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The Use of Sumudu Decomposition Method for Solving Predator-Prey Systems

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Abstract: In this paper, we use Sumudu Decomposition Method (SDM) ,which is combination of the Sumudu transform method and Adomian Decomposition Method, to obtain the approximate solutions of nonlinear systems of ordinary differential equations, particularly predator-prey systems. We can easily decompose the nonlinear terms by the help of Adomian polynomials. This technique provides a sequence of functions which converges fast to the accurate solution of the problems. Numerical illustrations are given to show the effectiveness and applicability of this method in solving these kind of systems.

Keywords: Sumudu transform, Adomian polynomials, predator-prey model, nonlinear systems.

1 Introduction

In recent years, a lot of scientists and engineers have focused on some numerical methods to solve plenty of nonlinear ordinary differential equations and partial differential equations that are not susceptible to analytical solution in any reasonably appropriate manner. These numerical methods give generally approximate solutions which converge fast to exact solutions easily and correctly. One of these numerical methods is Adomian decomposition method which has been introduced by George Adomian [1,2]. Adomian decomposition method has been applied to a large class of linear and nonlinear equations that are useful for many research work [3,4,5, 6,7]. The essential property of this method is the ability to solve these equations appropriately and accurately.

Systems of ordinary differential equations have been appeared frequently in a wide class of scientific applications in physics, engineering and other fields. So, their importance cannot be undervalued. Adomian decomposition method has been also used efficiently for solving systems of differential equations [8].

Sumudu transform has been introduced by Watagula and applied to the solution of ordinary differential equations in control engineering problems[9]. It has been also used to solve the systems of differential equations, PDEs and also for heat equations[10,11,12].

The predator-prey equations also known as the Lotka-Volterra equations, are a pair of first-order, nonlinear differential equations frequently used to model the dynamics of ecological systems. This model is one of the most interesting and important application of stability theory involves the interactions between two or more biological populations. It shows the situation in which one species (the predator) preys on the other species (the prey), while the prey lives on a different source of food. These kind of examples contain rather simple equations, but they characterize a wide class of problems of competing species [13,14].

In this paper we use Sumudu decomposition method to get the solutions of predator-prey systems.

2 Sumudu Decomposition Method

The sumudu transform is defined on the set of functions

$$A=\{f(t)|\exists M,\tau_1,\tau_2>0,|f(t)|<Me^{\frac{|t|}{\tau_j}},t\in(-1)^j\times[0,\infty]\} \quad (1)$$

by the following formula

$$F(u)=S[f(t)]=\int_0^\infty \frac{1}{u}f(t)e^{-\frac{t}{u}}dt$$
$$=\int_0^\infty f(ut)e^{-t}dt, u\in(-\tau_1,\tau_2). \quad (2)$$

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Theorem 1. $S[f^{(n)}(t)] = u^{-n}\left[F(u) - \sum_{k=0}^{n-1} u^k f^k(0)\right]$ for $n \geq 1$.

For instance for $n = 1$, $S[f'(t)] = \frac{1}{u}[F(u) - f(0)]$.

We use this theorem to get the approximate solution of given problems. Additionally, the properties and theorems of the Sumudu transform and its derivatives we refer to [9]. In order to get the basic idea of SDM, we consider the particular form of inhomogeneous nonlinear ordinary differential equation with initial condition given below:

$$Lu + Ru + Nu = g(t), u(0) = a \quad (3)$$

where L is the first order derivative and also is invertible, R is a linear differential operator, Nu represents the nonlinear terms and g(t) is the source term.

Applying the sumudu transform to the both sides of the Eq.(3), we get

$$S[Lu] + S[Ru] + S[Nu] = S[g(t)]. \quad (4)$$

Using the differentiation for sumudu transform having initial conditions above, we have

$$S[u(t)] = a + u\left\{S[g(t)] - S[Ru] - S[Nu]\right\}. \quad (5)$$

Now, applying the inverse sumudu transform on both sides of the Eq. (5), we obtain

$$u(t) = a + S^{-1}\left[u\left\{S[g(t)] - S[Ru] - S[Nu]\right\}\right]. \quad (6)$$

So, we may represent the solution as an infinite series:

$$u(t) = \sum_{n=0}^{\infty} u_n(t) \quad (7)$$

and the nonlinear term can be decomposed as

$$Nu(t) = \sum_{n=0}^{\infty} A_n \quad (8)$$

where A_n are Adomian polynomials of $u_0, u_1, u_2, \ldots$ and we can calculate

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[N\left(\sum_{i=0}^{\infty} \lambda^i u_i\right)\right]_{\lambda=0}, n = 0, 1, 2, \ldots \quad (9)$$

Substituting (7) and (8) into (6) , we get

$$\sum_{n=0}^{\infty} u_n(t) = Y(t) - S^{-1}\left[uS[R\sum_{n=0}^{\infty} u_n(t) + \sum_{n=0}^{\infty} A_n]\right] \quad (10)$$

where $Y(t)$ is the term arising from the source term and the given initial condition.

On comparing both sides of last equation and by using standard ADM we have:

$$u_0(t) = Y(t) \quad (11)$$

$$u_1(t) = -S^{-1}\left[uS[Ru_0(t) + A_0]\right] \quad (12)$$

$$u_2(t) = -S^{-1}\left[uS[Ru_1(t) + A_1]\right] \quad (13)$$

and the general relation is given by

$$u_{n+1}(t) = -S^{-1}\left[uS[Ru_n(t) + A_n]\right], n \geq 0. \quad (14)$$

Finally, applying the Sumudu transform of the right hand side of the last equation and then taking the inverse sumudu transform, we get $u_0, u_1, u_2, \ldots$ which are the series form of the desired solutions.

3 Numerical Examples

Let us now give the examples to illustrate the Sumudu Decomposition Method. We select small numbers for initial conditions to make the calculations easier. We also consider four terms approximation to the solutions. Here, we do not discuss the convergence of the solutions. For the convergence of the method the reader is referred to [15].

Example 1. Consider the Lotka-Volterra system

$$\begin{cases} \dot{x} = \frac{dx}{dt} = x(2-y) \\ \dot{y} = \frac{dy}{dt} = y(-3+x) \end{cases} \quad (15)$$

with initial conditions

$$x(0) = 1, y(0) = 2. \quad (16)$$

First let us take the sumudu transform of system as:

$$\begin{aligned} S[\dot{x}] &= \frac{S[x(t)] - x(0)}{u} = S[2x - xy] \\ S[\dot{y}] &= \frac{S[y(t)] - y(0)}{u} = S[-3y + yx] \end{aligned} \quad (17)$$

Taking the inverse Sumudu transform of last equations,then we get:

$$\begin{aligned} x(t) &= 1 + S^{-1}\left[uS[2x - xy]\right] \\ y(t) &= 2 + S^{-1}\left[uS[-3y + xy]\right] \end{aligned} \quad (18)$$

If we assume an infinite series solution of the unknown functions:

$$\begin{aligned} \sum_{n=0}^{\infty} x_n(t) &= 1 + S^{-1}\left[uS[2\sum_{n=0}^{\infty} x_n(t) - \sum_{n=0}^{\infty} A_n]\right] \\ \sum_{n=0}^{\infty} y_n(t) &= 2 + S^{-1}\left[uS[-3\sum_{n=0}^{\infty} y_n(t) + \sum_{n=0}^{\infty} A_n]\right] \end{aligned} \quad (19)$$

where A_n are Adomian polynomials that represents nonlinear term.

The first three components of the Adomian polynomials are given as follows:

$$A_0 = x_0y_0, \quad A_1 = x_1y_0 + x_0y_1, \quad A_2 = x_0y_2 + x_1y_1 + x_2y_0. \tag{20}$$

The recursive relation can be written as:

$$x_0 = 1$$
$$x_1 = S^{-1}[uS[2x_0 - A_0]]$$
$$x_{k+1} = S^{-1}[uS[2x_k - A_k]]$$

and

$$y_0 = 2$$
$$y_1 = S^{-1}[uS[-3y_0 + A_0]]$$
$$y_{k+1} = S^{-1}[uS[-3y_k + A_k]].$$

Calculating the necessary terms:

$$A_0 = 2$$
$$x_1 = S^{-1}[uS[2(1) - 2]] = 0$$
$$y_1 = S^{-1}[uS[-3(2) + 2]] = -4t$$
$$A_1 = -4t$$
$$x_2 = S^{-1}[uS[2(0) + 4t]] = 2t^2$$
$$y_2 = S^{-1}[uS[-3(-4t) - 4t]] = 4t^2.$$

Hence, we get

$$A_2 = 8t^2$$
$$x_3 = -\frac{4t^3}{3}$$
$$y_3 = -\frac{4t^3}{3}$$
$$\vdots$$

If we continue the iteration we get the series form of solutions as:

$$x(t) = 1 + 2t^2 - \frac{4t^3}{3} + \ldots \tag{21}$$

$$y(t) = 2 - 4t + 4t^2 - 4t^3 + \ldots \tag{22}$$

We sketch the following graph that shows the relations between the number of predators and the preys in time. As the graph states that the number of predators increase, as the number of preys decrease. Predators will reach their maximum as the preys reach their minimum. Since the number of preys has decreased, there are not enough food for predators. So their number would decrease and so on.

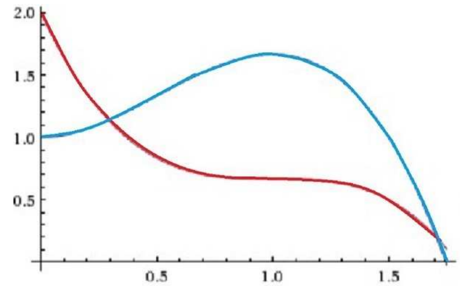

Fig. 1: Predator(red line)- Prey(blue line)

Example 2.
Consider the predator-prey model:

$$\begin{cases} \dot{x} = \frac{dx}{dt} = -x(1 + x + y) \\ \dot{y} = \frac{dy}{dt} = y(1 + y) \end{cases} \tag{23}$$

which is equivalent to Emden–Fowler equation of astrophysics

$$(\xi^2\eta')' + \xi^\lambda \eta^n = 0 \tag{24}$$

where $\lambda = n = 0$ with the initial conditions:

$$x(0) = 2, y(0) = 3. \tag{25}$$

By applying the previous method subject to the initial condition, we have

$$\begin{aligned} x(t) &= 2 + S^{-1}\left[uS[-x - x^2 - xy]\right] \\ y(t) &= 3 + S^{-1}\left[uS[y + y^2]\right]. \end{aligned} \tag{26}$$

If we assume an infinite series solution of the unknown functions:

$$\sum_{n=0}^{\infty} x_n(t) = 2 + S^{-1}\left[uS[-\sum_{n=0}^{\infty} x_n(t) - \sum_{n=0}^{\infty} A_n - \sum_{n=0}^{\infty} B_n]\right]$$
$$\sum_{n=0}^{\infty} y_n(t) = 3 + S^{-1}\left[uS[\sum_{n=0}^{\infty} y_n(t) + \sum_{n=0}^{\infty} C_n]\right] \tag{27}$$

where A_n, B_n and C_n are Adomian polynomials that represent the nonlinear terms.
Applying the same procedure to get the recursive relation:

$$x_0 = 2$$
$$x_1 = S^{-1}[uS[-x_0 - A_0 - B_0]]$$
$$x_{k+1} = S^{-1}[uS[-x_k - A_k - B_k]]$$
$$y_0 = 3$$
$$y_1 = S^{-1}[uS[y_0 + C_0]]$$
$$y_{k+1} = S^{-1}[uS[y_k + C_k]]$$

where Adomian polynomials are given by:

$$A_0 = x_0^2$$
$$B_0 = x_0 y_0$$
$$C_0 = y_0^2$$
$$A_1 = 2x_1 x_0$$
$$B_1 = x_1 y_0 + x_0 y_1$$
$$C_1 = 2y_1 y_0$$
$$A_2 = 2x_0 x_2 + x_1^2$$
$$B_2 = x_0 y_2 + x_1 y_1 + x_2 y_0$$
$$C_2 = 2y_0 y_2 + y_1.^2$$

yielding

$$A_0 = 4$$
$$B_0 = (2)(3) = 6$$
$$C_0 = 9$$
$$A_1 = 2(-12t)(2) = -48t$$
$$B_1 = (-12t)(3) + (2)(12t) = -12t$$
$$C_1 = 2(12t)(3) = 72t$$
$$x_1 = S^{-1}\left[uS[-2-4-6]\right] = -12t$$
$$x_2 = S^{-1}\left[uS[12t + 48t + 12t]\right] = 36t^2$$
$$y_1 = S^{-1}\left[uS[3+9]\right] = 12t$$
$$y_2 = S^{-1}\left[uS[12t + 72t]\right] = 42t^2.$$

continuing the iteration,we get:

$$A_2 = 288t^2 \qquad B_2 = 48t^2 \qquad C_2 = 396t^2$$

$$x_3 = -124t^3$$

$$y_3 = 146t^3.$$

Combining all of these solutions yields:

$$x(t) = 2 - 12t + 36t^2 - 124t^3 + \ldots \quad (28)$$

$$y(t) = 3 + 12t + 42t^2 + 146t^3 + \ldots \quad (29)$$

In Fig. 2, we can see that only the prey population progressively decreases and becomes extinct despite the inhesion of increasing predator population. This can be related to the effect of the prey death rate being greater than the conversion rate.

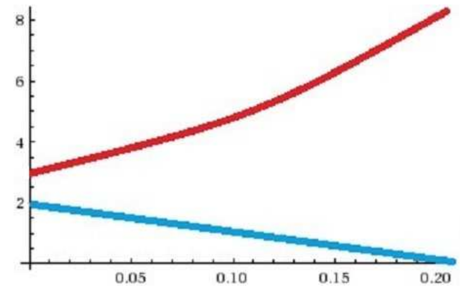

Fig. 2: Predator-Prey model which is equivalent to Emden–Fowler equation of astrophysics

4 Conclusions

In this paper, we present a mathematical model of the prey-predator problem which is very important in applied sciences and we employ Sumudu Decomposition Method to obtain the approximate solutions of these equations. Sumudu Decomposition Method gives a new approach to the solution of these kinds of problems.We solve two examples and we plot figures to illustrate this technique and it is seen that the Sumudu Decomposition Method is very useful and effective method to get the approximate solutions. Thus,it can be applied to many complicated linear and nonlinear equations reliably.

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