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On Exponentiated Skewed Student t Error Distribution on Some Heteroscedastic Models: Evidence of Nigeria Stock Exchange

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Abstract: In this paper, a new error innovation distribution was proposed in estimating some heteroscedasticity models. A new error innovation distribution was proposed called Exponentiated skewed student t distribution (ESSTD) and compared with the existing error distributions with an empirical dataset using daily returns on Nigeria Stock Exchange (NSE) index return from 30/08/2007 to 30/08/2017.The data shows stationarity at level without difference data and the ADF statistic shows evidence of stationarity, there is presence of ARCH effect. The estimate of the GARCH models and its extension shows a significant probability at 1%, 5% and 10% confident intervals forthe new error distribution and the existing distributions. The AIC and RMSE shows that the new error distributions outperformed in terms of fitness and forecasting evaluation with the smallest AIC and RMSE values respectively.
Keywords: RMSE, AIC, ESSTD, Volatility models, Error distributions, ADF and ARCH effect.

1 Introduction

The major assumption in the ordinary least square (OLS) in regression is that it has a constant error variance over time which is known as homoscedasticity. But however this assumption does not usually hold when dealing with financial series as they do exhibit heteroscedasticity. This problem led to the development of heteroscedastic models. Error innovation distribution plays a vital role in estimating the parameter of any volatility models. The introduction of error distribution was first adopted by Engle (1982) where he estimate the parameter of Autoregressive conditional heteroscedastic model ARCH using standard normal distribution as error distribution.

On these error distributions, the normal distribution proposed by Engle (1982) gained more ground in the estimation of the volatility models, followed by the student t distribution proposed by Bollerslev (1987) to fill the gap of the normal distribution which does not handle kurtosis of the error returns present in financial time series data and thereafter, just few studies have been done in the area of the error distributions.

In this article, we are proposing a new error distribution from the distribution proposed by Dikko and Agboola (2017) to account for the limitation of existing error distributions and used it to estimate some volatility models using Nigeria Stock Exchange (NSE) index returns daily data.

2 Literature Review

Engle (1982) was the first to propose volatility models by coming up with heteroscedasticity model called the Autoregressive Conditional Heteroscedasticity (ARCH) model to predict the variation on the conditional variance of any returns series where the parameter depends on the squared errors of the lag of the series while in 1983, Engle used the

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proposed model to model the U.S inflation. Lamoureux and Lastrapes (1990) applied the ARCH model to a daily trading volume of stock market returns. Nelson (1991) first proposed the Exponential GARCH models (EGARCH) to capture the leverage effect of financial time series which the original GARCH models did not give due consideration.Agboola et al., (2015) also worked on principle of parsimony in modelling time series using heteroscedasticity models by considering eight insurance stocks and using parsimonious volatility of ARCH, GARCH, EGARCH, and TGARCH and Power ARCH (PARCH) models. Dikko et al., (2015) also worked on modelling abrupt shift in time series using indicator variable where twelve volatility models and for each of the models an indicator variable was introduced to each of the models to take into account any major change in the stock market.

In 1982 when Engle proposed the ARCH model, he estimate the ARCH model using the distribution for error innovation for estimation of volatility models and the model of GARCH proposed by Bollerslev (1986) also adopted this normal distribution for the error innovation and used the distribution to estimate volatility models.Bollerslev (1987) used the student t distribution in the estimation of volatility models. The error innovation of this distribution was used to capture the limitation of the normal distribution.Generalized Error Distribution (GED) was proposed for error innovation by Nelson (1991) in estimating EGARCH model. Skewed normal distribution was introduced by O'Hagan and Leonard (1976) to capture the Skewedness of the normal distribution. The Skewed student t distribution of the error innovation was proposed by Hansen (1994). Skewed generalized error distribution was first used for error innovation by Theodossiou (1998) by adding a Skewed parameter for the generalized error distribution to capture the Skewedness of the GED

3 Methodology

3.1 Computation of return series from price

$$\text{Let } R_t = \ln\left(\frac{P_t}{P_{t-1}}\right) \tag{1}$$

where P_t and $P_{t-1}$are the present and previous closing prices and R_t the continuously compounded return serieswhich is the natural logarithm of the simple gross return.

3.2 Stationary Test

Stationarity of the return series of the Augmented Dickey–Fuller**(**ADF)test is given as:

$$\text{Let } y_t = \phi_1 y_{t-1} \tag{2}$$

$$y_t - y_{t-1} = \phi_1 y_t - y_{t-1}$$

$$\Delta y_t = (\phi_i - 1) y_{t-1}$$

$\Rightarrow \phi_1 - 1 = 0$ or $\phi_1 = 1$

Null hypothesis is $H_0 : \phi_1 = 1$

and alternative hypothesis is : $H_1 : \phi_1 < 1$

The Test Statistic (t-ratio): $$= \frac{\phi_1^n - 1}{std(\phi_1)} = \frac{\sum_{t=1}^{T} P_{t-1} e_t}{\sigma^2 \sqrt{\sum_{t=1}^{T} P_{t-1}^2}} \tag{3}$$

where $\phi_1 = \frac{\sum_{t=1}^{T} p_{t-1} p_t}{\sum_{t=1}^{T} p_{t-1}^2}$ and $\hat{\sigma}^2 = \frac{\sum_{t=1}^{T} (p_t - \hat{\phi}_1 p_{t-1})^2}{T-1}$

$P_0 = 0$, T is the sample size and ϕ_1 for each Insurance stock.

The null hypothesis is rejected if the calculated value of t is greater than t critical value.

3.3 Test for ARCH Effect

$$r_t = \phi_1 r_{t-1} + \varepsilon_t + \theta_1 \varepsilon_{t-1} \tag{4}$$

after obtaining the residuals e_t , the next step is regress the squared residual on a constant and its q lags as in the following equation:

$$\sigma_t^2 = \alpha_0 + \alpha_1 e_{t-1}^2 + \ldots + \alpha_q e_{t-q}^2 + r_t \tag{5}$$

The null hypothesis, that there is no ARCH effect up to order q can be formulated as:

$$H_0 : \alpha_1 = \ldots = \alpha_q = 0 \tag{6}$$

against the alternative

$$H_a : \alpha_i \neq 0 \text{ for some i} \in \{1,..,m\} \tag{7}$$

The test statistic for the joint significance of the q-lagged squared residuals is the number of observations times the R-squared (TR^2) from the regression. TR^2 is tested against $\chi^2_{(q)}$ distribution. This is asymptotically locally most powerful test.

3.4 Some Volatility Models

The GARCH (p, q) model was stated as follows:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \ldots + \alpha_q \varepsilon_{t-q}^2 + \beta_1 \varepsilon_{t-1}^2 + \ldots + \beta_p \varepsilon_{t-p}^2 + e_t \tag{8}$$

where $\alpha_i > 0$ and $\beta_i > 0$ for all i and j

The EGARCH (p, q) model was proposed by Nelson (1991) formulate the volatility as follows:

$$\ln(\sigma_t^2) = \alpha_0 + \sum_{i=1}^{p} \alpha_i \left[\lambda \varepsilon_{t-i} + \gamma \left\{ \left| \varepsilon_{t-i} \right| - \sqrt{\frac{2}{\pi}} \right\} \right] + \sum_{i}^{q} \beta_j \ln(\sigma_{t-j}^2) \tag{9}$$

$\alpha_0, \alpha_i, \gamma, \beta_j$ are the parameters of the model.

3.4.1Threshold Generalized Autoregressive Conditional Heteroscedasticity (TGARCH) Model

The Threshold GARCH model is similar to GJR-GARCH of Glosten, Jagannathan & Runkle (1993) stated as:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} (\alpha_i + \gamma_i N_{t-i}) \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}{}^2 \tag{10}$$

$$\alpha_0, \alpha_i, \gamma_i, \beta_j \geq 0$$

where, N_{t-i} is an indicator for negative ε_{t-i} that is N_{t-i} is 1 if $\varepsilon_{t-i} < 0$ and 0

3.4.2 Model Selection

Akaike Information Criteria (AIC,) is given as:

$$\text{AIC} = 2K - 2\ln(L) = 2K + \ln\left(\frac{RSS}{n}\right) \tag{11}$$

where k is the number of parameters in the model and L is the maximized value of the likelihood function for the model and $RSS = \sum e^2$ is the residual sum of squares.

3.4.3 Forecasting Evaluation

Evaluating the performance of different forecasting models plays a very important role in choosing the most accurate models.The most widely used evaluation measures is Root Mean Square Error (MSE) given as:

$$\text{RMSE} = \sqrt{\frac{\sum_{t=T+1}^{T+n}\left(\hat{\sigma}^2{}_t - \sigma^2{}_t\right)^2}{n}} \tag{12}$$

Where, n is the number of steps ahead, T is the sample size, $\hat{\sigma}_t$ and σ_t are the square root of the conditional forecasted volatility and the realized volatility respectively.

Maximum likelihood estimator of the error distributions

Exponentiated Skewed student-t Distribution (ESSTD)

The Exponentiated skewed student t distribution was derived by Dikko and Agboola (2017) and the distribution is given as: $x \sim ESSTD(u,\lambda)$, its PDF is given by;

$$g(x) = u\left\{\frac{1}{2}\left(1+\frac{x}{\sqrt{\lambda+x^2}}\right)\right\}^{u-1}\frac{\lambda}{2\left(\lambda+x^2\right)^{\frac{3}{2}}} \tag{13}$$

where $u>0, \lambda>0$ and

λ ,is Skewed parameter and u is a shape parameters

Now, the error distribution of the Exponentiated Skewed student-t distribution is given as

$$g(z_t,\alpha,\lambda) = \alpha\left\{\frac{1}{2}\left(1+\frac{z_t}{\sqrt{\lambda+z_t{}^2}}\right)\right\}^{\alpha-1}\frac{\lambda}{2\left(\lambda+z_t{}^2\right)^{\frac{3}{2}}} \tag{14}$$

$$g(z_t,\alpha,\lambda) = \alpha\left\{\frac{1}{2}\left(1+\frac{z_t}{\sqrt{\lambda+z_t{}^2}}\right)\right\}^{\alpha-1}\frac{\lambda}{2\left(\lambda+z_t{}^2\right)^{\frac{3}{2}}}$$

if $\varepsilon_t = z_t\sigma_2$

$$g(\varepsilon_t,\alpha,\lambda) = \alpha\left\{\frac{1}{2}\left(1+\frac{\frac{\varepsilon_t}{\sigma_t}}{\sqrt{\lambda+\left(\frac{\varepsilon_t}{\sigma_t}\right)^2}}\right)\right\}^{\alpha-1}\frac{\lambda}{2\left(\lambda+\left(\frac{\varepsilon_t}{\sigma_t}\right)^2\right)^{\frac{3}{2}}}\left(\frac{1}{\left(\sigma_t^2\right)^{\frac{1}{2}}}\right) \tag{15}$$

Log-likelihood function of error innovation of Exponentiated Skewed student-t distribution

$$L(\theta)=\prod_{i=1}^{n} g\left(\varepsilon_t,\alpha,\lambda\right)=L\left(\varepsilon_t,\alpha,\lambda\right)=\prod_{i=1}^{n} g\left(\varepsilon_t,\alpha,\lambda\right)$$

where $\theta=\left(\alpha,\lambda,\sigma_t\right)$ $and\ \sigma_t^2=\omega+\sum_{i=1}^{n} a_i\varepsilon_{t-i}^2+\sum_{i=1}^{n}\beta_i\sigma_{t-i}^2$

$$=\prod_{i=1}^{n}\left[\alpha\left\{\frac{1}{2}\left(1+\frac{\frac{\varepsilon_t}{\sigma_t}}{\sqrt{\lambda+\left(\frac{\varepsilon_t}{\sigma_t}\right)^2}}\right)\right\}^{\alpha-1}\frac{\lambda}{2\left(\lambda+\left(\frac{\varepsilon_t}{\sigma_t}\right)^2\right)^{\frac{3}{2}}}\left(\frac{1}{\left(\sigma_t^2\right)^{\frac{1}{2}}}\right)\right] \tag{16}$$

$$=\alpha^n\lambda^n\prod_{i=1}^{n}\left[\left\{\frac{1}{2}\left(1+\frac{\frac{\varepsilon_t}{\sigma_t}}{\sqrt{\lambda+\left(\frac{\varepsilon_t}{\sigma_t}\right)^2}}\right)\right\}^{\alpha-1}\frac{1}{2\left(\lambda+\left(\frac{\varepsilon_t}{\sigma_t}\right)^2\right)^{\frac{3}{2}}}\left(\frac{1}{\left(\sigma_t^2\right)^{\frac{1}{2}}}\right)\right] \tag{17}$$

Taking the log likelihood function of the above equation

$$InL\left(x;u,\lambda\right)=n\log\left(\alpha\right)+nt\log\left(\lambda\right)-n\alpha\log 2+$$

$$\left(\alpha-1\right)\sum_{i=1}^{n}\log\left(1+\frac{\varepsilon_t}{\sigma_t\sqrt{\lambda+\left(\frac{\varepsilon_t}{\sigma_t}\right)^2}}\right)-\frac{3}{2}\sum_{i=1}^{n}\log\left(\lambda+\left(\frac{\varepsilon_t}{\sigma_t}\right)^2\right)-0.5\log\left(\sigma_t^2\right) \tag{18}$$

Skewed Normal Distribution

$$f(z_t)=\frac{1}{\sigma\pi}e^{-\frac{(z_t-\varepsilon)^2}{2\sigma^2}}\int_{-\infty}^{\alpha}\frac{z_t-\varepsilon}{\sigma}e^{-\frac{t^2}{2}}dt,\qquad -\infty<z_t<\alpha \tag{19}$$

where ε is the location, σ is the scale and α denotes the shape parameter.

Standardized Skewed student t-distribution

$$f(z_t,\mu,\sigma,v,\lambda)=\begin{cases} bc\left[1+\frac{1}{v-2}\left(\frac{b\left(\frac{z_t-\mu}{\sigma}\right)+a}{1-\lambda}\right)^2\right]^{-\frac{v+1}{2}} & ,z_t<-\frac{a}{b} \\ bc\left[1+\frac{1}{v-2}\left(\frac{b\left(\frac{z_t-\mu}{\sigma}\right)+a}{1+\lambda}\right)^2\right]^{-\frac{v+1}{2}} & ,z_t\geq-\frac{a}{b} \end{cases} \quad (20)$$

where v is the shape parameter with $2<v<\infty$ and λ is the Skewedness parameters with $-1<\lambda<1$, μ and σ^2 are the mean and variance of the Skewed student t-distribution.

$$a=4\lambda c\left(\frac{v-2}{v-1}\right),\ b=1+3\lambda^2-a^2,\ c=\frac{\Gamma\left(\frac{v+1}{2}\right)}{\sqrt{\pi(v-2)\Gamma\left(\frac{V}{2}\right)}}$$

Standardized Skewed Generalized Error Distribution

$$f(z_t/v,\varepsilon,\theta,\delta)=\frac{v}{2\theta\Gamma\left(\frac{1}{v}\right)}\exp\left[-\frac{|z_t-\delta|^v}{[1+sign(z_t-\delta)\varepsilon]^v\theta^v}\right] \quad (21)$$

$$\theta>0,\ -\infty<z_t<\infty,\ v>0,\ -1<\varepsilon<1,\ -\infty<z_t<\infty$$

where

$$\theta=\Gamma\left(\frac{1}{v}\right)^{0.5}\Gamma\left(\frac{3}{V}\right)^{-0.5}S(\varepsilon)^{-1},$$

$$\delta=2\varepsilon S(\varepsilon)^{-1},$$

$$S(\varepsilon)=\sqrt{1+3\varepsilon^2-4A^2\varepsilon^2},$$

$$A=\Gamma\left(\frac{2}{v}\right)\Gamma\left(\frac{1}{v}\right)^{-0.5}\Gamma\left(\frac{3}{v}\right)^{-0.5}$$

where $v > 0$ is the shape parameter, ε is a Skewedness parameter with $-1 < \varepsilon < 1$.

4 Results

4.1 Empirical Results

An empirical analysis of the NSE index returns were carried out on index returns series. The obtained results as shown in Table 1 showed that the mean index return series was positive, positive skewed and very high kurtosis for NSE.The result of Jarque-Bera statistic revealed that the return series for NSE index returns was not normally distributed as the p-values were less than 1% and 5%.

Table 1. Descriptive Statistics of Nigeria Stock Exchange (NSE) index returns

Statistics	Returns of NSE
Mean	0.0001
Median	0.0000
Std. Dev.	0.1283
Skewedness	0.018767
Kurtosis	51.693
Jarque-Bera	2.6666e+05
Probability	0.001
Observations	3190

4.2 Stationary test

The stationarity of the index return series were investigated by observing the time plot of the series. The figure obtained as presented in 1 revealed that the price and return series of the stocks and the returns series were stationary. Also, a formal test of stationarity was also carried out using the Augmented Dickey- Fuller Test. The results obtained for NSE index returns showed that the Augmented Dickey- Fuller test statistic were all less than their critical values at 1%, 5% and 10% as shown in Table 2. Hence, there is no unit root. The return series were all stationary. Therefore, there is no need for transformation.

Table 1. Augmented Dickey-Fuller Test of stationarity test (ADF) of Nigeria Stock Exchange

Stocks	ADF Test Statistic	Comment
NSE index	-23.62148	Stationary at level without transformation

1% critical = -3.432219, 5% critical = -2.862251, 10% critical = -2.567193

4.3 ARCH Effect Test

The Lagrange Multiplier (LM) test) was applied. The F Statistic at different lag and the p-values obtained are summarized in Table 3. The results of F Statistic were significant at 1% for the NSE index returns which shows that there is ARCH effect.

Table 2. Lagrange Multiplier test of the presence of ARCH effect

	ARCH EFFECT	F-Statistic	P-value
NSE Index Returns	ARCH 1-2 test	F(2,2390) = 473.64	0.001
	ARCH 1-5 test	F(5,2384) = 231.85	0.001
	ARCH 1-10 test	F(10,2374)= 117.87	0.001

Estimates of the parameters of GARCH models and its extension based on Nigeria Stock Exchange (NSE)

Table 4A and Table 4B present the parameter estimates of GARCH model and its extension estimated at four (4) error distributions such as Skewed normal, Skewed Student- distribution and Skewed generalized error distribution and the proposed Exponentiated skewed student t distribution using returns from NSE. The result shows that the returns exhibit volatility clustering. This was concluded because the GARCH term was significant in most of the models considered ($p<0.05$) and ($p<0.01$) which means that small changes in volatility of both returns tends to be followed by large changes in volatility while small changes in volatility tends to be followed by small changes in volatility. In terms of leverage effect which measures whether there is a negative relationship between asset returns and volatility was found to be significant in GARCH, GJR-GARCH, EGARCH, TGARCH and APARCH models estimated at the four (4) distributions of error innovation($p<0.05$).

Table 4A. Estimates of the parameters of GARCH models and its extension based on Nigeria Stock Exchange (NSE) index returns using new classes of error distributions

Model	**Error**	ω **(p-value)**	α_1 **(p-value)**	β_1 **(p-value)**	γ_1 **(p-value)**	δ **(p-value)**	Skewed **(p-value)**	Shape(u) **(p-value)**
GARCH (1,1)	ESSTD	0.016676**	-00.2451**	.0011683**			0.00596	8.5634**
GJR-GARCH (1,1)	ESSTD	0.0029080**	0.000081**	0.00008**	-0.87643		0.4571	9.8900**
EGARCH (1,1)	ESSTD	0.094465**	0.038697*	0.026803	0.00105**		0.04258**	1.99821*
TGARCH (1,1)	ESSTD	0.095819**	-1.11642**	3.90917	-4.95819		5.56328	4.60107**
APARCH (1,1)	ESSTD	0.05931**	1.6709**	0.1976**	-1.646	-1.982*	0.2794	4.2030***

* at 5%, ** at 1% and *** at 10% significant

Fitness and Model selection of GARCH models and its extension based on Nigeria Stock Exchange (NSE)

The performance of GARCH model and its extension estimated at Skewed student-t distribution was compared with that of the proposed distributions. Table 5 shows the result of the fitness and model selection based on log- likelihood and Akaike Information Criteria (AIC) of GARCH, GJR-GARCH, EGARCH, TGARCH and APARCH models. The ESSTD performed better than the remaining distributions of error innovation. The Exponentiated Skewed Student-t distribution (ESSTD) was found to be the best for GARCH model and its' extension as reveal by its largest log- likelihood and least value of Akaike Information Criteria (AIC) for the NSE index returns.

Table 6 shows the forecasting performance of the estimated models using Root Mean Square Error (RMSE) and Mean Absolute Error (MAE). Model with the smallest RMSE was considered to most suitable for forecasting estimated at Skewed normal, Skewed student-t and Skewed generalized error distributions. Hence, from the results obtained showed that GJR-GARCH, EGARCH, TGARCH and APARCH models forecast evaluated at Skewed generalized error distributions than the remaining distributions of error GARCH models and its' extensions while GARCH model forecast evaluated at Skewed student-t distribution that the remaining error distributions.

Table 4B. Parameter estimation of GARCH and its extension on NSE index returns

Model	**Error**	ω **(p-value)**	α_1 **(p-value)**	β_1 **(p-value)**	γ_1 **(p-value)**	δ **(p-value)**	Skewed **(p-value)**	Shape **(p-value)**
GARCH (1,1)	SSTD	5.674 x10^{-10}	3.749 x10^{-01***}	1.00 x10^{-08**}			1.2460**	2.000***
	SNORM	7.875 x10^{-03*}	1.952 x10^{-01***}	6.864 x10^{-01***}			1.078**	
	SGED	2.091 x10^{-06***}	1.259 x10^{-01***}	8.624 x10^{-01***}			1.052***	1.173***
GJR-GARCH (1,1)	SSTD	0.0000	0.3923	0.0412	0.8992		1.1428	2.0173**
	SNORM	0.000006	0.1054***	0.9104***	-0.0577**		0.9710**	
	SGED	0.0000001**	0.0500**	0.9000**	0.4569**		1.0000**	2.0000**
EGARCH (1,1)	SSTD	-9.9999**	0.0001**	0.6656**	0.0001**		0.9208**	2.0100**
	SNORM	-1.3039**	-0.0323**	0.8153**	0.3605**		1.0718**	
	SGED	-1.1784**	0.0078**	0.8962**	0.0185**		1.0001**	0.2323**
TGARCH (1,1)	SSTD	0.0000	0.0830	0.1630			1.0199	2.0155
	SNORM	0.000004	0.0743	0.9213			0.9526	
	SGED	0.00001	0.0500	0.9000			1.0000	2.0000
APARCH (1,1)	SSTD	0.0000	0.1053	0.1916	-0.9052	2.000***	1.0109	2.0175
	SNORM	0.000006	0.0801	0.9072	-0.2285	2.000**	0.9787	
	SGED	0.000001	0.0500	0.9000	0.0500	2.000**	1.0000	2.0000

Table 5. shows the result of the fitness and model selection based on log- likelihood and Akaike Information Criteria (AIC). GARCH, GJR-GARCH, TGARCH and APARCH models

Model	**Error**	**LL**	**AIC**
GARCH (1,1)	SSTD	12848.89	-10.724
	SNORM	5830.812	-4.8650
	SGED	8476.708	-6.4909
	ESSTD	**1660000.899**	**-14.03949**
GJR-GARCH (1,1)	**SSTD**	**12412.8**	**-10.360**
	SNORM	6023.987	-5.0255
	SGED	4617.117	-3.8498
	ESSTD	**411000.8465**	**-13.85270**
EGARCH (1,1)	SSTD	10765.72	-8.9843
	SNORM	5853.97	-4.8835
	SGED	**14583.32**	**-12.172**
	ESSTD	**450000.3303**	**-9.42885**
TGARCH (1,1)	SSTD	12087.42	-10.089
	SNORM	6004.817	-5.0103
	SGED	4038.42	-3.3674
	ESSTD	**172000.5547**	**-12.110503**
APARCH (1,1)	SSTD	12470.73	-10.408
	SNORM	6026.297	-5.0297
	SGED	4041.617	-3.3692
	ESSTD	**4081000.134**	**-16.44370**

Bolded values are the highest value of likelihood function and the least value of AIC

Table 3. Forecasting Evaluation of GARCH models and its extension based on Nigeria Stock Exchange (NSE) index returns

Model	**Error**	**RMSE**
GARCH (1,1)	**SSTD**	**0.07109084**
	SNORM	5.758357
	SGED	0.4134991
	ESSTD	**0.00650665**
GJR-GARCH (1,1)	SSTD	0.071097
	SNORM	1.009956
	SGED	**$5.132486x10^{-31}$**
	ESSTD	**$5.132486x10^{-31}$**
EGARCH (1,1)	SSTD	0.07109883
	SNORM	4.239676
	SGED	**0.00001**
	ESSTD	**0.0000005913655**
TGARCH (1,1)	SSTD	0.0708405
	SNORM	6.892382
	SGED	**$5.132486x10^{-31}$**
	ESSTD	**$5.071885x10^{-31}$**
APARCH (1,1)	SSTD	0.071097
	SNORM	2.000931
	SGED	**$5.132486x10^{-31}$**
	ESSTD	**$5.0811329x10^{-34}$**

RMSE- Root Mean Square Error, Bolded values are the least Root Mean Square Error(RMSE)

5 Conclusion

In the article, we are able to come up with new error innovation distribution in estimating the parameters of GARCH models and its extension. We also compared the new proposed error distribution with the improved version of the standard Normal, Student-t and Generalized error distribution, i.e. skewed of the three (3) error distributions. The empirical result shows a positive returns, high kurtosis and skewness and it's was stationary at level without transformation. The data shows evidence of ARCH effect and most coefficients in the models were significant at 1%, 5% and 10%. From the results obtained shows on model selection using the AIC, the ESSTD fitted better in the GARCH model and is extension than the compared error distributions while the forecasting evaluation also shows that ESSTD outperformed the existing distributions with least RSME. The contribution of this article is proposing a new error distribution in estimating the GARCH models and its extension and compared the new error distribution with the improved error distribution in term of fitness and forecasting evaluation. Therefore, this new error distribution known Exponentiated Skewed Student t distribution outperformed the improved error distribution in term off fitness and forecasting. So therefore, in the estimation and forecasting of GARCH models and its extension, this new error distribution should be adopted in estimating and forecast of any financial returns.

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Appendix

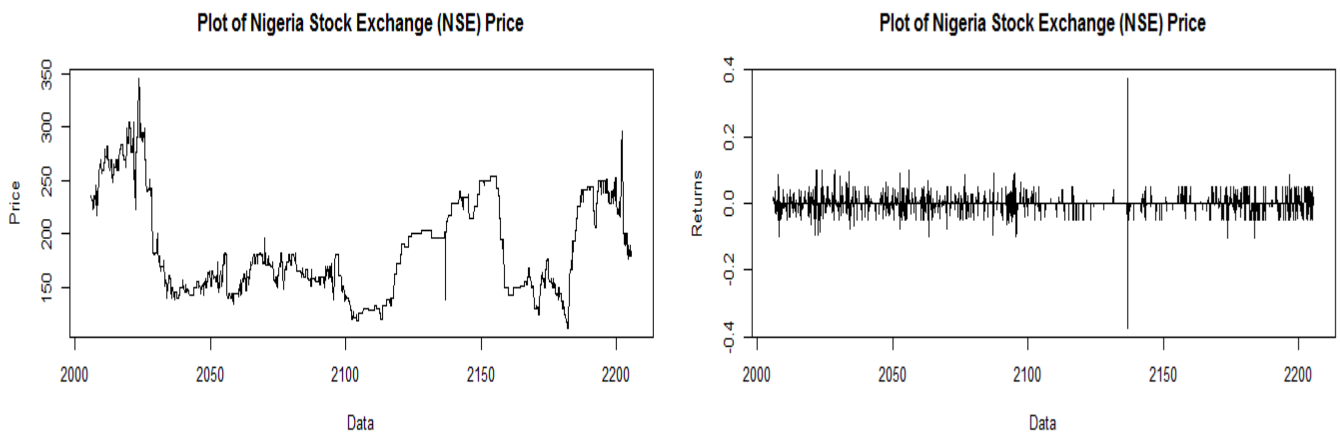

Figure 1. Raw plot of price index return and stationarity plot of NSE index returns