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Solving Fully Rough Interval Multi-level Multi-objective linear Fractional Programming Problems via FGP

Mohamed S. Osman[1], *Osama. E. Emam*[2], *Kamal. R. Raslan*[3] *and Farahat. A. Farahat*[4,*]

[1] Alasher University, 10^{th} of Ramadan city, Egypt
[2] Department of Information Systems, Faculty of Computers and Information, Helwan University, Egypt
[3] Mathematics Department, Faculty of Science, Al-Azhar University, Nasr-City, Cairo, Egypt
[4] Higher technological institute, $10^{th}$of Ramadan city, Egypt

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Abstract: This paper introduces an algorithm for solving fully rough intervalmulti-level multi-objective linear fractional programming problems where all of its coefficients in objective functionsand in constraints are rough intervals. At the first phases of the solution approach and to avoid the complexity of the problem, the shifting method proposed by Osman and El-sherbiny [20] will be used to split the rough problem into four crisp problems which will be solved simultaneously. At the second phase, for each problem, a membership function was constructed to develop a fuzzy goal programming model for obtaining the satisfactory solution of the multi-level multi-objective fractional programming problem. The linearization process introduced by Pal et. al [1] will be applied to linearize the membership functions. Finally, an illustrative numerical example is given to demonstrate the algorithm.

Keywords: Multi-level programming; Multi-objective programming; Fractional programming; rough intervals programming; fuzzy goal programming.

1 Introduction

The standard mathematical programming problem involves finding an optimal solution for just one decision maker. Nevertheless, many planning problems contain a hierarchical decision structure, each with independent and often conflicting objectives. These types of problems can be modeled using a multi-level mathematical programming (MLMP) approach. The basic concept of the MLMP technique is that the first-level decision maker (FLDM) sets his/her goal and/or decision, and then asks each subordinate level of the organization for their optima, that calculated in isolation. The lower level decision maker's decisions are then submitted and modified by the FLDM in consideration of the overall benefit for the organization. The process continues until a satisfactory solution is reached.Most of the developments in MLP problems focus on bi-level linear programming as a class of MLP [2,3,4].

In various areas of the real world, the problems are formulated as multi-objective programming problems. Many methodologies have been introduced for dealing with problems [1]. However, the issue of choosing a proper method in a given context is still a subject of active research.

Fractional programming deals with the optimization of one or more ratios of functions subject to set constraints. Recently, fractional programming has become one of the planning tools. It is applied in engineering, business, finance, economics and other disciplines [1,5,6,7]. Computer oriented technique was extended by Helmy et al. [8] to solve a special class of ML-MOFP problems.

Emam [9] introduced a bi-level integer non-linear programming problem with linear or non-linearconstraints, and in which the non-linear objective function at each level were maximized. It proposed a two planner integer model and a solution method for solving this problem. Therefore Emam proposed an interactive approach for solving bi-level integer multi-objective fractional programming problem [10]. Many fuzzy goal programming approaches have been introduced to solve multi-level programming problems [5,11].

* Corresponding author e-mail: farahat_math@yahoo.com

The rough set expressed by a boundary region of a set which is described by lower and upperapproximation sets where the set is considered as a crisp set if the boundary region is empty. This is exactly the idea of vagueness [12,13]. The approach for solving rough interval programming problem is to convert the objective function from rough interval to crisp using theorem of crisp evaluation. Roughness is a kind of uncertainty, another kind of uncertainty introduced in [14].

Hamzehee et al. [15] presented a linear programming (LP) problem which is considered where some or all of its coefficients in the objective function and /or constraints are rough intervals. In order to solve this problem, two LP problems with interval coefficients will be constructed. One of these problems is a LP where all of its coefficients are upper approximations of rough intervals and the other problem is a LP where all of its coefficients are lower approximations of rough intervals. Using these two LPs, two newly solutions are defined.

Many researches have been done in the area of rough set and rough intervals [16,17,18,19] Osman and El-Sherbiny [20] proposed a new method for solving rough interval programming problems called shifting method which will be used in this paper.

Multi-level programming problems (MLPPs) have recently increasingly appeared in decentralized management situations in the real world and have become highly complicated and large-scale, particularly with the development of economic integration and in the current age of big data; for example, business firms now a day's usually work in a decentralized manner in a complex commercial network comprised of suppliers, manufacturers, sales and logistics companies, customers and other specialized service functions. So (MLPPs) have many applications such as [21,22,23, 24] and we will have an application in our coming research.

The remaining of the paper is organized as follows: Section 2 introducesproblem formulation and solution concept. Section 3, introduces the solution algorithm. In section 4, an illustrative example will be introduced. Finally, in Section 5, conclusion and some open pointsfor future research work are stated in the field of rough intervalsmulti-level multi-objective fractional programming problems.

2 Problem Formulation and solution concept

Multi-level programming problems have more than one decision maker. A decision maker is located at each decision level and a vector of fractional objective functions needs to be optimized. Consider the hierarchical system be composed of a t-level decision makers. Let the decision maker at the i^{th}-level denoted by DM_i controls over the decision variable $x_i = (x_{i1}, x_{i2}, \cdots, x_{in_i}) \in R^{n_i},\ i = 1,2,\cdots,t$. where $x = (x_1, x_2, \cdots, x_t) \in R^n$ and $n = \sum_{i=1}^{t} n_i$.

Mathematically, ML-MOFP problem with rough intervals in objective functions and constraints of minimization-type may be formulated as follows:

$[1^{st}\ Level]$

$$\underbrace{min}_{x_1} F_1(x) = \underbrace{min}_{x_1} \left(f_{11}(x),\ f_{12}(x),\ \cdots, f_{1m_1}(x)\right), \tag{1}$$

where $x_2, x_3, \cdots,\ x_t$ solves

$[2^{nd}\ Level]$

$$\underbrace{min}_{x_2} F_2(x) = \underbrace{min}_{x_2} \left(f_{21}(x),\ f_{22}(x),\ \cdots, f_{2m_2}(x)\right), \tag{2}$$

$$\vdots$$

where x_t solves

$[\ t^{th}\ Level]$

$$\underbrace{min}_{x_t} F_t(x) = \underbrace{min}_{x_t} \left(f_{t1}(x),\ f_{t2}(x),\ \cdots, f_{tm_t}(x)\right), \tag{3}$$

subject to

$$x \in G = \left\{ x \in R^n \;\middle|\; \begin{array}{c} \sum_{j=1}^{n} \left(\left[\underline{a}_{ij}^L, \underline{a}_{ij}^U\right], \left[\overline{a}_{ij}^L, \overline{a}_{ij}^U\right] \right) x_j \le ([a,b],[c,d]),\ x = 0,\ a,b,c,d \in R^m,\ x_j = 0, \\ ,i = 1,2,\cdots,t \end{array} \right\} \tag{4}$$

where

$$f_{ij}(x) = \frac{N_{ij}(x)}{D_{ij}(x)} = \frac{\sum_{j=1}^{m_i} \left(\left[\underline{c}_{ij}^L, \underline{c}_{ij}^U\right], \left[\overline{c}_{ij}^L, \overline{c}_{ij}^U\right] \right) x_j + \left(\left[\underline{\alpha}_{ij}^L, \underline{\alpha}_{ij}^U\right], \left[\overline{\alpha}_{ij}^L, \overline{\alpha}_{ij}^U\right] \right)}{\sum_{j=1}^{m_i} \left(\left[\underline{d}_{ij}^L, \underline{d}_{ij}^U\right], \left[\overline{d}_{ij}^L, \overline{d}_{ij}^U\right] \right) x_j + \left(\left[\underline{\beta}_{ij}^L, \underline{\beta}_{ij}^U\right], \left[\overline{\beta}_{ij}^L, \overline{\beta}_{ij}^U\right] \right)},\quad i = 1,2,\cdots,t, \tag{5}$$

$F_1(x)$, $F_2(x)$ and $F_3(x)$ are the objective functions of the first level decision maker (FLDM), second level decision maker (SLDM) and the third level decision maker respectively.

G is the multi-level multi-objective convex constraint set.

$\left(\left[\underline{c}_{ij}^L, \underline{c}_{ij}^U\right], \left[\overline{c}_{ij}^L, \overline{c}_{ij}^U\right]\right)$ are rough intervals coefficients for x_j of the objective functions numerators, $\left(\left[\underline{d}_{ij}^L, \underline{d}_{ij}^U\right], \left[\overline{d}_{ij}^L, \overline{d}_{ij}^U\right]\right)$ are rough intervals coefficients for x_j in the objective functions denominators, $\left(\left[\underline{\alpha}_{ij}^L, \underline{\alpha}_{ij}^U\right], \left[\overline{\alpha}_{ij}^L, \overline{\alpha}_{ij}^U\right]\right)$ are rough intervals constants of the numerator, $\left(\left[\underline{\beta}_{ij}^L, \underline{\beta}_{ij}^U\right], \left[\overline{\beta}_{ij}^L, \overline{\beta}_{ij}^U\right]\right)$ are rough intervals constants of the numerator, $\left(\left[\underline{a}_{ij}^L, \underline{a}_{ij}^U\right], \left[\overline{a}_{ij}^L, \overline{a}_{ij}^U\right]\right)$are rough intervals coefficients for x_j in constraints, $([a,b],[c,d])\rightarrow$are rough intervals constants of the right hand side of the constraints.

It is customary to assume that $D_{ij}(x) > 0 \; \forall \; x \in G$,

Conversion of (MLMOFP) problem with rough coefficient in objective functions into upper and lower approximation is usually a hard work for many cases, but transformation process needs to know the following definitions [15]:

Definition 1[15] *Rough Interval (RI) can be considered as a qualitative value from vague concept defined on a variable xin R.*

Definition 2[15] *The qualitative value A is called a rough interval when one can assign two closed intervals A^* and A_* on R to it where $A_* \subseteq A^*$.*

Remark 1[15] *According to the rough interval properties we have:*

$\left[\underline{c}_{ij}^L, \underline{c}_{ij}^U\right] \subseteq \left[\overline{c}_{ij}^L, \overline{c}_{ij}^U\right] \rightarrow \overline{c}_{ij}^L \leq \underline{c}_{ij}^L \leq \underline{c}_{ij}^U \leq \overline{c}_{ij}^U$,

$\left[\underline{\alpha}_{ij}^L, \underline{\alpha}_{ij}^U\right] \subseteq \left[\overline{\alpha}_{ij}^L, \overline{\alpha}_{ij}^U\right] \rightarrow \overline{\alpha}_{ij}^L \leq \underline{\alpha}_{ij}^L \leq \underline{\alpha}_{ij}^U \leq \overline{\alpha}_{ij}^U$,

$([a,b],[c,d]) \rightarrow c \leq a \leq b \leq d$,

Now, the equivalent crisp problems of the (ML-MOFP) problem with fully rough intervals can be reformulated according to the shifting method in [20] as follows:

FP1:

[1^{st} *Level*]

$$\underbrace{min}_{x_1} F_1(x) = \underbrace{min}_{x_1} \left(f_{11}(x),\; f_{12}(x),\; \cdots,\; f_{1m_1}(x)\right) \quad (6)$$

where $x_2, x_3, \cdots, x_t$ solves

[2^{nd} *Level*]

$$\underbrace{min}_{x_2} F_2(x) = \underbrace{min}_{x_2} \left(f_{21}(x),\; f_{22}(x),\; \cdots,\; f_{2m_2}(x)\right), \quad (7)$$

$\vdots$

where x_t solves

[t^{th} *Level*]

$$\underbrace{min}_{x_t} F_t(x) = \underbrace{min}_{x_t} \left(f_{t1}(x),\; f_{t2}(x),\; \cdots,\; f_{tm_t}(x)\right), \quad (8)$$

subject to

$$x \in G = \left\{ x \in R^n \;\middle|\; \sum_{j=1}^{n} \overline{a}_{ij}^L x_j \leq c,\; x_j = 0,\;\; c \in R^m \right\} \quad (9)$$

where

$$f_{ij}(x) = \frac{N_{ij}(x)}{D_{ij}(x)} = \frac{\sum_{j=1}^{m_i} \overline{c}_{ij}^L x_j + \overline{\alpha}_{ij}^L}{\sum_{j=1}^{m_i} \overline{d}_{ij}^L x_j + \overline{\beta}_{ij}^L}, i = 1, 2, \cdots, t. \quad (10)$$

And find $X^{*c} = (x_1^{*c}, x_2^{*c}, \cdots, x_n^{*c})$

FP2:

[1^{st} *Level*]

$$\underbrace{min}_{x_1} F_1(x) = \underbrace{min}_{x_1} \left(f_{11}(x),\; f_{12}(x),\; \cdots,\; f_{1m_1}(x)\right), \quad (11)$$

where $x_2, x_3, \cdots, x_t$ solves

[2^{nd} *Level*]

$$\underbrace{min}_{x_2} F_2(x) = \underbrace{min}_{x_2} \left(f_{21}(x),\; f_{22}(x),\; \cdots,\; f_{2m_2}(x)\right), \quad (12)$$

$\vdots$

where x_t solves

[t^{th} *Level*]

$$\underbrace{min}_{x_t} F_t(x) = \underbrace{min}_{x_t} \left(f_{t1}(x),\; f_{t2}(x),\; \cdots,\; f_{tm_t}(x)\right), \quad (13)$$

subject to

$$x \in G = \left\{ x \in R^n \;\middle|\; \sum_{j=1}^{n} \underline{a}_{ij}^L x_j \leq (a-c),\; x_j = 0,\; a, c \in R^m \right\} \quad (14)$$

where

$$f_{ij}(x) = \frac{N_{ij}(x)}{D_{ij}(x)} = \frac{\sum_{j=1}^{m_i} (\underline{c}_{ij}^L - \overline{c}_{ij}^L) x_j + (\underline{\alpha}_{ij}^L - \overline{\alpha}_{ij}^L)}{\sum_{j=1}^{m_i} (\underline{d}_{ij}^L - \overline{d}_{ij}^L) x_j + \underline{\beta}_{ij}^L - \overline{\beta}_{ij}^L)}, \quad (15)$$

$i = 1, 2, \cdots, t.$

And find $X^{*(a-c)} = (x_1^{*(a-c)}, x_2^{*(a-c)}, \cdots, x_n^{*(a-c)})$

FP3:

$[1^{st}\ Level]$

$$\underbrace{min}_{x_1} F_1(x) = \underbrace{min}_{x_1}(f_{11}(x),\ f_{12}(x),\ \cdots,\ f_{1m_1}(x)), \tag{16}$$

where $x_2, x_3, \cdots, x_t$ *solves*

$[2^{nd}\ Level]$

$$\underbrace{min}_{x_2} F_2(x) = \underbrace{min}_{x_2}(f_{21}(x),\ f_{22}(x),\ \cdots,\ f_{2m_2}(x)), \tag{17}$$

$\vdots$

where x_t *solves*

$[\,t^{th}\ Level]$

$$\underbrace{min}_{x_t} F_t(x) = \underbrace{min}_{x_t}(f_{t1}(x),\ f_{t2}(x),\ \cdots,\ f_{tm_t}(x)), \tag{18}$$

subject to

$$x \in G = \left\{ x \in R^n \;\middle|\; \begin{array}{c} \sum_{j=1}^{n}\left(\underline{a}_{ij}^{U} - \underline{a}_{ij}^{L}\right)x_j \le (b-a),\ x_j = 0, \\ a, b \in R^m \end{array} \right\} \tag{19}$$

where

$$f_{ij}(x) = \frac{N_{ij}(x)}{D_{ij}(x)} = \frac{\sum_{j=1}^{m_i}\left(\underline{c}_{ij}^{U} - \underline{c}_{ij}^{L}\right)x_j + \left(\underline{\alpha}_{ij}^{U} - \underline{\alpha}_{ij}^{L}\right)}{\sum_{j=1}^{m_i}\left(\underline{d}_{ij}^{U} - \underline{d}_{ij}^{L}\right)x_j + \left(\underline{\beta}_{ij}^{U} - \underline{\beta}_{ij}^{L}\right)}, \tag{20}$$

$i = 1, 2, \cdots, t.$

$X^{*(b-a)} = (x_1^{*(b-a)}, x_2^{*(b-a)}, \cdots, x_n^{*(b-a)})$

FP4:

$[1^{st}\ Level]$

$$\underbrace{min}_{x_1} F_1(x) = \underbrace{min}_{x_1}(f_{11}(x),\ f_{12}(x),\ \cdots,\ f_{1m_1}(x)), \tag{21}$$

where $x_2, x_3, \cdots, x_t$ *solves*

$[2^{nd}\ Level]$

$$\underbrace{min}_{x_2} F_2(x) = \underbrace{min}_{x_2}(f_{21}(x),\ f_{22}(x),\ \cdots,\ f_{2m_2}(x)), \tag{22}$$

$\vdots$

where x_t *solves*

$[\,t^{th}\ Level]$

$$\underbrace{min}_{x_t} F_t(x) = \underbrace{min}_{x_t}(f_{t1}(x),\ f_{t2}(x),\ \cdots,\ f_{tm_t}(x)), \tag{23}$$

subject to

$$x \in G = \left\{ x \in R^n \;\middle|\; \begin{array}{c} \sum_{j=1}^{n}(\overline{a}_{ij}^{U} - \underline{a}_{ij}^{U})x_j \le (d-b),\ x_j = 0, \\ b, d \in R^m \end{array} \right\} \tag{24}$$

where

$$f_{ij}(x) = \frac{N_{ij}(x)}{D_{ij}(x)} = \frac{\sum_{j=1}^{m_i}(\overline{c}_{ij}^{U} - \underline{c}_{ij}^{U})x_j + (\overline{\alpha}_{ij}^{U} - \underline{\alpha}_{ij}^{U})}{\sum_{j=1}^{m_i}(\overline{d}_{ij}^{U} - \underline{d}_{ij}^{U})x_j + (\overline{\beta}_{ij}^{U} - \underline{\beta}_{ij}^{U})}, \tag{25}$$

$i = 1, 2, \cdots, t.$

And find $X^{*(d-b)} = (x_1^{*(d-b)}, x_2^{*(d-b)}, \cdots, x_n^{*(d-b)})$

Using the shifting method proposed by Osman and El-sherbiny in [20] then,

$$X^{*a} = X^{*c} + X^{*a-c}, \tag{26a}$$

$$X^{*b} = X^{*a} + X^{*b-a}, \tag{26b}$$

$$X^{*d} = X^{*??} + X^{*d-b} \tag{26c}$$

For solving the previous classical four (ML-MOFP) problems simultaneously, the fuzzy goal programming approach will be applied. The linearization procedure introduced by pal et.al in [1] will be applied to linearize the membership goals.

2.1 Fuzzy Goal Programming Approach for (ML-MOFP) Problems

The vector of objective functions for each decision maker is formulated as a fuzzy goal characterized by the membership function $\mu_{(f_{ij})}, (i = 1, 2, \cdots, t),\ (j = 1, 2, \cdots, m_i)$, at each level.

Characterization of Membership Functions

To define the membership functions of the fuzzy goals each objective function's individual maximum is taken as the corresponding aspiration level, as follows [5, 11]:

$$u_{ij} = \underbrace{max}_{x \in G}(f_{ij}(x)), \quad (i = 1, 2, \cdots, t), (\ j = 1, 2, \cdots, m_i). \tag{27}$$

where $u_{ij},\ (i = 1, 2, \cdots, t),\ \ (j = 1, 2, \cdots, m_i)$, give the upper tolerance limit or aspired level of achievement for the membership function of ij^{th} objective function. Similarly, each objective function's individual minimum is taken as the corresponding aspiration level, as follows:

$$g_{ij} = \underbrace{min}_{x \in G} \left(f_{ij}(x) \right), \quad (i = 1, 2, \cdots, t), (j = 1, 2, \cdots, m_i). \tag{28}$$

where $g_{?-j}$, $(i = 1, 2, \cdots, t)$, $(j = 1, 2, \cdots, m_i)$, give the lower tolerance limit or lowest acceptable level of achievement for the membership function of ij^{th} objective function. It can be assumed reasonably the values of $(f_{ij}(x)) = u_{ij}$, $(i = 1, 2, \cdots, t)$, $(j = 1, 2, \cdots, m_i)$, are acceptable and all values less than $g_{ij} = \underbrace{min}_{x \in G}(f_{ij}(x))$, are absolutely unacceptable. Then, the membership function $\mu_{ij}(f_{ij}(x))$, as shown in Fig(1.a), for the ij^{th} fuzzy goal can be formulated as [11]:

$$\mu_{f_{ij}}(f_{ij}(x)) = \begin{cases} 1, & if \ (f_{ij}(x)) \le g_{ij}, \\ \frac{u_{ij} - (f_{ij}(x))}{u_{ij} - -g_{ij}}, & if \ g_{ij} \le (f_{ij}(x)) \le u_{ij}, \\ 0, & if \ (f_{ij}(x)) \ge u_{ij}, \end{cases} \quad (i = 1, 2, \cdots, t), (j = 1, 2, \cdots, m_i), \tag{29}$$

2.2 Fuzzy Goal Programming Methodology

In the decision-making context, each decision maker is interested in maximizing his or her own objective function; the optimal solution of each DM, when calculated in isolation, would be considered as the best solution and the associated value of the objective function can be considered as the aspiration level of the corresponding fuzzy goal. In fuzzy programming approach, the highest degree of membership is one. For the defined membership functions in equation (29), the flexible membership goals having the aspired level unity can be represented as follows:

$$\mu_{f_{ij}}(f_{ij}(x)) + d_{ij}^- - d_{ij}^+ = 1, \quad (i = 1, 2, \cdots, t), \quad (j = 1, 2, \cdots, m_i), \tag{30}$$

or equivalently as:

$$\frac{u_{ij} - (f_{ij}(x))}{u_{ij} - -g_{ij}} + d_{ij}^- - d_{ij}^+ = 1, \quad (i = 1, 2, \cdots, t), \quad (j = 1, 2, \cdots, m_i), \tag{31}$$

where $d_{ij}^-, d_{ij}^+ \ge 0$ with $d_{ij}^- d_{ij}^+ = 0$, $(i = 1, 2, \cdots, t)$, $(j = 1, 2, \cdots, m_i)$ represent the under- and over- deviations, respectively, from the aspired levels [5].

In the methodology of goal programming, the under- and over-deviational variables are included in the achievement function for minimizing them depends on the type of the objective functions to be optimized. In the proposed FGP approach, the sum of under deviational variables is required to be minimized to achieve the aspired level. It may be noted that any over-deviation from a fuzzy goal indicates the full achievement of the membership value [5]. The equivalent proposed final (ML-MOFP) model for **(FP1)** can be formulated as follows:

$$min \ \ Z = \sum_{j=1}^{m_1} w_{1j}^+ d_{1j}^+ + \sum_{j=1}^{m_2} w_{2j}^+ d_{2j}^+ + \cdots + \sum_{j=1}^{m_t} w_{tj}^+ d_{tj}^+, \tag{32}$$

subject to

$$\frac{u_{ij} - (f_{ij}(x))}{u_{ij} - -g_{ij}} + d_{ij}^- - d_{ij}^+ = 1, \qquad (i = 1, 2, \cdots, t), (j = 1, 2, \cdots, m_i), \tag{33}$$

$$x_{ik} = x_{ik}^*, \qquad (i = 1, 2, \cdots, t-1), \quad (k = 1, 2, \cdots, n_i), \tag{34}$$

$$x \in G = \left\{ x \in R^n \,\middle|\, \sum_{j=1}^{n} \overline{a}_{ij}^L x_j \le c, \ x_j = 0, \ \ c \in R^m \right\} \tag{35}$$

$$d_{ij}^- d_{ij}^+ = 0, \ and \ d_{ij}^-, \ d_{ij}^+ \ge 0, \ (i = 1, 2, \cdots, t), \quad (j = 1, 2, \cdots, m_i), \tag{36}$$

where Z represents the achievement function consisting of the weighted under-deviational variables of the fuzzy goals. The numerical weights w_{ij}^- represent the relative importance of achieving the aspired levels of the respective fuzzy goals. To assess the relative importance of the fuzzy goals properly, the values of w_{ij}^- are determined as [5]:

$$w_{ij}^+ = \frac{1}{u_{ij} - g_{ij}}, \qquad (i = 1, 2, \cdots, t), (j = 1, 2, \cdots, m_i), \tag{37}$$

2.3 Linearization of Membership Goals

It can be easily noted that the membership goals in equations (33) are nonlinear in nature and this may needs difficult computational in the solution process. To avoid these problems, a linearization procedure is presented in this section as in [1]. The linearization process for the membership goals in (33) considering the expression of $f_{ij}(x)$ in equation (5) will be firstly introduced.

The ij^{th} membership goals can be presented as:

$$\mu_{f_{ij}}(f_{ij}(x)) + d_{ij}^{-} - d_{ij}^{+} = 1, \tag{38}$$

$$L_{ij}u_{ij} - L_{ij}f_{ij}(x) + d_{ij}^{-} - d_{ij}^{+} = 1, \quad \textit{where} \quad L_{ij} = \frac{1}{u_{ij} - g_{ij}}, \tag{39}$$

$$f_{ij}(x) = \frac{N_{ij}(x)}{D_{ij}(x)} = \frac{\sum_{j=1}^{m_i} \overline{c}_{ij}^{L} x_j + \overline{\alpha}_{ij}^{L}}{\sum_{j=1}^{m_i} \overline{d}_{ij}^{L} x_j + \overline{\beta}_{ij}^{L}}, \qquad i = 1, 2, \cdots, t. \tag{40}$$

using the expression of $f_{ij}(x)$, the above goal in equation (39) can be presented as:

$$L_{ij}u_{ij} - L_{ij}\frac{\left(\overline{c}_{ij}^{L}\right)x + \overline{\alpha}_{ij}^{L}}{\left(\overline{d}_{ij}^{L}\right)x + \overline{\beta}_{ij}^{L}} + d_{ij}^{-} - d_{ij}^{+} = 1, \tag{41}$$

$$L_{ij}u_{ij}\left[\left(\overline{d}_{ij}^{L}\right)x + \overline{\beta}_{ij}^{L}\right] - L_{ij}\left[\overline{c}_{ij}^{L}x + \overline{\alpha}_{ij}^{L}\right] + d_{ij}^{-}\left[\left(\overline{d}_{ij}^{L}\right)x + \overline{\beta}_{ij}^{L}\right] - d_{ij}^{+}\left[\left(\overline{d}_{ij}^{L}\right)x + \overline{\beta}_{ij}^{L}\right] = \left[\left(\overline{d}_{ij}^{L}\right)x + \overline{\beta}_{ij}^{L}\right],$$

$$-L_{ij}\left[\left(\overline{c}_{ij}^{L}\right)x + \overline{\alpha}_{ij}^{L}\right] + d_{ij}^{-}\left[\left(\overline{d}_{ij}^{L}\right)x + \overline{\beta}_{ij}^{L}\right] - d_{ij}^{+}\left[\left(\overline{d}_{ij}^{L}\right)x + \overline{\beta}_{ij}^{L}\right] = (1 - L_{ij}g_{ij})\left[\left(\overline{d}_{ij}^{L}\right)x + \overline{\beta}_{ij}^{L}\right],$$

$$-L_{ij}\left[\left(\overline{c}_{ij}^{L}\right)x + \overline{\alpha}_{ij}^{L}\right] + d_{ij}^{-}\left[\left(\overline{d}_{ij}^{L}\right)x + \overline{\beta}_{ij}^{L}\right] - d_{ij}^{+}\left[\left(\overline{d}_{ij}^{L}\right)x + \overline{\beta}_{ij}^{L}\right] = L_{ij}^{0}\left[\left(\overline{d}_{ij}^{L}\right)x + \overline{\beta}_{ij}^{L}\right],$$

where $L_{ij}^{0} = (1 - L_{ij}g_{ij})$,

$$\left[-L_{ij}\overline{c}_{ij}^{L} - L_{ij}^{0}(\overline{d}_{ij}^{L})\right]x + d_{ij}^{-}\left[\left(\overline{d}_{ij}^{L}\right)x + \overline{\beta}_{ij}^{L}\right] - d_{ij}^{+}\left[\left(\overline{d}_{ij}^{L}\right)x + \overline{\beta}_{ij}^{L}\right] = \left[L_{ij}\overline{\alpha}_{ij}^{L} + L_{ij}^{0}\overline{\beta}_{ij}^{L}\right],$$

$$C_{ij}x + d_{ij}^{-}\left[\left(\overline{d}_{ij}^{L}\right)x + \overline{\beta}_{ij}^{L}\right] - d_{ij}^{+}\left[\left(\overline{d}_{ij}^{L}\right)x + \overline{\beta}_{ij}^{L}\right] = G_{ij}, \tag{42}$$

Where

$$G_{ij} = L_{ij}\overline{\alpha}_{ij}^{L} + L_{ij}^{0}\overline{\beta}_{ij}^{L} \tag{43a}$$

and

$$C_{ij} = -L_{ij}\overline{c}_{ij}^{L} - L_{ij}^{0}(\overline{d}_{ij}^{L}),\ (i = 1, 2, \cdots, t),\ (j = 1, 2, \cdots, m_i) \tag{43b}$$

Thus, considering the method of variable change presented in [1] the goal expression in equation (40) can be linearized as follows.

By setting,

$$D_{ij}^{-} = d_{ij}^{-}\left[\left(\overline{d}_{ij}^{L}\right)x + \overline{\beta}_{ij}^{L}\right] \ \textit{and}\ D_{ij}^{+} = d_{ij}^{+}\left[\left(\overline{d}_{ij}^{L}\right)x + \overline{\beta}_{ij}^{L}\right], \tag{44}$$

Then the linear form of expression in equation (41) is obtained as:

$$C_{ij}x + D_{ij}^{-} - D_{ij}^{+} = G_{ij}, \tag{45}$$

with $D_{ij}^{-},\ D_{ij}^{+} \geq 0;\ \textit{and}\ D_{ij}^{-} \times\ D_{ij}^{+} = 0$ since $d_{ij}^{-},\ d_{ij}^{+} \geq 0$ and $\left(\overline{d}_{ij}^{L}\right)x + \overline{\beta}_{ij}^{L} > 0$. Now, it is noted that, minimization of d_{ij}^{+} means minimization of $D_{ij}^{+} = d_{ij}^{+}\left[\left(\overline{d}_{ij}^{L}\right)x + \overline{\beta}_{ij}^{L}\right]$ which is also a non-linear one. It may be noted that when the membership goal is fully achieved, $d_{ij}^{+} = 0$, and when its achievement is zero, $d_{ij}^{+} = 1$, are found in the solution [2,19]. So, involvement of $d_{ij}^{+} \leq 1$, in the solution leads to impose the following constraint to the model of the problem:

$$\frac{D_{ij}^{+}}{\left[\left(\overline{d}_{ij}^{L}\right)x + \overline{\beta}_{ij}^{L}\right]} \leq 1,\ \textit{that is} \qquad -\left(\overline{d}_{ij}^{L}\right)x + D_{ij}^{+} \leq \overline{\beta}_{ij}^{L} \tag{46}$$

Now, based on the simplest version of goal programming, the final proposed FGP model of the **(FP1)** becomes:

$$min \;\; Z = \sum_{j=1}^{m_1} w_{1j}^{+} d_{1j}^{+} + \sum_{j=1}^{m_2} w_{2j}^{+} d_{2j}^{+} + \cdots + \sum_{j=1}^{m_t} w_{tj}^{+} d_{tj}^{+}, \tag{47}$$

subject to

$$C_{ij}x + D_{ij}^{-} - D_{ij}^{+} = G_{ij}, \qquad (i = 1,2,\cdots,t), \; (j = 1,2,\cdots,m_i), \tag{48}$$

$$x_{ik} = x_{ik}^{*}, \qquad (i = 1,2,\cdots,t-1), \;\; (k = 1,2,\cdots,n_{\,i}), \tag{49}$$

$$-\left(\overline{d}_{ij}^{L}\right)x + D_{ij}^{+} \le \overline{\beta}_{ij}^{L}, \qquad (i = 1,2,\cdots,t), \;\; (j = 1,2,\cdots,m_i), \tag{50}$$

$$x \in G = \left\{ x \in R^n \;\middle|\; \sum_{j=1}^{n} \overline{a}_{ij}^{L} x_j \le c, \; x_j = 0, \;\; c \in R^m \right\} \tag{51}$$

$$D_{ij}^{-}, D_{ij}^{+} \ge 0, \qquad (i = 1,2,\cdots,t), \;\; (j = 1,2,\cdots,m_i), \tag{52}$$

Similarly, applying the linearization process of the membership goals considering the expression of $f_{ij}(x)$in equations (15),(20) and (25) and get the final proposed FGP model of the **(FP2),(FP3)** and **(FP4).**

3 Solution algorithm

Step (1): reformulate problem (1)-(5) into (FP1), (FP2), (FP3) and (FP4).

Step (2): For problem (FP1), Compute$u_{ij}, g_{ij}, w_{ij}^{-}, i = 1,2,\cdots,t, j = 1,\cdots,m_i$.

Step (3): Construct the membership function$\mu_{ij}(f_{ij}(x)), i = 1,2,\cdots,t, j = 1,\cdots,m_i$.

Step (4): Compute$C_{ij} and \;\; G_{ij}, i = 1,2,\cdots,t, j = 1,\cdots,m_i$ according to equation (42.a), (42.b).

Step (5): Do the linearization process for $\mu_{ij}(f_{ij}(x))$ according to equation (45).

Step (6): Put $i = 1$ in **FGP** model (46)-(52).

Step (7): Solve **FGP** model (46)-(52) to get $x_{1k} = x_{1k}^{*}, k = 1,2,\cdots,n_i$.

Step (8): put $i = i+1$ in **FGP** model (46)-(52) and go to step (7).

Step (9): If$i > t-1$, go to step (10), otherwise go to step (8).

Step (10): Solve **FGP** model (46)-(52) with $x_{ik} = x_{ik}^{*}, i = 1,2,\cdots,t-1, k = 1,2,\cdots,n_i$.

Step (11): If the DM solves (FP2), (FP3), and (FP4) go to step 13, otherwise go to step12.

Step (12): Repeat steps from (2) to (10) for (FP2), (FP3), and (FP4).

Step (13): Determine the solution according to the theorem proposed by Osman et al in[ ].

Step (14): Stop.

4 An Illustrative Example

To demonstrate the proposed FGP approach, consider the following $(ML-MOLFP)$ problem with fully rough intervals.

$[1^{st} \; Level]$

$$\underbrace{min}_{x_1}\left(\begin{array}{c} f_{11} = \dfrac{([2,3],[1,4])x_1 + ([5,7],[3,8])x_2 + ([3,5],[2,8])x_3 + ([5,7],[2,10])}{([4,5],[2,7])x_1 + ([7,9],[3,10])x_2 + ([2,5],[1,6])x_3 + ([5,9],[3,10])}, \\ f_{12} = \dfrac{([3,5],[2,7])x_1 + ([4,5],[3,6])x_2 + ([5,8],[3,9])x_3 + ([3,6],[1,8])}{([5,6],[3,8])x_1 + ([3,5],[2,6])x_2 + ([4,6],[3,7])x_3 + ([5,6],[2,9])} \end{array} \right),$$

$where \; x_2, x_3 solve \; [2^{nd} \; Level]$

$$\underbrace{min}_{x_2}\left(\begin{array}{c} f_{21} = \dfrac{([6,8],[3,10])x_1 + ([7,9],[4,11])x_2 + ([7,8],[5,12])x_3 + ([8,10],[5,13])}{([4,7],[2,9])x_1 + ([5,7],[3,10])x_2 + ([5,6],[2,9])x_3 + ([7,9],[3,12])}, \\ f_{22} = \dfrac{([5,7],[4,8])x_1 + ([6,8],[2,11])x_2 + ([3,5],[1,8])x_3 + ([5,7],[3,10])}{([5,7],[3,8])x_1 + ([3,4],[1,6])x_2 + ([2,3],[1,5])x_3 + ([6,8],[2,10])} \end{array} \right),$$

$where \; x_3 solves \; [3^{rd} \; Level]$

$$\underbrace{min}_{x_3}\left(\begin{array}{c} f_{21} = \dfrac{([3,5],[2,8])x_1 + ([5,7],[3,8])x_2 + ([2,3],[1,4])x_3 + ([8,9],[6,12])}{([2,3],[1,4])x_1 + ([3,5],[2,7])x_2 + ([4,5],[2,6])x_3 + ([6,8],[2,10])}, \\ f_{22} = \dfrac{([4,6],[3,8])x_1 + ([3,5],[2,9])x_2 + ([3,7],[2,10])x_3 + ([7,12],[4,16])}{([3,5],[1,8])x_1 + ([2,3],[1,5])x_2 + ([4,6],[3,7])x_3 + ([4,6],[2,9])} \end{array} \right),$$

subject to

$$([3,6],[1,7])x_1+([3,5],[2,6])x_2 \le ([12,14],[10,16]),$$
$$([3,5],[1,8])x_1-([5,7],[2,9])x_2+([6,7],[3,9])x_3 \le ([9,11],[8,14]),$$
$$([5,7],[2,9])x_1+([3,5],[1,8])x_2 \ge ([4,5],[3,6]),$$
$$x_1,x_2,x_3 \ge 0.$$

For solving the previous example, it will be reformulated into the following four linear fractional programming problems [20]:

FP1:	FP2:
$[1^{st}\ Level]$ $\underbrace{min}_{x_1}\left(\frac{x_1+3x_2+2x_3+2}{2x_1+3x_2+x_3+3},\frac{2x_1+3x_2+3x_3+1}{3x_1+2x_2+3x_3+2}\right)$, $where\ x_2,x_3\ \ solves$ $[2^{nd}\ Level]$ $\underbrace{min}_{x_2}\left(\frac{3x_1+4x_2+5x_3+5}{2x_1+3x_2+2x_3+3},\frac{4x_1+2x_2+x_3+3}{3x_1+x_2+x_3+2}\right)$, $where\ x_3\ \ solves$ $\underbrace{min}_{x_3}\left(\frac{2x_1+3x_2+x_3+6}{x_1+2x_2+2x_3+2},\frac{3x_1+2x_2+2x_3+4}{x_1+x_2+3x_3+2}\right)$,*subject to* $x_1+2x_2 \le 10,$ $x_1-2x_2+3x_3 \le 8,$ $2x_1+x_2 \ge 3,$ $x_1,x_2,x_3 \ge 0.$	$[1^{st}\ Level]$ $\underbrace{min}_{x_1}\left(\frac{x_1+2x_2+x_3+3}{2x_1+4x_2+x_3+2},\frac{x_1+x_2+2x_3+2}{2x_1+x_2+x_3+3}\right)$, $where\ x_2,x_3\ \ solves$ $[2^{nd}\ Level]$ $\underbrace{min}_{x_2}\left(\frac{3x_1+3x_2+2x_3+3}{2x_1+2x_2+3x_3+4},\frac{x_1+4x_2+2x_3+2}{2x_1+2x_2+x_3+4}\right)$, $where\ x_3\ \ solves$ $\underbrace{min}_{x_3}\left(\frac{x_1+2x_2+x_3+2}{x_1+x_2+2x_3+4},\frac{x_1+x_2+x_3+3}{2x_1+x_2+x_3+2}\right)$,*subject to* $2x_1+x_2 \le 2,$ $2x_1-3x_2+3x_3 \le 1,$ $3x_1+2x_2 \ge 1,$ $x_1,x_2,x_3 \ge 0.$

FP3:	FP4:
$[1^{st}\ Level]$ $\underbrace{min}_{x_1}\left(\frac{x_1+2x_2+2x_3+2}{x_1+2x_2+3x_3+4},\frac{2x_1+x_2+3x_3+3}{x_1+2x_2+2x_3+1}\right)$, $where\ x_2,x_3\ \ solves$ $[2^{nd}\ Level]$ $\underbrace{min}_{x_2}\left(\frac{2x_1+2x_2+x_3+2}{3x_1+2x_2+x_3+2},\frac{2x_1+2x_2+2x_3+2}{2x_1+x_2+x_3+2}\right)$, $where\ x_3\ \ solves$ $\underbrace{min}_{x_3}\left(\frac{2x_1+2x_2+x_3+1}{x_1+2x_2+x_3+2},\frac{2x_1+2x_2+4x_3+5}{2x_1+x_2+2x_3+2}\right)$,*subject to* $3x_1+2x_2 \le 2,$ $2x_1-2x_2+x_3 \le 2,$ $2x_1+2x_2 \ge 1,$ $x_1,x_2,x_3 \ge 0.$	$[1^{st}\ Level]$ $\underbrace{min}_{x_1}\left(\frac{x_1+x_2+3x_3+3}{2x_1+x_2+x_3+1},\frac{2x_1+x_2+x_3+2}{2x_1+x_2+x_3+3}\right)$, $where\ x_2,x_3\ \ solves$ $[2^{nd}\ Level]$ $\underbrace{min}_{x_2}\left(\frac{2x_1+2x_2+4x_3+3}{2x_1+3x_2+3x_3+3},\frac{x_1+3x_2+3x_3+3}{x_1+2x_2+2x_3+2}\right)$, $where\ x_3\ \ solves$ $\underbrace{min}_{x_3}\left(\frac{3x_1+x_2+x_3+3}{x_1+2x_2+x_3+2},\frac{2x_1+4x_2+3x_3+4}{3x_1+2x_2+x_3+3}\right)$,*subject to* $x_1+x_2 \le 2,$ $3x_1-2x_2+2x_3 \le 3$ $2x_1+3x_2 \ge 1,$ $x_1,x_2,x_3 \ge 0.$

For solving (FP1), the individual maximum and minimum values are summarized in Table 1. The decided aspiration levels, upper tolerance limitsand the weights w_{ij} are also considered.

Table 1: individual maximum, minimum values, $u_{ij}g_{ij}$ and weights w_{ij}.

	$f_{11}(x)$	$f_{12}(x)$	$f_{21}(x)$	$f_{22}(x)$	$f_{31}(x)$	$f_{32}(x)$
$max\ (f_{ij}(x))$	1.22	1.333333	1.967742	1.857143	2.571429	2.8
$min\ (f_{ij}(x))$	0.526315	0.6153846	1.3888889	1.288462	1.125	1.017544
u_{ij}	1.2	1.3	1.9	1.8	2.5	2.8
g_{ij}	0.5	0.6	1.3	1.2	1.1	1.01
w_{ij}	1.4	1.4	1.6	1.6	0.7	0.55

The coefficient of the linearized membership goals are presented in Table 2.

Table 2: the coefficient of the linearized membership goals $(C_{ij})^T$ *and* G_{ij}

	$f_{11}(x)$	$f_{12}(x)$	$f_{21}(x)$	$f_{22}(x)$	$f_{31}(x)$	$f_{32}(x)$
$(C_{ij})^T$	$\begin{pmatrix} -0.04 \\ -2.16 \\ -2.12 \end{pmatrix}^T$	$\begin{pmatrix} -0.34 \\ -2.56 \\ -1.74 \end{pmatrix}^T$	$\begin{pmatrix} -0.72 \\ -0.28 \\ -3.92 \end{pmatrix}^T$	$\begin{pmatrix} -0.76 \\ -1.32 \\ 0.28 \end{pmatrix}^T$	$\begin{pmatrix} -0.65 \\ -0.6 \\ 0.8 \end{pmatrix}^T$	$\begin{pmatrix} -1.11 \\ -0.56 \\ 0.52 \end{pmatrix}^T$
G_{ij}	0.76	-0.24	1.88	1.04	2.7	1.12

Solving the 1st level FGP model:

$$minZ = 1.4D_{11}^{+} + 1.4D_{12}^{+}$$

subject to

$$-0.04x_1 - 2.16x_2 - 2.12x_3 + D_{11}^{-} - D_{11}^{+} = 0.76,$$
$$-0.34x_1 - 2.56x_2 - 1.74x_3 + D_{12}^{-} - D_{12}^{+} = -0.24,$$
$$-2x_1 - 3x_2 - x_3 + D_{11}^{+} \leq 3,$$
$$-3x_1 - 2x_2 - 3x_3 + D_{12}^{+} \leq 2,$$
$$x_1 + 2x_2 \leq 10,$$
$$x_1 - 2x_2 + 3x_3 \leq 8,$$
$$2x_1 + x_2 \geq 3,$$
$$x_1, x_2, x_3, D_{11}^{-},\ D_{11}^{+},\ D_{12}^{-},\ D_{12}^{+} \geq 0$$

Using Lingo software package [25], the optimal solution of the upper level problem is obtained as; $(x_1^0, x_2^0, x_3^0) = (1.5, 4.25, 0)$.

Solving the 2nd level FGP model:

$$minZ = 1.4D_{11}^{+} + 1.4D_{12}^{+} + 1.6D_{21}^{+} + 1.6D_{22}^{+}$$

subject to

$$-0.04x_1 - 2.16x_2 - 2.12x_3 + D_{11}^{-} - D_{11}^{+} = 0.76,$$
$$-0.34x_1 - 2.56x_2 - 1.74x_3 + D_{12}^{-} - D_{12}^{+} = -0.24,$$
$$-0.72x_1 - 0.28x_2 - 3.92x_3 + D_{21}^{-} - D_{21}^{+} = 1.88,$$
$$-1.303x_1 - 1.716x_2 + 1.367x_3 + D_{22}^{-} - D_{22}^{+} = 1.04,$$
$$-2x_1 - 3x_2 - x_3 + D_{11}^{+} \leq 3,$$
$$-3x_1 - 2x_2 - 3x_3 + D_{12}^{+} \leq 2,$$
$$-2x_1 - 3x_2 - 2x_3 + D_{21}^{+} \leq 3,$$
$$-3x_1 - x_2 - x_3 + D_{12}^{+} \leq 2,$$
$$2x_1 + x_2 \leq 10,$$
$$x_1 - 2x_2 + 3x_3 \leq 8,$$
$$x_1 + 2x_2 \geq 3,$$
$$x_1 = 1.5,$$
$$x_2, x_3, D_{11}^{-},\ D_{11}^{+},\ D_{12}^{-},\ D_{12}^{+}, D_{21}^{-},\ D_{21}^{+}, D_{22}^{-},\ D_{22}^{+} \geq 0.$$

Using Lingo software package [25], the optimal solution of the second level problem is obtained as: $(x_1^0, x_2^0, x_3^0) = (1.5, 4.25, 0)$.

Solving the 3rd level FGP model:

$$minZ = 1.4D_{11}^{+} + 1.4D_{12}^{+} + 1.6D_{21}^{+} + 1.6D_{22}^{+} + 0.7D_{31}^{+} + 0.55D_{32}^{+}$$

subject to

$$-0.04x_1 - 2.16x_2 - 2.12x_3 + D_{11}^{-} - D_{11}^{+} = 0.76,$$

$$
\begin{aligned}
-0.34x_1-2.56x_2-1.74x_3+D_{12}^{-}-D_{12}^{+}&=-0.24,\\
-0.72x_1-0.28x_2-3.92x_3+D_{21}^{-}-D_{21}^{+}&=1.88,\\
-1.303x_1-1.716x_2+1.367x_3+D_{22}^{-}-D_{22}^{+}&=1.04,\\
-0.65x_1-0.6x_2+0.8x_3+D_{31}^{-}-D_{31}^{+}&=2.7,\\
-1.11x_1-0.56x_2+0.52x_3+D_{32}^{-}-D_{32}^{+}&=1.12,\\
-2x_1-3x_2-x_3+D_{11}^{+}&\le 3,\\
-3x_1-2x_2-3x_3+D_{12}^{+}&\le 2,\\
-2x_1-3x_2-2x_3+D_{21}^{+}&\le 3,\\
-3x_1-x_2-x_3+D_{22}^{+}&\le 2,\\
-x_1-2x_2-2x_3+D_{31}^{+}&\le 2,\\
-x_1-x_2-3x_3+D_{32}^{-}&\le 2,\\
2x_1+x_2&\le 10,\\
x_1-2x_2+3x_3&\le 8,\\
x_1+2x_2&\ge 3,\\
x_1&=1.5,\\
x_2&=4.25,\\
x_3, D_{11}^{-},\ D_{11}^{+},\ D_{12}^{-},\ D_{12}^{+}, D_{21}^{-},\ D_{21}^{+}, D_{22}^{-},\ D_{22}^{+}&\ge 0.
\end{aligned}
$$

Using Lingo software package [25], the optimal solution of the third level problem is obtained as: $\left(x_1^{*c},x_2^{*c},x_3^{*c}\right)=\left(\frac{3}{2},\frac{17}{4},0\right)$.

Similarly, applying the proposed algorithm to solve (FP2), (FP3) and (FP4), we get the following results:

$$\left(x_1^{*(a-c)},x_2^{*(a-c)},x_3^{*(a-c)}\right)=\left(\frac{1}{3},\frac{4}{3},0\right),$$

then, $\left(x_1^{*a},x_2^{*a},x_3^{*a}\right)=\left(\frac{11}{6},\frac{67}{12},0\right)$,

$$\left(x_1^{*(b-a)},x_2^{*(b-a)},x_3^{*(b-a)}\right)=(0.5,0,0),$$

then, $\left(x_1^{*b},x_2^{*b},x_3^{*b}\right)=\left(\frac{7}{3},\frac{67}{12},0\right)$,

$$\left(x_1^{*(d-b)},x_2^{*(d-b)},x_3^{*(d-b)}\right)=(0,2,0),$$

and $\left(x_1^{*d},x_2^{*d},x_3^{*d}\right)=\left(\frac{7}{3},\frac{91}{12},0\right)$,

Then the problem has a rough optimal solution in the form:

$$\left(\frac{3}{2},\frac{17}{4},0\right),\left(\frac{11}{6},\frac{67}{12},0\right),\left(\frac{7}{3},\frac{67}{12},0\right),\left(\frac{7}{3},\frac{91}{12},0\right).$$

and the following rough optimum values:

$$f_{11}=\{0.7115072934,\ 0.7485311398,\ 0.7830342577, 0.866666667\}$$

$$f_{12}=\{0.9513043478,\ 0.9544419143,\ 0.9571428571, 0.9973045822\}$$

$$f_{21}=\{1.003062787,\ 1.2250970246,\ 1.3747534517,\ 1.413333333\}$$

$$f_{22}=\{1.4571428571,\ 1.4934725849,\ 1.5056179775,\ 1.6279069767\}$$

$$f_{31}=\{1.2301495972,\ 1.3922330097,\ 1.5678233438,\ 1.8125\}$$

$$f_{32}=\{1.5040322581,\ 1.5665859564,\ 1.5921219822,\ 2.1935483871\}$$

5 Conclusion and summary

Multi-level multi-objective fractional programming problem (ML-MOFP) was considered where allthe coefficients in the objective functions and in constraints are rough intervals. Two FP problems with interval coefficients were constructed. One of these problems was a FP where all of its coefficients are lower approximations of the rough intervals and the other problem was a FP where all of its coefficients are upper approximations of rough intervals. A fuzzy goal programming model has been formulated to obtain the satisfactory solution of the multi-level multi-objective fractional programming problem. At the end, there exist many other open points for future work and research which should be explored and studied in the area of multi- level multi-objective rough interval optimization such as:

1. An algorithm is required for treating multi-level multi-objective integer fractional decision-making problems with rough parameters in the objective functions; in the constraints and in both.
2. An algorithm is needed for dealing with multi- level multi-objective mixed integer fractional decision-making problems with rough parameters in the objective functions; in the constraints and in both.
3. An algorithm must be investigated for treating multi- level multi-objectiveinteger quadratic decision-making problems with rough parameters in the objective functions; in the constraints and in both.

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Mohamed Sayed Ali Osman He is a professor of mathematics and operations research; he is a vice president of El Asher University. He has published several papers in both national and International journal. He is a referee and editor of several international journals in the field of Mathematical Programming and operations research. He interested in Convex and generalized convex Programming, parametric programming, Fuzzy and Rough mathematical Programming.

Osama Ezeldin Emam He is a Professor of operations research & Prof. of Mathematics and Vice Dean of Faculty of Computers and Information, Helwan University, Egypt. He has published several papers in both national and International journal. His research interest is in the Multi-Level Optimization field.

Kamal Raslan Mohamed He is a Professor of mathematics, Department of Mathematics, faculty of science Al-Azhar University, Cairo, Egypt. He has published several papers in both national and International journal. His research interest is in Applied Mathematics, Numerical Analysis.

Farahat Abd Allah Farahat He is an assistant lecturer at department of basic science, higher technological institute, 10th of Ramadan city. His research interest is in multi-level multi-objective rough fractional programming problems and goal programming problems.