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# Effects of viscosity at the interface of viscous fluid and micropolar elastic honeycomb

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The effect of viscosity of an infinite fluid lying over the surface of micropolar honeycomb half space has been investigated due to a concentrated source. The expressions of displacement components, microrotation, force stress and couple stress are obtained in the transformed domain after applying the integral transforms. A numerical inversion method has been used to obtain the resulting expressions in physical domain. As a special case the deformation caused due to a moving load has been deduced from the present problem by changing some parameters. The numerical results are presented graphically. Some particular cases have also been derived and these results have been compared with already established results.

**Keywords:** Micropolar, honey comb, microrotation, Couple stress, integral transforms.

## 1 Introduction

Materials with a cellular microstructure are frequently found in nature. In industry, synthetic cellular materials are increasingly used to make light and stiff structures or structures that need to absorb energy during their service lifetime. Cellular materials have several other attractive nonstructural features such as excellent dielectric properties and low thermal and electric conductivities. These features allow different conductive grades to be manufactured. For the purpose of analysing mechanical behaviour, these materials can be broadly categorized as two-dimensional cellular solids (honeycombs) or as three-dimensional cellular structures (foams).

Modern engineering structures are often made up of materials possessing an internal structure. Polycrystalline materials, materials with fibrous or coarse grain structure come in this category. Classical elasticity is inadequate to represent the behavior of such materials. The analysis of such materials requires incorporating the theory of oriented media. For this reason, micropolar theories were developed by Eringen (1966a, 1966b) for elastic solids, fluids and further for non-local polar. A micropolar continuum is a collection of interconnected particles in the form of small rigid bodies undergoing both translational and rotational motions.

The plane deformations of a honeycomb can involve both bending and stretching of cell walls. Thus at the perimeter of any cell in a regular honeycomb there are bending moments as well as normal and shear forces acting on the cell walls. For a homogenized continuum model of the honeycomb, these stress resultants on the cell boundary translate to couple stresses in addition to normal and shear stresses. Gibson and Ashby (1988) have summarized the mechanical properties of a variety of honeycombs and foams. Investigations into the effects of elastic properties of non-periodic honeycombs were discussed by Silva et al. (1995). Klintworth and Stronge 1988) studied elastic buckling and plastic collapse of metallic hexagonal honeycombs and concluded that yielding occurs at stresses below those given by the plastic analysis. Wang and Stronge (1999) developed a micropolar theory for two dimensional stresses in elastic honeycombs.

Papka and Kyriakides (1994, 1998) have examined the quasi-static compressive crushing and response of metallic and polycarbonate honeycombs subjected to both in-plane uniaxial and biaxial compressive loading. Triplett and Schonberg (1998) discussed the effect of differences in honeycomb material properties on static and dynamic response. Chung and Waas (2002) derived the sensitivities of the in-plane macroscopic linear stiffness of perfectly elliptical-cell honeycombs to geometric imperfections through an analytical method and a finite-element-based numerical solution. Huang, Yan and Yang (2002) investigated the relation between the Poisson's ratio of a re-entrant honeycomb structure by varying the micropolar material constants. Liang and Chen (2006) investigated the collapse of a sandwich panel or beam with a square cell honeycomb. Mora and Waas (2007) studied the micropolar elastic representation of a honeycomb structure using the configuration of a thick plate with a rigid circular inclusion, and in conjunction with experimental measurements of the deformation response. Chung and Waas (2009) derived a set of expressions for the characterization of circular cell honeycombs as micropolar elastic solids.

The deformation at any point of the medium is useful to analyze the deformation field around mining tremors and drilling into the crust of the earth. It can also contribute to the theoretical consideration of the seismic and volcanic sources since it can account for the deformation fields in the entire volume surrounding the source region.

In the present paper we determine the components of displacement, microrotation and stresses in a micropolar honeycomb half-space with an overlying viscous fluid due to concentrated source acting along the interface of two media. The solution is obtained by introducing potential functions after employing integral transformation technique. The problem of moving load is derived from the investigation by suitable change of parameters. The deformation due to other sources such as strip loads, time harmonic loads, etc. can also be similarly obtained.

## 2 Formulation of the problem

We consider a homogeneous, micropolar honeycomb solid half-space (medium I) with an overlying viscous fluid (medium II). A rectangular coordinate system $(x, y, z)$ having origin on the surface $y = 0$ and $y-$ axis pointing vertically into medium I is considered. A normal point force is assumed to be acting at the origin along the interface on the $y-$axis.

The stress-strain equations in two dimensional micropolar theory are written as (Eringen; 1966, 1968),

$$[\sigma] = \begin{bmatrix} t_{11} \\ t_{22} \\ t_{12} \\ t_{21} \\ m_{13} \\ m_{23} \end{bmatrix} = [D][\varepsilon] \tag{2.1}$$

where

$$[\varepsilon] = [\varepsilon_{11}, \varepsilon_{22}, \varepsilon_{12}, \varepsilon_{21}, \phi_{3,1}, \phi_{3,2}]^T, \tag{2.2}$$

$t_{11}, t_{22}, t_{12},$ and $t_{21}$ are the force stresses, $m_{13}$ and $m_{23}$ are couple stresses, $\varepsilon_{11}, \varepsilon_{22}, \varepsilon_{12}, \varepsilon_{21}$ are micro-strain tensor and $\phi_3$ is the microrotation vector.

The strain-displacement relations are as follows (Yang and Huang, 2001),

$$[\varepsilon] = \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{12} \\ \varepsilon_{21} \\ \phi_{3,1} \\ \phi_{3,2} \end{bmatrix} = \begin{bmatrix} \frac{\partial u_1}{\partial x} \\ \frac{\partial u_2}{\partial y} \\ \frac{\partial u_2}{\partial x} - \phi_3 \\ \frac{\partial u_1}{\partial y} + \phi_3 \\ \frac{\partial \phi_3}{\partial x} \\ \frac{\partial \phi_3}{\partial y} \end{bmatrix} \tag{2.3}$$

where where $u_1$ and $u_2$ are the displacements in the $x$ and $y$ directions.

The $[D]$ matrix, in equation (1) is defined as material property matrix expressed as follows:

In the micropolar plane strain problem, $\varepsilon_{11}, \varepsilon_{22}, \varepsilon_{12}, \varepsilon_{21}$ and $\phi_3$ are the only non-zero

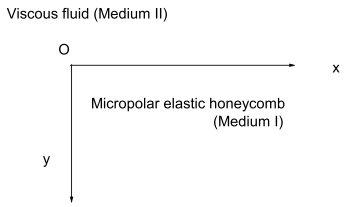


Figure 2.1: Geometry of the problem

strains in equation (3).

$$[D] = \begin{bmatrix} \lambda+2\mu^*+K & \lambda & 0 & 0 & 0 & 0 \\ \lambda & \lambda+2\mu^*+K & 0 & 0 & 0 & 0 \\ 0 & 0 & \mu^*+K & \mu^* & 0 & 0 \\ 0 & 0 & \mu^* & \mu^*+K & 0 & 0 \\ 0 & 0 & 0 & 0 & \gamma & 0 \\ 0 & 0 & 0 & 0 & 0 & \gamma \end{bmatrix} \tag{2.4}$$

where $\lambda, \mu^*, K, \alpha$ and $\beta$ are micropolar constants and have the relation $\mu = \mu^* + K/2$ (Cowin, 1970).

Following Fehler (1982), the equations of motion in a viscous medium are,

$$(K^0 + \frac{4}{3}\eta\frac{\partial}{\partial t})\nabla(\nabla.\overrightarrow{V}) - \eta\frac{\partial}{\partial t}\nabla(\nabla \times \overrightarrow{V}) = \rho^0\frac{\partial^2 \overrightarrow{V}}{\partial t^2}, \tag{2.5}$$

where $K^0$ is the bulk modulus, $\eta$ is the fluid viscosity, $\rho^0$ is the fluid density and $\overrightarrow{V}$ is the displacement in viscous medium.

The stress and displacement relation in viscous medium are given by

$$\tau_{mn} = (K^0 - \frac{2}{3}\eta\frac{\partial}{\partial t})V_{k,k}\delta_{mn} + \eta\frac{\partial}{\partial t}(V_{m,n} + V_{n,m}).m, n = 1, 2.3. \tag{2.6}$$

Using (3) and (4) in equation (1) and using the equation of motion,

$$t_{ji,j} = \rho\ddot{u}_i, \tag{2.7}$$

we obtain the equations of motion for a micropolar honeycomb structure in two dimensional form as,

$$(\lambda+2\mu^*+K)\frac{\partial^2 u_1}{\partial x^2} + (\mu^*+K)\frac{\partial^2 u_1}{\partial y^2} + (\lambda+\mu^*)\frac{\partial^2 u_2}{\partial x \partial y} + K\frac{\partial \phi_3}{\partial y} = \rho\frac{\partial^2 u_1}{\partial t^2}, \tag{2.8}$$

$$(\mu^*+K)\frac{\partial^2 u_2}{\partial x^2}+(\lambda+2\mu^*+K)\frac{\partial^2 u_2}{\partial y^2}+(\lambda+\mu^*)\frac{\partial^2 u_1}{\partial x\partial y}-K\frac{\partial\phi_3}{\partial x}=\rho\frac{\partial^2 u_2}{\partial t^2}, \quad (2.9)$$

$$\gamma\nabla^2\phi_3+K(\frac{\partial u_2}{\partial x}-\frac{\partial u_1}{\partial y})-2K\phi_3=\rho j\frac{\partial^2\phi_3}{\partial t^2}. \quad (2.10)$$

## 3 Solution of the Equations

The displacement components in medium I are related to the potential functions $q(x,y,t)$ and $\psi(x,y,t)$ as

$$u_1=\frac{\partial q}{\partial x}+\frac{\partial\psi}{\partial y}, \quad u_2=\frac{\partial q}{\partial y}-\frac{\partial\psi}{\partial x}. \quad (3.1)$$

Using (11) in equations (8)-(10), we obtain

$$[\nabla^2-\frac{\rho}{(\lambda+2\mu^*+K)}\frac{\partial^2}{\partial t^2}]q=0, \quad (3.2)$$

$$[\nabla^2-\frac{\rho}{(\mu^*+K)}\frac{\partial^2}{\partial t^2}]\psi+\frac{K}{(\mu^*+K)}\phi_3=0, \quad (3.3)$$

$$[\nabla^2-\frac{2K}{\gamma}-\frac{\rho j}{\gamma}\frac{\partial^2}{\partial t^2}]\phi_3-\frac{K}{\gamma}\nabla^2\psi=0, \quad (3.4)$$

Introducing dimensionless variables defined by

$$x'=\frac{x}{h}, \quad y'=\frac{y}{h}, \quad \phi_3'=\frac{j}{h^2}\phi_3, \quad q'=\frac{q}{h^2}, \quad \psi'=\frac{\psi}{h^2}, \quad [t_{ij},\tau_{ij}]'=\frac{[t_{ij},\tau_{ij}]}{\lambda},$$

$$m_{ij}'=\frac{m_{ij}}{\lambda h}, \quad t'=\frac{c_1}{h}t, \quad v'=\frac{v}{h^2}, \quad \varphi=\frac{\varphi}{h^2}, \quad F'=\frac{F}{\lambda}, \quad (3.5)$$

where $h$ is a parameter having dimension of length $c_1^2=\frac{\lambda^*+2\mu^*+K}{\rho}$, in equations (12)-(14) and then applying the Laplace transform with respect to time '$t$' defined by

$$[\bar{q},\bar{\psi},\bar{\phi_3}](x,y,p)=\int_o^\infty e^{-pt}[q,\psi,\phi_3](x,y,t)dt, \quad (3.6)$$

and then the Fourier transform with respect to '$x$' defined by

$$[\tilde{q},\tilde{\psi},\tilde{\phi_3}](\xi,y,p)=\int_{-\infty}^\infty e^{\imath\xi x}[\bar{q},\bar{\psi},\bar{\phi_3}](x,y,p)dx, \quad (3.7)$$

on the resulting equations we get,

$$[\frac{d^2}{dy^2}-(\xi^2+p^2)]\tilde{q}=0, \quad (3.8)$$

$$[\frac{d^2}{dy^2}-(\xi^2+\frac{\rho c_1^2}{\mu^*+K}p^2)]\tilde{\psi}+\frac{Kh^2}{j(\mu^*+K)}\tilde{\phi_3}=0, \quad (3.9)$$

$$[\frac{d^2}{dy^2} - (\xi^2 + \frac{2Kh^2}{\gamma} + \frac{\rho j c_1^2}{\gamma} p^2)]\tilde{\phi_3} - \frac{Kj}{\gamma}[\frac{d^2}{dy^2} - \xi^2]\tilde{\psi} = 0, \tag{3.10}$$

Eliminating $\tilde{\phi_3}$ from (19) and (20), we get

$$[\frac{d^4}{dy^4} + A^* \frac{d^2}{dy^2} + B^*]\tilde{\psi} = 0, \tag{3.11}$$

where

$$A^* = -(a_{11} + a_{13} - a_{12}a_{14}), \quad B^* = a_{11}a_{13} - a_{12}a_{14}\xi^2,$$

$$a_{11} = \xi^2 + \frac{\rho c_1^2}{\mu^* + K} p^2. \quad a_{12} = \frac{Kh^2}{j(\mu^* + K)},$$

$$a_{13} = \xi^2 + \frac{2Kh^2}{\gamma} + \frac{\rho j c_1^2}{\gamma} p^2, \quad a_{14} = \frac{Kj}{\gamma}. \tag{3.12}$$

The solutions of equations (18) and (21) satisfying the radiation conditions are

$$\tilde{q} = D_1 exp(-q_1 y), \tag{3.13}$$

$$\tilde{\psi} = D_2 exp(-q_2 y) + D_3 exp(-q_3 y), \tag{3.14}$$

$$\tilde{\phi_3} = a_2^* D_2 exp(-q_2 y) + a_3^* D_3 exp(-q_3 y), \tag{3.15}$$

where

$$q_1^2 = \xi^2 + p^2, \quad q_{2,3}^2 = \frac{-A^* \pm \sqrt{A^{*2} - 4B^*}}{2},$$

$$a_{2,3}^* = \frac{1}{a_{12}}(a_{11} - q_{2,3}^2). \tag{3.16}$$

Similarly we obtain the solution for equations of viscous medium (medium II) as

$$\tilde{v} = D_4 exp(q_4 y), \tag{3.17}$$

$$\tilde{\varphi} = D_5 exp(q_5 y), \tag{3.18}$$

where $\tilde{v}(x, y, t)$ and $\tilde{\varphi}(x, y, t)$ are potential functions in the viscous medium such that

$$V_1 = \frac{\partial v}{\partial x} + \frac{\partial \varphi}{\partial y}, \quad V_2 = \frac{\partial v}{\partial y} - \frac{\partial \varphi}{\partial x}. \tag{3.19}$$

and

$$q_4^2 = \xi^2 + \frac{3\rho^0 c_1^2 p^2 h}{3K_0 h + 4\eta c_1 p}, \quad q_5^2 = \xi^2 + \frac{\rho^0 c_1^2 p^2 h}{\eta c_1}. \tag{3.20}$$

## 4 Boundary conditions

We consider viscous fluid/micropolar elastic honeycomb interface at which a concentrated normal load is acting at the origin of the Cartesian coordinate system. Mathematically the boundary conditions at the interface $y = 0$ are given by,

$$t_{22} = \tau_{22} - F\delta(x)\delta(t). \quad t_{21} = \tau_{21}, \quad m_{23} = 0, \quad u_1 = V_1, \quad u_2 = V_2. \tag{4.1}$$

where $\delta()$ is Dirac delta function.

Using the dimensionless quantities defined by (15) on the boundary conditions (31), we obtain the boundary conditions in dimensionless form with primes. After suppressing the primes for convenience and applying Laplace and Fourier transform defined by (16) and (17) on the resulting dimensionless boundary conditions, we obtain the boundary conditions in the transformed domain.

With the help of (1)-(4), (6), (11), (15), (23)-(25), (27) and (28) in the transformed boundary conditions, we obtain the expressions for displacement components, microrotation, force stress and tangential couple stress as,

$$\tilde{u_1} = \frac{F}{\Delta}[i\xi\Delta_1 e^{-q_1 y} - q_2\Delta_2 e^{-q_2 y} + q_3\Delta_3 e^{-q_3 y}] \tag{4.2}$$

$$\tilde{u_2} = \frac{F}{\Delta}[q_1\Delta_1 e^{-q_1 y} + i\xi(\Delta_2 e^{-q_2 y} - \Delta_3 e^{-q_3 y})] \tag{4.3}$$

$$\tilde{\phi_3} = \frac{F}{\Delta}[a_2^*\Delta_2 e^{-q_2 y} - a_3^*\Delta_3 e^{-q_3 y})] \tag{4.4}$$

$$\tilde{t_{21}} = -\frac{F}{\Delta}[s_1\Delta_1 e^{-q_1 y} - s_2\Delta_2 e^{-q_2 y} + s_3\Delta_3 e^{-q_3 y}] \tag{4.5}$$

$$\tilde{t_{22}} = -\frac{F}{\Delta}[r_1\Delta_1 e^{-q_1 y} - r_2\Delta_2 e^{-q_2 y} + r_3\Delta_3 e^{-q_3 y}] \tag{4.6}$$

$$\tilde{m_{23}} = -\frac{F\gamma}{j\lambda\Delta}[a_2^* q_2\Delta_2 e^{-q_2 y} - a_3^* q_3\Delta_3 e^{-q_3 y}] \tag{4.7}$$

where

$$\Delta = \sum_{i=1}^{4}(-1)^{i+1} f_i g_i, \quad \Delta_1 = a_3^* q_3 f_{12} - a_2^* q_2 f_{11}, \quad \Delta_2 = a_3^* q_3 f_{21},$$

$$\Delta_3 = a_2^* q_2 f_{21}, \quad f_1 = a_2^* q_2 r_3 - a_3^* q_3 r_2, \quad f_2 = a_2^* q_2 s_3 - a_3^* q_3 s_2,$$

$$f_3 = q_2 q_3 (a_2^* - a_3^*), \quad f_4 = i\xi(a_2^* q_2 - a_3^* q_3),$$

$$g_1 = s_1(\xi^2 - q_4 q_5) + s_4(\xi^2 + q_1 q_5) + i\xi s_5(q_1 + q_4),$$

$$g_2 = r_1(\xi^2 - q_4 q_5) + r_4(\xi^2 + q_1 q_5) + i\xi r_5(q_1 + q_4),$$

$$g_3 = r_1(i\xi s_4 - q_4 s_5) - r_4(i\xi s_1 - q_1 s_5) - r_5(q_1 s_4 - q_4 s_1),$$

$$g_4 = r_1(i\xi s_5 + q_5 s_4) - r_4(s_1 q_5 - i\xi s_5) - i\xi r_5(s_1 + s_4),$$

$$f_{11} = s_3(\xi^2 - q_4 q_5) - q_3(i\xi s_4 - q_4 s_5) - i\xi(s_4 q_5 + i\xi s_5),$$

$$f_{12} = s_2(\xi^2 - q_4 q_5) - q_2(i\xi s_4 - q_4 s_5) - i\xi(s_4 q_5 + i\xi s_5),$$

$$f_{21} = s_1(\xi^2 - q_4 q_5) + s_4(\xi^2 + q_1 q_5) + i\xi s_5(q_1 + q_4),$$

$$r_1 = -\xi^2 + q_1^2 \frac{\lambda + 2\mu^* + K}{\lambda}, \quad r_{2,3} = -i\xi \frac{(2\mu^* + K)}{\lambda} q_{2,3},$$

$$r_4 = \frac{1}{\lambda}[(K^0 + \frac{2}{3h}\eta c_1 p)q_4^2 + \xi^2(\frac{2}{3h}\eta c_1 p - K^0)]$$

$$r_5 = -\frac{2i\xi\eta c_1 p}{\lambda h} q_5, \quad s_1 = i\xi q_1 \frac{(2\mu^* + K)}{\lambda},$$

$$s_{2,3} = \frac{1}{\lambda}[\mu^* \xi^2 + (\mu^* + K)q_{2,3}^2 + \frac{Kh^2}{j} a_{2,3}^*]$$

$$s_4 = \frac{2i\xi\eta c_1 p}{\lambda h} q_4, \quad s_5 = -\frac{\eta c_1 p}{\lambda h}(\xi^2 + q_5^2). \tag{4.8}$$

## 5 Particular cases

### 5.1

Neglecting micropolarity effect in medium I i.e $\alpha = \beta = \gamma = K = j = 0, \mu^* = \mu$ ,we obtain the components of displacement and stresses for an elastic medium at viscous fluid/elastic solid half-space interface as,

$$\tilde{u_1} = \frac{F}{\Delta^*}[i\xi \Delta_1^* e^{-q_1' y} - q_2' \Delta_2^* e^{-q_2' y}] \tag{5.1}$$

$$\tilde{u_2} = \frac{F}{\Delta^*}[q_1' \Delta_1^* e^{-q_1' y} + i\xi \Delta_2^* e^{-q_2' y}] \tag{5.2}$$

$$\tilde{t_{21}} = -\frac{F}{\Delta^*}[s_1^* \Delta_1^* e^{-q_1' y} - s_2^* \Delta_2^* e^{-q_2' y}] \tag{5.3}$$

$$\tilde{t_{22}} = -\frac{F}{\Delta^*}[r_1^* \Delta_1^* e^{-q_1' y} - r_2^* \Delta_2^* e^{-q_2' y}] \tag{5.4}$$

where

$$\Delta^* = \sum_{i=1}^{6}(-1)^{i+1} f_i^*, \quad \Delta_1^* = s_2^*(\xi^2 - q_4 q_5) - q_2'(i\xi s_4 - q_4 s_5) - i\xi(s_4 q_5 + i\xi s_5),$$

$$\Delta_2^* = s_1^*(\xi^2 - q_4 q_5) + q_1'(i\xi s_5 + s_4 q_5) - i\xi(i\xi s_4 - s_5 q_4),$$

$$f_1^* = (\xi^2 - q_4 q_5)(r_1^* s_2^* - r_2^* s_1^*), \quad f_2^* = (i\xi s_4 - q_4 s_5)(r_1^* q_2' - i\xi r_2^*),$$

$$f_3^* = (r_2^* q_1' - i\xi r_1^*)(s_4 q_5 + i\xi s_5), \quad f_4^* = (\xi^2 - q_1' q_2')(s_4 r_5 - s_5 r_4),$$

$$f_5^* = (i\xi s_1^* + q_1' s_2^*)(r_4 q_5 + i\xi r_5), \quad f_6^* = (q_2' s_1^* - i\xi s_2^*)(r_5 q_4 - i\xi r_4),$$
$$q_1'^2 = \xi^2 + \frac{\rho c_1^2 p^2}{\lambda + 2\mu}, \quad q_2'^2 = \xi^2 + \frac{\rho c_1^2 p^2}{\mu}, \quad s_1^* = \frac{2i\xi\mu}{\lambda} q_1',$$
$$s_2^* = \frac{\mu}{\lambda}(\xi^2 + q_2'^2), \quad r_1^* = \frac{(\lambda + 2\mu)}{\lambda} q_1'^2 - \xi^2, \quad r_2^* = -\frac{2i\xi\mu}{\lambda} q_2'. \tag{5.5}$$

**5.2**

Neglecting viscous effect in medium II, we obtain the components of displacement, microrotation and stresses for a micropolar elastic honeycomb at non-viscous fluid/ micropolar elastic honeycomb half-space interface as,

$$\tilde{u_1} = \frac{F}{\Delta^{**}}[i\xi\Delta_1^{**}e^{-q_1 y} - q_2\Delta_2^{**}e^{-q_2 y} + q_3\Delta_3^{**}e^{-q_3 y}] \tag{5.6}$$

$$\tilde{u_2} = \frac{F}{\Delta^{**}}[q_1\Delta_1^{**}e^{-q_1 y} + i\xi(\Delta_2^{**}e^{-q_2 y} - \Delta_3^{**}e^{-q_3 y})] \tag{5.7}$$

$$\tilde{\phi}_3 = \frac{F}{\Delta^{**}}[a_2^*\Delta_2^{**}e^{-q_2 y} - a_3^*\Delta_3^{**}e^{-q_3 y})] \tag{5.8}$$

$$\tilde{t_{21}} = -\frac{F}{\Delta^{**}}[s_1\Delta_1^{**}e^{-q_1 y} - s_2\Delta_2^{**}e^{-q_2 y} + s_3\Delta_3^{**}e^{-q_3 y}] \tag{5.9}$$

$$\tilde{t_{22}} = -\frac{F}{\Delta^{**}}[r_1\Delta_1^{**}e^{-q_1 y} - r_2\Delta_2^{**}e^{-q_2 y} + r_3\Delta_3^{**}e^{-q_3 y}] \tag{5.10}$$

$$\tilde{m_{23}} = -\frac{F\gamma}{j\lambda\Delta^{**}}[a_2^* q_2\Delta_2^{**}e^{-q_2 y} - a_3^* q_3\Delta_3^{**}e^{-q_3 y}] \tag{5.11}$$

where

$$\Delta^{**} = \sum_{i=1}^{3}(-1)^{i+1} f_i^{**}, \quad \Delta_1^{**} = q_4'(a_2^* q_2 s_3 - a_3^* q_3 s_2),$$
$$\Delta_2^{**} = -a_3^* q_3 s_1 q_4', \quad \Delta_3^{**} = -a_2^* q_2 s_1 q_4',$$
$$f_1^{**} = s_1 q_4'(a_3^* q_3 r_2 - a_2^* q_2 r_3), \quad f_2^{**} = (r_1 q_4' + r_4^* q_1)(a_3^* q_3 s_2 - a_2^* q_2 s_3),$$
$$f_3^{**} = i\xi s_1 r_4^*(a_2^* q_2 - a_3^* q_3), \quad q_4'^2 = \xi^2 + \frac{\rho^0 c_1^2 p^2}{K^0}, \quad r_4^* = \frac{K^0}{\lambda}(\xi^2 - q_4'^2). \tag{5.12}$$

**5.3**

Neglecting micropolarity effect in medium I and viscous effect in medium II, we obtain the components of displacement and stresses for an elastic solid at non-viscous fluid/ elastic solid half-space interface as,

$$\tilde{u_1} = \frac{F}{\Delta^{***}}[i\xi\Delta_1^{***}e^{-q_1' y} - q_2'\Delta_2^{***}e^{-q_2' y}] \tag{5.13}$$

$$\tilde{u_2} = \frac{F}{\Delta^{***}}[q_1'\Delta_1^{***}e^{-q_1' y} + i\xi\Delta_2^{***}e^{-q_2' y}] \tag{5.14}$$

$$\tilde{t_{21}} = -\frac{F}{\Delta^{***}}[s_1^* \Delta_1^{***} e^{-q_1' y} - s_2^* \Delta_2^{***} e^{-q_2' y}] \tag{5.15}$$

$$\tilde{t_{22}} = -\frac{F}{\Delta^{***}}[r_1^* \Delta_1^{***} e^{-q_1' y} - r_2^* \Delta_2^{***} e^{-q_2' y}] \tag{5.16}$$

where

$$\Delta^{***} = s_1^*(r_2^* q_4' - i\xi r_4^*) - s_2^*(r_1^* q_4' + r_4^* q_1'), \quad \Delta_1^{***} = -s_2^* q_4'. \quad \Delta_2^{***} = s_1^* q_4'. \tag{5.17}$$

**5.4**

Neglecting micropolarity effect in medium I and letting $K^0, \eta, \rho^0 \rightarrow 0$ in medium II, we obtain the transformed components due to a concentrated source acting on the free surface of elastic half-space.

## 6 Steady state response due to moving load at the interface of viscous fluid and micropolar elastic honeycomb.

We consider a concentrated normal point load moving along the interface of viscous fluid (medium II) and micropolar elastic honeycomb (medium I). The rectangular cartesian coordinates are introduced having origin on the surface $y = 0$ and $y-$ axis pointing vertically into medium I. Let us consider a pressure pulse $P(x + Ut)$ which is moving with a constant velocity $U$ in the negative $x$ direction for an infinite long time so that a steady state prevails in the neighbourhood of the loading as seen by the observer moving with the load (Figure 2).

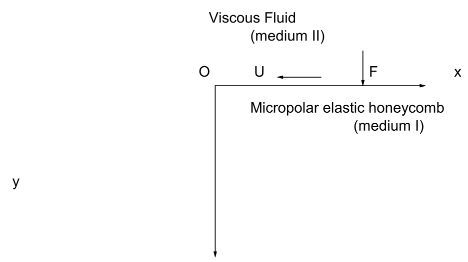


Figure 6.2: Steady state response at the interface

Using Galilean transformations (Fung, 1968) $x^* = x + Ut$, $y^* = y$, $t^* = t$ where $U$ is the magnitude of moving load velocity at the interface of viscous fluid and micropolar elastic honeycomb and introducing dimensionless quantities defined by (15) and applying Fourier transforms defined by (17) in the resulting equations, we obtain the results in case of moving load at the interface of viscous fluid and micropolar elastic honeycomb. The boundary conditions in this case are

$$t_{22} = \tau_{22} - F\delta(x^*)\delta(t). \quad t_{21} = \tau_{21}, \quad m_{23} = 0, \quad u_1 = V_1, \quad u_2 = V_2. \tag{6.1}$$

where $P(x + Ut) = F\delta(x^*)$.

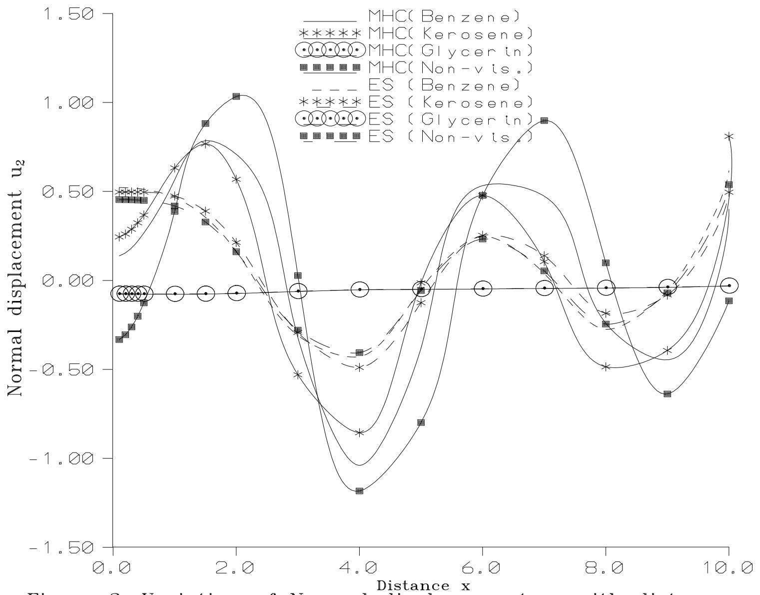

Figure 3: Variation of Normal displacement $u_2$ with distance x.
(Dynamic concentrated force)

By changing $p \rightarrow -i\xi\frac{U}{c_1}$ in the expressions (22), (26), (30), (38), (43) and (50), we obtain

(a) The expressions given by (32)-(37) for displacement, microrotation, stresses and couple stress for micropolar elastic honeycomb at viscous fluid/micropolar elastic honeycomb interface in case of moving normal point load. Kumar and Ailawalia (2005) obtained these expressions (taking $\mu^* = \mu$) for different load velocities.

(b) The transformed components given by (39)-(42) for displacement and stresses for an elastic medium at viscous fluid/elastic solid half-space interface due to moving point load at the interface.

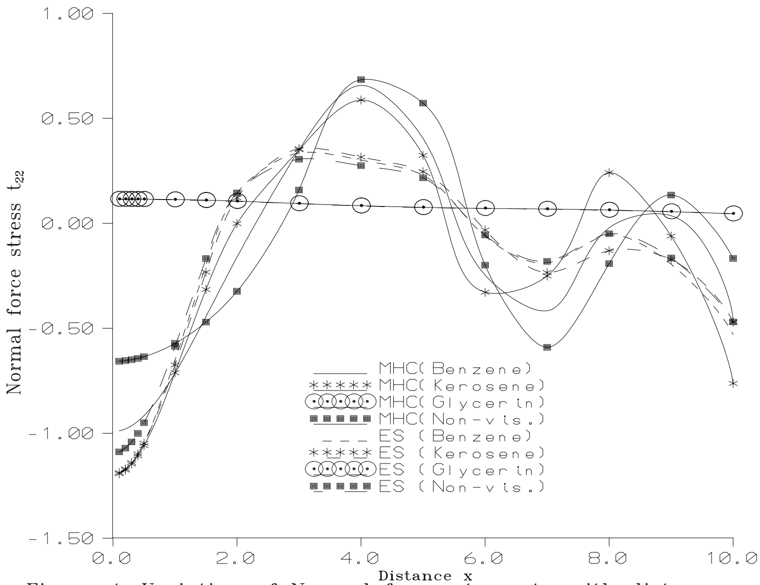


Figure 4: Variation of Normal force stress $t_{22}$ with distance x.
(Dynamic concentrated force)

(c) The expressions given by (44)-(49) for displacement, microrotation, stresses and couple stress for micropolar elastic honeycomb at non-viscous fluid/micropolar elastic honeycomb interface for a moving normal point load. Kumar and Ailawalia (2004) derived these expressions (taking $\mu^* = \mu$) for subsonic, transonic and supersonic load velocities.

(d) The components given by (51)-(54) for displacement and stresses for an elastic medium at non-viscous fluid/elastic solid half-space interface due to a moving point load along the interface of two media (Kennedy and Hermann, 1973).

(e) The transformed components due to a moving concentrated load acting on the free surface of elastic half-space after neglecting micropolarity effect in medium I and letting $K^0, \eta, \rho^0 \to 0$ in medium II (Cole and Huth, 1958).

## 7 Inversion of the transform

The transformed displacements, microrotation, stresses and couple stress are functions of $y$, the parameters of Laplace and Fourier transforms $p$ and $\xi$ respectively, and hence are of the form $\tilde{f}(\xi, y, p)$ . To get the function in the physical domain, we first invert the

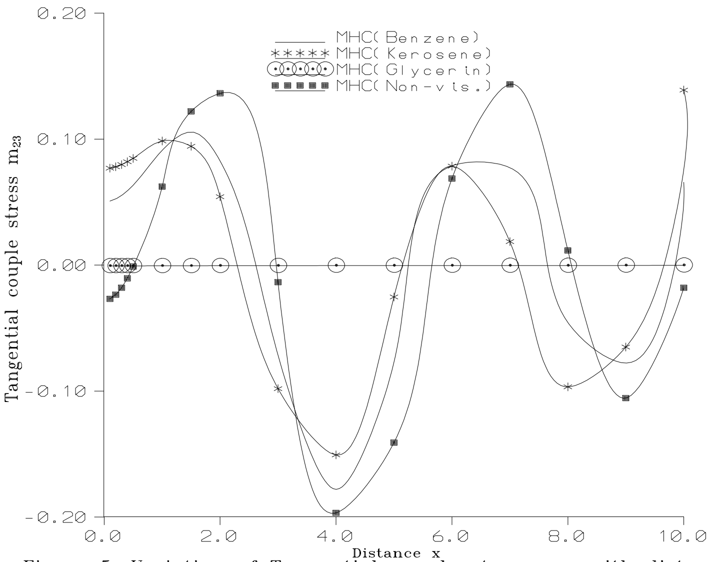


Figure 5: Variation of Tangential couple stress $m_{23}$ with distance x. (Dynamic concentrated force)

Fourier transform and then Laplace transform by using the method applied by Sharma and Kumar (1997).

## 8 Numerical results and discussions

For numerical calculations we take aluminium epoxy composite (Gauthier, 1982, pp.459) as micropolar elastic solid (medium I).

$$\rho = 2.19 \times 10^3 Kg/m^3. \quad \lambda = 7.59 \times 10^9 N/m^2, \quad \mu = 1.89 \times 10^9 N/m^2,$$

$$K = 0.0149 \times 10^9 N/m^2, \quad \gamma = 0.0268 \times 10^5 N, \quad j = 0.00196 \times 10^{-4} m^2,$$

Following White (2003) we take the physical constants for a viscous fluid as,

| Fluid | Density $\rho^0 (Kg/m^3)$ | Viscosity $\eta (Kg/msec)$ | Bulk Modulus $K^0 (N/m^2)$ |
|---|---|---|---|
| Benzene | 881 | 6.51x$10^{-4}$ | 1.4x$10^9$ |
| Kerosene | 804 | 1.92x$10^{-3}$ | 1.6x$10^9$ |
| Glycerin | 1260 | 1.49 | 4.34x$10^9$ |

The values of normal displacement $u_2$ , normal force stress $t_{22}$ and tangential couple stress $m_{23}$ for a micropolar elastic honeycomb(MHC) and elastic solid(ES) have been studied and the variations of these components with distance $x$ at the plane $y = 1.0$ , $F = 1.0$ and $h^2 = 1.0 \times 10^{-19} m^2$, for

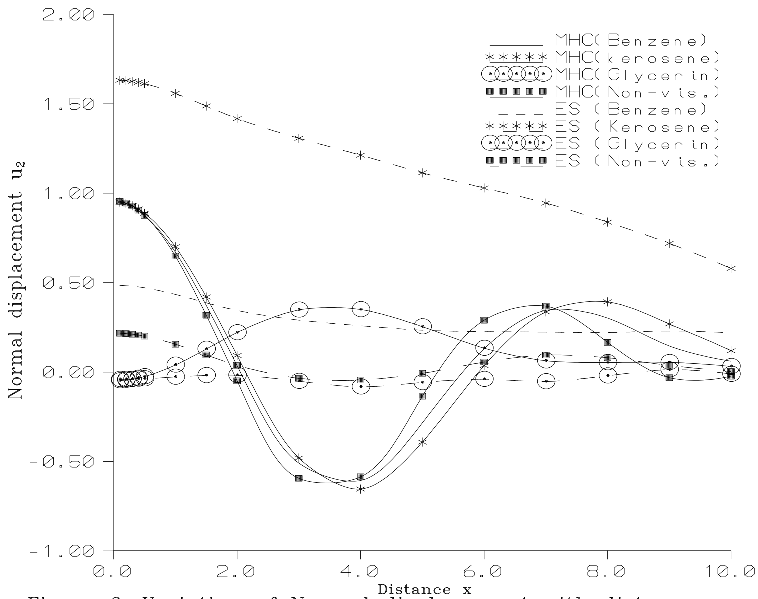


Figure 6: Variation of Normal displacement with distance x.
(Concentrated moving force)

(i) Micropolar elastic honeycomb (MHC) are shown by (a) solid line (—) with benzene as viscous fluid. (b) solid line with centered symbols (*) with kerosene as viscous fluid. (c) solid line with centered symbols ($\odot$) with glycerin as viscous fluid. (d) solid line with centered symbols ($\bullet$) with non-viscous fluid.

(ii) Elastic solid (ES) are shown by (a) dashed line (....) with benzene as viscous fluid. (b) dashed line with centered symbols (*) with kerosene as viscous fluid. (c) dashed line with centered symbols ($\odot$) with glycerin as viscous fluid. (d) dashed line with centered symbols ($\bullet$) with non-viscous fluid.

These variations are shown in Figure 3-8. The computations are carried out for $y = 1.0$ in the range $0 \le x \le 10.0$. In case of moving load, the calculations are carried out for the case when $U < c_1$.

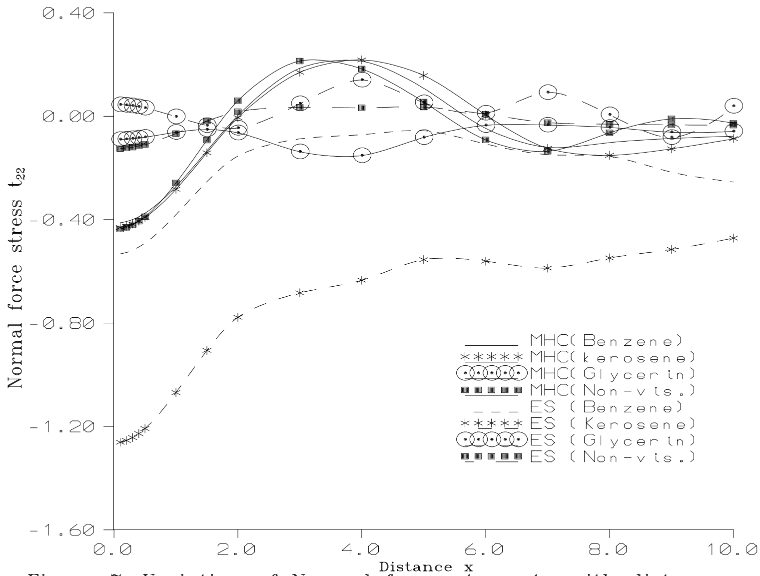


Figure 7: Variation of Normal force stress $t_{22}$ with distance x.
(Concentrated moving force)

# 9 Discussions for different cases

## 9.1 Dynamic load

The values of normal displacement for both MHC and ES lie in a very short range when a non-viscous fluid lies over the surface of solid. With increase in viscosity of fluid the variations becomes oscillatory in nature. There is not much difference in the values of normal displacement for the three types of viscous fluids considered in the problem. It could however be observed that the variations of normal displacement for an ES are less oscillatory as compared to the variations obtained for MHC. These variations of normal displacement are shown in figure 3. Similar to the discussions for normal displacement, the values of normal force stress are also less in magnitude for a non-viscous fluid lying over the surface of the medium (MHC and ES). The variations obtained in this case are however less oscillatory for a particular medium in comparison to the variations obtained for normal displacement. For an ES the values of normal force stress are very close to each other for different viscous fluids considered in the problem but this difference is somewhat significant in nature for MHC. These variations of normal force stress are shown in figure

4. It could easily be observed from figure 5 that the variations of tangential couple stress are quite similar to the variations of normal displacement with difference in magnitude.

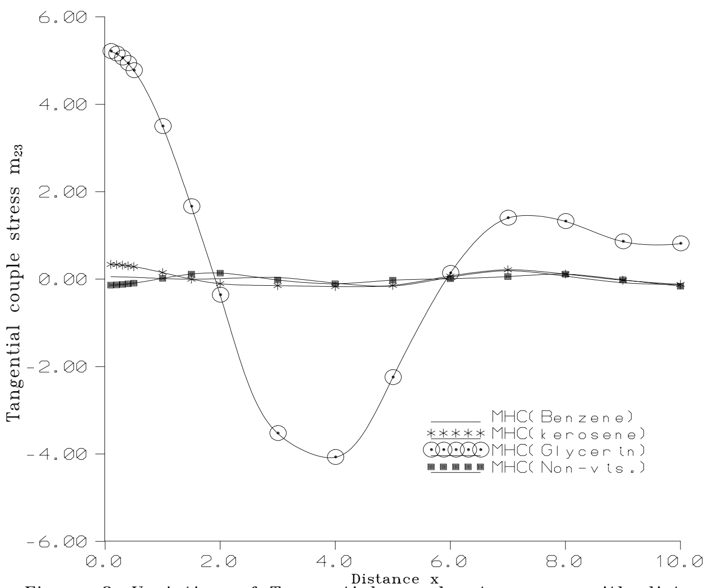


Figure 8: Variation of Tangential couple stress $m_{23}$ with distance x. (Concentrated moving force)

### 9.2 Moving load

The variations of normal displacement for ES are almost linear in nature when a low viscosity fluid lies over the surface of solid (Benzene and Kerosene). For MHC, the value of normal displacement, near the point of application of source, is almost identical for low viscosity fluids (Benzene, Kerosene and non-viscous fluid) but as the source moves away from the point of application, the difference between these values becomes significant. For a high viscous fluid (Glycerin) the variation of normal displacement for MHC is less oscillatory. These variations of normal displacement on application of moving load are shown in figure 6. The variations of normal force stress are similar to the variations of normal displacement but with opposite nature. The values of normal force stress for MHC (with low viscous fluid over the surface) first increase and then oscillate with increase in horizontal distance. Here also the variation of normal force stress with high viscous fluid lying over the solid (MHC) is less oscillatory. These variations of normal force stress are shown in figure 7.We may observe from figure 8 that the values of tangential couple stress lie in a short range with a non-viscous fluid and a low viscous fluid overlying the half-space. These variations are however highly oscillatory in nature when the source is moving with a constant velocity along the interface of a high viscous fluid (Glycerin) and elastic honeycomb.

## 10 Conclusion

The variations of all the quantities vary significantly with the viscosity of the fluid lying over the surface of solid. Also micropolarity effect and the nature of source applied along the interface of two media plays an important role in the study of deformation of a solid.

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