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Estimations in Step-stress Partially Accelerated Life Tests for the Compound Weibull-Gamma Distribution Under Type-I Censoring

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Abstract: In this paper, step-stress partially accelerated life tests SSPALT are applied when the lifetime of a product under design stress follows a 3-parameter compound Weibull-gamma distribution based on type-I censored samples. Maximum likelihood estimators MLEs for the considered parameters obtained for the distribution parameter and acceleration factor. Also, asymptotic variance and covariance matrix of the estimators given. Furthermore, confidence intervals of the estimators presented. Bayes estimators for the parameters are carried out based on (a) non-informative prior NIP for each parameter represented, (b) both symmetric loss (squared error loss) function and asymmetric LINEX loss function. The Bayes estimates cannot be obtained in explicit form, so we propose to use MCMC algorithm. Analysis of a simulated data set presented for illustrative purposes. Finally, a Monte Carlo simulation study is carried out to investigate the precision of the Bayes estimates with MLEs and to compare the performance of different corresponding confidence intervals considered.

Keywords: Step stress-Partially accelerated life test; 3-parameter compound Weibull-gamma distribution; Maximum likelihood method; Fisher information matrix; Type-I censoring; Bayes estimations; MCMC algorithm; Simulation.

1 Introduction

n : A total number of test items in a PALT.
T : A lifetime of an item at normal conditions.
Y : Total lifetime of an item in an SS-PALT.
y_i : The observed value of the total lifetime Y_i of item $i, i = 1, , n$.
η : The censoring time of a PALT.
β: Acceleration factor ($\beta > 1$).
τ: Stress change time in an SS-PALT ($\tau < \eta$).
δ_{1i}, δ_{2i}: Indicator functions in an SS-PALT $\delta_{1i} = I(Y_i \le \tau), \delta_{2i} = I(\tau < Y_i \le \eta)$.
n_u, n_a: The number of items failed at normal and accelerated conditions respectively.
$\lfloor$ (.): Evaluated at (.)
ALT: Accelerated life test.
OALT: Ordinary accelerated life test.
PALT: Partially accelerated life testing.
SSPALT: Step stress partially accelerated life test.
MCMC: Markov chain Monte Carlo.
MLE: Maximum likelihood estimate.
MSE: Mean square error.
RAB: Relative absolute bias.
CI: Confidence interval.

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BE: Bayes estimate.
NIP: Non-informative prior.

Under the continual improvement in the manufacturing design, it is difficult to obtain information about the lifetime of products with high reliability at the time of testing under normal conditions. They make the lifetime testing under normal conditions very costly and take a long time. For this reason, ALTs are preferred to be used in manufacturing industries to obtain enough failure data, in a short period. ALT is achieved by subjecting the test units to conditions that are more severe than the normal ones, such as higher levels of temperature, pressure, voltage, vibration, cycling rate, load, etc. Research on ALT has commenced in the 1950s to develop a more effective testing technique. Chernoff (1962) and Bessler et al (1962) introduced and studied the concept of ALT. Data collected at such accelerated conditions are then extrapolated through a physically appropriate statistical model to estimate the lifetime distribution under normal use conditions.

In ALT, the experiment can be started either at higher stresses than the normal ones and continued at these conditions or it can be stated in the normal conditions. So, there are two types of ALT. The first one is said to be the OALT, and the second one is called PALT. The major assumption in OALT is that the mathematical model relating the lifetime of the unit to the stress must be known or can be assumed. In some cases, such life stress relationships are not known and cannot be assumed, i.e. OALT data cannot be extrapolated to normal condition. So, in such cases, PALT is a more sui test to be performed for which tested units ws subjected to both normal and accelerated conditions. PALTs include two types, one is called step PALTs (see Abdel-Hamid and AL-Hussaini (2008)) and the other is called constant PALTs (see Abdel-Hamid (2009), Abushal and Soliman(2015).

The SSPALT (which considered in this research) permits the test to be changed from normal use condition to accelerated condition at a pre-specified time. Baia and Chung (1992) used the MLE to estimate the scale parameters and the acceleration factor for the exponential distribution lifetime using type-I censoring data, and in (1993) Bai et al. obtained the same results when the lifetime subjected to the log-normal distribution. SSPALT is used to get quick information for the lifetime of product with high reliability; specially, when the mathematical model related to test conditions of mean lifetime of the product is unknown and can't assume. The step stress scheme applies stress to test units in the way that the stress will be change in a prespecified time. Test unit starts at a specified low stress. If the unit does not fail at a specified time, stress on it raised and held a specified time. Stress is repeatedly increasing until test unit fails or censoring scheme reached.

There is some research regarding Bayesian and non-Bayesian estimation based on ALT, for example see Ahmad and Islam (1996), Attia et al (1996), Abdel-Ghaly et al (1997), Abdel-Ghani (1998), Abdel-Ghaly et al (2002a, 2002b), Abdel-Ghaly et al (2003), Abdel-Ghani (2004), Ismail (2004, 2006), Abd-Elfattah et al. (2008, 2009), Abushal and Soliman (2015), Abdel-Hamid and Abushal (2015). Recently there has been a considerable amount of interest in the predication problems, for more details, Abd-Elfattah and Al-Harbey (2010), Ismail (2010, 2014).

A wide variety of loss functions has been developed in literature to describe various types of loss structures. The first type is the squared error loss (SEL) function which is classified as a symmetric function and associates equal importance to the losses for overestimation and underestimation of equal magnitude. The second type was introduced by Varian (1975) is the linear exponential (LINEX) loss function. The LINEX loss function with parameter k and c is given by

$$L(u^{\approx}(\phi); u(\phi)) = k[e^{a(u^{\approx}(\phi)-u(\phi))} - c(u^{\approx}(\phi) - u(\phi)) - 1],$$

where k and c are constants. The sign and magnitude of c represent the direction and degree of symmetry, respectively. ($a > 0$ means overestimation is more serious than underestimation, and $a < 0$ means the opposite). Under LINEX loss function, the Bayes estimator $u^{\approx}(\phi)$ of $u(\phi)$ is given by

$$u^{\approx}(\phi) = -\frac{1}{a}\ln E(e^{-au(\phi)}), a \neq 0,$$

For a closed to zero, the LINEX loss is approximately squared error loss and therefore almost symmetric [Zellner (1986)].
The novelty of this research is to apply the SSPALTs to the 3-parameter compound Weibull-gamma distribution using type-I censored data and then estimate the parameters using ML and Bayes methods.

The organization of the rest of this paper is as follows. In section 2 the 3-parameter compound Weibull-gamma distribution is introduced as a lifetime model and the test method is also described. Section 3 presents point and interval estimates of parameters and acceleration factor for the 3-parameter compound Weibull-gamma distribution under type-I censoring using maximum likelihood method. Based on the SEL and LINEX loss functions, Bayesian estimation of the parameters obtained in Section 4. A simulation study and an illustrative example presented in section 5. Finally, conclusions included in section 6.

2 The Model Assumptions

This section introduces the assumed model for product life and also fully describes the test method.

2.1 The 3-parameter Compound WeibullGamma Distribution: As a Failure Time Model

The 3-parameter compound Weibull-gamma distribution was first proposed as a lifetime model by Mead (2007). This distribution can be specialized to many distributions such as 3-parameter Pareto, 2-paraemetr Burr XII, Lomax, Fisk and beta type II. Also, this distribution can be transformed to different distributions such as 3-parameter generalized Burr type III, 3-parameter Weibull, 3-parameter generalized logistic type I, 2-parameter Burr type II, 2-parameter extreme value, 2-parameter exponential, 2-parameter Rayleigh, beta type I and F-distribution. It has also been applied in areas of quality control, reliability studies, duration, and failure time modeling. The probability density function of the 3-parameter compound Weibull-gamma distribution is given as

$$f(x;c,q,\alpha) = c\alpha\frac{x^{c-1}}{q}[1+\frac{x^c}{q}]^{-(\alpha+1)}, x>0, c,q,\alpha>0 \tag{1}$$

Where,q is the scale parameter and (c,α) are the shape parameters. The cumulative distribution function(cdf)is

$$F(x;c,q,\alpha) = 1-[1+\frac{x^c}{q}]^{-\alpha}, x>0, c,q,\alpha>0, \tag{2}$$

The corresponding reliability function $R(x)$ and hazard rate $H(x)$ functions are

$$R(x;c,q,\alpha) = [1+\frac{x^c}{q}]^{-\alpha}, \tag{3}$$

and

$$H(x;c,q,\alpha) = c\alpha\frac{x^{c-1}}{q}[1+\frac{x^c}{q}]^{-1}. \tag{4}$$

2.2 The Test Method

In SSPALT, all of the n units are tested first under normal condition, if the unit does not fail for a pre-specified time τ, then it runs at accelerated condition until failure. This means that if the item has not failed by some pre-specified time τ, the test is switched to the higher level of stress and it is continued until items fail. The effect of this switch is to multiply the remaining lifetime of the item by the inverse of the acceleration factor β. In this case, switching to the higher stress level will shorten the life of the test item. Thus the total lifetime of a test item, denoted by Y, passes through two stages, which are the normal and accelerated conditions. Then the lifetime of the unit in SSPALT is given as follows:

$$Y = \begin{cases} T, & if \quad T\le\tau, \\ \tau+\frac{T-\tau}{\beta}, & if \quad T>\tau, \end{cases} \tag{5}$$

Where, T is the lifetime of an item at use condition, τ is the stress change time and β is the acceleration factor which is the ratio of mean life at use condition to that at accelerated condition, usually $\beta>1$. Assume that the lifetime of the test item follows the 3-parameter compound Weibull gamma distribution with q is the scale parameter and (c,α) are the shape parameters. Therefore, the probability density function of total lifetime Y of an item is given by:

$$f(y) = \begin{cases} 0, & if \quad y<0, \\ f_1(y), & if \quad 0\le y\le\tau, \\ f_2(y), & if \quad \tau<y<\infty, \end{cases} \tag{6}$$

where, $f_1(y;c,q,\alpha) = c\alpha\frac{y^{c-1}}{q}[1+\frac{y^c}{q}]^{-(\alpha+1)}, 0<y<\tau, c,q,\alpha>0$, is the equivalent form to equation (1), and, $f_2(y;c,q,\alpha) = c\alpha\beta\frac{[\beta(y-\tau)+\tau]^{c-1}}{q}[1+\frac{[\beta(y-\tau)+\tau]^c}{q}]^{-(\alpha+1)}, \tau<y<\infty, c,q,\alpha>0, \beta>1$, is obtained by the transformation variable technique using equations (1) and (5).

3 Maximum Likelihood Estimation

In this section point and interval estimation for the parameters and acceleration factor of 3-parameter compound Weibull-gamma distribution based on type-I censoring are evaluated using ML method.

3.1 Point Estimates

In type-I censoring, the test terminates when the censoring time η is reached. The observed values of the total lifetime Y are $y_1 < \ldots < y_{n_u} < \tau < y_{n_u+1} < \ldots < y_{(n_u+n_a)} \leq \eta$, where, n_u and n_a are the number of items failed at normal conditions and accelerated conditions respectively. Let δ_{1i} ,δ_{2i} be indictor function, such that

$$\delta_{1i} = \begin{cases} 1, & y_i \leq \tau, \qquad i = 1, 2, , n \\ 0, & otherwise, \end{cases}$$

and,

$$\delta_{2i} = \begin{cases} 1, & \tau < y_i \leq \eta, \qquad i = 1, 2, , n \\ 0, & otherwise, \end{cases}$$

For simplifying $y_{(i)}$ can be expressed by y_i. Since the lifetimes $y_1, \ldots., y_n$ of n items are independent and identically distributed random variables, then their likelihood function is given by

$$L(y|\beta, c, q, \alpha) = \prod_{i=1}^{n} [f_1(y_i)]^{\delta_{1i}} [f_2(y_i)]^{\delta_{2i}} [R_2(\eta)]^{\tilde{\delta_{1i}}\tilde{\delta_{2i}}}, \tag{7}$$

$$L(y|\beta, c, q, \alpha) = \prod_{i=1}^{n} [c\alpha \frac{(y_i)^{c-1}}{q} [1 + \frac{(y_i)^c}{q}]^{-(\alpha+1)}]^{\delta_{1i}} [c\alpha\beta \frac{(\beta(y_i - \tau) + \tau)^{c-1}}{q}$$
$$\times [1 + \frac{(\beta(y_i - \tau) + \tau)^c}{q}]^{-(\alpha+1)}]^{\delta_{2i}} [1 + \frac{(\beta(\eta - \tau) + \tau)^c}{q}]^{-\alpha\tilde{\delta_{1i}}\tilde{\delta_{2i}}}$$

where, $\tilde{\delta_{1i}} = 1 - \delta_{1i}$ and $\tilde{\delta_{2i}} = 1 - \delta_{2i}$
The logarithm of the likelihood function is

$$lnL = n_0 lnc + n_0 ln\alpha - n_0 lnq + n_a ln\beta + (c-1)[\sum_{i=1}^{n} \delta_{1i} ln(y_i) + \sum_{i=1}^{n} \delta_{2i} ln(A)]$$
$$-(\alpha+1)[\sum_{i=1}^{n} \delta_{1i} ln[1 + \frac{(y_i)^c}{q}] + \sum_{i=1}^{n} \delta_{2i} ln(A_1)] - \alpha(n - n_0) ln(D_1), \tag{8}$$

Where, $\sum_{i=1}^{n} \delta_{1i} = n_u$, $\sum_{i=1}^{n} \delta_{2i} = n_a$, $n_a + n_u = n_0$, $\sum_{i=1}^{n} \tilde{\delta_{1i}}\tilde{\delta_{2i}} = n - n_u - n_a = (n - n_0)$,
$A = [\tau + \beta(y_i - \tau)]$, $D = [\tau + \beta(\eta - \tau)]$, $D_1 = [1 + \frac{D^c}{q}]$ and $A_1 = [1 + \frac{A^c}{q}]$.
MLE of β, c, α and q are solutions to the system of equations obtained by letting the first partial derivatives of the total log likelihood to be zero with respect to β,c,α and q respectively. Therefore, the system of equations is as follows:

$$\frac{\partial lnL}{\partial\beta} = \frac{n_a}{\beta} + (c-1)[\sum_{i=1}^{n} \delta_{2i}(y_i - \tau)A^{-1} - c(\alpha+1)\sum_{i=1}^{n} \delta_{2i}(y_i - \tau)A^{c-1}(A_1 q)^{-1}$$
$$-\alpha c(n - n_0)(\eta - \tau)D^{c-1}(D_1 q)^{-1}, \tag{9}$$

$$\frac{\partial lnL}{\partial c} = \frac{n_0}{c} + \sum_{i=1}^{n} \delta_{1i} ln(y_i) + \sum_{i=1}^{n} \delta_{2i} ln(A) - \alpha(n - n_0)D^c ln(D)(D_1 q)^{-1}$$
$$-(\alpha+1)[\sum_{i=1}^{n} \delta_{1i}(y_i)^c ln(y_i)(Hq)^{-1} + \sum_{i=1}^{n} \delta_{2i} A^c ln(A)(A_1 q)^{-1}], \tag{10}$$

$$\frac{\partial lnL}{\partial q} = -\frac{n_0}{q} + (\alpha+1)[\sum_{i=1}^{n}\delta_{1i}(y_i)^c(Hq^2)^{-1} + \sum_{i=1}^{n}\delta_{2i}A^c(A_1q^2)^{-1}]$$
$$+\alpha(n-n_0)D^c(D_1q^2)^{-1}, \quad (11)$$

and

$$\frac{\partial lnL}{\partial \alpha} = \frac{n_0}{\alpha} - \sum_{i=1}^{n}\delta_{1i}ln(H) - \sum_{i=1}^{n}\delta_{2i}ln(A_1) - (n-n_0)ln(D_1), \quad (12)$$

From equation (12) the MLE of α is expressed by

$$\hat{\alpha} = \frac{n_0}{a_1} \quad (13)$$

where, $H = [1 + \frac{(y_i)^c}{q}]$ and $a_1 = \sum_{i=1}^{n}\delta_{1i}ln(H) - \sum_{i=1}^{n}\delta_{2i}ln(A_1) - (n-n_0)ln(D_1)$.
Consequently, by substituting for $\hat{\alpha}$ into equation (9), (10) and (11), the system equations are reduced into three nonlinear equation as follows:

$$\frac{n_a}{\hat{\beta}} + (\hat{c}-1)\sum_{i=1}^{n}\delta_{2i}(y_i-\tau)A^{-1} - \hat{c}(\frac{n_0}{a_1}+1)a_2 - a_3\hat{c}(\frac{n_0}{a_1})D^{\hat{c}-1}(D_1\hat{q})^{-1} = 0, \quad (14)$$

$$\frac{n_0}{\hat{c}} + \sum_{i=1}^{n}\delta_{1i}ln(y_i) + \sum_{i=1}^{n}\delta_{2i}ln(A) - (\frac{n_0}{a_1}+1)a_4 - (\frac{n_0}{a_1})(n-n_0)D^{\hat{c}}ln(D)(D_1\hat{q})^{-1} = 0, \quad (15)$$

$$-\frac{n_0}{\hat{q}} + (\frac{n_0}{a_1}+1)a_5 + (\frac{n_0}{a_1})(n-n_0)D^{\hat{c}}(D_1\hat{q}^2)^{-1} = 0, \quad (16)$$

where, $a_2 = \sum_{i=1}^{n}\delta_{2i}(y_i - \tau)A^{c-1}(A_1q)^{-1}$, $a_3 = (n - n_0)(\eta - \tau)$,
$a_4 = \sum_{i=1}^{n}\delta_{1i}(y_i)^c ln(y_i)(Hq)^{-1} + \sum_{i=1}^{n}\delta_{2i}A^c ln(A)(A_1q)^{-1}$ and
$a_5 = \sum_{i=1}^{n}\delta_{1i}(y_i)^c(Hq^2)^{-1} + \sum_{i=1}^{n}\delta_{2i}A^c(A_1q^2)^{-1}$
Equations (14) to (16) can't be solved analytically; statistical software can be used to solve these equations numerically. The asymptotic variance and covariance matrix of the MLE parameters can be approximated by numerically inverting the asymptotic Fisher-information matrix (F). The asymptotic Fisher information matrix (F) can be written as follows:

$$F = \begin{bmatrix} f_{11} & f_{12} & f_{13} & f_{14} \\ f_{21} & f_{22} & f_{23} & f_{24} \\ f_{31} & f_{32} & f_{33} & f_{34} \\ f_{41} & f_{42} & f_{43} & f_{44} \end{bmatrix} = \begin{bmatrix} \frac{-\partial^2 lnL}{\partial\beta^2} & \frac{-\partial^2 lnL}{\partial\beta\partial c} & \frac{-\partial^2 lnL}{\partial\beta\partial q} & \frac{-\partial^2 lnL}{\partial\beta\partial\alpha} \\ \frac{-\partial^2 lnL}{\partial c\partial\beta} & \frac{-\partial^2 lnL}{\partial c^2} & \frac{-\partial^2 lnL}{\partial c\partial q} & \frac{-\partial^2 lnL}{\partial c\partial\alpha} \\ \frac{-\partial^2 lnL}{\partial q\partial\beta} & \frac{-\partial^2 lnL}{\partial q\partial c} & \frac{-\partial^2 lnL}{\partial q^2} & \frac{-\partial^2 lnL}{\partial q\partial\alpha} \\ \frac{-\partial^2 lnL}{\partial\alpha\partial\beta} & \frac{-\partial^2 lnL}{\partial\alpha\partial c} & \frac{-\partial^2 lnL}{\partial\alpha\partial q} & \frac{-\partial^2 lnL}{\partial\alpha^2} \end{bmatrix} \downarrow (\hat{\beta}, \hat{c}, \hat{q}, \hat{\alpha}) \quad (1)$$

3.2 Approximate Confidence Interval

If $L_\phi = L_\phi(y_1, ..., y_n)$ and $U_\phi = U_\phi(y_1, ..., y_n)$ are function of the sample data $y_1, ..., y_n$ then a CI for a population parameter ϕ is given by

$$P[L_\phi \le \phi \le U_\phi] = \gamma,$$

where, L_ϕ and U_ϕ are the lower and upper confidence limits which enclose ϕ with probability γ. The interval $[L_\phi, U_\phi]$ is called a 100γ % CI for ϕ.
For large sample size, the MLE, under appropriate regularity conditions, are consistent and asymptotically normally distributed. Therefore, the approximate 100γ% confidence limits for the MLE $\hat{\phi}$ of a population parameter ϕ can be constructed, such that

$$P[-Z \le \frac{\hat{\phi}-\phi}{\sigma(\hat{\phi})} \le Z] = \gamma,$$

where, Z is the $[\frac{100(1-\gamma)}{2}]$ standard normal percentile. Therefore, the approximate 100γ % confidence limits for a population parameter ϕ can be obtained, such that

$$P[\hat{\phi} - Z\sigma(\hat{\phi}) \le \phi \le \hat{\phi} + Z\sigma(\hat{\phi})] = \gamma, \quad (17)$$

Then, the approximate confidence limits for β,c,q and α will be constructed using equation (17) with confidence levels 95% and 99%.

4 Bayes Estimation

Considering the case of type-I censoring, then the likelihood function of the total lifetimes $y_1, ..., y_n$, of n items independent and identically distributed random variables takes the following form

$$L(\underline{y}|\beta, c, q, \alpha) = \prod_{i=1}^{n}[f_1(y_i)]^{\delta_{1i}}[f_2(y_i)]^{\delta_{2i}}[R(\eta)]^{\tilde{\delta_{1i}}\tilde{\delta_{2i}}},$$

$$L(\underline{y}|\beta, c, q, \alpha) = c^{n_0}\alpha^{n_0}\beta^{n_a}q^{-n_0}[\prod_{i=1}^{n_u}[(y_i)^{c-1}][\prod_{i=1}^{n_u}(H)^{-(\alpha+1)}]$$

$$\times[D_1]^{-\alpha(n-n_0)}[\prod_{i=1}^{n_a}A^{c-1}(A_1)^{-(\alpha+1)}] \tag{18}$$

It assumed that the parameters are a prior independent and let the NIP for each parameter be represented. It follows that a NIP for α,c,q and β are respectively given by

$$g_1(\alpha) \propto \alpha^{-1}, \quad \alpha > 0, \qquad g_2(q) \propto q^{-1}, \quad q > 0,$$

$$g_3(\beta) \propto \beta^{-1}, \quad \beta > 0, \qquad g_4(c) \propto c^{-1}, \quad c > 0, \tag{19}$$

Consequently, the joint NIP will be as follows:

$$g(\alpha, \beta, c, q) \propto (\alpha\beta cq)^{-1}, \quad \alpha, q, c > 0, \quad \beta > 1, \tag{20}$$

Combining the joint prior density of $u(\phi) = u(\alpha, \beta, c, q)$ in (20) and likelihood function (18) to obtain the joint posterior density of $u(\phi)$ given the data as follows:

$$\pi(u(\phi)|\underline{y}) = \frac{c^{n_0-1}\alpha^{n_0-1}\beta^{n_a-1}q^{-n_0-1}J[u(\phi)]}{J_1}; \alpha, c, q > 0, \beta > 1, \tag{21}$$

where, $J[u(\phi)] = \prod_{i=1}^{n_u}[(y_i)^{c-1}][\prod_{i=1}^{n_u}(H)^{-(\alpha+1)}][\prod_{i=1}^{n_a}A^{c-1}(A_1)^{-(\alpha+1)}][D_1]^{-\alpha(n-n_0)}$
and $J_1 = \int_0^\infty\int_0^\infty\int_0^\infty\int_1^\infty c^{n_0}\alpha^{n_0}\beta^{n_a}q^{-n_0}J[u(\phi)]d\alpha dc dq d\beta$
Therefore, the BE of the unknown parameters $u(\phi) = u(\alpha, \beta, c, q)$ based on the posterior density function under SEL and LINEX loss functions; denoted by

$$\tilde{u}_{(SEL)}(\phi) \quad and \quad H\tilde{u}_{(LINEX)}(\phi)$$

respectively; can be calculated through the following equations as follows:

$$\tilde{u}_{(SEL)}(\phi) = E[u(\phi)|\underline{y}] = \int_0^\infty\int_0^\infty\int_0^\infty\int_1^\infty u(\phi)\pi(u(\phi)|\underline{y})d\alpha dc dq d\beta, \tag{22}$$

and

$$H\tilde{u}_{(LINEX)}(\phi) = -\frac{1}{a}log[E(e^{-au(\phi)}|\underline{y})] = -\frac{1}{a}log[\int_0^\infty\int_0^\infty\int_0^\infty\int_1^\infty e^{-au(\phi)}\pi(u(\phi)|\underline{y})d\alpha dc dq d\beta] \tag{23}$$

The integrals involved in (22) and (23) cant be solved analytically, so its necessary to apply a numerical technique and computer facilities to evaluate the BE of unknown parameters.

4.1 MCMC Algorithm

A wide variety of MCMC schemes are available, and it can be difficult to choose among them. An important sub-class of MCMC methods are Gibbs sampling and more general Metropolis within-Gibbs samplers. The advantage of using the MCMC method over the MLE method is that we can always obtain a reasonable interval estimate of the parameters by constructing the probability intervals based on the empirical posterior distribution. It is often unavailable in maximum likelihood estimation. Indeed, the MCMC samples may be used to completely summarize the posterior uncertainty about the parameters β,c,q and α, through a kernel estimate of the posterior distribution. This is also true of any function of the parameters. The joint posterior density function of α,c,q and β can be written as

$$\pi(u(\phi)|\underline{y}) \propto c^{n_0-1}\alpha^{n_0-1}\beta^{n_a-1}q^{-n_0-1}[\prod_{i=1}^{n_u}[(y_i)^{c-1}][\prod_{i=1}^{n_u}(H)^{-(\alpha+1)}][\prod_{i=1}^{n_a}A^{c-1}(A_1)^{-(\alpha+1)}][D_1]^{-\alpha(n-n_0)}, \tag{24}$$

The conditional posterior PDF's of α,c,q and β are as follows

$$\pi_1(\alpha|\beta,c,q,\underline{y}) \propto \alpha^{n_0-1}[\prod_{i=1}^{n_a}(A_1)^{-(\alpha+1)}][\prod_{i=1}^{n_u}(H)^{-(\alpha+1)}][D_1]^{-\alpha(n-n_0)}, \alpha > 0 \tag{25}$$

$$\pi_2(c|\alpha,q,\beta,\underline{y}) \propto c^{n_0-1}[\prod_{i=1}^{n_u}(y_1)^{c-1}][\prod_{i=1}^{n_u}(H)^{-(\alpha+1)}][\prod_{i=1}^{n_a}A^{c-1}(A_1)^{-(\alpha+1)}][D_1]^{-\alpha(n-n_0)}, c > 0 \tag{26}$$

$$\pi_3(q|\alpha,c,\beta,\underline{y}) \propto q^{-n_0-1}[\prod_{i=1}^{n_a}(A_1)^{-(\alpha+1)}][\prod_{i=1}^{n_u}(H)^{-(\alpha+1)}][D_1]^{-\alpha(n-n_0)}, q > 0 \tag{27}$$

and

$$\pi_4(\beta|\alpha,c,q,\underline{y}) \propto \beta^{n_a-1}[\prod_{i=1}^{n_a}(A^{c-1})(A_1)^{-(\alpha+1)}][D_1]^{-\alpha(n-n_0)}, \beta > 1 \tag{28}$$

For details regarding the implementation of Metropolis–Hastings algorithm, the readers may refer to Robert and Casella (2004). To run the Gibbs sampler algorithm we started with the ML estimates. We then drew samples from various full conditionals, in turn, using the most recent values of all other conditioning variables unless some systematic pattern of convergence achieved.

The algorithm of Metropolis–Hastings is as follows:

Step 1: Start with an $(\beta^{(0)} = \beta, \alpha^{(0)} = \alpha, c^{(0)} = c)$ and $(q^{(0)} = q)$ and set $I = 1$.

Step 2: Generate $\alpha^{(I)}$ from the Uniform distribution $\pi_1(\alpha|\beta,c,q,\underline{y})$.

Step 3: Generate $c^{(I)}$ from the Uniform distribution $\pi_2(c|\alpha,q,\beta,\underline{y})$.

Step 4: Generate $q^{(I)}$ from the Uniform distribution $\pi_3(q|\alpha,c,\beta,\underline{y})$.

Step 5: Generate $\beta^{(I)}$ from the Uniform distribution $\pi_4(\beta|\alpha,c,q,\underline{y})$.

Step 6: Compute $\alpha^{(I)}$, $\beta^{(I)}$, $c^{(I)}$ and $q^{(I)}$. Step 7: Set $I = I + 1$.

Step 8: Repeat steps 2-6 N times.

Step 9: We obtain the Bayes MCMC point estimate of $\phi_I(\phi_1 = \alpha, \phi_2 = c, \phi_3 = q, \phi_4 = \beta)$ as

$$E(\phi_I|data) \propto \frac{1}{N-M}\sum_{i=M+1}^{N}\phi_I^{(i)}, \tag{29}$$

Where, M is the burn-in period (that is, a number of iterations before the stationary distribution is achieved) and the posterior variance of ϕ becomes

$$\hat{V}(\phi_I|data) \propto \frac{1}{N-M}\sum_{i=M+1}^{N}(\phi_I^{(i)} - \hat{E}(\phi_I|data))^2, \tag{30}$$

Step 10: To compute the credible intervals of ϕ_I, we usually take the quantiles of the sample as the endpoints of the interval. Order as $\phi_I^{M+1}, \phi_I^{M+2}, \phi_I^{M+3}, ..., \phi_I^{N}$ as $\phi_{I(1)}, \phi_{I(2)}, \phi_{I(3)}, ..., \phi_{I(N-M)}$. Then the $100(1-2\gamma)$ % symmetric credible interval is

$$(\phi_{I[\gamma(N-M)]}, \phi_{I[(1-\gamma)(N-M)]}), \tag{31}$$

5 Simulation Studies

Due to the complicated expressions of the estimators, an analytical comparison of these estimators is impossible. Therefore, a Monte Carlo simulation study is carried out to calculate the MLEs, BEs, MSEs and RABs, based on r=1000

Monte Carlo simulations. Some computations were performed using (MATHEMATICA ver. 7.0). The performance of the resulting estimators of the acceleration factor (β), the scale parameter (q) and the shape parameters (c, α) has been considered in terms of RAB; which is the absolute difference between the mean estimates and its true value divided by the true value of the parameter (i.e. $RAB(\hat{\theta}) = \frac{\hat{\theta}-\theta}{\theta}$) and MSE; which is the sum squares of the difference between the estimated parameter and its true value divided by the number of the sample (i.e. $MSE(\hat{\theta}) = E[(\hat{\theta}-\theta)^2]$).

Furthermore, the two-sided confidence intervals of the acceleration factor, the scale parameter, and the shape parameters obtained. The Simulation study performed according to the following steps
Step 1: Generate random samples of sizes $100, 150, 250, 350, 450$ and 500 from the 3-parameter compound Weibull-gamma distribution. The generation of the 3-parameter compound Weibull-gamma distribution is very simple, if U has a uniform (0,1) random number, then $Y = [q[(1-U)^{-(\frac{1}{\alpha})} - 1]]^{\frac{1}{c}}$ follows a 3-parameter compound Weibull-gamma distribution. The true parameters values selected as
$Case.1 : (\beta = 1.75, c = 0.75, \alpha = 1.25, q = 1.5)$
and $Case.2 : (\beta = 1.5, c = 1.45, \alpha = 0.75, q = 2)$.
Step 2: Choosing the censoring time τ at the normal condition to be $\tau = 3$ and censoring time of a PALT to be $\eta = 6$.
Step 3: For each sample and in the two sets of parameters, the acceleration factor and the parameters of distribution were estimated in SSPALT under type-I censored sample.
Step 4: Iteration technique used for solving the two nonlinear likelihood for β, c and q given in equations (14), (15) and (16), respectively. The estimate of the shape parameter α was easily obtained from equation (13).
Step 5: The MSE and RABias of the estimators for the acceleration factor, the scale parameter and the shape parameters for all sample sizes and in two sets of parameters tabulated.
Step 6: The confidence limit with level confidence $\gamma = 0.95$ and $\gamma = 0.99$ of the acceleration factor, the scale parameter and the shape parameters were constructed.
Simulation results summarized in Tables 1, 2 and 3. The asymptotic variances and covariance matrix of the estimators displayed in Table 1. Table 2 presents the approximated confidence limits at 95% and 99% for the parameters and acceleration factor. Table 3 gives the MSE and RAB of the estimators.

Table 1: Asymptotic variances and covariances of estimators under type-I censoring.

	Case 1				Case 2			
n	$\hat{\beta}$	$\hat{c}$	$\hat{\alpha}$	$\hat{q}$	$\hat{\beta}$	$\hat{c}$	$\hat{\alpha}$	$\hat{q}$
	0.758	-0.083	0.029	0.154	0.293	-0.061	0.020	0.0073
100	-0.083	0.00641	-0.020	-0.065	-0.061	0.049	-0.021	-0.0072
	0.029	-0.020	0.085	0.232	0.020	-0.021	0.027	0.0091
	0.154	-0.065	0.232	0.536	0.0073	-0.0072	0.0091	0.0045
	0.806	-0.076	0.038	0.222	0.129	-0.027	0.0077	0.0031
150	-0.076	0.00429	-0.011	-0.045	-0.027	0.026	-0.0091	-0.0034
	0.038	-0.011	0.062	0.167	0.0077	-0.0091	0.011	0.0034
	0.222	-0.045	0.167	0.432	0.0031	-0.0034	0.0034	0.0021
	0.745	-0.073	0.099	0.127	0.048	-0.015	0.0043	0.0017
250	-0.073	0.0038	-0.016	-0.037	-0.015	0.014	-0.005	-0.0019
	0.099	-0.016	0.064	0.128	0.0043	-0.005	0.0058	0.002
	0.127	-0.037	0.128	0.307	0.0017	-0.0019	0.002	0.001
	0.705	-0.070	0.023	0.153	0.028	-0.008	0.002	0.0009
350	-0.070	0.0043	-0.0097	-0.039	-0.008	0.008	-0.003	-0.001
	0.023	-0.0097	0.045	0.131	0.002	-0.003	0.003	0.001
	0.153	-0.039	0.131	0.360	0.0009	-0.001	0.001	0.0006
	0.629	-0.094	0.013	0.164	0.024	-0.0077	0.0023	0.0009
450	-0.094	0.0035	-0.015	-0.056	-0.0077	0.0072	-0.0025	-0.0009
	0.013	-0.015	0.022	0.064	0.0023	-0.0025	0.0027	0.0009
	0.164	-0.056	0.064	0.273	0.0009	-0.0009	0.0009	0.0005
	0.492	-0.191	0.021	0.118	0.030	-0.0092	0.0029	0.0012
500	-0.191	0.010	-0.046	-0.149	-0.0092	0.0083	-0.0031	-0.0012
	0.021	-0.046	0.039	0.101	0.0029	-0.0031	0.0033	0.0012
	0.118	-0.149	0.101	0.266	0.0012	-0.0012	0.0012	0.00061

Table 2: Confidence bounds of the parameters (c, q, α, β) at confidence level 0.95 and 0.99 under type-I censoring.

		Case1				Case 2			
n	parameters	Standar deviation	Lower Bound	Upper Bound	Length	Standar deviation	Lower Bound	Upper Bound	Length
100	c	0.025	0.7123	1.1208	0.4085	0.123	1.3074	1.9861	0.6787
			0.5037	1.8294	1.3258		1.3298	1.9637	0.6339
	q	0.059	0.3674	3.4327	3.0654	0.732	1.8362	3.1638	1.3276
			-2.6981	6.4981	9.1962		1.7429	3.0571	1.3142
	α	0.292	1.1303	1.7229	0.5710	0.2495	0.4409	1.1914	0.7505
			0.5376	2.3157	1.778		0.3292	1.1612	0.8320
	β	0.871	-0.1903	1.513	1.6392	0.4770	1.0784	1.7688	0.6903
			-0.4777	1.9266	2.4043		1.1340	1.9133	0.7793
150	c	0.065	0.8045	1.1755	0.3710	0.160	1.3747	1.8972	0.5225
			0.3822	1.6178	1.2357		1.3846	1.8873	0.5028
	q	0.052	0.6997	2.8777	2.1779	0.657	1.6873	3.0127	1.3254
			-1.4782	5.05563	6.5338		1.4697	2.9303	1.4606
	α	0.250	0.9972	1.5608	0.5636	0.107	0.3322	1.0747	0.7425
			0.4836	2.2243	1.7408		0.4001	1.2068	0.8067
	β	0.863	-0.0395	1.3567	1.3962	0.220	1.3215	1.8389	0.5175
			-0.2588	1.5761	1.8349		1.3631	1.9974	0.6343
250	c	0.061	0.7400	1.1099	0.3699	0.120	1.3147	1.8237	0.5090
			0.6700	1.3299	0.6600		1.3235	1.8149	0.5514
	q	0.033	0.0957	2.6134	1.5177	0.554	1.3525	2.6475	1.2950
			-0.4220	4.1311	4.5531		1.3245	2.7755	1.4510
	α	0.254	1.1493	1.7054	0.5561	0.076	0.4229	1.0987	0.6759
			0.5931	2.2616	1.6685		0.4772	1.2444	0.7672
	β	0.840	0.2189	1.6047	1.3858	0.167	1.2084	1.9689	0.7605
			0.2641	1.8596	1.5955		1.2455	1.9319	0.6864
350	c	0.066	0.7256	0.8919	0.1662	0.098	1.2337	1.7397	0.5061
			0.5594	1.0581	0.4987		1.3330	1.7304	0.4974
	q	0.025	1.2718	2.5282	1.2564	0.600	1.4030	2.7969	1.2939
			0.0154	3.7846	3.7692		1.3508	2.7492	1.3984
	α	0.213	1.2517	1.8018	0.5501	0.057	0.3728	0.9740	0.6013
			0.9615	2.4319	1.4703		0.2130	0.9337	0.7207
	β	0.793	0.4923	1.8012	1.2089	0.156	1.3931	1.73347	0.3404
			0.3181	1.7754	1.4573		1.42046	1.9601	0.5356
450	c	0.059	0.6788	0.8155	0.1367	0.085	1.3690	1.7817	0.4127
			0.5421	0.9522	0.4102		1.3781	1.7726	0.3945
	q	0.022	1.3383	2.4617	1.1234	0.523	1.5065	2.7435	1.2371
			0.2149	3.5851	3.3702		1.3956	2.7044	1.3088
	α	0.147	1.1272	1.6955	0.5683	0.052	0.3890	0.9706	0.5816
			0.7589	2.1638	1.4050		0.2221	0.8675	0.6454
	β	0.701	0.6073	1.6316	1.0244	0.172	1.4280	1.7084	0.2803
			0.4463	1.7926	1.3462		1.4506	1.8858	0.4353
500	c	0.011	0.7196	0.8204	0.1008	0.091	1.3296	1.7298	0.4002
			0.6388	0.9612	0.3224		1.3377	1.7217	0.3841
	q	0.025	0.9863	2.1561	1.1698	0.516	1.6522	2.7479	1.1958
			-0.3836	2.9259	3.3095		1.4320	2.7080	1.2760
	α	0.198	1.0031	1.4439	0.4408	0.057	0.5132	1.0426	0.5294
			1.1500	2.4908	1.3408		0.5477	1.1081	0.5604
	β	0.701	0.9645	1.8694	0.9049	0.172	1.4877	1.7601	0.2725
			0.8223	2.0116	1.1892		1.5096	1.8383	0.3287

Table 3: MLEs and BEs (using SEL and LINEX) for the parameters based on 1000 simulation for two parameters cases and the censoring times are $\tau = 3$ and $\eta = 6$.

		Case1				Case 2			
n	Methods	c MSE(c) RAB(c)	α MSE(α) RAB(α)	q MSE(q) RAB(q)	β MSE(β) RAB(β)	c MSE(c) RAB(c)	α MSE(α) RAB(α)	q MSE(q) RAB(q)	β MSE(β) RAB(β)
100	ML	0.9569 0.0493 0.2761	1.4066 0.1649 0.1825	1.3608 0.2021 0.2872	1.9070 0.4014 0.3539	1.8500 0.3133 0.2683	0.6951 0.0169 0.0759	1.4923 0.5438 0.2861	1.7092 0.2260 0.1087
	SEL	0.8967 0.0399 0.2183	1.2645 0.0014 0.0060	1.54181 0.01221 0.02729	1.85726 0.12274 0.08842	1.6958 0.2688 0.2184	0.7199 0.0057 0.0720	1.5802 0.4198 0.2301	1.6602 0.1709 0.0981
	LINEX a_1=-3	0.7839 0.0081 0.0156	1.2774 0.0029 0.0058	1.58176 0.08175 0.03395	1.82045 0.08955 0.06958	1.6125 0.2140 0.2903	0.7123 0.0055 0.0707	1.6171 0.3829 0.2108	1.6609 0.1702 0.1142
	LINEX a_2=-1	0.8067 0.0129 0.0683	1.6272 0.1904 0.2558	1.54181 0.01221 0.02922	1.92726 0.12274 0.05288	1.6402 0.2017 0.2092	0.7231 0.0045 0.0600	1.5890 0.4107 0.2227	1.6702 0.1602 0.11488
	LINEX a_3=1	0.8213 0.0466 0.1689	1.5892 0.2192 0.1851	1.4996 0.01388 0.3395	1.93928 0.46072 0.3695	1.6049 0.1903 0.2090	0.6179 0.1800 0.1800	1.5409 0.4591 0.2608	1.6549 0.1049 0.0942
150	ML	0.9261 0.0426 0.2483	1.3755 0.1568 0.1657	1.3989 0.2965 0.2856	1.8947 0.3841 0.3277	1.8048 0.2986 0.2547	0.6925 0.0151 0.0735	1.5304 0.5300 0.2782	1.6996 0.21429 0.1073
	SEL	0.8869 0.0389 0.2007	1.2664 0.0015 0.0070	1.5859 0.0759 0.0422	1.8437 0.1063 0.0759	1.6971 0.2669 0.2105	0.7195 0.0056 0.0633	1.5886 0.4114 0.2299	1.6619 0.1619 0.0842
	LINEX a_1=-3	0.7802 0.0070 0.0131	1.2786 0.0029 0.0058	1.5936 0.0936 0.0487	1.8741 0.0759 0.0613	1.6943 0.2680 0.2168	0.7109 0.0902 0.0750	1.6238 0.3762 0.2113	1.6809 0.1809 0.0961
	LINEX a_2=-1	0.8007 0.0122 0.0601	1.6438 0.2054 0.2153	1.5559 0.0559 0.0733	1.8437 0.1063 0.0431	1.6971 0.2269 0.2005	0.6519 0.1684 0.1479	1.6586 0.3114 0.1937	1.6619 0.1519 0.1153
	LINEX a_3=1	0.8209 0.0468 0.1214	1.6089 0.08472 0.2052	1.5160 0.0196 0.0487	1.8087 0.1413 0.0613	1.6209 0.2875 0.2763	0.7220 0.0038 0.0602	1.5511 0.4489 0.2113	1.6385 0.1185 0.0961
250	ML	0.8811 0.0380 0.1950	1.3505 0.1413 0.1578	1.4295 0.2496 0.2498	1.8051 0.0822 0.0644	1.7564 0.2915 0.2320	0.7051 0.0069 0.0903	1.5895 0.5196 0.2698	1.5951 0.1122 0.1644
	SEL	0.8581 0.0347 0.1887	1.2822 0.00318 0.00836	1.5660 0.06602 0.0311	1.7847 0.0654 0.0521	1.6973 0.2667 0.2234	0.7092 0.0076 0.0851	1.5956 0.4044 0.2263	1.6516 0.1516 0.0767
	LINEX a_1=-3	0.7800 0.0068 0.01300	1.2892 0.00320 0.0093	1.5618 0.0686 0.1524	1.7834 0.0666 0.0532	1.6445 0.2977 0.2904	0.7285 0.0030 0.0917	1.6596 0.3704 0.2082	1.6893 0.1893 0.1070
	LINEX a_2= -1	0.7970 0.0097 0.0587	1.6547 0.2662 0.2112	1.5683 0.0760 0.1732	1.8147 0.0954 0.0372	1.6973 0.2667 0.2234	0.6616 0.1956 0.3188	1.6956 0.3244 0.1909	1.6616 0.1616 0.1246
	LINEX a_3=1	0.8125 0.0423 0.1166	1.6215 0.1932 0.1833	1.5783 0.0284 0.1524	1.8214 0.1286 0.0532	1.6210 0.2874 0.2743	0.6296 0.1850 0.3017	1.5596 0.4904 0.3082	1.6296 0.1296 0.1070

		Case1				Case 2			
n	Methods	c MSE(c) RAB(c)	α MSE(α) RAB(α)	q MSE(q) RAB(q)	β MSE(β) RAB(β)	c MSE(c) RAB(c)	α MSE(α) RAB(α)	q MSE(q) RAB(q)	β MSE(β) RAB(β)
	ML	0.86323	1.31607	1.4470	1.7850	1.6922	0.7203	1.7187	1.5843
350		0.0367	0.0891	0.1114	0.0159	0.2292	0.0032	0.2789	0.0413
		0.1801	0.1094	0.2080	0.0587	0.1960	0.0449	0.2094	0.1601
	SEL	0.8097	1.2728	1.5613	1.7922	1.5875	0.7191	1.7977	1.6661
		0.0079	0.0025	0.0213	0.0878	0.1564	0.0041	0.2023	0.1681
		0.0720	0.0057	0.1311	0.0989	0.1312	0.0457	0.2235	0.0912
	LINEX	0.77487	1.2838	1.5461	1.7899	1.5746	0.7204	1.8312	1.5933
	a_1=-3	0.0065	0.0029	0.0071	0.0601	0.1477	0.0057	0.0970	0.0933
		0.0102	0.0058	0.0169	0.0504	0.1209	0.0606	0.1048	0.1107
	LINEX	0.7976	1.5921	1.5713	1.7922	1.6075	0.7204	1.8977	1.6662
	a_2=-1	0.0098	0.1871	0.0713	0.0878	0.1663	0.0045	0.0923	0.1662
		0.07208	0.2577	0.1789	0.0347	0.1317	0.0617	0.0870	0.1277
	LINEX	0.70109	1.5899	1.5348	1.7901	1.6217	0.6347	1.7622	1.6347
	a_3=1	0.01671	0.1536	0.0348	0.1199	0.1862	0.1840	0.2078	0.1347
		0.16887	0.1568	0.1556	0.0504	0.1616	0.6006	0.2948	0.1107
	ML	0.8103	1.2960	1.4602	1.7228	1.6670	0.7258	1.7500	1.5815
		0.0221	0.0083	0.0107	0.0069	0.1750	0.0033	0.1800	0.0394
450		0.1522	0.0096	0.0873	0.0055	0.1624	0.0361	0.2000	0.0560
	SEL	0.7898	1.2700	1.5549	1.7654	1.5578	0.7191	1.8696	1.5679
		0.00273	0.0013	0.0064	0.0086	0.0985	0.0042	0.0804	0.0980
		0.05934	0.0042	0.0026	0.0718	0.0745	0.0528	0.1014	0.0906
	LINEX	0.7685	1.2779	1.5394	1.7605	1.5448	0.7203	1.9327	1.5248
	a_1=-3	0.0008	0.0014	0.0049	0.0075	0.0974	0.0054	0.0473	0.0294
		0.00155	0.0028	0.0012	0.0123	0.0642	0.0569	0.0640	0.1107
	LINEX	0.7898	1.5654	1.5750	1.7745	1.5978	0.6680	1.8396	1.5679
	a_2=-1	0.0028	0.1208	0.0248	0.0246	0.1258	0.1942	0.1104	0.0680
		0.0593	0.1513	0.0697	0.0355	0.1150	0.0840	0.1676	0.1282
	LINEX	0.71207	1.5940	1.5385	1.7839	1.6221	0.6367	1.8646	1.5467
	a_3=1	0.01646	0.1934	0.0385	0.0360	0.1855	0.1837	0.0854	0.0867
		0.15757	0.2127	0.0462	0.0523	0.1733	0.5679	0.1040	0.0907
	ML	0.7924	1.2852	1.4817	1.7333	1.5821	0.7288	1.8288	1.5804
		0.0198	0.0112	0.0070	0.0106	0.1187	0.0023	0.0813	0.0081
500		0.0397	0.0191	0.0729	0.0164	0.1289	0.0336	0.1036	0.0547
	SEL	0.7683	1.2647	1.5360	1.7651	1.5375	0.7633	1.9007	1.569
		0.0007	0.0007	0.0008	0.0029	0.0163	0.0013	0.0593	0.0069
		0.0352	0.0050	0.0011	0.0199	0.0176	0.0061	0.0704	0.0840
	LINEX	0.7603	1.2635	1.5102	1.7523	1.4447	0.7602	1.9537	1.4957
	a_1=-3	0.0005	0.0005	0.0002	0.0007	0.0005	0.0010	0.0063	0.0058
		0.0063	0.0048	0.0005	0.0056	0.0138	0.0020	0.0373	0.0056
	LINEX	0.7819	1.3651	1.5760	1.7651	1.5975	0.6990	1.8907	1.669
	a_2=-1	0.0027	0.0147	0.0760	0.0049	0.1963	0.0343	0.0903	0.0690
		0.06533	0.0245	0.0811	0.0159	0.1676	0.1187	0.0414	0.1246
	LINEX	0.7222	1.5838	1.5403	1.7335	1.6211	0.6879	1.8656	1.6379
	a_3=1	0.0164	0.1836	0.0403	0.0115	0.2072	0.0339	0.1344	0.0379
		0.1548	0.2058	0.0585	0.0516	0.1772	0.1420	0.1973	0.1056

6 Concluding Remarks

In this article, we have investigated some inferences on SSPALT model when the observed data follows the 3-parameter compound Weibull-gamma distribution. Under type-I censoring, the test unit first run at normal use condition, and if it does not fail for a specified time τ, then it is run at accelerated condition until the censoring time η reached. The full Bayes and non-Bayes methods used for estimating the acceleration factor and parameters of 3-parameter compound Weibull gamma distribution under type-I censoring. The classical Bayes estimates cannot be obtained in explicit form. One can clearly see the scope of MCMC based on Bayesian solutions which make every inferential development routinely available. A

simulation study was conducted to examine and compare the performance of the proposed methods for different sample sizes and different parameter values. It may be noticed, from Tables 1, 2 and 3 that
1. For the first set of parameters ($c = 0.75, q = 1.5, \alpha = 1.25, \beta = 1.75$), the maximum likelihood estimators have good statistical properties than the second set of parameters ($c = 1.45, q = 2, \alpha = 0.75, \beta = 1.5$) for all sample sizes (see Table 1).
2. Maximum likelihood estimators are consistent and asymptotically normally distributed (see Table 2).
3. The asymptotic variance and covariance of estimators decrease regarding the interval of estimators when the sample size n increases, it can be noted that the interval of the estimators at $\gamma = 0.99$ is greater than the corresponding at $\gamma = 0.95$. Also, as the sample size increases the interval of the estimators decreases for the two confidence level (see Table 2).
4. The MSEs and RABs decrease when the sample size n increases (see Table 3).
5. In general, we observed from Table 3 that the BEs are better than MLEs for small sample sizes and by increasing the sample size the MLEs gives better values and nearly approach to the BEs.

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