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A Generalized Class of Ratio-Cum-Dual to Ratio Estimators of Finite Population Mean Using Auxiliary Information in Sample Surveys

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Abstract: In this paper, we have suggested a general class of ratio-cum-dual to ratio type estimators of finite population mean using an auxiliary variable, (say x) that is correlated with the variable of interest (say y). The proposed class of estimators includes several known estimators based on transformation in auxiliary variable x. The bias and mean squared error (*MSE*) expressions of the proposed class of estimators have been obtained to the first degree of approximation. We have compared the generalized ratio-cum-dual to ratio type estimators of finite population mean to the usual unbiased estimator and various existing ratio, product and ratio-cum-product type estimators. It is found that the suggested estimators are better than other existing estimators under some realistic conditions. Numerical illustrations are given in support of the present study.

Keywords: Auxiliary variable, Ratio-cum-dual to ratio type estimator, Finite population mean, Simple random sampling, Bias, Mean squared error.

1 Introduction

In sample surveys, auxiliary information is used at the selection stage as well as estimation stage to improve the efficiency of the estimators. The use of auxiliary information at the estimation stage appears to have started with the work of Cochran (1940). When the correlation between study variate y and auxiliary variate x is positive (high), the ratio method of estimation is used for estimating the population mean. The ratio method is most effective if $\rho \frac{C_y}{C_x} > \frac{1}{2}$, where (C_y, C_x) are coefficients of variation of (y, x) respectively and ρ is the correlation coefficient between y and x. On the other hand, if the correlation is negative, the product method of estimation envisaged by Robson (1957) and revisited by Murthy (1964) is used and this is most effective if $\rho \frac{C_y}{C_x} < -\frac{1}{2}$.

Srivenkataramana (1980) first proposed dual to ratio estimator and Bandyopadhyaya (1980) proposed dual to product estimator. Singh and Tailor (2005), Singh and Espejo (2003), Tailor and Sharma (2009) worked on ratio-cum-product estimators. Sharma and Tailor (2010), Chaudhary and Singh (2012) worked on ratio, dual to ratio and dual to product estimators to estimate the population mean $\bar{Y}$ of the study variable y.

Consider a finite population $U = (u_1, u_2, ..., u_N)$ of size N units. A sample of size $n(n < N)$ is drawn using simple random sampling without replacement (*SRSWOR*) method to estimate the population mean $\bar{Y} = N^{-1}\sum_{i=1}^{N} y_i$ of the study variate y.

Let the sample means $(\bar{x}, \bar{y})$ be the unbiased estimators of the population means respectively $(\bar{X}, \bar{Y})$ based on n observations. The classical ratio and product estimators of population mean $\bar{Y}$ of the study variable y are respectively given as

$$\bar{y}_R = \bar{y}\left(\frac{\bar{X}}{\bar{x}}\right) \tag{1.1}$$

and

$$\bar{y}_P = \bar{y}\left(\frac{\bar{x}}{\bar{X}}\right). \tag{1.2}$$

The biases and mean squared errors (*MSEs*) of $\bar{y}_R$ and $\bar{y}_P$ to the first degree of approximation are respectively given as

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$$B(\bar{y}_R)=\frac{(1-f)}{n}\bar{Y}C_x^2(1-K)\,, \tag{1.3}$$

$$B(\bar{y}_P)=\frac{(1-f)}{n}\bar{Y}C_x^2K\,, \tag{1.4}$$

$$MSE(\bar{y}_R)=\frac{(1-f)}{n}\bar{Y}^2\left[C_y^2+C_x^2(1-2K)\right], \tag{1.5}$$

And

$$MSE(\bar{y}_P)=\frac{(1-f)}{n}\bar{Y}^2\left[C_y^2+C_x^2(1+2K)\right], \tag{1.6}$$

Where

$$f=\frac{n}{N},K=\rho\frac{C_y}{C_x},\rho=\frac{S_{xy}}{S_xS_y},C_y=\frac{S_y}{\bar{Y}},C_x=\frac{S_x}{\bar{X}},$$

$$S_{xy}=\frac{\sum_{i=1}^{N}(x_i-\bar{X})(y_i-\bar{Y})}{(N-1)},$$

$$S_x^2=\frac{\sum_{i=1}^{N}(x_i-\bar{X})^2}{(N-1)}\ and\ S_y^2=\frac{\sum_{i=1}^{N}(y_i-\bar{Y})^2}{(N-1)}.$$

Consider a transformation

$$x_i^*=\frac{N\bar{X}-nx_i}{N-n}=(1+g)\bar{X}-gx_i\,,i=1,2,...,N,$$

Where $g=\frac{n}{(N-n)}$.

Then $\bar{x}^*=\{(1+g)\bar{X}-g\bar{x}\}$ is an unbiased estimator of the population mean $\bar{X}=N^{-1}\sum_{i=1}^{N}x_i$ of the auxiliary variable x and the correlation between $\bar{y}$ and $\bar{x}^*$ is negative.

Using the transformation x_i^* on the auxiliary variable x, Srivenkataramana (1980) and Bandyopadhyaya (1980) obtained dual to ratio and dual to product type estimators as

$$\bar{y}_R^*=\bar{y}\left(\frac{\bar{x}^*}{\bar{X}}\right) \tag{1.7}$$

And

$$\bar{y}_P^*=\bar{y}\left(\frac{\bar{X}}{\bar{x}^*}\right). \tag{1.8}$$

The biases and mean squared errors (*MSEs*) of $\bar{y}_R^*$ and $\bar{y}_P^*$ to the first degree of approximation are respectively given as

$$B(\bar{y}_R^*)=-\frac{(1-f)}{n}\bar{Y}C_x^2gK\,, \tag{1.9}$$

$$B(\bar{y}_P^*)=\frac{(1-f)}{n}\bar{Y}C_x^2g(g+K), \tag{1.10}$$

$$MSE(\bar{y}_R^*)=\frac{(1-f)}{n}\bar{Y}^2\left[C_y^2+C_x^2g(g-2K)\right] \tag{1.11}$$

And

$$MSE(\bar{y}_P^*)=\frac{(1-f)}{n}\bar{Y}^2\left[C_y^2+C_x^2g(g+2K)\right]. \tag{1.12}$$

Singh and Agnihotri (2008) defined a family of ratio-product estimators of population mean $\bar{Y}$ in simple random sampling (*SRS*) as

$$\bar{y}_{RP}=\alpha\bar{y}\left(\frac{a\bar{X}+b}{a\bar{x}+b}\right)+(1-\alpha)\bar{y}\left(\frac{a\bar{x}+b}{a\bar{X}+b}\right), \tag{1.13}$$

Where 'a' and 'b' are known characterizing positive scalars and 'α' is a real constant to be determined such that the *MSE* of $\bar{y}_{RP}$ is minimum.

The bias and *MSE* of $\bar{y}_{RP}$ to the first degree of approximation are respectively given as

$$B(\bar{y}_{RP})=\frac{(1-f)}{n}\bar{Y}C_x^2\delta[K+\alpha(\delta-2K)] \tag{1.14}$$

And

$$MSE(\bar{y}_{RP})=\frac{(1-f)}{n}\bar{Y}^2\left[C_y^2+\delta(1-2\alpha)C_x^2\{\delta(1-2\alpha)+2K\}\right], \tag{1.15}$$

Where $\delta=\frac{a\bar{X}}{(a\bar{X}+b)}$.

The aim of this paper is to suggest a generalized class of ratio-cum-dual to ratio type estimators for population mean $\bar{Y}$ in *SRSWOR* and their properties are studied under large sample approximation. It is interesting to note that the proposed generalized class of ratio-cum-dual to ratio type estimators includes several known estimators based on transformation on auxiliary variable x.

2 A Generalized Class of Ratio-Cum-Dual to Ratio Type Estimators of Finite Population Mean

We suggest a family of ratio-cum-dual to ratio type estimators in *SRSWOR* for population mean $\bar{Y}$ as

$$\bar{y}_{RP}^*=\eta\bar{y}\left(\frac{a\bar{X}+b}{a\bar{x}+b}\right)+(1-\eta)\bar{y}\left(\frac{a\bar{x}^*+b}{a\bar{X}+b}\right), \tag{2.1}$$

Where (a,b) are same as defined earlier, η being a suitably chosen scalar and

$\bar{x}^* = \dfrac{N\bar{X} - n\bar{x}}{N-n} = \{(1+g)\bar{X} - g\bar{x}\}$ With $g = \dfrac{n}{N-n}$.

To obtain the bias and MSE of $\bar{y}_R^{(d)}$ the first degree of approximation, we write

$e_0 = \dfrac{(\bar{y} - \bar{Y})}{\bar{Y}}$ And $e_1 = \dfrac{(\bar{x} - \bar{X})}{\bar{X}}$,

Such that

$$E(e_0) = 0$$
$$E(e_1) = 0$$

And

$$\left.\begin{aligned} E(e_0^2) &= \frac{(1-f)}{n} C_y^2 \\ E(e_1^2) &= \frac{(1-f)}{n} C_x^2 \\ E(e_0 e_1) &= \frac{(1-f)}{n} K C_x^2 \end{aligned}\right\}. \tag{2.2}$$

Expressing (2.1) in terms of e's, we have

$$\bar{y}_{RP}^* = \bar{Y}(1+e_0)[\eta(1+\delta e_1)^{-1} + (1-\eta)(1-\delta g e_1)]. \tag{2.3}$$

We assume that $|\delta e_1| < 1$ so that $(1+\delta e_1)^{-1}$ is expandable. From (2.3) we have

$$\bar{y}_{RP}^* = \bar{Y}(1+e_0)[\eta(1-\delta e_1 + \delta^2 e_1^2 - \ldots) + (1-\eta)(1-\delta g e_1)]$$
$$= \bar{Y}(1+e_0)[1 - \delta g e_1 + \eta\delta e_1(g-1) + \eta\delta^2 e_1^2 - \ldots]$$
$$= \bar{Y}[1 + e_0 - \delta e_1\{\eta(1-g)+g\} - \delta e_1 e_0\{\eta(1-g)+g\} + \eta\delta^2 e_1^2 - \ldots]$$

We assume that the contribution of terms involving powers in e_0 and e_1 higher than the second is negligible. Thus, from the above expression we write to a first approximation,

$$\bar{y}_{RP}^* \cong \bar{Y}[1 + e_0 - \delta e_1\{\eta(1-g)+g\} - \delta e_1 e_0\{\eta(1-g)+g\} + \eta\delta^2 e_1^2]$$

or

$$(\bar{y}_{RP}^* - \bar{Y}) = \bar{Y}[e_0 - \delta e_1\{\eta(1-g)+g\} - \delta e_1 e_0\{\eta(1-g)+g\} + \eta\delta^2 e_1^2] \tag{2.4}$$

Taking expectation of both the sides of (2.4) we obtain the bias of $\bar{y}_{RP}^*$ to the first degree of approximation, as

$$B(\bar{y}_{RP}^*) = \bar{Y}\frac{(1-f)}{n}\left[-\delta K C_x^2\{\eta(1-g)+g\} + \eta\delta^2 C_x^2\right]$$
$$= \bar{Y}\frac{(1-f)}{n}\delta C_x^2[-K\{\eta(1-g)+g\} + \eta\delta]. \tag{2.5}$$

The bias of $\bar{y}_{RP}^*$ is almost unbiased when

$$B(\bar{y}_{RP}^*) = \bar{Y}\frac{(1-f)}{n}\delta C_x^2[-K\{\eta(1-g)+g\} + \eta\delta] = 0$$

i.e. $\eta = \dfrac{gK}{\{\delta - K(1-g)\}}$.

Squaring both sides of (2.4) and neglecting terms of e's having power greater than two we have

$$(\bar{y}_{RP}^* - \bar{Y})^2 = \bar{Y}^2\left[e_0^2 + \delta^2 e_1^2\{\eta(1-g)+g\}^2 - 2\delta e_0 e_1\{\eta(1-g)+g\}\right] \tag{2.6}$$

Taking expectation of both sides of (2.6) we get the MSE of $\bar{y}_{RP}^*$ to the first degree of approximation as

$$MSE(\bar{y}_{RP}^*) = \bar{Y}^2\frac{(1-f)}{n}\left[C_y^2 + \delta C_x^2\{\eta(1-g)+g\}\{\delta\{\eta(1-g)+g\} - 2K\}\right] \tag{2.7}$$

Assuming $(1-g) > 0$ (which is typical situation in sample surveys), minimizing (2.7) with respect to η, we get

$$\eta = \frac{K - \delta g}{\delta(1-g)} = \eta_{opt.}(say). \tag{2.8}$$

Substituting the value of $\eta_{opt.}$ in (2.1) yields the asymptotically optimum estimator (AOE) as

$$\bar{y}_{RP_{opt.}}^* = \eta_{opt.}\bar{y}\left(\frac{a\bar{X}+b}{a\bar{x}+b}\right) + (1-\eta_{opt.})\bar{y}\left(\frac{a\bar{x}^*+b}{a\bar{X}+b}\right). \tag{2.9}$$

Thus, the resulting bias and MSE of $\bar{y}_{RP_{opt.}}^*$ respectively are given as

$$B(\bar{y}_{RP_{opt.}}^*) = \bar{Y}\frac{(1-f)}{n}\delta C_x^2\left[\eta_{opt.}\{\delta - K(1-g)\} - Kg\right], \tag{2.10}$$

and

$$\min.MSE(\bar{y}_{RP}^*) = MSE(\bar{y}_{RP_{opt.}}^*) = \bar{Y}^2\frac{(1-f)}{n}C_y^2(1-\rho^2). \tag{2.11}$$

Table 2.1 shows members of the proposed class of estimators $\bar{y}_{RP}^*$ for different choices of (a, b, δ, η).

In the Table 2.1, coefficient of variation C_x and coefficient of kurtosis $\beta_2(x)$ of an auxiliary variable x are known.

Table 2. 1: Members of the estimator $\bar{y}^{*}_{RP}$ for different choices of (a,b,δ,η)

S. No.	Estimators	Values of constants (a,b,δ,η)			
		a	b	η	δ
1.	$\bar{y}_R=\bar{y}\left(\frac{\bar{X}}{\bar{x}}\right)$ Cochran (1940) estimator	1	0	1	1
2.	$\bar{y}^{*}_R=\bar{y}\left(\frac{\bar{x}^{*}}{\bar{X}}\right)$ Srivenkataramana (1980)	1	0	0	1
3.	$\bar{y}_{SD}=\bar{y}\left(\frac{\bar{X}+C_x}{\bar{x}+C_x}\right)$ Sisodia and Dwivedi (1981) estimator	1	C_x	1	$\frac{\bar{X}}{\bar{X}+C_x}$
4.	$\bar{y}^{*}_{SD}=\bar{y}\left(\frac{\bar{x}^{*}+C_x}{\bar{X}+C_x}\right)$ Shah and Patel (1986) and Singh and Upadhyaya (1999) estimator	1	C_x	0	$\frac{\bar{X}}{\bar{X}+C_x}$
5.	$\bar{y}_{SK}=\bar{y}\left(\frac{\bar{X}+\beta_2(x)}{\bar{x}+\beta_2(x)}\right)$ Singh et al. (2004) estimator	1	$\beta_2(x)$	1	$\frac{\bar{X}}{\bar{X}+\beta_2(x)}$
6.	$\bar{y}^{*}_{SK}=\bar{y}\left(\frac{\bar{x}^{*}+\beta_2(x)}{\bar{X}+\beta_2(x)}\right)$ Dual to Singh et al (2004) estimator	1	$\beta_2(x)$	0	$\frac{\bar{X}}{\bar{X}+\beta_2(x)}$
7.	$\bar{y}_{UP1}=\bar{y}\left(\frac{\bar{X}C_x+\beta_2(x)}{\bar{x}C_x+\beta_2(x)}\right)$ Upadhyaya and Singh (1999) estimator	C_x	$\beta_2(x)$	1	$\frac{\bar{X}C_x}{\bar{X}C_x+\beta_2(x)}$
8.	$\bar{y}^{*}_{UP1}=\bar{y}\left(\frac{\bar{x}^{*}C_x+\beta_2(x)}{\bar{X}C_x+\beta_2(x)}\right)$ Dual to Upadhyaya and Singh (1999) ratio type estimator	C_x	$\beta_2(x)$	0	$\frac{\bar{X}C_x}{\bar{X}C_x+\beta_2(x)}$
9.	$\bar{y}_{UP2}=\bar{y}\left(\frac{\bar{X}\beta_2(x)+C_x}{\bar{x}\beta_2(x)+C_x}\right)$ Upadhyaya and Singh (1999) estimator	$\beta_2(x)$	C_x	1	$\frac{\bar{X}\beta_2(x)}{\bar{X}\beta_2(x)+C_x}$
10.	$\bar{y}^{*}_{UP2}=\bar{y}\left(\frac{\bar{x}^{*}\beta_2(x)+C_x}{\bar{X}\beta_2(x)+C_x}\right)$ Dual to Upadhyaya and Singh (1999) ratio type estimator	$\beta_2(x)$	C_x	0	$\frac{\bar{X}\beta_2(x)}{\bar{X}\beta_2(x)+C_x}$
11.	$\bar{y}_{ST}=\bar{y}\left[\eta\left(\frac{\bar{X}}{\bar{x}}\right)+(1-\eta)\left(\frac{\bar{x}^{*}}{\bar{X}}\right)\right]$ Sharma and Tailor (2010) estimator	1	0	η	1
12.	$\bar{y}_{SA}=\bar{y}\left(\frac{a\bar{X}+b}{a\bar{x}+b}\right)$ Singh and Agnihotri (2008) estimator	a	b	1	$\frac{a\bar{X}}{a\bar{X}+b}$
13.	$\bar{y}^{*}_{SA}=\bar{y}\left(\frac{a\bar{x}^{*}+b}{a\bar{X}+b}\right)$ Dual to Singh and Agnihotri (2008) ratio type estimator	a	b	0	$\frac{a\bar{X}}{a\bar{X}+b}$

3 Efficiency Comparisons

Under *SRSWOR*, variance of sample mean $\bar{y}$ is

$$Var(\bar{y}) = \frac{(1-f)}{n}\bar{Y}^2 C_y^2 . \tag{3.1}$$

From (2.7) and (3.1), it is found that the proposed dual to product-cum-dual to ratio type estimators $\bar{y}_{RP}^*$ is more efficient than $\bar{y}$ if

$$Var(\bar{y}) - MSE(\bar{y}_{RP}^*)$$
$$= \frac{(1-f)}{n}\bar{Y}^2 C_y^2 - \bar{Y}^2 \frac{(1-f)}{n}\left[C_y^2 + \delta C_x^2\{\eta(1-g)+g\}\{\delta\{\eta(1-g)+g\} - 2K\}\right] > 0$$
$$= -\bar{Y}^2 \frac{(1-f)}{n}\left[\delta C_x^2\{\eta(1-g)+g\}[\delta\{\eta(1-g)+g\} - 2K]\right] > 0.$$

This condition holds if

$$\left.\begin{array}{ll} either & \dfrac{g}{g-1} < \eta < \dfrac{2K-g}{1-g} \\ or & \dfrac{2K-g}{1-g} < \eta < \dfrac{g}{g-1} \end{array}\right\}$$

Or equivalently,

$$\min.\left\{\frac{g}{g-1}, \frac{(2K-g)}{(1-g)}\right\} < \eta < \max.\left\{\frac{g}{g-1}, \frac{(2K-g)}{(1-g)}\right\}. \tag{3.2}$$

From (1.5) and (2.7) we have that

$MSE(\bar{y}_{RP}^*) - Var(\bar{y}_R)$ if

$$\left.\begin{array}{ll} either & \dfrac{(2K-1-\delta g)}{\delta(1-g)} < \eta < \dfrac{(1-\delta g)}{\delta(1-g)} \\ or & \dfrac{(1-\delta g)}{\delta(1-g)} < \eta < \dfrac{(2K-1-\delta g)}{\delta(1-g)} \end{array}\right\}$$

Or equivalently,

$$\min.\left\{\frac{(2K-1-\delta g)}{\delta(1-g)}, \frac{(1-\delta g)}{\delta(1-g)}\right\} < \eta < \max.\left\{\frac{(2K-1-\delta g)}{\delta(1-g)}, \frac{(1-\delta g)}{\delta(1-g)}\right\}. \tag{3.3}$$

It is observed from (1.11) and (2.7) that the proposed class of estimators $\bar{y}_{RP}^*$ is more efficient than the Srivenkataramana's (1980) and Bandyopadhyaya's (1980) estimator $\bar{y}_R^*$ if

$$\left.\begin{array}{ll} either & -\dfrac{g(1+\delta)}{\delta(1-g)} < \eta < \dfrac{g(1-\delta)}{\delta(1-g)} \\ or & \dfrac{g(1-\delta)}{\delta(1-g)} < \eta < -\dfrac{g(1+\delta)}{\delta(1-g)} \end{array}\right\}$$

or equivalently,

$$\min.\left\{\frac{-g(1+\delta)}{\delta(1-g)}, \frac{g(1-\delta)}{\delta(1-g)}\right\} < \eta < \max.\left\{\frac{-g(1+\delta)}{\delta(1-g)}, \frac{g(1-\delta)}{\delta(1-g)}\right\} \tag{3.4}$$

It follows from (1.6) and (2.7) that the proposed class of estimators $\bar{y}_{RP}^*$ is more efficient than classical product estimator $\bar{y}_P$ if

$$\left.\begin{array}{ll} either & -\dfrac{(1+\delta g)}{\delta(1-g)} < \eta < \dfrac{(1+2K+\delta g)}{\delta(1-g)} \\ or & \dfrac{(1+2K+\delta g)}{\delta(1-g)} < \eta < -\dfrac{(1+\delta g)}{\delta(1-g)} \end{array}\right\}$$

Or equivalently

$$\min.\left\{-\frac{(1+\delta g)}{\delta(1-g)}, \frac{(1+2K+\delta g)}{\delta(1-g)}\right\} < \eta < \max.\left\{-\frac{(1+\delta g)}{\delta(1-g)}, \frac{(1+2K+\delta g)}{\delta(1-g)}\right\}. \tag{3.5}$$

From (1.12) and (2.7) we have that

$$MSE(\bar{y}_{RP}^*) < Var(\bar{y}_P^*)$$

if

$$\left.\begin{array}{ll} either & -\dfrac{g(1+\delta)}{\delta(1-g)} < \eta < \dfrac{\{g(1-g)+2K\}}{\delta(1-g)} \\ or & \dfrac{\{g(1-g)+2K\}}{\delta(1-g)} < \eta < -\dfrac{g(1+\delta)}{\delta(1-g)} \end{array}\right\}$$

or equivalently

$$\min.\left\{-\frac{g(1+\delta)}{\delta(1-g)}, \frac{\{g(1-g)+2K\}}{\delta(1-g)}\right\} < \eta < \max.\left\{-\frac{g(1+\delta)}{\delta(1-g)}, \frac{\{g(1-g)+2K\}}{\delta(1-g)}\right\}. \tag{3.6}$$

If we set $\eta = 1$ in (2.1), then $\bar{y}_{RP}^*$ reduces to the estimator, as

$$\bar{y}_{SA} = \bar{y}\left(\frac{a\bar{X}+b}{a\bar{x}+b}\right) \tag{3.7}$$

Which is due to Singh and Agnihotri (2008).

Putting $\eta = 1$ in (2.7), we get the mean squared error of the estimator $\bar{y}_{SA}$ to the first degree of approximation, as

$$MSE(\bar{y}_{SA}) = \frac{(1-f)}{n}\bar{Y}^2\left[C_y^2 + \delta C_x^2(\delta - 2K)\right] \tag{3.8}$$

From (2.7) and (3.8) we have that

$MSE(\bar{y}_{RP}^*) < MSE(\bar{y}_{SA}^*)$ if

$$\left.\begin{array}{ll} either & \dfrac{\{2K-g(1-g)\}}{\delta(1-g)} < \eta < 1 \\ or & 1 < \eta < \dfrac{\{2K-g(1-g)\}}{\delta(1-g)} \end{array}\right\}$$

Or equivalently

$$\min .\left\{\frac{\{2K-g(1-g)\}}{\delta(1-g)},1\right\}<\eta<\max .\left\{\frac{\{2K-g(1-g)\}}{\delta(1-g)},1\right\}. \quad (3.9)$$

Interesting $\eta=0$ in (2.1), we get the estimator

$$\bar{y}_{SA}^{*}=\bar{y}\left(\frac{a\bar{x}^{*}+b}{a\bar{x}+b}\right) \quad (3.10)$$

Which is due to Singh and Agnihotri (2008) ratio type estimator $\bar{y}_{SA}$.

Putting $\eta=0$ in (2.7), we get the mean squared error of the $\bar{y}_{SA}^{*}$ to the first degree of approximation, as

$$MSE(\bar{y}_{SA}^{*})=\frac{(1-f)}{n}\bar{Y}^{2}\left[C_{y}^{2}+\delta g C_{x}^{2}(\delta g-2K)\right], \quad (3.11)$$

We note from (2.7) and (3.11) that the suggested class of estimators $\bar{y}_{RP}^{*}$ will dominate over the estimator $\bar{y}_{SA}^{*}$ if

$$\left.\begin{array}{l} either\, 0<\eta<\dfrac{2(K-\delta g)}{\delta(1-g)} \\ or \quad \dfrac{2(K-\delta g)}{\delta(1-g)}<\eta<0 \end{array}\right\}$$

or equivalently,

$$\min .\left\{0,\frac{2(K-\delta g)}{\delta(1-g)}\right\}<\eta<\max .\left\{0,\frac{2(K-\delta g)}{\delta(1-g)}\right\}. \quad (3.12)$$

From (1.15) and (2.7) we note that if the two constants α and η are different, then the proposed class of estimators $\bar{y}_{RP}^{*}$ will dominate over Singh and Agnihotri's (2008) estimator $\bar{y}_{R}$ if

$$\left.\begin{array}{l} either\, \dfrac{\{2(\alpha\delta-K)-\delta(1+g)\}}{\delta(1-g)}<\eta<\dfrac{(1-g-2\alpha)}{(1-g)} \\ or \quad \dfrac{(1-g-2\alpha)}{(1-g)}<\eta<\dfrac{\{2(\alpha\delta-K)-\delta(1+g)\}}{\delta(1-g)} \end{array}\right\}$$

or equivalently

$$\min .\left\{\frac{(1-g-2\alpha)}{(1-g)},\frac{\{2(\alpha\delta-K)-\delta(1+g)\}}{\delta(1-g)}\right\}<\eta<\max .\left\{\frac{(1-g-2\alpha)}{(1-g)},\frac{\{2(\alpha\delta-K)-\delta(1+g)\}}{\delta(1-g)}\right\} \quad (3.13)$$

Let $\alpha=\eta$ in (1.13), and then the mean squared error of the estimator $\bar{y}_{RP}$ to the first degree of approximation is given by

$$MSE(\bar{y}_{RP})=\frac{(1-f)}{n}\bar{Y}^{2}\left[C_{y}^{2}+\delta(1-2\eta)C_{x}^{2}\{\delta(1-2\eta)+2K)\}\right] \quad (3.14)$$

If two constants (α,η) are same (i.e. $\alpha=\eta$), then from (2.7) and (3.14) it is observed that the suggested class of estimators $\bar{y}_{RP}^{*}$ is better than Singh and Agnihotri's (2008) estimator $\bar{y}_{RP}$ if

$$\left.\begin{array}{ll} either & \eta<1, K>\delta \\ or & \eta<\dfrac{\{2K+\delta(1-g)\}}{\delta(3-g)}, K<\delta \end{array}\right\} \quad (3.15)$$

4 Particular Case

To illustrate our general results, we consider an estimator which utilizes information on $\bar{X}$ and correlation coefficient ρ .When information on both ($\bar{X}$, ρ) are available; we define the following class of estimators (just by putting $a=1$ and $b=\rho$ in (2.1)) for population mean $\bar{Y}$:

$$\bar{y}_{RP(1)}^{*}=\eta\bar{y}\left(\frac{\bar{X}+\rho}{\bar{x}+\rho}\right)+(1-\eta)\bar{y}\left(\frac{\bar{x}^{*}+\rho}{\bar{X}+\rho}\right) \quad (4.1)$$

For $\eta=1$, the classes of estimators $\bar{y}_{RP(1)}^{*}$ reduce to the estimator

$$\bar{y}_{RP(1)1}^{*}=\bar{y}\left(\frac{\bar{X}+\rho}{\bar{x}+\rho}\right) \quad (4.2)$$

Which is due to Singh and Tailor (2003).

If we set $\eta=0$ in (4.1), the class of estimators $\bar{y}_{RP(1)}^{*}$ boils down to the estimator

$$\bar{y}_{RP(1)2}^{*}=\bar{y}\left(\frac{\bar{x}^{*}+\rho}{\bar{X}+\rho}\right) \quad (4.3)$$

Which is dual's to Singh and Tailor's (2003) estimator $\bar{y}_{RP(1)1}^{*}$.

To the first degree of approximation, the mean squared errors of $\bar{y}_{RP(1)}^{*}$, $\bar{y}_{RP(1)1}^{*}$ and $\bar{y}_{RP(1)2}^{*}$ are respectively given by

$$MSE(\bar{y}_{RP(1)}^{*})=\frac{(1-f)}{n}\bar{Y}^{2}\left[C_{y}^{2}+\delta_{1}C_{x}^{2}\{\eta(1-g)+g\}\{\delta_{1}(\eta(1-g)+g)-2K\}\right], \quad (4.4)$$

$$MSE(\bar{y}_{RP(1)1}^{*})=\frac{(1-f)}{n}\bar{Y}^{2}\left[C_{y}^{2}+\delta_{1}C_{x}^{2}(\delta_{1}-2K)\right] \quad (4.5)$$

And

$$MSE(\bar{y}_{PR(1)2}^{*})=\frac{(1-f)}{n}\bar{Y}^{2}\left[C_{y}^{2}+\delta_{1}C_{x}^{2}g(\delta_{1}g-2K)\right] \quad (4.6)$$

where $\delta_{1}=\dfrac{\bar{X}}{\bar{X}+\rho}$.

4.1 Efficiency Comparison

In this section, we have presented the comparison of the suggested class of estimators $\bar{y}^*_{RP(1)}$ with usual unbiased estimator $\bar{y}$, ratio estimator $\bar{y}_R$, dual to ratio estimator $\bar{y}^*_R$, Singh and Tailor's (2003) estimator $\bar{y}^*_{RP(1)1}$ and dual to Singh and Tailor's (2003) $\bar{y}^*_{RP(1)2}$.

From (3.1) and (4.4) we note that

$MSE(\bar{y}^*_{RP(1)}) < Var(\bar{y})$ If

$$\left.\begin{array}{l} either\ \dfrac{g}{(g-1)} < \eta < \dfrac{(2K-\delta_1 g)}{\delta_1(1-g)} \\ or\quad \dfrac{(2K-\delta_1 g)}{\delta_1(1-g)} < \eta < \dfrac{g}{(g-1)} \end{array}\right\}$$

or equivalently,

$$\min.\left\{\frac{g}{(g-1)},\frac{(2K-\delta_1 g)}{\delta_1(1-g)}\right\} < \eta < \max.\left\{\frac{g}{(g-1)},\frac{(2K-\delta_1 g)}{\delta_1(1-g)}\right\}. \quad (4.7)$$

It is observed from (1.5) and (4.4) that the class of estimators $\bar{y}^*_{RP(1)}$ is more efficient than usual ratio estimator $\bar{y}_R$ if

$$\left.\begin{array}{l} either\ \dfrac{(2K-\delta_1 g-1)}{\delta_1(1-g)} < \eta < \dfrac{(1-\delta_1 g)}{\delta_1(1-g)} \\ or\quad \dfrac{(1-\delta_1 g)}{\delta_1(1-g)} < \eta < \dfrac{(2K-\delta_1 g-1)}{\delta_1(1-g)} \end{array}\right\}$$

or equivalently,

$$\min.\left\{\frac{(2K-\delta_1 g-1)}{\delta_1(1-g)},\frac{(1-\delta_1 g)}{\delta_1(1-g)}\right\} < \eta < \max.\left\{\frac{(2K-\delta_1 g-1)}{\delta_1(1-g)},\frac{(1-\delta_1 g)}{\delta_1(1-g)}\right\}. \quad (4.8)$$

We note from (1.11) and (4.4)

That $MSE(\bar{y}^*_{RP(1)}) < MSE(\bar{y}^*_R)$ if

$$\left.\begin{array}{l} either\ \dfrac{(2K-g-\delta_1 g)}{\delta_1(1-g)} < \eta < \dfrac{g(1-\delta_1)}{\delta_1(1-g)} \\ or\quad \dfrac{g(1-\delta_1)}{\delta_1(1-g)} < \eta < \dfrac{(2K-g-\delta_1 g)}{\delta_1(1-g)} \end{array}\right\}$$

Or equivalently

$$\min.\left\{\frac{g(1-\delta_1)}{\delta_1(1-g)},\frac{(2K-g-\delta_1 g)}{\delta_1(1-g)}\right\} < \eta < \max.\left\{\frac{g(1-\delta_1)}{\delta_1(1-g)},\frac{(2K-g-\delta_1 g)}{\delta_1(1-g)}\right\}. \quad (4.9)$$

From (4.4) and (4.5) we note that the proposed class of estimators $\bar{y}^*_{RP(1)}$ is more efficient than Singh and Tailor's estimator $\bar{y}^*_{RP(1)1}$ if

$$\left.\begin{array}{l} either\ \dfrac{(1+g)}{(g-1)} < \eta < \dfrac{(1-g+2K)}{(1-g)} \\ or\quad \dfrac{(1-g+2K)}{(1-g)} < \eta < \dfrac{(1+g)}{(g-1)} \end{array}\right\}$$

Or equivalently

$$\min.\left\{\frac{(1+g)}{(g-1)},\frac{(1-g+2K)}{(1-g)}\right\} < \eta < \max.\left\{\frac{(1+g)}{(g-1)},\frac{(1-g+2K)}{(1-g)}\right\}. \quad (4.10)$$

It is observed from (4.4) and (4.6) that the proposed class of estimators $\bar{y}^*_{RP(1)}$ is better than the estimator $\bar{y}^*_{RP(1)2}$ if

$$\left.\begin{array}{l} either\ \dfrac{(2K-\delta_1-\delta_1 g)}{\delta_1(1-g)} < \eta < 1 \\ or\quad 1 < \eta < \dfrac{(2K-\delta_1-\delta_1 g)}{\delta_1(1-g)} \end{array}\right\}$$

or equivalently

$$\min.\left\{1,\frac{(2K-g-\delta_1 g)}{\delta_1(1-g)}\right\} < \eta < \max.\left\{1,\frac{(2K-g-\delta_1 g)}{\delta_1(1-g)}\right\} \quad (4.11)$$

5 Empirical Study

To see the performance of the proposed class of estimators $\bar{y}^*_{RP(1)}$ over the estimator $\bar{y}$, $\bar{y}_R$, $\bar{y}^*_R$, $\bar{y}^*_{RP(1)1}$ and $\bar{y}^*_{RP(1)2}$, we consider a natural population data set. The description of the population is given below:

Population: Murthy (1967)

Y = Output for 80 factories in a region and X = Fixed Capital

$N = 80$, $n = 20$, $\bar{Y} = 51.8264$, $\bar{X} = 11.2646$, $\rho = 0.9413$, $C_y = 0.3542$, $C_x = 0.7507$.

We have computed the range of η and findings are shown in Table 5.1. We have further computed the percent relative efficiency of the proposed class of estimators $\bar{y}^*_{RP(1)}$ relative to $\bar{y}$, $\bar{y}_R$, $\bar{y}^*_R$, $\bar{y}^*_{RP(1)1}$ and $\bar{y}^*_{RP(1)2}$, for different values of η by using the following formulae :

$$E_1 = PRE(\bar{y}^*_{RP(1)},\bar{y}) = \frac{C_y^2}{\left[\begin{array}{l} C_y^2+\delta_1 C_x^2\{\eta(1-g)+g\} \\ \{\delta_1(\eta(1-g)+g)-2K\} \end{array}\right]}\times 100 \quad (5.1)$$

$$E_2 = PRE(\bar{y}^*_{RP(1)},\bar{y}_R) = \frac{[C_y^2+C_x^2(1-2K)]}{\left[\begin{array}{l} C_y^2+\delta_1 C_x^2\{\eta(1-g)+g\} \\ \{\delta_1(\eta(1-g)+g)-2K\} \end{array}\right]}\times 100 \quad (5.2)$$

Table 5.1: Range of η in which the proposed class of estimators $\bar{y}^*_{RP(1)}$ is better than $\bar{y}$, $\bar{y}_R$ $\bar{y}^*_R$, $\bar{y}^*_{RP(1)1}$ and $\bar{y}^*_{RP(1)2}$.

Estimator					
Range of η	$\bar{y}$	$\bar{y}_R$	$\bar{y}^*_R$	$\bar{y}^*_{RP(1)1}$	$\bar{y}^*_{RP(1)2}$
	(-0.5000, 0.9384)	(-0.6810, 1.1194)	(0.0398,0.3986)	(-2.0000, 2.3324)	(-0.5616, 1.000)

Table 5.2: *PREs* of the proposed class of estimators $\bar{y}^*_{RP(1)}$ with respect to $\bar{y}$, $\bar{y}_R$ $\bar{y}^*_R$, $\bar{y}^*_{RP(1)1}$ and $\bar{y}^*_{RP(1)2}$ for different values of η .

Population					
η	E_1	E_2	E_3	E_4	E_5
-0.6810	*	*	*	*	*
-0.5616	*	129.6694	*	*	*
-0.2500	203.6331	305.8427	*	235.8419	*
0.0000	509.4976	765.2300	*	590.0854	100.0000
0.0398	591.3831	888.2164	100.0013	684.9229	116.0718
0.1000	723.0638	1085.9916	122.2681	837.4316	141.9170
0.2000	872.6990	1310.7333	147.5710	1010.7348	171.2862
0.2500	865.2228	1299.5046	146.3068	1002.0761	169.8188
0.3986	591.4677	888.3435	100.0156	685.0209	116.0884
0.9386	*	150.1336	*	115.7713	*
0.2225	877.4028	1317.7981	148.3664	1016.1826	172.2094
0.2192	877.5447	1318.0112	148.3904	1016.3470	172.2373

**Stands for PREs less than 100%.*

$$E_3 = PRE(\bar{y}^*_{RP(1)}, \bar{y}^*_R) = \frac{[C_y^2 + gC_x^2(g-2K)]}{\begin{bmatrix} C_y^2 + \delta_1 C_x^2\{\eta(1-g)+g\} \\ \{\delta_1(\eta(1-g)+g)-2K\} \end{bmatrix}} \times 100 \quad (5.3)$$

$$E_4 = PRE(\bar{y}^*_{RP(1)}, \bar{y}^*_{RP(1)1}) = \frac{[C_y^2 + \delta_1 C_x^2(\delta_1-2K)]}{\begin{bmatrix} C_y^2 + \delta_1 C_x^2\{\eta(1-g)+g\} \\ \{\delta_1(\eta(1-g)+g)-2K\} \end{bmatrix}} \times 100 \quad (5.4)$$

$$E_5 = PRE(\bar{y}^*_{RP(1)}, \bar{y}^*_{RP(1)2}) = \frac{[C_y^2 + g\delta_1 C_x^2(\delta_1 g-2K)]}{\begin{bmatrix} C_y^2 + \delta_1 C_x^2\{\eta(1-g)+g\} \\ \{\delta_1(\eta(1-g)+g)-2K\} \end{bmatrix}} \times 100 \quad (5.5)$$

Table 5.1 exhibits that there is enough scope of choosing the value of scalar 'η' for obtaining better estimators than usual unbiased estimator $\bar{y}$, ratio estimator $\bar{y}_R$, dual to ratio estimator $\bar{y}^*_R$, Singh and Tailor's (2003) estimator $\bar{y}^*_{RP(1)1}$ and dual to Singh and Tailor's (2003) $\bar{y}^*_{RP(1)2}$. It is observed from Table 5.2 that larger gain in efficiency can be observed by using the proposed class of estimators $\bar{y}^*_{RP(1)}$ over $\bar{y}$, $\bar{y}_R$, $\bar{y}^*_R$, $\bar{y}^*_{RP(1)1}$ and $\bar{y}^*_{RP(1)2}$ even when the scalar 'η' departs from its true optimum value 'η_{opt}'. Largest gain in efficacy is observed at the optimum value η_{opt} of η.

Finally our recommendation is to use the proposed class of estimators $\bar{y}^*_{RP(1)}$ in practice.

6 Conclusion

This paper addresses the problem of estimating the population mean $\bar{Y}$ of the study variable y using information on an auxiliary variable x. A class of ratio-cum-dual to ratio estimators has been proposed and its properties are studied under large sample approximation. Different estimators of the population mean $\bar{Y}$ have been identified as a member of the proposed class of ratio-cum-dual to ratio estimators. Regions of performance have been obtained in which suggested class of estimators perform better than other existing estimators. It is a unified approach. Properties of several estimators belonging to the proposed class of estimators can be studied easily. In particular, we have studied the properties of a class of estimators $\bar{y}^*_{RP(1)}$ based on known correlation coefficient ρ between the study variable y and the auxiliary variable x. An empirical study is carried out to judge the merits of the proposed class of estimators $\bar{y}^*_{RP(1)}$ over other estimators. It has been shown empirically that the suggested class of estimators $\bar{y}^*_{RP(1)}$ is more efficient than some other existing estimators.

Finally, we conclude that the proposed class of estimators $\bar{y}^*_{RP}$ and its member $\bar{y}^*_{RP(1)}$ are immense useful to the researchers and practitioners engaged in this area.

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