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Some New I-Lacunary Generalized Difference Convergent Sequence Spaces in 2-Normed Spaces

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Abstract: In this paper, we introduce and examine the properties of some new class of ideal lacunary convergent sequence spaces using, an infinite matrix with respect to a modulus function $F = (f_k)$ in 2-normed linear space. We study these spaces for some topological structures and algebraic properties. We also give some relations related to these sequence spaces.

Keywords: Difference sequence, lacunary sequence, I-convergent, infinite matrix, 2-normed space. **AMS subject classification** (2000)**: 40A05, 46B70,46A45**

1 Introduction and Preliminaries

The idea of difference sequence spaces was introduced by Kizmaz [9]. Who studied the difference sequence spaces $\ell_\infty(\Delta), c(\Delta)$ and $c_0(\Delta)$. The the idea was further generalized by Et and Çolak [4] for $\ell_\infty(\Delta^n), c(\Delta^n)$ and $c_0(\Delta^n)$. Let ω be the space of all complex or real sequence $x = (x_k)$ and m, s be non-negative integers, then $Z = \ell_\infty, c$ and c_0, we have sequence spaces,

$$Z(\Delta_s^m x_k) = \left\{x = (x_k) \in \omega : (\Delta_s^m x_k) \in Z\right\},$$

where $(\Delta_s^m x) = (\Delta_s^m x_k) = \left(\Delta_s^{m-1} x_k - \Delta_s^{m-1} x_{k+1}\right)$ and $\Delta_s^0 x_k = (x_k)$ for all $k \in \mathbb{N}$, which is equivalent to the following binomial representation

$$\Delta_s^m x_k = \sum_{v=0}^{m} (-1)^v \binom{m}{v} x_{k+sv}$$

P.Kostyrko et al [10] introduced the concept of I-convergence of sequence in metric space and studied some properties of such convergence. Since then many author have been studied these subject and obtained various results [29,30,31,32,**?**] Note that I-convergence is an interesting generalization of statistical convergence.

The concept of 2-normed space was initially introduced by Gähler [7] as an interesting nonlinear generalization of a normed linear space which was subsequently studied by many authors, [8]. Recently a lot of activities have been started to study summability, sequence spaces and related topics in these nonlinear space. Sahiner et al., [23] introduce I-convergence in 2-normed space.

Give that $I \subset 2^{\mathbb{N}}$ be trivial ideal in $\mathbb{N}$. The sequence $(x_n)_{n\in\mathbb{N}}$ in X is said to be I-convergent to $x \in X$, if for each $\varepsilon > 0$ then the set,

$$A(\varepsilon) = \left\{n \in \mathbb{N} : \| x_n - x \| \geq \varepsilon\right\} \in I \quad [10,11]$$

Let X be a real vector space of dimension d, where $2 \leq d < \infty$. A 2-norm on X is a function $\|.,.\| : X \times X \to R$. Which satisfies:

$(i)\| x,y \| = 0$ if and only if x and y are linearly dependent,
$(ii)\| x,y \| = \| y,x \|$,
$(iii)\| \alpha x,y \| = |\alpha| \ \| x,y \|,\ \alpha \in \mathbb{R}$,
$(iv)\| x,y+z \| \leq \ \| x,y \| + \| x,z \|$.

The pair $(X, \| .,. \|)$ is called a 2- normed spaces [8]. As an example of a 2-normed space we may take $X = \mathbb{R}^2$ being equipped with standard and Euclid 2-norm on $\mathbb{R}^2$

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are given by,

$$||x_1,x_2||_E = abs\left(\begin{vmatrix} x_{11} & x_{22} \\ x_{21} & x_{22} \end{vmatrix}\right)$$

We know that $\Big(X,\|\ .,.\ \|\Big)$ is 2-Banach space if every Cauchy sequence in X is convergent to some $x \in X$.

By an ideal we mean a family $I \subset 2^X$ of subsets a non-empty set X satisfying:

$(i)\phi \in I$
$(ii)A,B \in I$ implies $A \cup B \in I$
$(iii)A \in I,\ B \subset A$ impliy $B \in I$

While an admissible ideal I of X further satisfies $y \in I$ for each $y \in X$ [10]. By Lacunary sequence we mean an increasing sequence $\theta = k_r$ of positive integers satisfying; $k_0 = 0$ and $h_r = k_r - k_{r-1} \to \infty$ as $r \to \infty$. We denote the intervals, which θ determines by $I_r = (k_{r-1} - k_r]$.

A sequence space X is said to be solid or normal if $(\alpha_k x_k) \in X$ whenever $(x_k) \in X$ and for all sequences of scalar (α_k) with $|\alpha_k| \le 1$.

We recall that a modulus f is a function from $[0,\infty) \to [0,\infty)$ such that

$(i)\,f(x) = 0$ if and only if $x = 0$,
$(ii)\,f(x+y) \le f(x) + f(y)$ for all $x \ge 0,\ \ y \ge 0$,
$(iii)\,f$ is increasing,
$(iv)\,f$ is continuous from right at 0.

It follows from (i) and (iv) that (f) must be continuous every where on $[0,\ \infty)$. For a sequence of moduli $F = (f_k)$, we give the following conditions.

$(v)\sup_k f_k(x) < \infty$ for all $x > 0$
$(vi)\lim_{x\to o} f_k(x) = 0$, uniformly in $k \ge I$.

We remark that in case $f_k = f$ for all k, where f is a modulus, the conditions (v) and (vi) are automatically fulfilled.

2 Main Results

In this article using the lacunary sequence and notion of ideal, we aimed to introduced some new ideal convergent sequence space by combining an infinite matrix with respect to a modulus function $F = (f_k)$ and study their linear topological structures. Also we give some relations related to these sequence spaces.

Let I be an admissible ideal, $F = (f_k)$ be a sequence of moduli,$\Big(X,\|\ .,\|\ \Big)$ be 2-normed space, $p = (p_k)$ be a sequence of strictly positive real numbers and $A = (a_{nk})$ be an infinite matrix of complex numbers. We write $Ax = (A_n(x))_{n=1}^{\infty}$ if $A_n(x) = \sum_{n=1}^{\infty}(a_{nk}(x))$ converges for each $n \in \mathbb{N}$. By $\omega(2-X)$ we denotes the space of all sequences defined over 2-normed space $\Big(X,\|\ .,\|\ \Big)$. Now we define the following sequence spaces:

$$N_\theta^I[A,\Delta_s^m,F,p,\|,.,\|]_0 = \Big\{x=(x_k) \in \omega(2-X) : \forall\varepsilon > 0\ , \Big\{r \in \mathbb{N} : \frac{1}{h_r}\sum_{k\in I_r}\Big[f_k\Big(\ \|\ A_k(\Delta_s^m x_k),z\ \|\ \Big)\Big]^{p_k} \ge \varepsilon\Big\} \in I \text{ each } z \in X\Big\}$$

$$N_\theta^I[A,\Delta_s^m,F,p,\|,.,\|] = \Big\{x=(x_k) \in \omega(2-X) : \forall\varepsilon > 0, \Big\{r \in \mathbb{N} : \frac{1}{h_r}\sum_{k\in I_r}\Big[f_k\Big(\ \|\ A_k(\Delta_s^m x_k) - L,z\ \|\ \Big)\Big]^{p_k} \ge \varepsilon\Big\} \in I \text{ for some } L > 0 \text{ and each } z \in X\Big\},$$

Where $A_k(\Delta_s^m x_k) = \sum_{k=1}^{\infty} a_{nk}\Delta_s^m x_k$ for all $n \in \mathbb{N}$

If $F(x) = x$, we get

$$N_\theta^I[A,\Delta_s^m p,\|,.,\|]_0 = \Big\{x=(x_k) \in \omega(2-X) : \forall\varepsilon > 0, \Big\{r \in \mathbb{N} : \frac{1}{h_r}\sum_{k\in I_r}\Big[\Big(\ \|\ A_k(\Delta_s^m x_k),z\ \|\ \Big)\Big]^{p_k} \ge \varepsilon\Big\} \in I \text{ each } z \in X\Big\},$$

$$N_\theta^I[A,\Delta_s^m,p,\|,.,\|] = \Big\{x=(x_k) \in \omega(2-X) : \forall\varepsilon > 0, \Big\{r \in \mathbb{N} : \frac{1}{h_r}\sum_{k\in I_r}\Big[\Big(\ \|\ A_k(\Delta_s^m x_k) - L,z\ \|\ \Big)\Big]^{p_k} \ge \varepsilon\Big\} \in I \text{ for some } L > 0 \text{ and each } z \in X\Big\},$$

If $p = (p_k) = 1$ for all $k \in \mathbb{N}$

$$N_\theta^I[A,\Delta_s^m,F,\|,.,\|]_0 = \Big\{x=(x_k) \in \omega(2-X) : \forall\varepsilon > 0, \Big\{r \in \mathbb{N} : \frac{1}{h_r}\sum_{k\in I_r}\Big[\Big(\ \|\ A_k(\Delta_s^m x_k),z\ \|\ \Big)\Big] \ge \varepsilon\Big\} \in I \text{ each } z \in X\Big\},$$

$$N_\theta^I[A,\Delta_s^m,F,\|,.,\|] = \Big\{x=(x_k) \in \omega(2-X) : \forall\varepsilon > 0, \Big\{r \in \mathbb{N} : \frac{1}{h_r}\sum_{k\in I_r}\Big[\Big(\ \|\ A_k(\Delta_s^m x_k) - L,z\ \|\ \Big)\Big] \ge \varepsilon\Big\} \in I \text{ for some } L > 0 \text{ and each } z \in X\Big\}$$

The following inequality will be used throughout the paper. Let $p = (p_k)$ be a positive sequence of real numbers with $0 < p_k \leq \sup p_k = G,\ D = \max\left(1, 2^{G-1}\right)$. Then for all $a_k, b_k \in \mathbb{C}$ for all $k \in \mathbb{N}$. We have

$$|a_k + b_k|^{p_k} \leq D(|a_k|^{p_k} + |b_k|^{p_k}) \tag{1}$$

Theorem 2.1. Let $F = (f_k)$ be a sequence of modulus functions, $p = (p_k)$ is bounded then, $N_\theta^I[A, \Delta_s^m, F, p, \|,.,\|]$ and $N_\theta^I[A, \Delta_s^m, F, p, \|,.,\|]_0$ are linear space over the complex field $\mathbb{C}$.

Proof. We shall give the proof only for $N_\theta^I[A, \Delta_s^m, F, p, \|,.,\|]_0$ and other can be proved by the same technique. Let $x, y \in N_\theta^I[A, \Delta_s^m, F, p, \|,.,\|]_0$ and $\alpha, \beta \in \mathbb{C}$, there exist M_α and N_β such that $|\alpha| \leq M_\alpha$ and $|\beta| \leq N_\beta$. Since $\|,.,\|$ is 2-norm and $F = (f_k)$ is a modulus function for all k from equation (1),the following inequality holds:

$$\frac{1}{h_r}\sum_{k \in I_r}\left[f_k\left(\| A_k \Delta_s^m(\alpha x_k + \beta y_k), z \|\right)\right]^{p_k}$$

$$\leq D.(M_\alpha)^H \frac{1}{h_r}\sum_{k \in I_r}\left[\left(\| A_k \Delta_s^m x_k), z \|\right)\right]^{p_k}$$

$$+D.(N_\beta)^H \frac{1}{h_r}\sum_{k \in I_r}\left[f_k\left(\| A_k \Delta_s^m(\beta y_k), z \|\right)\right]^{p_k}$$

On the other hand from the above inequality we get.

$$\left\{r \in \mathbb{N} : \frac{1}{h_r}\sum_{k \in I_r}\left[f_k\left(\| A_k \Delta_s^m(\alpha x_k + \beta y_k), z \|\right)\right]^{p_k} \geq \varepsilon\right\}$$

$$\subset \left\{r \in \mathbb{N} : D.(M_\alpha)^H \frac{1}{h_r}\sum_{k \in I_r}\left[f_k\left(\| A_k(\Delta_s^m x_k), z \|\right)\right]^{p_k} \geq \varepsilon\right\}$$

$$\cup \left\{r \in \mathbb{N} : D.(N_\beta)^H \frac{1}{h_r}\sum_{k \in I_r}\left[f_k\left(\| A_k(\Delta_s^m y_k), z \|\right)\right]^{p_k} \geq \varepsilon\right\}.$$

Two sets on the right side belongs to I, so this completes the proof.

Lemma 1. Let f be a modulus function ,and let $0 < \delta < 1$. Then for each $x > \delta$ we have $f(x) \leq 2f(1)\delta^{-1}x$ [16]

Theorem 2.2. Let $F = (f_k)$ be sequence of a moduli and $0 < \inf_k p_k = h \leq p_k \leq \sup_k p_k = H < \infty$.

$$N_\theta^I[A, \Delta_s^m, p, \|,.,\|, z] \subset N_\theta^I[A, \Delta_s^m, F, p, \|,.,\|, z]$$

and

$$N_\theta^I[A, \Delta_s^m, p, \|,.,\|, z]_0 \subset N_\theta^I[A, \Delta_s^m, F, p, \|,.,\|, z]_0$$

Proof. Let $x \in N_\theta^I[A, \Delta_s^m, p, \|,.,\|, z]$, then for some $L > 0$ and for each $z \in X$

$$\left\{r \in \mathbb{N} : \frac{1}{h_r}\sum_{k \in I_r}\left[f_k\left(\| A_k(\Delta_s^m x_k - L), z \|\right)\right]^{p_k} \geq \varepsilon\right\}$$

Now let $\varepsilon > 0$, and we choose $0 < \delta < 1$ such that for every t with $0 \leq t \leq \delta$, we have $f_k(t) < \varepsilon$ for all k. Now by using Lemma (1), we get

$$\left\{r \in \mathbb{N} : \frac{1}{h_r}\sum_{k \in I_r}\left[f_k\left(\| A_k(\Delta_s^m x_k - L), z \|\right)\right]^{p_k} \geq \varepsilon\right\}$$

$$= \left\{r \in \mathbb{N} : \frac{1}{h_r}\left(h_r.\max[\varepsilon^h, \varepsilon^H]\right) \geq \varepsilon\right\}$$

$$\cup \left\{r \in \mathbb{N} : \frac{1}{h_r}\left(\max\left\{\left(2f_k(1)\delta^{-1}\right)^h, \left(2f_k(1)\delta^{-1}\right)^H\right.\right.$$

$$\times \sum_{k \in I_r}\left[\left(\| A_k(\Delta_s^m x_k - L), z \|\right)\right]^{p_k} \geq \varepsilon\right\}$$

This completes the proof. The other case can be proved similarly.

Theorem 2.3. Let $F = (f_k)$ be a sequence of moduli. If $\lim_t \sup \frac{f_k(t)}{t} = \eta > 0$ for all k then

$$N_\theta^I[A, \Delta_s^m, p, \|,.,\|, z]_0 = N_\theta^I[A, \Delta_s^m, F, p, \|,.,\|, z]_0$$

and

$$N_\theta^I[A, \Delta_s^m, p, \|,.,\|, z] = N_\theta^I[A, \Delta_s^m, F, p, \|,.,\|, z]$$

Proof. In Theorem 2.2, it is shown that,

$$N_\theta^I[A, \Delta_s^m, F, p, \|,.,\|, z] \subset N_\theta^I[A, \Delta_s^m, p, \|,.,\|, z]$$

we must show that $N_\theta^I[A, \Delta_s^m, p, \|,.,\|, z] \subset N_\theta^I[A, \Delta_s^m, F, p, \|,.,\|, z]$ For any modulus function there exist a positive limit $B > 0$ and $x \in N_\theta^I[A, \Delta_s^m, F, p, \|,.,\|, z]$. Since $B > 0$ for every $f_k(t) \geq Bt$ for all k. Hence

$$\left\{r \in \mathbb{N} : \frac{1}{h_r}\sum_{k \in I_r}\left[f_k\left(\| A_k(\Delta_s^m x_k - L), z \|\right)\right]^{p_k}\right\}$$

$$\geq \left\{r \in \mathbb{N} : B^H \frac{1}{h_r}\sum_{k \in I_r}\left[\left(\| A_k(\Delta_s^m x_k - L), z \|\right)\right]^{p_k} \geq \varepsilon\right\}$$

This completes the proof.

Corollary 1. Let $F_1 = (f_k)$ and $F_2 = (g_k)$ be sequences of moduli. If

$$\lim_t \sup \frac{f_{k(t)}}{g_{k(t)}} < \infty$$

implies that

$$N_\theta^I[A,\Delta_s^m,F_2,p,\|,.,\|,z]_0 = N_\theta^I[A,\Delta_s^m,F_2,p,\|,.,\|,z]_0$$

and

$$N_\theta^I[A,\Delta_s^m,F_1,p,\|,.,\|,z] = N_\theta^I[A,\Delta_s^m,F_2,p,\|,.,\|,z]$$

Theorem 2.4. Let $\left(X,\|.,.\|_S\right)$ and $\left(X,\|.,.\|_E\right)$ be standard Euclid 2- normed spaces respectively then

$$N_\theta^I[A,\Delta_s^m,F,p,\|,.,\|_S]\cap N_\theta^I[A,\Delta_s^m,F,p,\|,.,\|_E]$$

$$\subset \left[N_\theta^I A,\Delta_s^m,F,p\right]\left(\|,.,\|_S+\|,.,\|_E\right)$$

Proof. We have the following inclusion.

$$\left\{r\in\mathbb{N}:\frac{1}{h_r}\sum_{k\in I_r}\left[f_k A_k\left(\|.,.\|_S+\|.,.\|_E\,(\Delta_s^m x_k-L),z\right)\right]^{p_k}\geq\varepsilon\right\}$$

$$\subset\left\{r\in\mathbb{N}:D\frac{1}{h_r}\sum_{k\in I_r}\left[f_k\left(A_k\,\|\,(\Delta_s^m x_k-L),z\,\|_S\right)\right]^{p_k}\geq\varepsilon\right\}$$

$$\cup\left\{r\in\mathbb{N}:D\frac{1}{h_r}\sum_{k\in I_r}\left[f_k\left(A_k\,\|\,(\Delta_s^m x_k-L),z\,\|_E\right)\right]^{p_k}\geq\varepsilon\right\}$$

by using equation (1). This complete the proof.

Theorem 2.5. Let $F_1 = (f_k)$ and $F_2 = (g_k)$ be sequences of moduli. Then

(i)

$$N_\theta^I[A,\Delta_s^m,F_1,p,\|,.,\|,z]_0\subset N_\theta^I[A,\Delta_s^m,F_1oF_2,p,\|,.,\|,z]_0$$

and

$$N_\theta^I[A,\Delta_s^m,F_1,p,\|,.,\|,z]\subset N_\theta^I[A,\Delta_s^m,F_1oF_2,p,\|,.,\|,z]$$

(ii)

$$N_\theta^I[A,\Delta_s^m,F_1,p,\|,.,\|,z]_0\cap N_\theta^I[A,\Delta_s^m,F_2,p,\|,.,\|,z]_0\subset N_\theta^I[A,\Delta_s^m,F_1+F_2,p,\|,.,\|,z]_0$$

and

$$N_\theta^I[A,\Delta_s^m,F_1,p,\|,.,\|,z]\cap N_\theta^I[A,\Delta_s^m,F_2,p,\|,.,\|,z]\subset N_\theta^I[A,\Delta_s^m,F_1+F_2,p,\|,.,\|,z]$$

Proof. (i) Let $x_k\in N_\theta^I[A,\Delta_s^m,F_1,p,\|,.,\|,z]$. Let $0<\varepsilon<1$ and $0<\delta<1$, such that $f_k(t)<\varepsilon$ for $0<t<\delta$. Let $y_k=g_k\left(\|A_k(\Delta_s^m x_k-L),z\,\|\right)$. Let

$$\frac{1}{h_r}\sum_{k\in I_r}\left[f_k(y_k)\right]^{p_k}=\frac{1}{h_r}\sum_1\left[f_k(y_k)\right]^{p_k}+\frac{1}{h_r}\sum_2\left[f_k(y_k)\right]^{p_k}$$

where the first summation is over $y_k\leq\delta$ and the second summation is over $y_k>\delta$. Then $\frac{1}{h_r}\sum_1\left[f_k(y_k)\right]^{p_k}\leq\varepsilon^H$ and for $y_k>\delta$, we use the fact that,

$$y_k<\frac{y_k}{\delta}<1+(|\frac{y_k}{\delta}|)$$

where $|z|$ denotes the integer part of z. From the properties of modulus function, we have for $y_k>\delta$

$$f_k(y_k)<\left(1+(|\frac{y_k}{\delta}|)\right)f_k(1)\leq 2f_k\frac{y_k}{\delta}$$

Hence.

$$\frac{1}{h_r}\sum_2\left[f_k(y_k)\right]^{p_k}\leq\left[2\frac{f_k(1)}{\delta}\right]^H\frac{1}{h_r}\sum_2\left[y_k\right]^{p_k}$$

which together with $\frac{1}{h_r}\sum_1\left[f_k(y_k)\right]^{p_k}\leq\varepsilon^H$ yields.

$$\frac{1}{h_r}\sum_{k\in I_r}\left[f_k(y_k)\right]^{p_k}\leq\varepsilon^H+\max\left(1,[2\frac{f_k(1)}{\delta}]^H\right)\sum_{k\in I_r}\left[f_k y_k\right]^{p_k}$$

This completes the proof.

(ii) Let $x_k\in N_\theta^I[A,\Delta_s^m,F_1,p,\|,.,\|,z]\cap N_\theta^I[A,\Delta_s^m,F_2,p,\|,.,\|,z]$. The fact is that

$$\frac{1}{h_r}\sum_{k\in I_r}\left[(f_k+g_k)\left(\|A_k(\Delta_s^m x_k-L),z\,\|\right)\right]^{p_k}$$

$$\leq D.\frac{1}{h_r}\sum_{k\in I_r}\left[f_k\left(\|A_k(\Delta_s^m x_k-L),z\,\|\right)\right]^{p_k}$$

$$+D\frac{1}{h_r}\sum_{k\in I_r}\left[g_k\left(\|A_k(\Delta_s^m x_k-L),z\,\|\right)\right]^{p_k}$$

gives the results

3 Statistical Convergent

The notion of statistical convergence of sequences was introduced by Fast [5]. Later on it was studied from sequence space and linked with summability theory by Fridy [6], Salat [28] and many others. The notion depends on the density of subsets of the set $\mathbb{N}$ of natural numbers. A subset E of $\mathbb{N}$ is said to have density $\delta(E)$ if $\delta(E) = \lim_{n\to\infty} \frac{1}{n}\sum_{\infty}^{n} \tau_E(k)$ exist, where τ_E is the characteristics function of E.

A complex number sequence $x = (x_k)$ is said to be statistically convergent to the number L if for every $\varepsilon > 0$, $\lim_{n\to\infty} \frac{|K(\varepsilon)|}{n} = 0$, where $|K(\varepsilon)|$ denotes the number of elements in the set $K(\varepsilon) = \left\{k \in \mathbb{N} : |x_k - L| \geq \varepsilon\right\}$.

A complex number sequence $x = (x_k)$ is said to be strongly generalized difference $S^{\lambda}(A,\Delta_s^m)$-statistically convergent to the number L if for every $\varepsilon > 0$, $\lim_{n\to\infty} \frac{1}{\lambda_n}|KA(\Delta_s^m,\varepsilon)| = 0$, where $|KA(\Delta_s^m,\varepsilon)|$ denotes the number of elements in the set $KA(\Delta_s^m,\varepsilon) = \left\{k \in I_n : |A_k(\Delta_s^m x_k) - L| \geq \varepsilon\right\}$.

The set of all strongly generalized difference statistically convergent sequences is denoted by $S^{\lambda}(A,\Delta_s^m)$, If $m = 0$, $\Delta = 0$, then $S^{\lambda}(A,\Delta_s^m)$ reduces to $S^{\lambda}(A)$, which was defined and studied by Bilgin and Altin [1]. If A is identity matrix, and $\lambda_n = n$, $s = 0$, $S^{\lambda}(A,\Delta_s^m)$ reduces to $S^{\lambda}(\Delta^m)$ which was defined by Et and Nuray [3]. If $m = 0$, $s = 0$ and $\lambda_n = n$ then $S^{\lambda}(A,\Delta_s^m)$ reduces to S_A, which was defined by Esi [2]. If $m = 0$, $s = 0$ and A is identity matrix and $\lambda_n = n$, strongly generalized difference $S^{\lambda}(A,\Delta_s^m)$-statistically convergent sequences reduces to ordinary statistically convergent sequences.

Theorem 3.1 Let $F = (f_k)$ be a sequence of modulus functions. Then

$$N_\theta^I[A,\Delta_s^m,F,p,\|,.,\|,z] \subset S^{\lambda}(A,\Delta_s^m)$$

Proof. Let $x \in N_\theta^I[A,\Delta_s^m,F,p,\|,.,\|,z]$, then

$$\frac{1}{h_r}\sum_{k\in I_r}\left[f_k\left(\parallel A_k(\Delta_s^m x_k - L),z\parallel\right)\right]^{p_k}$$

$$\geq \frac{1}{h_r}\sum_{k\in I_r,\|A_k(\Delta_s^m x_k - L),z\|>S}\left[f_k\left(\parallel A_k\Delta_s^m(x_k - L),z\parallel\right)\right]^{p_k}$$
$$\geq \frac{1}{h_r}\sum_{k\in I_r,\|A_k(\Delta_s^m x_k - L),z\|>S}\left[f_k(\varepsilon)\right]^{p_k}$$
$$\geq \frac{1}{h_r}\sum_{k\in I_r,\|A_k(\Delta_s^m x_k - L),z\|>S}\min\left(f_k(\varepsilon)^h, f_k(\varepsilon)^H\right)$$
$$\geq \min\left(f_k(\varepsilon)^h, f_k(\varepsilon)^H\right)\frac{1}{h_r}|KA(\Delta_s^m,\varepsilon)|$$

Hence $x \in S^{\lambda}(A,\Delta_s^m)$

Theorem 3.2 Let $F = (f_k)$ be a sequence of modulus functions. Then

$$N_\theta^I[A,\Delta_s^m,F,p,\|,.,\|,z] = S^{\lambda}(A,\Delta_s^m)$$

Proof. By Theorem 3.1, it is sufficient to show that

$$N_\theta^I[A,\Delta_s^m,F,p,\|,.,\|,z] \supset S^{\lambda}(A,\Delta_s^m).$$

Let $x_k \in S^{\lambda}(A,\Delta_s^m)$. Since $F = (f_k)$ is bounded so that there exist an integer $K > 0$, such that $f_k\left(\parallel A_k(\Delta_s^m x_k - L),z\parallel\right) \leq K$. Then for given $\varepsilon > 0$, we have

$$\frac{1}{h_r}\sum_{k\in I_r}\left[f_k\left(\parallel A_k(\Delta_s^m x_k - L),z\parallel\right)\right]^{p_k}$$

$$= \frac{1}{h_r}\sum_{k\in I_r,\|A_k(\Delta_s^m x_k - L),z\|\leq S}\left[f_k\left(\parallel A_k(\Delta_s^m x_k - L),z\parallel\right)\right]^{p_k}$$
$$+ \frac{1}{h_r}\sum_{k\in I_r,\|A_k(\Delta_s^m x_k - L),z\|>S}\left[f_k\left(\parallel A_k(\Delta_s^m x_k - L),z\parallel\right)\right]^{p_k}$$
$$\leq \max\left(f_k(\varepsilon)^h, f_k(\varepsilon)^H\right) + K^H\frac{1}{h_r}|KA(\Delta_s^m,\varepsilon)|$$

Taking limit as $\varepsilon \to 0$ and $n \to \infty$, it follows that $x \in N_\theta^I[A,\Delta_s^m,F,p,\|,.,\|,z]$

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