

On Some Real Valued I –Convergent Λ –Summable Difference Sequence Spaces Defined by Sequences of Orlicz Functions

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Abstract: In this article we introduce some new difference sequence spaces using I –convergent and Orlicz function, and study some basic topological and algebraic properties of these spaces. Also we investigate the relations between these spaces.

Keywords: Ideal, I -convergent, Orlicz function, difference sequence

1 Introduction

The notion of I –convergence initially introduced by Kostyrko, Salat and Wilczynski [1]. Later on, it was further investigated from the sequence space point of view and linked with the summability theory by Salat, Tripathy and Ziman [2,3], Tripathy and Hazarika [4,5,6] and Kumar and Kumar [7], Aiyub [26], Khan et.al [27] and many others authors.

Let X be a non-empty set, then a family of sets $I \subset 2^X$ (the class of all subsets of X) is called an *ideal* if and only if for each $A, B \in I$, we have $A \cup B \in I$ and for each $A \in I$ and each $B \subset A$, we have $B \in I$. A non-empty family of sets $F \subset 2^X$ is a *filter* on X if and only if $\emptyset \notin F$, for each $A, B \in F$, we have $A \cap B \in F$ and each $A \in F$ and each $A \subset B$, we have $B \in F$. An ideal I is called *non-trivial ideal* if $I \neq \emptyset$ and $X \notin I$. Clearly $I \subset 2^X$ is a non-trivial ideal if and only if $F = F(I) = \{X - A : A \in I\}$ is a filter on X . A non-trivial ideal $I \subset 2^X$ is called *admissible* if and only if $\{\{x\} : x \in X\} \subset I$. A non-trivial ideal I is *maximal* if there cannot exist any non-trivial ideal $J \neq I$ containing I as a subset. Further details on ideals of 2^X can be found in Kostyrko, et.al [1].

Lemma 1.1. ([1, Lemma 5.1]) If $I \subset 2^N$ is a maximal ideal, then for each $A \subset N$ we have either $A \in I$ or $N - A \in I$.

Example 1.2. If we take $I = I_f = \{A \subseteq N : A \text{ is a finite subset}\}$. Then I_f is a non-trivial admissible ideal of N and

the corresponding convergence coincide with the usual convergence.

Example 1.3. If we take $I = I_\delta = \{A \subseteq N : \delta(A) = 0\}$ where $\delta(A)$ denote the asymptotic density of the set A . Then I_δ is a non-trivial admissible ideal of N and the corresponding convergence coincide with the statistical convergence.

Kizmaz [8] defined the difference sequence spaces $\ell_\infty(\Delta)$, $c(\Delta)$ and $c_0(\Delta)$ as follows: For $Z = \ell_\infty, c$ and c_0

$$Z(\Delta) = \{x = (x_k) : (\Delta x_k) \in \Delta Z\},$$

where $\Delta x = (\Delta x_k) = (x_k - x_{k+1})$, for all $k \in N$. The above spaces are Banach spaces, normed by

$$\|x\| = |x_1| + \sup_k \|\Delta x_k\|.$$

The idea of Kizmaz [8] was applied to introduce different type of difference sequence spaces and study their different properties by Tripathy ([9,10]), Et and Esi [11] and many others.

Recall in [12] that an *Orlicz function* M is continuous, convex, nondecreasing function is defined for $x > 0$ such that $M(0) = 0$ and $M(x) > 0$. If convexity of Orlicz function is replaced by $M(x+y) \leq M(x) + M(y)$ then this function is called the *modulus function* and characterized by Ruckle [13]. An Orlicz function M is said to satisfy

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Δ_2 -condition for all values of u , if there exists $K > 0$ such that $M(2u) \leq KM(u)$, $u \geq 0$.

Lemma 1.4. Let M be an Orlicz function which satisfies Δ_2 -condition and let $0 < \delta < 1$. Then for each $t \geq \delta$, we have $M(t) < K\delta^{-1}tM(2)$ for some constant $K > 0$.

Two Orlicz functions M_1 and M_2 are said to be *equivalent* if there exist positive constants α, β and x_0 such that

$$M_1(\alpha) \leq M_2(x) \leq M_1(\beta)$$

for all x with $0 \leq x < x_0$.

Lindenstrauss and Tzafriri [14] studied some Orlicz type sequence spaces defined as follows:

$$\ell_M = \left\{ (x_k) \in w : \sum_{k=1}^{\infty} M\left(\frac{|x_k|}{\rho}\right) < \infty, \text{ for some } \rho > 0 \right\}.$$

The space ℓ_M with the norm

$$\|x\| = \inf \left\{ \rho > 0 : \sum_{k=1}^{\infty} M\left(\frac{|x_k|}{\rho}\right) \leq 1 \right\}$$

becomes a Banach space which is called an *Orlicz sequence space*. The space ℓ_M is closely related to the space ℓ_p which is an Orlicz sequence space with $M(t) = |t|^p$, for $1 \leq p < \infty$.

In the later stage, different classes of Orlicz sequence spaces were introduced and studied by Parashar and Choudhary [15], Esi and Et [16], Et, Altin, Choudhary and Tripathy [17], Altinok, Altin and Isik [18], Gungor, Et and Altin [19], Hazarika et.al [24], Hazarika and Esi [25], Esi and Ozdemir [28] and many others.

Throughout the article N and R denote the set of positive integers and set of real numbers, respectively. The zero sequence is denoted by θ .

A sequence space E is said to be *solid (or normal)* if $(y_k) \in E$ whenever $(x_k) \in E$ and $|y_k| \leq |x_k|$ for all $k \in N$.

Lemma 1.5. ([20, page 53]) A sequence space E is normal implies E is monotone.

Let $n \in N$ and X^n be a real vector space of dimension n . A real-valued function $\|\cdot, \dots, \cdot\|$ on X^n satisfying the following four conditions:

(i) $\|x_1, x_2, \dots, x_n\| = 0$ if and only if x_1, x_2, \dots, x_n are linearly dependent,

(ii) $\|x_1, x_2, \dots, x_n\|$ is invariant under permutation,

(iii) $\|\alpha x_1, x_2, \dots, x_n\| = |\alpha| \|x_1, x_2, \dots, x_n\|$, $\alpha \in R$,

(iv) $\|x_1 + x_1', x_2, \dots, x_n\| \leq \|x_1, x_2, \dots, x_n\| + \|x_1', x_2, \dots, x_n\|$ is called an *n -norm* on X^n , and the pair $(X^n, \|\cdot, \dots, \cdot\|)$ is called an *n -normed space* [21].

A trivial example of n -normed space is $X^n = R^n$ equipped with the following Euclidean n -norm:

$$\|x_1, x_2, \dots, x_n\|_E = \text{abs} \left(\begin{pmatrix} x_{11} \dots x_{1n} \\ \dots \\ x_{n1} \dots x_{nn} \end{pmatrix} \right)$$

where $x_i = (x_{i1}, \dots, x_{in}) \in R^n$ for each $i = 1, 2, \dots, n$.

Lemma 1.6. Every n -normed space is an $(n-r)$ -normed space for all $r = 1, 2, \dots, n-1$. In particular every n -normed space is a normed space.

Lemma 1.7. A standard n -normed space is complete if and only if it is complete with respect to usual norm $\|\cdot\| = \sqrt{\langle \cdot, \cdot \rangle}$.

Lemma 1.8. On a standard n -normed space X , the derived from $(n-1)$ -norm $\|\cdot, \dots, \cdot\|_{\infty}$ defined with respect to the orthogonal set $\{e_1, e_2, \dots, e_n\}$ is equivalent to the standard $(n-1)$ -norm $\|\cdot, \dots, \cdot\|_S$. To be precise, for all $z_1, z_2, \dots, z_{n-1} \in X$, we have

$$\begin{aligned} \|z_1, z_2, \dots, z_{n-1}\|_{\infty} &\leq \|z_1, z_2, \dots, z_{n-1}\|_S \\ &\leq \sqrt{n} \|z_1, z_2, \dots, z_{n-1}\|_{\infty}, \end{aligned}$$

where

$$\|z_1, z_2, \dots, z_{n-1}\|_{\infty} = \max_{1 \leq i \leq n} \{\|z_1, z_2, \dots, z_{n-1}, e_i\|_S\}.$$

Some counter examples for n -normed spaces can be found in Dutta et.al [29].

2 Some New Sequence Spaces

The following well-known inequality will be used throughout the article. Let $p = (p_k)$ be any sequence of positive real numbers with $0 \leq p_k \leq \sup_k p_k = G$, $D = \max\{1, 2^{G-1}\}$ then

$$|a_k + b_k|^{p_k} \leq D(|a_k|^{p_k} + |b_k|^{p_k})$$

for all $k \in N$ and $a_k, b_k \in C$. Also $|a_k|^{p_k} \leq \max\{1, |a_k|^G\}$ for all $a_k \in C$.

The main aim of this article is to introduce the following sequence spaces and examine topological and algebraic properties of the resulting sequence spaces.

In paper [22], Mursaleen and Noman introduced the notion of λ -convergent and λ -bounded sequences as follows: Let $\lambda = (\lambda_k)_{k=0}^{\infty}$ be a strictly increasing sequence of positive real numbers tending to infinity, that is

$$0 < \lambda_0 < \lambda_1 < \dots \text{ and } \lambda_k \rightarrow \infty \text{ as } k \rightarrow \infty$$

and let

$$\Lambda_k(x) = \frac{1}{\lambda_k} \sum_{m=1}^k (\lambda_m - \lambda_{m-1}) x_m.$$

Let I be an admissible ideal of N and let $p = (p_k)$ be a bounded sequence of positive real numbers for all $k \in N$. Let $\mathbf{M} = (M_k)$ be a sequence of Orlicz functions and $(X, \|\cdot, \dots, \cdot\|)$ be an n -normed space. Further $w(n-X)$ denotes X -valued sequence space, we define the following sequence spaces as follows:

$$c^I(\mathbf{M}, \Lambda, p, \|\dots, \dots\|)_\Delta$$

$$= \left\{ (x_k) \in w(n-X) : \left\{ k \in N : \left[M_k \left(\left\| \frac{\Lambda_k(\Delta x) - L}{\rho}, z_1, z_2, \dots, z_{n-1} \right\| \right) \right]^{p_k} \geq \varepsilon \right\} \in I, \right.$$

$$\left. \text{for some } \rho > 0 \text{ and } L \text{ and for every } z_1, z_2, \dots, z_{n-1} \in X \right\},$$

$$c^I_o(\mathbf{M}, \Lambda, p, \|\dots, \dots\|)_\Delta$$

$$= \left\{ (x_k) \in w(n-X) : \left\{ k \in N : \left[M_k \left(\left\| \frac{\Lambda_k(\Delta x)}{\rho}, z_1, z_2, \dots, z_{n-1} \right\| \right) \right]^{p_k} \geq \varepsilon \right\} \in I, \right.$$

$$\left. \text{for some } \rho > 0 \text{ and for every } z_1, z_2, \dots, z_{n-1} \in X \right\},$$

$$\ell^I_\infty(\mathbf{M}, \Lambda, p, \|\dots, \dots\|)_\Delta$$

$$= \left\{ (x_k) \in w(n-X) : \exists K > 0 \text{ such that } \left\{ k \in N : \left[M_k \left(\left\| \frac{\Lambda_k(\Delta x)}{\rho}, z_1, z_2, \dots, z_{n-1} \right\| \right) \right]^{p_k} \geq K \right\} \in I, \right.$$

$$\left. \text{for some } \rho > 0 \text{ and for every } z_1, z_2, \dots, z_{n-1} \in X \right\}$$

and

$$\ell_\infty(\mathbf{M}, \Lambda, p, \|\dots, \dots\|)_\Delta$$

$$= \left\{ (x_k) \in w(n-X) : \exists K > 0 \text{ such that } \sup_k \left[M_k \left(\left\| \frac{\Lambda_k(\Delta x)}{\rho}, z_1, z_2, \dots, z_{n-1} \right\| \right) \right]^{p_k} \leq K, \right.$$

$$\left. \text{for some } \rho > 0 \text{ and for every } z_1, z_2, \dots, z_{n-1} \in X \right\}.$$

3 Main Results

In this section we examine the basic topological and algebraic properties of these spaces and obtain the inclusion relation between these spaces.

Theorem 3.1. If $\{\Lambda_k(\Delta x), z_1, z_2, \dots, z_{n-1}\}$ is a linearly dependent set in $(X, \|\dots, \dots\|)$ for all but finite k , where $x = (x_k) \in w(n-X)$ and $\inf_k p_k > 0$, then

(a) $I\text{-}\lim_{k \rightarrow \infty} \left[M_k \left(\left\| \frac{\Lambda_k(\Delta x)}{\rho}, z_1, z_2, \dots, z_{n-1} \right\| \right) \right]^{p_k} = 0,$
for some $\rho > 0$,

(b) $\sup_k \left[M_k \left(\left\| \frac{\Lambda_k(\Delta x)}{\rho}, z_1, z_2, \dots, z_{n-1} \right\| \right) \right]^{p_k} < \infty,$ for some $\rho > 0$.

Proof. (a) Suppose that $\{\Lambda_k(\Delta x), z_1, z_2, \dots, z_{n-1}\}$ is a linearly dependent set in $(X, \|\dots, \dots\|)$ for all but finite k . Then we have

$$\|\Lambda_k(\Delta x), z_1, z_2, \dots, z_{n-1}\| \rightarrow 0 \text{ as } k \rightarrow \infty.$$

Since M_k is continuous for all k and $0 \leq p_k \leq \sup_k p_k = G < \infty$ for each k , then we have

$$I\text{-}\lim_{k \rightarrow \infty} \left[M_k \left(\left\| \frac{\Lambda_k(\Delta x)}{\rho}, z_1, z_2, \dots, z_{n-1} \right\| \right) \right]^{p_k} = 0,$$

for some $\rho > 0$.

(b) The proof of this part is similar to part (a).

Theorem 3.2. $c^I_o(\mathbf{M}, \Lambda, p, \|\dots, \dots\|)_\Delta, c^I(\mathbf{M}, \Lambda, p, \|\dots, \dots\|)_\Delta$ and $\ell^I_\infty(\mathbf{M}, \Lambda, p, \|\dots, \dots\|)_\Delta$ are linear spaces.

Proof. We will prove the result for the space $c^I_o(\mathbf{M}, \Lambda, p, \|\dots, \dots\|)_\Delta$ only, and the others can be proved in similar way. Let $x = (x_k)$ and $y = (y_k)$ be two elements in $c^I_o(\mathbf{M}, \Lambda, p, \|\dots, \dots\|)_\Delta$. Then for every $z_1, z_2, \dots, z_{n-1} \in X$ there exist $\rho_1 > 0$ and $\rho_2 > 0$ such that

$$A_{\frac{\varepsilon}{2}} = \left\{ k \in N : \left[M_k \left(\left\| \frac{\Lambda_k(\Delta x)}{\rho}, z_1, z_2, \dots, z_{n-1} \right\| \right) \right]^{p_k} \geq \frac{\varepsilon}{2} \right\} \in I$$

and

$$B_{\frac{\varepsilon}{2}} = \left\{ k \in N : \left[M_k \left(\left\| \frac{\Lambda_k(\Delta y)}{\rho}, z_1, z_2, \dots, z_{n-1} \right\| \right) \right]^{p_k} \geq \frac{\varepsilon}{2} \right\} \in I.$$

Let α, β be two scalars. Since $\mathbf{M} = (M_k)$ is a sequence of continuous functions, the following inequality holds:

$$\left[M_k \left(\left\| \frac{\Lambda_k(\Delta(\alpha x + \beta y))}{|\alpha|\rho_1 + |\beta|\rho_2}, z_1, z_2, \dots, z_{n-1} \right\| \right) \right]^{p_k}$$

$$\leq D \frac{|\alpha|\rho_1}{|\alpha|\rho_1 + |\beta|\rho_2} \left[M_k \left(\left\| \frac{\Lambda_k(\Delta(x))}{\rho_1}, z_1, z_2, \dots, z_{n-1} \right\| \right) \right]^{p_k}$$

$$+ D \frac{|\beta|\rho_2}{|\alpha|\rho_1 + |\beta|\rho_2} \left[M_k \left(\left\| \frac{\Lambda_k(\Delta(y))}{\rho_2}, z_1, z_2, \dots, z_{n-1} \right\| \right) \right]^{p_k}$$

$$\leq DK \left[M_k \left(\left\| \frac{\Lambda_k(\Delta(x))}{\rho_1}, z_1, z_2, \dots, z_{n-1} \right\| \right) \right]^{p_k}$$

$$+ DK \left[M_k \left(\left\| \frac{\Lambda_k(\Delta(y))}{\rho_2}, z_1, z_2, \dots, z_{n-1} \right\| \right) \right]^{p_k},$$

where $K = \max \left\{ 1, \left(\frac{|\alpha|\rho_1}{|\alpha|\rho_1 + |\beta|\rho_2} \right)^G, \left(\frac{|\beta|\rho_2}{|\alpha|\rho_1 + |\beta|\rho_2} \right)^G \right\}.$

From the above relation we obtain the following:

$$\left\{ k \in N : \left[M_k \left(\left\| \frac{\Lambda_k(\Delta(\alpha x + \beta y))}{|\alpha|\rho_1 + |\beta|\rho_2}, z_1, z_2, \dots, z_{n-1} \right\| \right) \right]^{p_k} \geq \varepsilon \right\} \subseteq$$

$$\left\{ k \in N : DK \left[M_k \left(\left\| \frac{\Lambda_k(\Delta(x))}{\rho_1}, z_1, z_2, \dots, z_{n-1} \right\| \right) \right]^{p_k} \geq \frac{\varepsilon}{2} \right\}$$

$$\cup \left\{ k \in N : DK \left[M_k \left(\left\| \frac{\Lambda_k(\Delta(y))}{\rho_2}, z_1, z_2, \dots, z_{n-1} \right\| \right) \right]^{p_k} \geq \frac{\varepsilon}{2} \right\} \in I.$$

This completes the proof.

Remark 3.3. It is easy to verify that the space $\ell_\infty(\mathbf{M}, \Lambda, p, \|\dots, \dots\|)_\Delta$ is a linear space.

The proof of the following theorem is similar to [23, Theorem 2.2], therefore we omit it.

Theorem 3.4. The space $\ell_\infty(\mathbf{M}, \Lambda, p, \|\dots, \dots\|)_\Delta$ is a paranormed space (not totally paranormed) with the paranorm g defined by

$$g(x) = \inf \left\{ \rho^{\frac{p_k}{G}} : \sup_k \left[M_k \left(\left\| \frac{\Lambda_k(\Delta x)}{\rho}, z_1, z_2, \dots, z_{n-1} \right\| \right) \right]^{p_k} \leq 1, \right. \\ \left. \text{for some } \rho > 0 \text{ and for every } z_1, z_2, \dots, z_{n-1} \in X \right\},$$

where $G = \max \{1, \sup p_k\}$.

Theorem 3.5. Let $\mathbf{M} = (M_k)$ and $\mathbf{S} = (S_k)$ be sequences of Orlicz functions. Then the following hold:

(i) $c_o^I(\mathbf{S}, \Lambda, p, \|\dots, \dots\|)_\Delta \subseteq c_o^I(\mathbf{M} \circ \mathbf{S}, \Lambda, p, \|\dots, \dots\|)_\Delta$, provided $p = (p_k)$ be such that $G_0 = \inf p_k > 0$.

(ii) $c_o^I(\mathbf{M}, \Lambda, p, \|\dots, \dots\|)_\Delta \cap c_o^I(\mathbf{S}, \Lambda, p, \|\dots, \dots\|)_\Delta \subseteq c_o^I(\mathbf{M} + \mathbf{S}, \Lambda, p, \|\dots, \dots\|)_\Delta$.

Proof. (i) Let $\varepsilon > 0$ be given. Choose $\varepsilon_1 > 0$ such that $\max \{ \varepsilon_1^G, \varepsilon_1^{G_0} \} < \varepsilon$. Choose $0 < \delta < 1$ such that $0 < t < \delta$ implies that $M_k(t) < \varepsilon_1$ for each $k \in N$. Let $x = (x_k)$ be any element in $c_o^I(\mathbf{S}, \Lambda, p, \|\dots, \dots\|)_\Delta$. For every $z_1, z_2, \dots, z_{n-1} \in X$, put

$$A_\delta = \left\{ k \in N : \left[S_k \left(\left\| \frac{\Lambda_k(\Delta x)}{\rho}, z_1, z_2, \dots, z_{n-1} \right\| \right) \right]^{p_k} \geq \delta^G \right\}.$$

Then by the definition of ideal we have $A_\delta \in I$. If $k \notin A_\delta$ we have

$$\left[S_k \left(\left\| \frac{\Lambda_k(\Delta x)}{\rho}, z_1, z_2, \dots, z_{n-1} \right\| \right) \right]^{p_k} < \delta^G, \text{ for } k = 1, 2, \dots, n \\ \Rightarrow S_k \left(\left\| \frac{\Lambda_k(\Delta x)}{\rho}, z_1, z_2, \dots, z_{n-1} \right\| \right) < \delta, \quad (3.1)$$

Since $\mathbf{M} = (M_k)$ is a sequence of continuous functions, from the relation (3.1) we have

$$M_k \left(S_k \left(\left\| \frac{\Lambda_k(\Delta x)}{\rho}, z_1, z_2, \dots, z_{n-1} \right\| \right) \right) < \varepsilon_1,$$

for $k = 1, 2, 3, \dots, n$. Consequently we get

$$\left[M_k \left(S_k \left(\left\| \frac{\Lambda_k(\Delta x)}{\rho}, z_1, z_2, \dots, z_{n-1} \right\| \right) \right) \right]^{p_k} < \max \{ \varepsilon_1^G, \varepsilon_1^{G_0} \} < \varepsilon \\ \Rightarrow \left[M_k \left(S_k \left(\left\| \frac{\Lambda_k(\Delta x)}{\rho}, z_1, z_2, \dots, z_{n-1} \right\| \right) \right) \right]^{p_k} < \varepsilon.$$

This implies that

$$\left\{ k \in N : \left[M_k \left(S_k \left(\left\| \frac{\Lambda_k(\Delta x)}{\rho}, z_1, z_2, \dots, z_{n-1} \right\| \right) \right) \right]^{p_k} \geq \varepsilon \right\} \\ \subseteq A_\delta \in I.$$

This completes the proof.

(ii) Let $x = (x_k) \in c_o^I(\mathbf{M}, \Lambda, p, \|\dots, \dots\|)_\Delta \cap c_o^I(\mathbf{S}, \Lambda, p, \|\dots, \dots\|)_\Delta$. Then by the following inequality

the result follows:

$$\left[(M_k + S_k) \left(\left\| \frac{\Lambda_k(\Delta x)}{\rho}, z_1, z_2, \dots, z_{n-1} \right\| \right) \right]^{p_k} \\ \leq D \left[M_k \left(\left\| \frac{\Lambda_k(\Delta x)}{\rho}, z_1, z_2, \dots, z_{n-1} \right\| \right) \right]^{p_k} \\ + D \left[S_k \left(\left\| \frac{\Lambda_k(\Delta x)}{\rho}, z_1, z_2, \dots, z_{n-1} \right\| \right) \right]^{p_k}.$$

The proof of the following theorems are easy and so omitted.

Theorem 3.6. Let $0 < p_k \leq q_k$ and $\left(\frac{q_k}{p_k} \right)$ is bounded, then

$$c_o^I(\mathbf{M}, \Lambda, q, \|\dots, \dots\|)_\Delta \subseteq c_o^I(\mathbf{M}, \Lambda, p, \|\dots, \dots\|)_\Delta.$$

Theorem 3.7. For any two sequences $p = (p_k)$ and $q = (q_k)$ of positive real numbers, then the following holds:

$$Z(\mathbf{M}, \Lambda, p, \|\dots, \dots\|)_\Delta \cap Z(\mathbf{M}, \Lambda, q, \|\dots, \dots\|)_\Delta \neq \emptyset,$$

for $Z = c^I, c_o^I, \ell_\infty^I$ and ℓ_∞ .

Proposition 3.8. The sequence spaces $Z(\mathbf{M}, \Lambda, p, \|\dots, \dots\|)_\Delta$ are normal as well as monotone for $Z = c_o^I$ and ℓ_∞^I .

Proof. We shall give the prove of the proposition for $c_o^I(\mathbf{M}, \Lambda, p, \|\dots, \dots\|)_\Delta$ only. Let $x = (x_k) \in c_o^I(\mathbf{M}, \Lambda, p, \|\dots, \dots\|)_\Delta$ and $y = (y_k)$ be such that $|y_k| \leq |x_k|$ for all $k \in N$. Then for given $\varepsilon > 0$ we have

$$B = \left\{ k \in N : \left[M_k \left(\left\| \frac{\Lambda_k(\Delta x)}{\rho}, z_1, z_2, \dots, z_{n-1} \right\| \right) \right]^{p_k} \geq \varepsilon \right\} \in I.$$

Again the set

$$E = \left\{ k \in N : \left[M_k \left(\left\| \frac{\Lambda_k(\Delta y)}{\rho}, z_1, z_2, \dots, z_{n-1} \right\| \right) \right]^{p_k} \geq \varepsilon \right\} \subseteq B.$$

Hence $E \in I$ and so $y = (y_k) \in c_o^I(\mathbf{M}, \Lambda, p, \|\dots, \dots\|)_\Delta$. Thus the space $c_o^I(\mathbf{M}, \Lambda, p, \|\dots, \dots\|)_\Delta$ is normal. Also from the Lemma 1.5., it follows that $c_o^I(\mathbf{M}, \Lambda, p, \|\dots, \dots\|)_\Delta$ is monotone.

In view of Lemma 1.8., we state the following theorem.

Theorem 3.9. Let X be a standard n -normed space and $\{e_1, e_2, \dots, e_n\}$ be an orthonormal set in X . Then the following holds:

- $c^I(\mathbf{M}, \Lambda, p, \|\dots, \dots\|_\infty)_\Delta = c^I(\mathbf{M}, \Lambda, p, \|\dots, \dots\|_{n-1})_\Delta$,
- $c_o^I(\mathbf{M}, \Lambda, p, \|\dots, \dots\|_\infty)_\Delta = c_o^I(\mathbf{M}, \Lambda, p, \|\dots, \dots\|_{n-1})_\Delta$,
- $\ell_\infty^I(\mathbf{M}, \Lambda, p, \|\dots, \dots\|_\infty)_\Delta = \ell_\infty^I(\mathbf{M}, \Lambda, p, \|\dots, \dots\|_{n-1})_\Delta$,
- $\ell_\infty(\mathbf{M}, \Lambda, p, \|\dots, \dots\|_\infty)_\Delta = \ell_\infty(\mathbf{M}, \Lambda, p, \|\dots, \dots\|_{n-1})_\Delta$,

where $\|\dots, \dots\|_\infty$ is derived $(n-1)$ -norm defined with respect to the set $\{e_1, e_2, \dots, e_n\}$ and $\|\dots, \dots\|_{n-1}$ is the standard $(n-1)$ -norm on X .

Theorem 3.10. The spaces $c^I(\mathbf{M}, \Lambda, p, \|\dots, \dots\|)_\Delta$ and $c^I(\mathbf{M}, p, \|\dots, \dots\|_\infty)_\Delta$ are equivalent as topological spaces.

Proof. Consider the mapping $T : c^I(\mathbf{M}, \Lambda, p, \|\dots, \dots\|)_\Delta \rightarrow c^I(\mathbf{M}, p, \|\dots, \dots\|_\infty)_\Delta$ defined by $T(x) = (\Lambda_k(\Delta x))$ for each $x = (x_k) \in c^I(\mathbf{M}, \Lambda, p, \|\dots, \dots\|)_\Delta$. Then clearly T is a linear homeomorphism and the proof follows.

4 Conclusion:

In this paper defined some new difference sequence spaces combining the concepts of Orlicz function and I-convergence. Further, we proved some topological and algebraic properties of resulting spaces. This notion can be used for further generalization of such spaces.

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