



Adomian Decomposition Method for Nonlinear Age-Structured Population Models with Using Trapezoidal and Simpson Rules

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Abstract: In this paper, the Adomian decomposition method is used for solving nonlinear age-structured population models to obtain approximate-exact solutions. The results show that the Adomian decomposition method is very accuracy and efficiency.

Keywords: Age-structured population models; Integral boundary condition; Adomian decomposition method; Adomian's polynomials; Trapezoidal rule; Simpson rule

1 Introduction

In this paper, the Adomian decomposition method is introduced for solving approximately the following nonlinear age-structured population models [1].

$$\begin{aligned} \frac{\partial p(t, x)}{\partial t} + \frac{\partial p(t, x)}{\partial x} &= -[d_1(x) + d_2(x)P(t)]p(t, x), \quad t \geq 0, \quad 0 \leq x \leq A, \\ p(0, x) &= p_0(x), \quad 0 \leq x \leq A, \\ p(t, 0) &= \int_a^A [b_1(\xi) + b_2(\xi)P(t)]p(t, \xi) d\xi, \quad t \geq 0, \\ P(t) &= \int_0^A p(t, x) dx, \quad t \geq 0, \end{aligned} \quad (1)$$

where t, x denote time and age, respectively, $P(t)$ denotes the total population number at time t , $p(t, x)$ is the age-specific density of individuals of age x at time t , which means that $\int_a^{a+\Delta a} p(t, x) dx$ gives the number of individuals that have age between a and $a + \Delta a$ at time t , $d_1(x)$ is the natural death rate (without considering competition), $d_2(x)$ is the increase of death rate considering competition, $b_1(x)$ is the natural fertility rate (without considering competition), $b_2(x)$ is the decrease of fertility rate considering competition, a denotes the lowest age when an individual can bear, and A is the maximum age that an individual of the population may reach.

Recently, Li [2] used the Variational Iteration Method (VIM) for solving Eq. (1) and Ghoreishi et al. [3] applied the Homotopy Analysis Method (HAM) and Optimal Homotopy Asymptotic Method (OHAM) to solve it.

For nonlinear age-structured population model (1), Adomian decomposition method has not yet been applied. The aim of this paper is to fill this gap. In the beginning of the 1980's, Adomian [4-7] proposed a new and fruitful method (hereafter called the Adomian Decomposition Method or ADM) for solving linear and nonlinear (algebraic, differential, partial differential, integral, etc.) equations. It has been shown that this method yields a rapid convergence of the solutions series to linear and nonlinear deterministic and stochastic equations.

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The main objective of this paper is to apply ADM to solve nonlinear age-structured population model to obtain the approximate-exact solutions.

2 Analysis of the method

In an operator form, Eq. (1) can be written as

$$L_t p + L_x p = -d_1(x)p - d_2(x)Np, \quad (2)$$

where the differential operators L_t and L_x are defined by

$$L_t = \frac{\partial}{\partial t}, \quad L_x = \frac{\partial}{\partial x},$$

and $Np = P(t)p(t, x)$ is the nonlinear operator. Assuming L_t is invertible, therefore the inverse operator L_t^{-1} is given by the integral $L_t^{-1}[\cdot] = \int_0^t [\cdot] dt$.

on both sides of Eq. (2), and using the initial condition, we obtain

$$p(t, x) = p(0, x) - L_t^{-1} [L_x p + d_1(x)p + d_2(x)Np]. \quad (3)$$

The details of the ADM are well known now in the literature and can be found in [4-7]. The Adomian's technique consists of approximating the solution $p(t, x)$ of Eq. (2) as an infinite series

$$p(t, x) = \sum_{n=0}^{\infty} \hat{a}_n p_n(t, x), \quad (4)$$

where the components $p_n(t, x)$ of the solution $p(t, x)$ will be determined in a recursive manner, and decomposing the nonlinear operator N as

$$Np = \sum_{n=0}^{\infty} \hat{a}_n P_n(t) = \sum_{n=0}^{\infty} \hat{a}_n p_n(t, x) = \sum_{n=0}^{\infty} \hat{a}_n A_n, \quad (5)$$

where A_n are Adomian polynomials of p_0, \dots, p_n [4-7] given by

$$A_n = \frac{1}{n!} \frac{d^n}{d\lambda^n} N \sum_{i=0}^{\infty} \lambda^i p_i, \quad n = 0, 1, \dots \quad (6)$$

The proofs of the convergence of the series $\sum_{n=0}^{\infty} \hat{a}_n p_n$ and $\sum_{n=0}^{\infty} \hat{a}_n A_n$ are given in [6,8-12]. Substituting (4) and (5) into both sides of (3) yields

$$\sum_{n=0}^{\infty} \hat{a}_n p_n = p(0, x) - L_t^{-1} \left[L_x \sum_{n=0}^{\infty} \hat{a}_n p_n + d_1(x) \sum_{n=0}^{\infty} \hat{a}_n p_n + d_2(x) \sum_{n=0}^{\infty} \hat{a}_n A_n \right]. \quad (7)$$

From Eq. (7), the iterates are then determined in the following recursive way:

$$p_0 = p(0, x), \quad p_{n+1} = -L_t^{-1} [L_x p_n + d_1(x)p_n + d_2(x)A_n], \quad n = 0, 1, \dots \quad (8)$$

For the nonlinear term $Np = P(t)p(t, x) = \sum_{n=0}^{\infty} \hat{a}_n A_n$, the corresponding Adomian polynomials are given by

$$A_0 = P_0 p_0,$$

$$A_1 = P_1 p_0 + P_0 p_1,$$

$$A_2 = P_2 p_0 + P_1 p_1 + P_0 p_2,$$

$$A_3 = P_3 p_0 + P_2 p_1 + P_1 p_2 + P_0 p_3,$$

$$A_4 = P_4 p_0 + P_3 p_1 + P_2 p_2 + P_1 p_3 + P_0 p_4,$$

$$\vdots$$

$$A_n = \sum_{i=0}^n \hat{a}_i P_{n-i} p_i, \quad n \geq 0, \quad n = 0, 1, \dots$$

Thus all components of p can be calculated once the p_n and A_n are given. We then define the n -term approximant to the solution p by $\phi_n[p] = \sum_{i=0}^{n-1} p_i$ with $\lim_{n \rightarrow \infty} \phi_n[p] = p$.

3 Applications and numerical results

In this section, the ADM for solving nonlinear age-structured population models is illustrated in the following two examples. The computations associated with the examples were performed using a Maple 13 package with a precision of 30 digits.

Example 1 We consider the nonlinear age-structured population model (1) with $d_1(x) = 0$, $d_2(x) = 1$ and $p_0(x) = \frac{e^{-x}}{2}$. Thus the model can be written as [1-3]

$$\begin{aligned} \frac{\partial p(t, x)}{\partial t} + \frac{\partial p(t, x)}{\partial x} &= -P(t) p(t, x), \quad t \geq 0, 0 \leq x \leq A, \\ p(0, x) &= \frac{e^{-x}}{2}, \quad 0 \leq x \leq A, \\ p(t, 0) &= P(t), \quad t \geq 0, \\ P(t) &= \int_0^A p(t, x) dx, \quad t \geq 0, \end{aligned} \quad (9)$$

Where $A \rightarrow +\infty$. And it is easy to verify that $p(t, x) = 1 + e^{-t-x}$, $t \geq 0, x \geq 0$, is the exact solution of the problem (9).

Solution: Take $\Omega = [0, 1] \times [0, 100]$, which denotes (1 unit time) \times (100 unit age). Applying the ADM for this problem, we obtain the following recursion scheme

$$\begin{aligned} p_0(t, x) &= \frac{e^{-x}}{2}, \\ p_{n+1}(t, x) &= -L_t^{-1} [L_x p_n + A_n], \quad n = 0, 1, \dots \end{aligned} \quad (10)$$

Following the algorithm (10), we find that the even iterates $p_n(t, x) = 0$, $n = 2, 4, 6, \dots$ and the odd iterates are given by

$$p_1(t, x) = \frac{1}{4} t e^{-x},$$

$$p_3(t, x) = -\frac{1}{48} t^3 e^{-x},$$

$$p_5(t, x) = \frac{1}{480} t^5 e^{-x},$$

$$p_7(t, x) = -\frac{17}{80640} t^7 e^{-x},$$

$$p_9(t, x) = \frac{31}{1451520} t^9 e^{-x},$$

$$p_{11}(t, x) = -\frac{691}{319334400} t^{11} e^{-x},$$

$$\vdots$$

Thus, the approximate solution in a series form is given by

$$\begin{aligned} \phi_n(t, x) = e^{-x} &+ \frac{1}{4} t e^{-x} - \frac{1}{48} t^3 e^{-x} + \frac{1}{480} t^5 e^{-x} - \frac{17}{80640} t^7 e^{-x} + \frac{31}{1451520} t^9 e^{-x} - \frac{691}{319334400} t^{11} e^{-x} + \dots + O(t^{n-1}), \quad n \geq 1. \end{aligned}$$



This series has the closed form as $n \rightarrow \infty$

$$p(t, x) = \lim_{n \rightarrow \infty} \phi_n(t, x) = \frac{e^{-x}}{1 + e^{-t}}$$

which is the exact solution of the problem (9) compatible with VIM and HAM.

In Tables 1-4, we list the absolute errors using the ADM (4-term approximation), the numerical solution of (10) with the trapezoidal rule (TRAP) p_T and the numerical solution of (10) with the Simpson rule (SIMP) p_S for various values of x at time $t = 0.001, 0.01, 0.1$ and $t = 1.0$. Twenty points have been used in the trapezoidal and Simpson rules. Comparing them with the HAM (4-term approximation) [3] results. At the end of all tables, the $k \cdot k_2$ has been calculated. Table 5 reproduces the residual error obtained by ADM (4-term approximation). In figure 1 we represent the contour plot of both the exact solution and the approximate solution of the problem (9) by using ADM (4-term approximation).

Table 1: Comparison of the absolute errors for example 1 with $t = 0.001$

x	ADM: $ p - \phi_4 $	TRAP: $ p - p_T $	SIMP: $ p - p_S $	HAM: $ p - \psi_4 $
0	2.08333×10^{-18}	2.60416×10^{-14}	3.89935×10^{-25}	0
5	1.40374×10^{-20}	1.75467×10^{-16}	2.62736×10^{-27}	0
10	9.45832×10^{-23}	1.18228×10^{-18}	1.77030×10^{-29}	0
20	4.29407×10^{-27}	5.36758×10^{-23}	8.03717×10^{-34}	2.06795×10^{-25}
30	1.94950×10^{-31}	2.43688×10^{-27}	3.64887×10^{-38}	0
40	8.85074×10^{-36}	1.10634×10^{-31}	1.65658×10^{-42}	0
50	4.01823×10^{-40}	5.02278×10^{-36}	7.52088×10^{-47}	0
$k \cdot k_2$	1.07820×10^{-24}	1.84142×10^{-14}	2.75726×10^{-25}	2.06795×10^{-25}

Table 2: Comparison of the absolute errors for example 1 with $t = 0.01$

x	ADM: $ p - \phi_4 $	TRAP: $ p - p_T $	SIMP: $ p - p_S $	HAM: $ p - \psi_4 $
0	2.08331×10^{-13}	2.60407×10^{-11}	1.51218×10^{-18}	2.08278×10^{-13}
5	1.40372×10^{-15}	1.75461×10^{-13}	1.01890×10^{-20}	1.40339×10^{-15}
10	9.45822×10^{-18}	1.18225×10^{-15}	6.86531×10^{-23}	9.45628×10^{-18}
20	4.29403×10^{-22}	5.36740×10^{-20}	3.11684×10^{-27}	4.29307×10^{-22}
30	1.94949×10^{-26}	2.43679×10^{-24}	1.41504×10^{-31}	1.94943×10^{-26}
40	8.85065×10^{-31}	1.10630×10^{-28}	6.42430×10^{-36}	8.84772×10^{-31}
50	4.01819×10^{-35}	5.02261×10^{-33}	2.91662×10^{-40}	4.01666×10^{-35}
$k \cdot k_2$	1.47312×10^{-13}	1.84136×10^{-11}	1.06927×10^{-18}	2.08283×10^{-13}

Table 3: Comparison of the absolute errors for example 1 with $t = 0.1$

x	ADM: $ p - \phi_4 $	TRAP: $ p - p_T $	SIMP: $ p - p_S $	HAM: $ p - \psi_4 $
0	2.08123×10^{-8}	2.59400×10^{-8}	1.48620×10^{-11}	2.08123×10^{-8}
5	1.40232×10^{-10}	1.74782×10^{-10}	1.00139×10^{-13}	1.40232×10^{-10}
10	9.44876×10^{-13}	1.17767×10^{-12}	6.74734×10^{-16}	9.44876×10^{-13}
20	4.28973×10^{-17}	5.34664×10^{-17}	3.06329×10^{-20}	4.28973×10^{-17}
30	1.94753×10^{-21}	2.42737×10^{-21}	1.39073×10^{-24}	1.94753×10^{-21}
40	8.84179×10^{-26}	1.10202×10^{-25}	6.31391×10^{-29}	8.84179×10^{-26}
50	4.01417×10^{-30}	5.00319×10^{-30}	2.86651×10^{-33}	4.01417×10^{-30}
$k \cdot k_2$	1.47165×10^{-8}	1.83424×10^{-8}	1.05090×10^{-11}	2.08127×10^{-8}

Table 4: Comparison of the absolute errors for example 1 with $t = 1$

x	ADM: $ p - \phi_4 $	TRAP: $ p - p_T $	SIMP: $ p - p_S $	HAM: $ p - \psi_4 $
0	1.89191×10^{-3}	1.11512×10^{-4}	1.29417×10^{-4}	1.89191×10^{-3}
5	1.27476×10^{-5}	7.51362×10^{-7}	8.72008×10^{-7}	1.27476×10^{-5}
10	8.58927×10^{-8}	5.06263×10^{-9}	5.87554×10^{-9}	8.58927×10^{-8}
20	3.89952×10^{-12}	2.29843×10^{-13}	2.66749×10^{-13}	3.89952×10^{-12}
30	1.77038×10^{-16}	1.04348×10^{-17}	1.21104×10^{-17}	1.77038×10^{-16}
40	8.03751×10^{-21}	4.73742×10^{-22}	5.49811×10^{-22}	8.03751×10^{-21}
50	3.64902×10^{-25}	2.15078×10^{-26}	2.49614×10^{-26}	3.64902×10^{-25}
$k \cdot k_2$	1.33778×10^{-3}	7.88509×10^{-5}	9.15120×10^{-5}	1.89195×10^{-3}

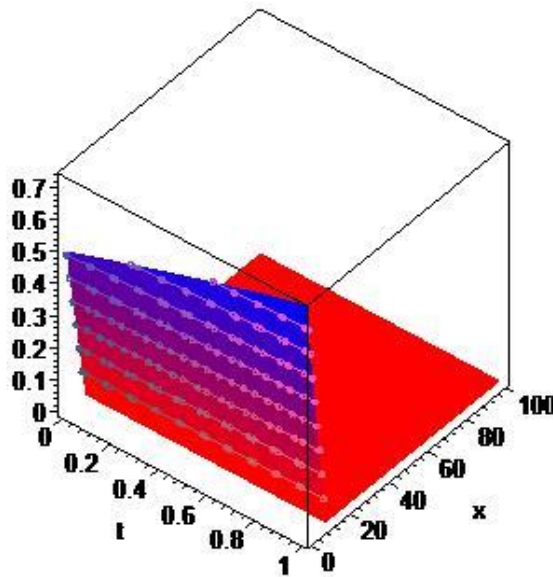


Fig. 1: Continuous line: Exact solution ADM $\phi_4(t, x)$

Table 5: Residual error for example 1

x	$t = 0.001$	$t = 0.01$	$t = 0.1$	$t = 1$
0	1.04167×10^{-14}	1.04166×10^{-10}	1.04123×10^{-6}	9.98264×10^{-3}
5	7.01869×10^{-17}	7.01867×10^{-13}	7.01577×10^{-9}	6.72625×10^{-5}
10	4.72916×10^{-19}	4.72914×10^{-15}	4.72719×10^{-11}	4.53211×10^{-7}
20	2.14703×10^{-23}	2.14703×10^{-19}	2.14614×10^{-15}	2.05758×10^{-11}
30	9.74752×10^{-28}	9.74748×10^{-24}	9.74346×10^{-20}	9.34138×10^{-16}
40	4.42537×10^{-32}	4.42535×10^{-28}	4.42353×10^{-24}	4.24098×10^{-20}
50	2.00911×10^{-36}	2.00911×10^{-32}	2.00828×10^{-28}	1.92540×10^{-24}
$k \cdot k_2$	7.36570×10^{-15}	7.36566×10^{-11}	7.36263×10^{-7}	7.05879×10^{-3}

Example 2 We consider the nonlinear age-structured population model (1) with $d_1(x) = 1$, $d_2(x) = 1$ and $p_0(x) = \frac{e^{-x}}{2}$. Thus the model can be written as [1-3]

$$\begin{aligned}
 \frac{\partial p(t, x)}{\partial t} + \frac{\partial p(t, x)}{\partial x} &= -[1 + P(t)] p(t, x), \quad t \geq 0, 0 \leq x \leq A, \\
 p(0, x) &= \frac{e^{-x}}{2}, \quad 0 \leq x \leq A, \\
 p(t, 0) &= P(t), \quad t \geq 0, \\
 P(t) &= \int_0^A p(t, x) dx, \quad t \geq 0,
 \end{aligned} \tag{11}$$

where $A \rightarrow +\infty$. And it is easy to verify that $p(t, x) = \frac{e^{-x}}{2 + t}$, $t \geq 0, x \geq 0$, is the exact solution of the problem (11).



Solution: Take $\Omega = [0, 1] \times [0, 100]$, which denotes (1 unit time) \times (100 unit age). On applying the ADM for this problem, we obtain the following recursion scheme

$$p_0(t, x) = \frac{e^{-x}}{2}, \quad [L_\omega p_n + p_n + A_n], n = 0, 1, \dots \quad (12)$$

Using the algorithm (12), we find that the iterations as $A \rightarrow +\infty$ are given by

$$\begin{aligned} p_0(t, x) &= \frac{1}{2} e^{-x}, \\ p_1(t, x) &= -\frac{1}{4} t e^{-x}, \\ p_2(t, x) &= \frac{1}{8} t^2 e^{-x}, \\ p_3(t, x) &= -\frac{1}{16} t^3 e^{-x}, \\ p_4(t, x) &= \frac{1}{32} t^4 e^{-x}, \\ &\vdots \\ p_n(t, x) &= \frac{(-1)^n}{2^{n+1}} t^n e^{-x}, n = 0, 1, \dots \end{aligned}$$

Thus, the approximate solution in a series form is given by

$$\phi_n(t, x) = e^{-x} \sum_{i=0}^{n-1} \frac{(-1)^i}{2^{i+1}} t^i, n \geq 1.$$

which is a geometric series. This series has the closed form as $n \rightarrow \infty$

$$p(t, x) = \lim_{n \rightarrow \infty} \phi_n(t, x) = \frac{e^{-x}}{2+t}.$$

which is the exact solution of the problem (11) compatible with VIM and HAM.

In Tables 6-9, we list the absolute errors using the ADM (4-term approximation), the numerical solution of (12) with the trapezoidal rule (TRAP) p_T and the numerical solution of (12) with the Simpson rule (SIMP) p_S for various values of x at time $t = 0.001, 0.01, 0.1$ and $t = 1.0$. Twenty points have been used in the trapezoidal and Simpson rules. Comparing them with the HAM (4-term approximation) [3] results. At the end of all tables, the $k \cdot k_2$ has been calculated. Table 10 reproduces the residual error obtained by ADM (4-term approximation). In figure 2 we represent the contour plot of both the exact solution and the approximate solution of the problem (11) by using ADM (4-term approximation).

Table 6: Comparison of the absolute errors for example 2 with $t = 0.001$

x	ADM: $ p - \phi_4 $	TRAP: $ p - p_T $	SIMP: $ p - p_S $	HAM: $ p - \psi_4 $
0	3.12344×10^{-14}	7.80406×10^{-14}	6.24479×10^{-18}	3.11973×10^{-14}
5	2.10456×10^{-16}	5.25833×10^{-16}	4.20770×10^{-20}	2.10335×10^{-16}
10	1.41804×10^{-18}	3.54304×10^{-18}	2.83513×10^{-22}	1.41624×10^{-18}
20	6.43789×10^{-23}	1.60853×10^{-22}	1.28714×10^{-26}	6.43133×10^{-23}
30	2.92280×10^{-27}	7.30275×10^{-27}	5.84364×10^{-31}	2.92194×10^{-27}
40	1.32695×10^{-31}	3.31544×10^{-31}	2.65300×10^{-35}	1.32504×10^{-31}
50	6.02433×10^{-36}	1.50520×10^{-35}	1.20446×10^{-39}	6.03029×10^{-36}
$k \cdot k_2$	2.20860×10^{-14}	5.51830×10^{-14}	4.41573×10^{-18}	3.11980×10^{-14}

Table 7: Comparison of the absolute errors for example 2 with $t = 0.01$

x	ADM: $ p - \phi_4 $	TRAP: $ p - p_T $	SIMP: $ p - p_S $	HAM: $ p - \psi_4 $
0	3.10945×10^{-10}	7.67278×10^{-11}	6.19824×10^{-13}	3.10945×10^{-10}
5	2.09513×10^{-12}	5.16987×10^{-13}	4.17634×10^{-15}	2.09513×10^{-12}
10	1.41169×10^{-14}	3.48343×10^{-15}	2.81400×10^{-17}	1.41169×10^{-14}
20	6.40906×10^{-19}	1.58147×10^{-19}	1.27755×10^{-21}	6.40906×10^{-19}
30	2.90971×10^{-23}	7.17989×10^{-24}	5.80008×10^{-26}	2.90971×10^{-23}
40	1.32101×10^{-27}	3.25966×10^{-28}	2.63323×10^{-30}	1.32101×10^{-27}
50	5.99736×10^{-32}	1.47988×10^{-32}	1.19548×10^{-34}	5.99736×10^{-32}
k-k ₂	2.19872×10^{-10}	5.42547×10^{-11}	4.38282×10^{-13}	3.10952×10^{-10}

Table 8: Comparison of the absolute errors for example 2 with $t = 0.1$

x	ADM: $ p - \phi_4 $	TRAP: $ p - p_T $	SIMP: $ p - p_S $	HAM: $ p - \psi_4 $
0	2.97619×10^{-6}	1.30793×10^{-8}	5.76078×10^{-10}	2.97619×10^{-6}
5	2.00534×10^{-8}	8.81280×10^{-11}	3.88158×10^{-10}	2.00534×10^{-8}
10	1.35119×10^{-10}	5.93802×10^{-13}	2.61539×10^{-12}	1.35119×10^{-10}
20	6.13439×10^{-15}	2.69585×10^{-17}	1.18738×10^{-16}	6.13439×10^{-15}
30	2.78501×10^{-19}	1.22391×10^{-21}	5.39072×10^{-21}	2.78501×10^{-19}
40	1.26439×10^{-23}	5.55657×10^{-26}	2.44738×10^{-25}	1.26439×10^{-23}
50	5.74033×10^{-28}	2.52268×10^{-30}	1.11111×10^{-29}	5.74033×10^{-28}
k-k ₂	2.10448×10^{-6}	9.24850×10^{-9}	4.07349×10^{-8}	2.97626×10^{-6}

Table 9: Comparison of the absolute errors for example 2 with $t = 1$

x	ADM: $ p - \phi_4 $	TRAP: $ p - p_T $	SIMP: $ p - p_S $	HAM: $ p - \psi_4 $
0	2.08333×10^{-2}	3.06013×10^{-3}	3.08779×10^{-3}	2.08333×10^{-2}
5	1.40374×10^{-4}	2.06190×10^{-5}	2.08054×10^{-5}	1.40374×10^{-4}
10	9.45832×10^{-7}	1.38929×10^{-7}	1.40185×10^{-7}	9.45832×10^{-7}
20	4.29407×10^{-11}	6.30740×10^{-12}	6.36442×10^{-12}	4.29407×10^{-11}
30	1.94950×10^{-15}	2.86355×10^{-16}	2.88944×10^{-16}	1.94950×10^{-15}
40	8.85074×10^{-20}	1.30005×10^{-20}	1.31180×10^{-20}	8.85074×10^{-20}
50	4.01823×10^{-24}	5.90223×10^{-25}	5.95558×10^{-25}	4.01823×10^{-24}
k-k ₂	1.47314×10^{-2}	2.16384×10^{-3}	2.18340×10^{-3}	2.08338×10^{-2}

Table 10: Residual error for example 2

x	$t = 0.001$	$t = 0.01$	$t = 0.1$	$t = 1$
0	1.24953×10^{-10}	1.24533×10^{-7}	1.20465×10^{-4}	8.98438×10^{-2}
5	8.41928×10^{-13}	8.39095×10^{-10}	8.11686×10^{-7}	6.05362×10^{-4}
10	5.67286×10^{-15}	5.65378×10^{-12}	5.46910×10^{-9}	4.07890×10^{-6}
20	2.57548×10^{-19}	2.56681×10^{-16}	2.48297×10^{-13}	1.85182×10^{-10}
30	1.16926×10^{-23}	1.16533×10^{-20}	1.12726×10^{-17}	8.40724×10^{-15}
40	5.30845×10^{-28}	5.29059×10^{-25}	5.11777×10^{-22}	3.81688×10^{-19}
50	2.41003×10^{-32}	2.40193×10^{-29}	2.32347×10^{-26}	1.73286×10^{-23}
k-k ₂	8.83552×10^{-11}	8.80580×10^{-8}	8.51815×10^{-5}	6.35291×10^{-2}

4 Conclusions

In this work ADM has been successfully used to solve nonlinear age-structured population models giving it a wider applicability. The proposed scheme was applied directly without any need for transformation formulae or restrictive assumptions. Results have shown that the analytical approximate solution process of ADM is compatible with those methods in the literature providing analytical approximation such as VIM and HAM. The results obtained in the two cases demonstrate the reliability and the efficiency of this method.

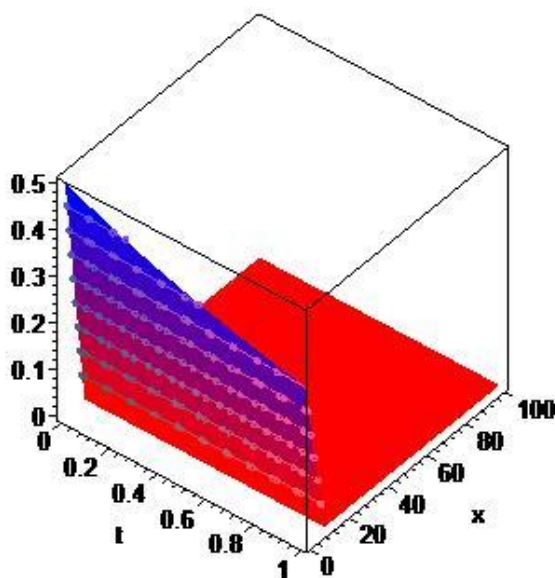


Fig. 2: Continuous line: Exact solution ADM $\phi_4(t, x)$

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