143	.2235	.2323	.8816
		2	.2022	.2111	.2197	.2280	.8868
		3	.1836	.1919	.2000	.2078	.8835
		4	.1680	.1758	.1834	.1908	.8805
		Min/Max	.8203	.8203	.8206	.8214	

Table 2: Bias and MSEs (in parentheses) of the maximum likelihood estimates of R using record values for different values of α_2

n	m	α_1	α_2 = 0.2	1	3	5
10	10	.2	.1449(.0691)	.1568(.0762)	.0986(.0362)	.0718(.0210)
	25	.2	.1369(.0724)	.1597(.0773)	.1014(.0339)	.0721(.0171)
	50	.2	.1402(.0691)	.1639(.0763)	.1044(.0334)	.0755(.0182)
	10	1	.0454(.0130)	.1449(.0691)	.1732(.0888)	.1568(.0762)
	25	1	.0371(.0161)	.1369(.0724)	.1717(.0896)	.1597(.0773)
	50	1	.0427(.0152)	.1402(.0691)	.1750(.0872)	.1639(.0763)
	10	5	.0091(.0009)	.0454(.0130)	.1103(.0469)	.1449(.0691)
	25	5	.0066(.0012)	.0371(.0161)	.1005(.0512)	.1369(.0724)
	50	5	.1402(.0691)	.1056(.0491)	.0427(.0152)	.0084(.0011)
25	10	.2	.1449(.0691)	.1568(.0762)	.0986(.0362)	.0718(.0210)
	25	.2	.1369(.0724)	.1597(.0773)	.1014(.0339)	.0721(.0177)
	50	.2	.1402(.0691)	.1639(.0763)	.1044(.0334)	.0755(.0182)
	10	1	.0454(.0130)	.1449(.0691)	.1732(.0888)	.1568(.0762)
	25	1	.0371(.0161)	.1369(.0724)	.1717(.0896)	.1597(.0773)
	50	1	.0427(.0152)	.1402(.0691)	.1750(.0872)	.1639(.0763)
	10	5	.0091(.0009)	.0454(.0130)	.1103(.0469)	.1449(.0691)
	25	5	.0066(.0012)	.0371(.0161)	.1005(.0512)	.1369(.0724)
	50	5	.0084(.0011)	.0427(.0152)	.1056(.0491)	.1402(.0691)

Table 3: Bias and MSEs (in parentheses) of the Bayesian estimates of R under squared error loss function using record values when β is known for different values of α_2

n	m	α_1	α_2 = 0.2	1	3	5
10	10	.2	.0196(.0134)	.0669(.0132)	.0662(.0083)	.0606(.0063)
	25	.2	.0783(.0209)	.1426(.0320)	.1047(.0162)	.0833(.0101)
	50	.2	.0229(.0221)	.0948(.0245)	.0710(.0114)	.0549(.0066)
	10	1	.1805(.0365)	.1075(.0165)	.0042(.0032)	.0311(.0033)
	25	1	.1271(.0194)	.0585(.0081)	.0158(.0033)	.0341(.0032)
	50	1	.1535(.0287)	.0936(.0159)	.0185(.0048)	.0013(.0028)
	10	5	.2407(.0590)	.3597(.1310)	.3093(.0968)	.2406(.0587)
	25	5	.2099(.0446)	.3339(.1124)	.3096(.0966)	.2583(.0672)
	50	5	.2209(.0499)	.3470(.1217)	.3290(.1092)	.2812(.0798)
25	10	.2	.0832(.0279)	.1660(.0464)	.1385(.0291)	.1199(.0210)
	25	.2	.0223(.0101)	.0867(.0149)	.0658(.0075)	.0529(.0047)
	50	.2	.0547(.0173)	.0732(.0111)	.0516(.0046)	.0391(.0025)
	10	1	.1267(.0216)	.0419(.0100)	.0507(.0084)	.0779(.0103)
	25	1	.1508(.0248)	.0895(.0100)	.0107(.0020)	.0119(.0014)
	50	1	.1461(.0237)	.0889(.0112)	.0175(.0024)	.0018(.0013)
	10	5	.2106(.0462)	.3201(.1055)	.2754(.0781)	.2120(.0465)
	25	5	.2203(.0490)	.3481(.1220)	.3222(.1045)	.2695(.0731)
	50	5	.2078(.0434)	.3361(.1133)	.3197(.1026)	.2726(.0745)
50	10	.2	.0595(.0110)	.0306(.0046)	.0402(.0031)	.0392(.0025)
	25	.2	.0375(.0131)	.0997(.0180)	.0721(.0082)	.0564(.0048)
	50	.2	.0191(.0065)	.0447(.0054)	.0339(.0022)	.0261(.0012)
	10	1	.1965(.0408)	.1288(.0192)	.0241(.0024)	.0130(.0016)
	25	1	.1276(.0194)	.0608(.0086)	.0118(.0035)	.0303(.0032)
	50	1	.1601(.0276)	.1063(.0141)	.0317(.0026)	.0095(.0011)
	10	5	.2404(.0581)	.3613(.1311)	.3128(.0982)	.2447(.0601)
	25	5	.1923(.0374)	.3097(.0969)	.2885(.0841)	.2391(.0578)
	50	5	.2134(.0459)	.3436(.1188)	.3262(.1070)	.2782(.0779)

Table 4: Bias and MSEs (in parentheses) of the Bayesian estimates of R under linex loss function using record values when β is known for different values of α_2

n	m	α_1	α_2 = 0.2	1	3	5
10	10	.2	.0658(.0193)	.0584(.0298)	.0537(.0258)	.1059(.0306)
	25	.2	.1569(.0053)	.1636(.0591)	.0175(.0286)	.0615(.0266)
	50	.2	.0910(.0345)	.0712(.0519)	.0727(.0422)	.1495(.0504)
	10	1	.0947(.0118)	.0241(.0083)	.0146(.0099)	.0107(.0093)
	25	1	.0538(.0053)	.0233(.0080)	.0306(.0109)	.0044(.0089)
	50	1	.0784(.0102)	.0278(.0134)	.0402(.0175)	.0832(.0202)
	10	5	.1654(.0315)	.2587(.0687)	.2431(.0613)	.2044(.0439)
	25	5	.1441(.0211)	.2406(.0589)	.2611(.0698)	.2592(.0688)
	50	5	.1501(.0229)	.2585(.0685)	.2975(.0909)	.3114(.0993)
25	10	.2	.1656(.0468)	.2043(.0837)	.0941(.0516)	.0332(.0387)
	25	.2	.1046(.0216)	.0801(.0277)	.0664(.0227)	.1375(.0334)
	50	.2	.1303(.0333)	.1152(.0449)	.0404(.0270)	.1221(.0343)
	10	1	.0507(.0062)	.0504(.0129)	.1001(.0239)	.0945(.0225)
	25	1	.0728(.0068)	.0120(.0049)	.0126(.0065)	.0468(.0078)
	50	1	.0602(.0058)	.0034(.0072)	.0070(.0095)	.0532(.0107)
	10	5	.1425(.0214)	.2192(.0511)	.1978(.0430)	.1605(.0294)
	25	5	.1522(.0234)	.2556(.0663)	.2800(.0799)	.2790(.0796)
	50	5	.1434(.0207)	.2455(.0609)	.2813(.0802)	.2951(.0881)
50	10	.2	.1031(.0330)	.1456(.0572)	.0064(.0427)	.0491(.0403)
	25	.2	.1448(.0440)	.1517(.0657)	.0086(.0359)	.1113(.0435)
	50	.2	.0903(.0229)	.0549(.0357)	.0924(.0403)	.1647(.0539)
	10	1	.0786(.0101)	.0026(.0116)	.0439(.0179)	.0383(.0174)
	25	1	.0526(.0070)	.0264(.0164)	.0365(.0174)	.0037(.0145)
	50	1	.0711(.0072)	.0173(.0077)	.0315(.0118)	.0737(.0156)
	10	5	.1542(.0284)	.3009(.0936)	.2215(.0538)	.1837(.0383)
	25	5	.1370(.0197)	.2267(.0544)	.2423(.0631)	.2375(.0611)
	50	5	.1434(.0209)	.2459(.0617)	.2810(.0811)	.2917(.0883)

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