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Some Improved Estimators for Estimating Population Variance in the Presence of Measurement Errors

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Abstract: In this paper the problem of estimating finite population variance under measurement errors is discussed. Some estimators based on arithmetic mean, geometric mean and harmonic mean under measurement errors are proposed. Biases and mean square errors of proposed estimators are calculated to the first order of approximation. A comparative study is made among the usual unbiased estimator, usual ratio estimator and Kadilar and Cingi(2006a) estimator. Hypothetical study is also given at the end of the paper to support the theoretical findings.

Keywords: Auxiliary Information, Bias, Efficiency, Measurement Errors, Mean square error, Population Variance, Simple Random Sampling.

1 Introduction

Generally in statistical analysis it is assumed that observations are recorded without any error. However, in practice, this assumption may not be true and the data may be influenced by measurement errors due to various reasons, Cochran [1], Sukhatme et. al.[9] and Biemer et.al. (1991)[7]. When the observations are influenced by measurement errors then the estimates of population parameters (Mean, Variance, Total etc.) based on that values leeds to the incorrect estimates. So the study of these effects is essential.

Measurement errors are generally taken as the difference between true and observed values on any desirable Characteristic. Measurement errors are generally taken as normally distributed with mean zero implies that average effect of measurement errors on respondents answer is zero, chapter1 by Robert M. Groves(Measurement Errors in Surveys)[7]. But it will increase the Variability, so estimation of effect of these errors needs attention. Many authors including Das and Tripathi (1978) [2], Srivastava and Jhajj(1980)[15], Singh and Karpe (2009)[12], [13],[14] and Diana and Giordan(2012)[3], studied the effect of measurement errors on estimation of population parameters. In the present article we study the estimation of finite population variance in the presence of measurement errors.

2 Notations

Let us consider Y and X are the study and auxiliary variables defined on a finite population $U = (U_1, U_2, ...U_N)$ of size N and a sample of size n is taken by simple random sampling without replacement(SRSWOR) on these two characteristics Y and X. Here it is assumed that y_i and x_i are recorded instead of true values Y_i and X_i respectively. The observational errors /measurement errors are defined as

$$u_i = y_i - Y_i \tag{1}$$

$$v_i = x_i - X_i \tag{2}$$

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u_i and v_i are random in nature with mean zero and different variances σ_u^2 and σ_v^2 respectively. It is assumed that $u_i's$ and $v_i's$ are uncorrelated although $Y_i's$ and $X_i's$ are correlated. It is also assumed that $u_i's$ and $v_i's$ are uncorrelated with $Y_i's$ and $X_i's$ respectively.

Let (μ_Y,μ_X) and (σ_Y^2,σ_X^2) are mean and variances of (Y,X) ,i.e, study and auxiliary variables. ρ is the correlation coefficient between X and Y. Let $\bar{y}=\frac{1}{n}\sum_{i=1}^{n}y_i$,$\bar{x}=\frac{1}{n}\sum_{i=1}^{n}x_i$ be the unbiased estimators of the population means μ_Y and μ_X respectively.

$$s_y^2=\frac{1}{n-1}\sum_{i=1}^{n}(y_i-\bar{y})^2$$

and

$$s_x^2=\frac{1}{n-1}\sum_{i=1}^{n}(x_i-\bar{x})^2$$

are the expected values of s_y^2 and s_x^2 under measurement errors are

$E(s_y^2)=\sigma_Y^2+\sigma_u^2$ and $E(s_x^2)=\sigma_X^2+\sigma_v^2$

Let error variances σ_u^2 and σ_v^2 are known a prior than unbiased estimators of population variance under measurement errors are

$$\hat{\sigma}_y^2=s_y^2-\sigma_u^2>0$$

$$\hat{\sigma}_x^2=s_x^2-\sigma_v^2>0$$

Now, let us define

$$\hat{\sigma}_y^2=\sigma_Y^2(1+e_0)$$

$$\hat{\sigma}_x^2=\sigma_X^2(1+e_1)$$

$$E(e_0)=E(e_1)=0$$

$$E(e_0^2)=\frac{A_y}{n}$$

,

$$E(e_1^2)=\frac{A_x}{n}$$

$$E(e_0e_1)=\frac{\delta-1}{n}$$

Where,

$$A_y=\gamma_{2Y}+\gamma_{2u}\frac{\sigma_u^2}{\sigma_Y^2}+2(1+\frac{\sigma_u^2}{\sigma_Y^2})^2$$

$$A_x=\gamma_{2X}+\gamma_{2v}\frac{\sigma_v^2}{\sigma_X^2}+2(1+\frac{\sigma_v^2}{\sigma_X^2})^2$$

$$\delta=\frac{\mu_{22}(X,Y)}{\sigma_X^2\sigma_Y^2}$$

$$\gamma_{2z}=\beta_{2z}-3$$

,

$$\beta_{2z}=\mu_{4z}/(\mu_{2z}^2)$$

and

$$\mu_{rz}=E(z_i-\mu_z)^2;z=X,Y,U,V$$

$$\mu_{22}(XY) = E\{(X_i - \mu_X)^2 (Y_i - \mu_Y)^2\}$$

And

$$B = \frac{\sigma_X^2}{\sigma_X^2 - C_X}$$

The usual unbiased estimator of the population variance of the study variable Y under measurement errors is defined by

$$t_0 = \hat{\sigma}_y^2 \tag{3}$$

$$= \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2$$

$$Bias(t_0) = 0 \tag{4}$$

$$MSE(t_0) = \sigma_Y^4 \frac{A_y}{n} \tag{5}$$

Isaki (1983)[5] estimator under measurement errors

$$t_1 = \hat{\sigma}_y^2 \left(\frac{\sigma_X^2}{\hat{\sigma}_x^2}\right) \tag{6}$$

$$Bias(t_1) = \frac{\sigma_Y^2}{n}(1 + A_x - \delta) \tag{7}$$

$$MSE(t_1) = \frac{\sigma_Y^4}{n}(2 + A_y + A_x - 2\delta) \tag{8}$$

Kadilar and Cingi (2006a)[6] estimator under measurement errors is

$$t_2 = \hat{\sigma}_y^2 \left(\frac{\sigma_X^2 - C_X}{\hat{\sigma}_x^2 - C_X}\right) \tag{9}$$

$$Bias(t_2) = \frac{\sigma_Y^2}{n} B(1 + BA_x - \delta) \tag{10}$$

$$MSE(t_2) = \frac{\sigma_Y^4}{n}[A_y + B^2 A_x - 2B(\delta - 1)] \tag{11}$$

where,

$$B = \frac{\sigma_X^2}{\sigma_X^2 - C_X}$$

3 Suggested Estimations

3.1 The Estimator Based on t_0 and t_1

Taking the arithmetic mean (AM), geometric mean(GM) and harmonic mean(HM) of the estimators t_0 and t_1 we get the following estimator of the population variance under measurement errors respectively as

$$t_3 = \frac{1}{2}(t_0 + t_1) = \frac{\hat{\sigma}_y^2}{2}\left(1 + \frac{\sigma_X^2}{\hat{\sigma}_x^2}\right) \tag{12}$$

$$t_4 = (t_0 t_1)^{\frac{1}{2}} = \hat{\sigma}_y^2 \left(\frac{\sigma_X^2}{\hat{\sigma}_x^2}\right)^{\frac{1}{2}} \tag{13}$$

$$t_5 = \frac{2}{\frac{1}{t_0}+\frac{1}{t_1}} = \frac{2\hat{\sigma}_y^2}{1+\frac{\hat{\sigma_X^2}}{\sigma_X^2}} \tag{14}$$

The biases and mean squared errors of t_3, t_4 and t_5 up to first degree of approximation are given by

$$Bias(t_3) = \frac{\sigma_Y^2}{2n}[A_X-(\delta-1)] \tag{15}$$

$$Bias(t_4) = \frac{\sigma_Y^2}{8n}[3A_X-4(\delta-1)] \tag{16}$$

$$Bias(t_5) = \frac{\sigma_Y^2}{4n}[A_X-2(\delta-1)] \tag{17}$$

$$MSE(t_3) = MSE(t_4) = MSE(t_5) = \frac{\sigma_Y^4}{4n}[A_X+4(A_Y-(\delta-1))] \tag{18}$$

3.2 The Estimator Based on t_0 and t_2

The estimators of population variance under measurement errors based on arithmetic mean (AM), geometric mean (GM) and harmonic mean(HM) of the estimators t_0 and t_2 are respectively defined as

$$t_6 = \frac{t_0+t_2}{2} = \frac{\hat{\sigma}_y^2}{2}\left[1+\frac{\sigma_X^2-C_X}{\hat{\sigma}_x^2-C_X}\right] \tag{19}$$

$$t_7 = (t_0 t_1)^{\frac{1}{2}} = \hat{\sigma}_y^2\left[\frac{\sigma_X^2-C_X}{\hat{\sigma}_x^2-C_X}\right]^{\frac{1}{2}} \tag{20}$$

$$t_8 = \frac{2}{\frac{1}{t_0}+\frac{1}{t_2}} = \frac{2\hat{\sigma}_y^2}{1+\frac{\hat{\sigma}_x^2-C_X}{\sigma_X^2-C_X}} \tag{21}$$

The biases and mean squared errors of t_6, t_7 and t_8 up to first degree of approximation are given by

$$Bias(t_6) = \frac{\sigma_Y^2}{2n}[B^2A_X-B(\delta-1)] \tag{22}$$

$$Bias(t_7) = \frac{B\sigma_Y^2}{8n}[3BA_X-4(\delta-1)] \tag{23}$$

$$Bias(t_8) = \frac{B\sigma_Y^2}{4n}[BA_X-2(\delta-1)] \tag{24}$$

$$MSE(t_6) = MSE(t_7) = MSE(t_8) = \frac{\sigma_Y^4}{4n}[4A_Y+B\{BA_X-4(\delta-1)\}] \tag{25}$$

3.3 The estimators based on t_1 and t_2

We proposed the following estimators of population variance based on arithmetic mean (AM), geometric mean (GM) and harmonic mean (HM) of the estimators t_1 and t_2 are respectively defined as

$$t_9 = \frac{1}{2}(t_1+t_2) = \frac{\hat{\sigma}_y^2}{2}\left[\frac{\sigma_X^2}{\hat{\sigma}_x^2}+\frac{\sigma_X^2-C_X}{\hat{\sigma}_x^2-C_X}\right] \tag{26}$$

$$t_{10} = (t_1 t_2)^{\frac{1}{2}} = \hat{\sigma}_y^2 \left(\frac{\sigma_X}{\hat{\sigma}_x}\right)\left(\frac{\sigma_X^2 - C_X}{\hat{\sigma}_x^2 - C_X}\right)^{\frac{1}{2}} \tag{27}$$

$$t_{11} = \frac{2}{\left(\frac{1}{t_1} + \frac{1}{t_2}\right)} = \frac{2\hat{\sigma}_y^2}{\left(\frac{\hat{\sigma}_x^2}{\sigma_X^2}\right) + \left(\frac{\hat{\sigma}_x^2 - C_X}{\sigma_X^2 - C_X}\right)} \tag{28}$$

The biases and mean squared errors of t_9, t_{10} and t_{11} up to first degree of approximation are given by

$$Bias(t_9) = \frac{(1+B)\sigma_Y^2}{2n}[1 + (1+B)A_X - \delta] \tag{29}$$

$$Bias(t_{10}) = \frac{\sigma_Y^2}{64n}[41A_X - 32(1+B)(\delta - 1)] \tag{30}$$

$$Bias(t_{11}) = \frac{(1+B)\sigma_Y^2}{4n}[4 + (1+B) - 4\delta] \tag{31}$$

$$MSE(t_9) = MSE(t_{10}) = MSE(t_{11}) = \frac{\sigma_Y^4}{4n}[4A_Y + (1+B)\{4 + (1+B)A_X - 4\delta\}] \tag{32}$$

3.4 The estimators based on t_1, t_2 and t_3

We define the following estimators of population variance based on arithmetic mean (AM), geometric mean (GM) and harmonic mean (HM) of the estimators t_1, t_2 and t_3 (to the first degree of approximation) are respectively as

$$t_{12} = \frac{1}{3}(t_0 + t_1 + t_2) = \frac{\hat{\sigma}_y^2}{3}\left[1 + \frac{\sigma_X^2}{\hat{\sigma}_x^2} + \frac{\sigma_X^2 - C_X}{\hat{\sigma}_x^2 - C_X}\right] \tag{33}$$

$$t_{13} = (t_0 t_1 t_2)^{\frac{1}{3}} = \hat{\sigma}_y^2 \left[\left(\frac{\sigma_X^2}{\hat{\sigma}_x^2}\right)\left(\frac{\sigma_X^2 - C_X}{\hat{\sigma}_x^2 - C_X}\right)\right]^{\frac{1}{3}} \tag{34}$$

$$t_{14} = \left(\frac{3}{\frac{1}{t_0} + \frac{1}{t_1} + \frac{1}{t_2}}\right) = \frac{3\hat{\sigma}_y^2}{1 + \frac{\hat{\sigma}_x^2}{\sigma_X^2} + \frac{\hat{\sigma}_x^2 - C_X}{\sigma_X^2 - C_X}} \tag{35}$$

The biases and mean squared errors of t_{12}, t_{13} and t_{14} up to first degree of approximation are given by

$$Bias(t_{12}) = \frac{\sigma_Y^2}{3n}(1+B)[1 + (1+B)A_X - \delta] \tag{36}$$

$$Bias(t_{13}) = \frac{\sigma_Y^2}{9n}(1+B)[3 + 5(1+B)A_X - 3\delta] \tag{37}$$

$$Bias(t_{14}) = \frac{\sigma_Y^2}{9n}(1+B)[3 + (1+B)A_X - 3\delta] \tag{38}$$

$$MSE(t_{12}) = MSE(t_{13}) = MSE(t_{14}) = \frac{\sigma_Y^4}{9n}[9A_Y + (1+B)\{6 - (1+B)A_X - 6\delta\}] \tag{39}$$

4 Efficiency Comparisons

The efficiency comparison of t_1, t_2, t_3, t_4, t_5, t_6, t_7, t_8, t_9, t_{10}, t_{11}, t_{12}, t_{13} and t_{14} with respect to t_0 respectively are given by

$$MSE(t_1) - MSE(t_0) < 0$$
$$\frac{\sigma_Y^4}{n}(2 + A_Y + A_X - 2\delta) - \frac{\sigma_Y^4}{n}A_Y < 0$$

$$2\delta - A_X > 2 \tag{40}$$

$$MSE(t_2) - MSE(t_0) < 0 \; \frac{\sigma_Y^4}{n}[A_Y + B^2A_X - 2B(\delta - 1)] - \frac{\sigma_Y^4}{n}A_Y < 0$$

$$2\delta - BA_X > 2 \tag{41}$$

$$MSE(t_3) - MSE(t_0) = MSE(t_4) - MSE(t_0) = MSE(t_5) - MSE(t_0) < 0$$
$$\frac{\sigma_Y^4}{4n}[A_X + 4\{A_Y - (\delta - 1)\}] - \frac{\sigma_Y^4}{n}A_Y < 0$$

$$4\delta - A_X > 4 \tag{42}$$

$$MSE(t_6) - MSE(t_0) = MSE(t_7) - MSE(t_0) = MSE(t_8) - MSE(t_0) < 0$$
$$\frac{\sigma_Y^4}{4n}[4A_Y + B\{BA_X - 4(\delta - 1)\}] - \frac{\sigma_Y^4}{n}A_Y < 0$$

$$4\delta - BA_X > 4 \tag{43}$$

$$MSE(t_9) - MSE(t_0) = MSE(t_{10}) - MSE(t_0) = MSE(t_{11}) - MSE(t_0) < 0$$
$$\frac{\sigma_Y^4}{4n}[4A_Y + (1+B)\{4 + (1+B)A_X - 4\delta\}] - \frac{\sigma_Y^4}{n}A_Y < 0$$

$$4\delta - (1+B)A_X > 4 \tag{44}$$

$$MSE(t_{12}) - MSE(t_0) = MSE(t_{13}) - MSE(t_0) = MSE(t_{14}) - MSE(t_0) < 0$$
$$\frac{\sigma_Y^4}{9n}[9A_Y + (1+B)\{6 - (1+B)A_X - 6\delta\}] - \frac{\sigma_Y^4}{n}A_Y < 0$$

$$6\delta - (1+B)A_X > 6 \tag{45}$$

The efficiency comparison of t_2, t_3, t_4, t_5, t_6, t_7, t_8, t_9, t_{10}, t_{11}, t_{12}, t_{13} and $t_{14}$with respect to t_1 respectively are given by

$$MSE(t_2) - MSE(t_1) < 0$$
$$\frac{\sigma_Y^4}{n}[A_Y + B^2A_X - 2B(\delta - 1)] - \frac{\sigma_Y^4}{n}(2 + A_Y + A_X - 2\delta) < 0$$

$$2\delta - (1+B)A_X > 2 \tag{46}$$

$$MSE(t_3) - MSE(t_1) = MSE(t_4) - MSE(t_1) = MSE(t_5) - MSE(t_1) < 0$$
$$\frac{\sigma_Y^4}{4n}[A_X + 4\{A_Y - (\delta - 1)\}] - \frac{\sigma_Y^4}{n}(2 + A_Y + A_X - 2\delta) < 0$$

$$4\delta - 3A_X > 4 \tag{47}$$

$$MSE(t_6) - MSE(t_1) = MSE(t_7) - MSE(t_1) = MSE(t_8) - MSE(t_1) < 0$$
$$\frac{\sigma_Y^4}{4n}[4A_Y + B\{BA_X - 4(\delta - 1)\}] - \frac{\sigma_Y^4}{n}(2 + A_Y + A_X - 2\delta) < 0$$

$$4\delta - (B+2)A_X > 4 \tag{48}$$

$$MSE(t_9) - MSE(t_1) = MSE(t_{10}) - MSE(t_1) = MSE(t_{11}) - MSE(t_1) < 0$$

$$\frac{\sigma_Y^4}{4n}[4A_Y+(1+B)\{4+(1+B)A_X-4\delta\}]-\frac{\sigma_Y^4}{n}(2+A_Y+A_X-2\delta)<0$$

$$4\delta-(3+B)A_X>4 \tag{49}$$

$$MSE(t_{12})-MSE(t_1)=MSE(t_{13})-MSE(t_1)=MSE(t_{14})-MSE(t_1)<0$$
$$\frac{\sigma_Y^4}{9n}[9A_Y+(1+B)\{6-(1+B)A_X-6\delta\}]-\frac{\sigma_Y^4}{n}(2+A_Y+A_X-2\delta)<0$$

$$18\delta-(4+B)A_X>6 \tag{50}$$

The efficiency comparison of t_3, t_4, t_5, t_6, t_7, t_8, t_9, t_{10},t_{11}, t_{12},t_{13} and t_{14} with respect to t_2 respectively are given by

$$MSE(t_3)-MSE(t_2)=MSE(t_4)-MSE(t_2)=MSE(t_5)-MSE(t_2)<0$$
$$\frac{\sigma_Y^4}{4n}[A_X+4\{A_Y-(\delta-1)\}]-\frac{\sigma_Y^4}{n}[A_Y+B^2A_X-2B(\delta-1)]<0$$

$$4\delta-(1-2B)A_X>4 \tag{51}$$

$$MSE(t_6)-MSE(t_2)=MSE(t_7)-MSE(t_2)=MSE(t_8)-MSE(t_2)<0$$
$$\frac{\sigma_Y^4}{4n}[4A_Y+B\{BA_X-4(\delta-1)\}]-\frac{\sigma_Y^4}{n}[A_Y+B^2A_X-2B(\delta-1)]<0$$

$$4\delta-3BA_X>4 \tag{52}$$

$$MSE(t_9)-MSE(t_2)=MSE(t_{10})-MSE(t_2)=MSE(t_{11})-MSE(t_2)<0$$
$$\frac{\sigma_Y^4}{4n}[4A_Y+(1+B)\{4+(1+B)A_X-4\delta\}]-\frac{\sigma_Y^4}{n}[A_Y+B^2A_X-2B(\delta-1)]<0$$

$$4\delta-A_X(1+3B)>4 \tag{53}$$

$$MSE(t_{12})-MSE(t_2)=MSE(t_{13})-MSE(t_2)=MSE(t_{14})-MSE(t_2)<0$$
$$\frac{\sigma_Y^4}{9n}[9A_Y+(1+B)\{6-(1+B)A_X-6\delta\}]-\frac{\sigma_Y^4}{n}[A_Y+B^2A_X-2B(\delta-1)]<0$$

$$6\delta-A_X(1+4B)>6 \tag{54}$$

The efficiency comparison of t_6, t_7, t_8, t_9, t_{10}, t_{11}, t_{12}, t_{13} and t_{14} with respect to t_3, t_4, t_5 respectively are given by

$$MSE(t_6)/MSE(t_7)/MSE(t_8)-MSE(t_3)/MSE(t_4)/MSE(t_5)<0$$
$$\frac{\sigma_Y^4}{4n}[4A_Y+B\{BA_X-4(\delta-1)\}]-\frac{\sigma_Y^4}{4n}[A_X+4\{A_Y-(\delta-1)\}]<0$$

$$4\delta-(1+B)A_X>0 \tag{55}$$

$$MSE(t_9)/MSE(t_{10})/MSE(t_{11})-MSE(t_3)/MSE(t_4)/MSE(t_5)<0$$
$$\frac{\sigma_Y^4}{4n}[4A_Y+(1+B)\{4+(1+B)A_X-4\delta\}]-\frac{\sigma_Y^4}{4n}[A_X+4\{A_Y-(\delta-1)\}]<0$$

$$4\delta-(2+B)A_X>4 \tag{56}$$

$$MSE(t_{12})/MSE(t_{13})/MSE(t_{14})-MSE(t_3)/MSE(t_4)/MSE(t_5)<0$$
$$\frac{\sigma_Y^4}{9n}[9A_Y+(1+B)\{6-(1+B)A_X-6\delta\}]-\frac{\sigma_Y^4}{4n}[A_X+4\{A_Y-(\delta-1)\}]<0$$

$$12\delta-(5+2B)A_X>12 \tag{57}$$

The efficiency comparison of t_9, t_{10}, t_{11}, t_{12},t_{13} and t_{14} with respect to t_6, t_7, and t_8 respectively are given by

$$MSE(t_9)/MSE(t_{10})/MSE(t_{11}) - MSE(t_6)/MSE(t_7)/MSE(t_8) < 0$$
$$\frac{\sigma_Y^4}{4n}[4A_Y + (1+B)\{4+(1+B)A_X - 4\delta\}] - \frac{\sigma_Y^4}{4n}[4A_Y + B\{BA_X - 4(\delta-1)\}] < 0$$

$$16\delta - (1+2B)A_X > 16 \tag{58}$$

$$MSE(t_{12})/MSE(t_{13})/MSE(t_{14}) - MSE(t_6)/MSE(t_7)/MSE(t_8) < 0$$
$$\frac{\sigma_Y^4}{9n}[9A_Y + (1+B)\{6-(1+B)A_X - 6\delta\}] - \frac{\sigma_Y^4}{4n}[4A_Y + B\{BA_X - 4(\delta-1)\}] < 0$$

$$12\delta - A_X(2+5B) > 12 \tag{59}$$

The efficiency comparison of t_{12}, t_{13} and t_{14} with respect to t_9, t_{10} and t_{11} respectively are given by

$$MSE(t_{12})/MSE(t_{13})/MSE(t_{14}) - MSE(t_9)/MSE(t_{10})/MSE(t_{11}) < 0$$
$$\frac{\sigma_Y^4}{9n}[9A_Y + (1+B)\{6-(1+B)A_X - 6\delta\}] - \frac{\sigma_Y^4}{4n}[4A_Y + (1+B)\{4+(1+B)A_X - 4\delta\}] < 0$$

$$12\delta - 5(1+B)A_X > 12 \tag{60}$$

Conditions in which proposed estimators are efficient than others are defined in $(40)-(60)$.

5 Simulation Study

In this section, we demonstrate the performance of adopted estimators over other competitors, generating population from normal distribution by using R programme. The description of this data is as follows $X = N(5,10), Y = X + N(0,1), y = Y + N(1,3), x = X + N(1,3), n = 5000, \mu_X = 4.95, \mu_Y = 4.93, \sigma_X^2 = 99.38, \sigma_Y^2 = 100.12, \sigma_u^2 = 25.57, \sigma_v^2 = 24.28, \rho_{XY} = 0.99$

Table 1: MSE's of estimators (with and without measurement errors)

Estimator	MSE with measurement errors	MSE without measurement errors
MSE (t_0)	6.25	3.93
MSE(t_1)	4.60	0.97
MSE(t_2)	4.68	0.09
MSE(t_3)/MSE(t_4)/MSE(t_5)	3.90	1.02
MSE(t_6)/$MSE(t_7)$/$MSE(t_8)$	3.88	0.98
MSE(t_9)/MSE(t_{10})/MSE(t_{11})	4.78	2.02
MSE(t_{12})/MSE(t_{13})/MSE(t_{14})	3.85	0.473

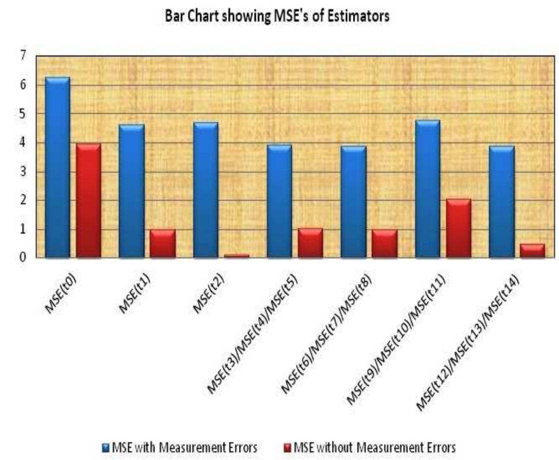

Fig. 1: Bar Graph showing MSEs of estimators

6 Conclusion

In this article we have suggested some estimators of population variance under measurement errors which are based on arithmetic mean, geometric mean and harmonic mean of the usual unbiased, usual ratio and Kadilar and Cingi(2006a) estimators. The expressions of bias and mean squared error of proposed estimators have been derived up to first degree of approximation. The theoretical conditions under which the proposed estimators are more efficient than usual unbiased usual ratio and Kadilar and Cingi(2006a) estimators have been obtained in section 4.Proposed estimators are better than the previous estimators if the dataset satisfies the condition obtained in equations $(40)-(60)$ under section 4. In numerical findings the data set does not satisfies the conditions derived in equations (49), (53) and (58) hence the performance of estimator (t_9), (t_{10}) and (t_{11}) are unsatisfactory. And performance of Estimators (t_3), (t_4), (t_5), (t_6), (t_7), (t_8), (t_{12}), (t_{13}) and (t_{14}) are better than the (t_0), (t_1) and (t_2). Thus the proposed estimators have been recommended for its use in practice if the data is satisfying the condition mentioned in section 4. Since the estimators are based on Arithmetic mean, Geometric Mean and Harmonic Mean of estimators t_0, t_1 and t_2. Hence if the Characteristic understudy is normally distributed then the estimator based on Arithmetic Mean should be used, when the distribution of Population is skewed then the estimators based on Geometric Mean must be used and if the observations are skewed and are of per unit change type then the estimators based on Harmonic Mean must be used.

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