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Chaos Theory and Lorenz Attractors

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Abstract: Chaos theory is one of the fundamental theories in our lives. It ended the so-called "deterministic era" where everything is predictable. It was thought that the behavior, whether in the future or the past, of all the physical systems is known and that reaching perfect prediction is a matter of precision and accuracy. In this review, chaos theory is introduced along with its origin and history. In addition, Lorenz attractors are also introduced with the famous butterfly representation of Lorenz. Moreover, applications of the chaos theory are included here. These applications are in the field of economics, circuits and meteorology.

Keywords: Chaos, Lorenz, Butterfly.

1 Introduction

Chaos theory is one of the revolutionary theories that isn't of less importance than the theory of evolution and the special theory of relativity. It is a combination of a number of fields including mathematics and philosophy and that was developed for describing highly complex, unstable and unpredictable systems. These systems include weather models, the stock market, bird migration patterns, behavior of boiling water, neural networks and systems related to quantum phenomena. This theory is based on two main components; the first one is that systems, regardless of their degree of complexity, depend on an underlying overall equation or a principle that governs their behavior thus making it deterministic, theoretically, which is not due to its instability and the presence of a large number of contributing factors. The second main component is the high sensitivity to initial conditions, that a minute change in the initial conditions, such as rounding errors in numerical computation [2] of a certain dynamical system can produce cataclysmic and unpredictable outcomes for that dynamical system. Chaos theory has ended the era of deterministic systems. In the past, it was thought that every event taking place is a direct consequence of another event, that we can predict the upcoming events from the current ones. For example, it was believed that having more accurate measurements related to a certain physical system can lead to almost perfect prediction of the system's behavior, either in the future or in the past. Nowadays, with quantum mechanics put to work, almost nothing is predictable. In 1860, James Clerk Maxwell was the first one to introduce the idea of chaos. When Maxwell tried to explain how changes in hard molecules in the sphere introduce great microscopic chaos in gases, this was the first trial recorded in the history that someone discussed chaos. When Lorenz was working on predicting the weather through a simulation on the computer, he found a very interesting phenomenon that contributed greatly to understanding the chaos theory. The computer that he was using used a six-digit precision. However, when he tried to use only the first three digits, he found that the results have changed tremendously. For example, the original number is 1.876596 and he used 1.876. He worked on understanding this phenomenon, until he presented his paper **"**Predictability: Does the Flap of a Butterfly's Wings in Brazil set off a Tornado in Texas?**"** (Lorenz)

Then the phenomenon of the butterfly started to emerge. However, some scientists have also worked on the chaos theory or on creating the basics that Lorenz built his foundation on. For example, Leibniz assumed the possibility of fractional derivatives in 1695, and David Ruelle and Floris Takens elucidated the strange attractors' phenomena.

2 Lorenz Attractors

For each chaotic system there exist a point of equilibrium, called an attractor, where the system is in a state of rest. Due the instability of the chaotic system, it never reaches such stable state, but instead possesses a series of states leading to what's called dynamic equilibrium. In such a dynamic equilibrium state, where a system approaches an attractor, another attractor is created under the effect of forces that are applied to the system which can be mechanical, electrical, or any other type of

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forces. The attractors in a two-attractor-system are called Lorenz attractors as Lorenz was the first one to study such attractors. He derived a simplified model of convection in the earth's atmosphere that is described through the following differential equations:

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Where x, y, z and t are parameters that describe the system's state and , and are constants related to the system. In **Lorenz model** these constants were chosen to be 28, 10, 8/3 respectively. The solution for such differential equations with such parameters was drawn on Matlab and resulted in the following chaotic system:

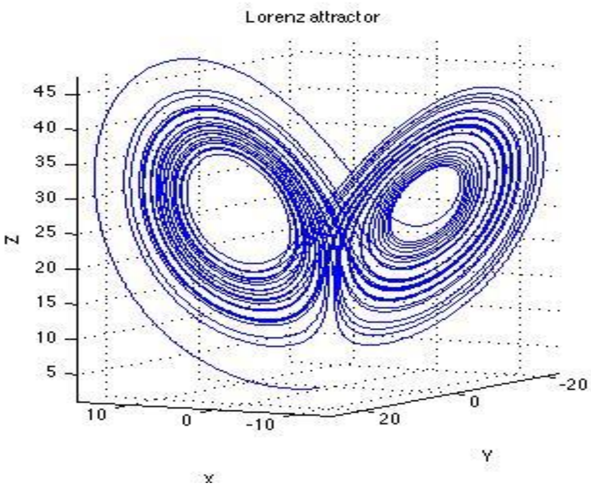

Figure (1): Lorenz attractors

Where the two attractors are quite clear. As any other chaotic systems, as mentioned before, a minute change in the initial conditions can produce cataclysmic and unpredictable outcomes and this can be inferred from the following graphs drawn on Matlab:

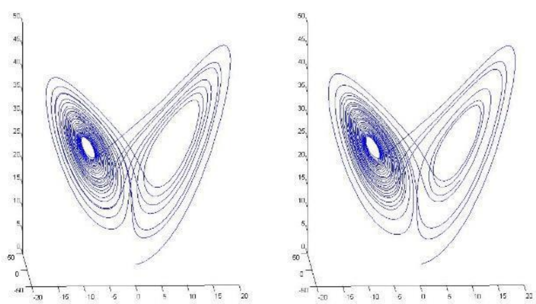

Figure (2): The effect of a minute change in the initial conditions (1,1,1) vs (1.1,1.1,1.1) attractors

3 Applications

- Meteorology

When considering the graph of the Euclidean distance between two cases (with two different initial values) versus time (Figure (3)). It is obvious that at the beginning there is no difference but after some time the difference is clear and the function is unpredictable. If we apply this to the convection currents in the weather prediction, it is impossible to predict the weather more than 5-10 days according to the chosen parameters. In case of convection currents in weather prediction, these parameters are defined as:

is a control parameter, representing the temperature difference between the top and bottom of the tank, is called the Prandtl number (it involves the viscosity and thermal conductivity of the fluid), and is the ratio of width-to-height of the convection layer.

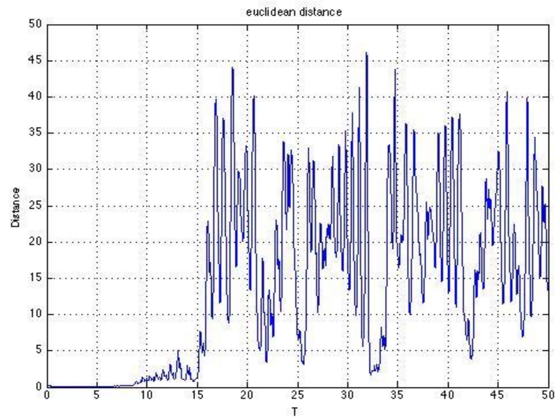

Figure (3): Graph of the Euclidean distance between two cases versus time.

- Circuits

One of the applications that proves Lorenz attractors physically is the Chua circuit. The Chua's circuit was invented in 1983. It consists of five components; one resistor, two capacitors, one inductor, and one Chua's diode. All these components are passive except the Chua's diode. Passive components are those components that do not need a power source. Furthermore, all the components are linear, except for the Chua's diode which is non-linear. When the researchers tried to represent the current versus the nonlinear voltage function (Figure (5)), they found that

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Where *g(x)*=I (current), is the voltage and both m_0 and m_1 are the slopes. From the diagram, it is clear that this system is chaotic. Moreover, the researchers found that there is a great similarity between the equations of the Chua's circuit and those of Lorenz. Not only this, but also when they tried to represent the behavior of this circuit on the oscilloscope, they found that there is a great similarity between Lorenz's representations and those of the circuit (Figure (4)). This application proves that Lorenz's representation can be used to predict the behavior of systems; Lorenz representations are not just abstract data, and they can be used to represent systems.

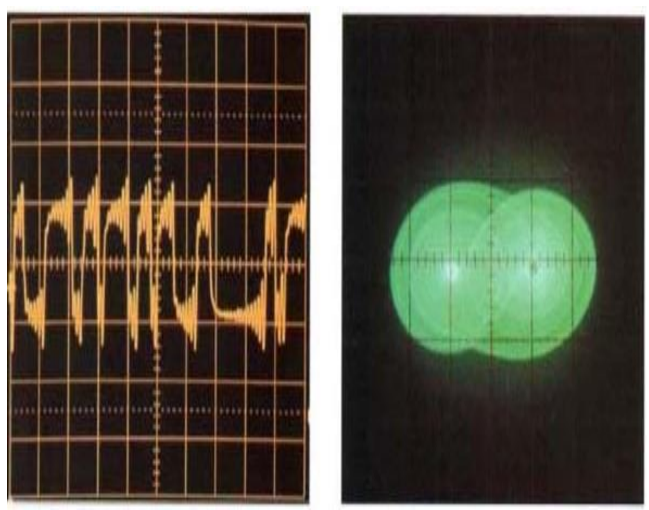

Figure (4): representations of the circuit using the oscilloscope. The Wave form on the Left, while the Lissajous Figure on the right.

(D. L. O. Chua)

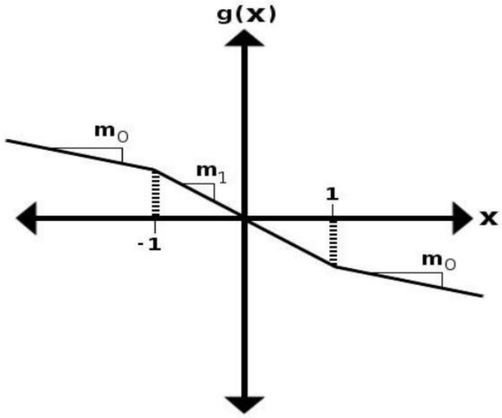

Figure (5): plot of current versus voltage.

(D. L. O. Chua)

- Economics

In economics, the system of exchange rate which we deal with as the prices of the currency in the financial treatment is highly influenced by news which are unpredictable. This nonlinear behavior get approachably controlled by chaos theory in sense of some researchers having many studies which are positive to chaotic dynamics (Fernández-Rodríguez et al. 2002,Westerhoff, Darvas 1998 and Hommes 2005,) . On the other side, some researchers reject treating exchange rate as chaotic dynamic system with number of studies (Hsieh 1989, Brooks C. 1996 and Serletis et al. 2000). Chaotic theory supposes that for a system to be chaotic, it should exhibit complex behavior at some point which is highly influenced by the smallest disturbances and that what happens due to the vibrations of exchange rates. Thus it ought to be treated as a chaotic system. Foreign currency demand is determined by equation (1).

(—) ()

Where S_t is current exchange rate, e_t is the current price of the foreign currency, e_e is the future estimated exchange rate and α is the sensitivity parameter. Introducing another linear function, the trade balance (Ti), which is the calculation of a country's exports minus its imports. Trade balance depending on the current exchange rate (e_t) and last exchange rate (e_{t-1}) is given by the following:

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The expected exchange rate represents the stable state at which the speculators on the market do not wish to sell nor buy. This implies as following:

(3)

Replacing equations (1) and (2) in (3) we get

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By solving this equation using quadratic formula, we get two roots for the solution. The Positive one is

[( )] √[( )] ()

In this case, the constants will be (, β = 4) and (= 26) .The graphical representation of the solution has a maximum value of 2.76 and a minimum value of 0.091. Any other value from outside the interval represented by these two values is attracted. This system with these specific parameters represents a chaotic system. The following figures show a graphical representation using two slightly different initial values: 0.2 and 0.2005.

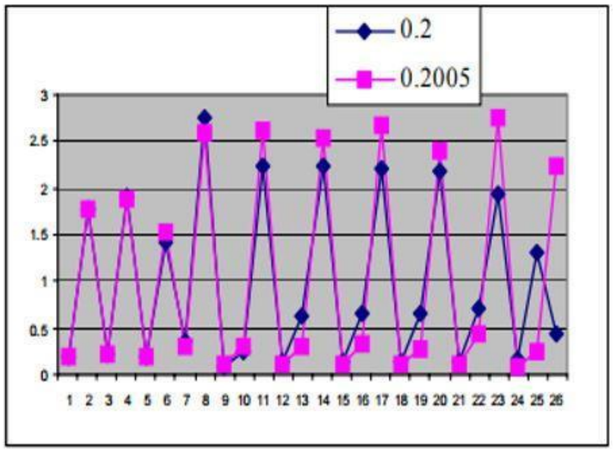

Figure (6): A graphical representation using two slightly different initial values: 0.2 and 0.2005. Chaos and nonlinear model. (Vlad el al.)

6 Conclusions

As an axiom, each physical system which can be described should be understandable and predictable. This dominant thought was altered by a paradigm shift which is supported by the fact that the real systems have unpredictable nonlinear behavior that differs from the linear behavior used simple schools' models which are used to elaborate the concepts. Namely, simulations and schools' models are based upon simplifications, through eliminating many variables which have a little effect on the real system, but nowadays there are rapidly developing simulations, using more powerful computers and numerical solutions, making it possible to uncover more complex behaviors as well as accounting for different variables.

This utterly changes the lives of humans, and this's the significance of that review. Thus our future models and plans are no longer valid, we can no longer predict the state of the economy, the state of the stock exchange or even the weather on the long run. Nowadays, chaos theory branches to take part in classical and quantum mechanics, thermal reactions, physiological control systems, evolution, computer science, circuit implementation, communication, applied engineering and many other fields including physics and metaphysics aspects. More research should be conducted on that field in order to find out how to control the chaotic behavior of different systems in order to increase the validity of our future models and plans especially in the field of economics, where future models and plans can give crucial information about the general state of the economy in different countries.

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