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Deformation in Porous Thermoelastic Material with Temperature Dependent Properties

R. Kumar[1] and S. Devi[2]

[1]Department of Mathematics, Kurukshetra University, Kurukshetra-136 119, India

Email Address: rajneesh_kuk@rediffmail.com

[2]Dayanand College, Hisar, Haryana-125001, India.

Email Address: savita.007@rediffmail.com

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A general solution to the field equations of thermoelastic material with one relaxation time (Lord and Shulman theory) with voids under the dependence of modulus of elasticity and thermal conductivity on reference temperature has been obtained in the transformed domain using Laplace and Fourier transforms due to mechanical and thermal sources.The uniformly or linearly (instantaneous or continuous)distributed sources have been taken to show the utility of the solution obtained. The transformed solutions are inverted using a numerical inversion technique.The effect of dependence of modulus of elasticity on the normal stress, changes in volume fraction field and temperature distribution have been depicted graphically for Lord and Shulman theory(L-S)and coupled theory(CT) of thermoelasticity, with voids for a particular model.Some particular cases are also deduced from the present formulation

Keywords: Thermoelasticity, generalized thermoelasticity, modulus of elasticity, thermal conductivity, instantaneous source, uniformly/linearly distributed source, integral transforms.

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1 Introduction

Biot (1956) formulated the theory of coupled thermoelasticity to eliminate the paradox inherent in the classical uncoupled theory that elastic changes have no effect on the temperature. The heat equations for both the theories, however, are of the diffusion type, predicting infinite speeds of propagation for heat waves contrary to physical observations. Lord and Shulman (1967) introduced the theory of generalized thermoelasticity with one ralaxation time by postulating new law of heat conduction to replace the classical Fourier law. This law contains the heat flux vector as well as its time derivative. It contains also a new constant that act as a relaxation time. The heat equation of this theory is of the wave type, ensuring finite speeds of propagation for heat and elastic waves. The remaining governing equations for this theory, namely, the equation of motion and the constitutive relations remain the same as those for the coupled and uncoupled theories. This theory was extended by Dhaliwal and Sherief (1980) to general anisotropic media in the presence of heat sources.

The theory of linear elastic materials with voids is one of the most important generalizations of the classical theory of elasticity. This theory has practical use for investigating various types of geological and biological materials for which elastic theory is inadequate. This theory is concerned with elastic materials consisting of a distribution of small pores (voids), in which the voids volume is included among the kinematics variables and in the limiting case of volume tending to zero, the theory reduces to the classical theory of elasticity.

A non-linear theory of elastic materials with voids was developed by Nunziato and Cowin (1979). Later, Cowin and Nunziato (1983) developed a theory of linear elastic materials with voids for the mathematical study of the mechanical behavior of porous solids. They considered several applications of the linear theory by investigating the response of the materials to homogeneous deformations, pure bending of beams and small amplitudes of acoustic waves. Considerable amount of work has been done in the linear theory of elastic materials with voids.

The first investigation in the theory of thermoelastic materials with voids are due to Nunziato and Cowin (1979) and Jaric and Golubovic (1979). Iesan (1986)developed the theory of thermoelastic material with voids and established uniqueness, reciprocal and variational theorem. Different authors has been discussed different types of problem in linear thermoelastic materials with voids (1987, 1990, 2001, 2002, 2004, 2005).

Most of the investigation were done under the assumption of temperature-independent material properties, which limit the applicability of the solutions obtained to certain ranges of temperature. Modern structural elements are often subjected to temperature change of such magnitude that their material properties may be longer be regarded as having constant values even in an approximate sense. At high temperature the materials characteristics such as modulus of elasticity,thermal conductivity and the coefficient of linear thermal expansion are no longer constants.The thermal and mechanical properties of the materials vary with temperature, so the temperature-dependent of the material properties must be taken into consideration in the thermal stress analysis of these elements. Tanigawa (1995)investigated thermoelastic problems for non-homogeneous structural material. Ezzat et al (2004, 2001) investigated the dependence of modulus of elasticity on reference temperature in generalized thermoelasticity and obtained interesting results. Youssef (2005) used the equation of generalized thermoelasticity with one relaxation time with variable modulus of elasticity and the thermal conductivity to solve a problem of an infinite material with spherical cavity.

In the present investigation the equations of generalized thermoelastic with voids, with the dependence of modulus of elasticity and thermal conductivity on the reference temperature are used to obtain the components of displacement, stress, change in volume fraction field and temperature distribution due to distributed(instantaneous) sources.

2 Basic Equations

Following Lord and shulman (1967), Cowin and Nunziato (1983), the field equations and constitutive relations in thermoelastic body with voids without body forces, heat sources and extrinsic equilibrated body force can be written as:

$$(\lambda+2\mu)\nabla(\nabla\cdot\overrightarrow{u})-\mu(\nabla\times\nabla\times\overrightarrow{u})+b\nabla\phi-\beta\nabla T=\rho\frac{\partial^2\overrightarrow{u}}{\partial t^2}, \tag{2.1}$$

$$\alpha\nabla^2\phi-b\nabla\cdot\overrightarrow{u}-\xi_1\phi-\omega_0\frac{\partial\phi}{\partial t}+mT=\rho\psi\frac{\partial^2\phi}{\partial t^2}, \tag{2.2}$$

$$K\nabla^2T-\beta T_0(\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2})\nabla\cdot\overrightarrow{u}-mT_0(\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2})\phi=\rho C_e(\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2})T, \tag{2.3}$$

$$t_{ij}=\lambda\, u_{k,k}\delta_{ij}+\mu(u_{i,j}+u_{j,i})+b\phi\delta_{ij}-\beta T\delta_{ij}, \tag{2.4}$$

where λ, μ- Lame's constants,α,b,ξ_1,ω_0,m,ψ-material constants due to presence of voids, T- the temperature distribution $\overrightarrow{u}$- displacement vector, $\beta=(3\lambda+2\mu)\alpha_t$, α_t- coefficient of linear thermal expansion, ρ, C_e- density and specific heat respectively, K- thermal conductivity,ϕ- change in volume fraction field, T_0- reference temperature, t_{ij} -components of stress tensor, τ_0- the relaxation time, δ_{ij}- Kronecker delta,

$$\nabla=\hat{i}\frac{\partial}{\partial x}+\hat{j}\frac{\partial}{\partial y}+\hat{k}\frac{\partial}{\partial z},\quad \nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}.$$

3 Formulation And Solution of The Problem

We consider a homogeneous, isotropic, generalized thermoelastic half space with voids in the undeformed temperature T_0. The rectangular cartesian co-ordinate system (x,y,z) having origin on the surface $z=0$ with $z-$ axis pointing normally in to the medium is introduced.For two dimensional problem, we assume the displacement vector as

$$\overrightarrow{u}=(u,0,w) \tag{3.1}$$

Our aim is to investigate the effect of temperature dependence of modulus of elasticity keeping the other elastic and thermal parameters as constant. Therefore we may assume

$$\lambda = \lambda_0(1-\alpha^*T_0), \mu = \mu_0(1-\alpha^*T_0), \beta = \beta_0(1-\alpha^*T_0), \xi_1 = \xi_{10}(1-\alpha^*T_0),$$

$$m = m_0(1-\alpha^*T_0), b = b_0(1-\alpha^*T_0), \alpha = \alpha_0(1-\alpha^*T_0), \psi = \psi_0(1-\alpha^*T_0),$$

$$K = K_0(1-\alpha^*T_0), \omega_0 = \omega_{10}(1-\alpha^*T_0), \tag{3.2}$$

where λ_0, μ_0, β_0, ξ_{10}, m_0, b_0, α_0, ψ_0, K_0, ω_{10} are considered constants, α^* is called empirical material constant, in case of the reference temperature independent of modulus of elasticity and thermal conductivity $\alpha^* = 0$. To facilitate the solution, following dimensionless quantities are introduced:

$$x' = \frac{\omega_1^*}{c_1}x, \quad z' = \frac{\omega_1^*}{c_1}z, \quad u' = \frac{\omega_1^*}{c_1}u, \quad w' = \frac{\omega_1^*}{c_1}w,$$

$$t'_{33} = \frac{t_{33}}{\mu_0}, \quad t'_{31} = \frac{t_{31}}{\mu_0}, \quad \phi' = \frac{\omega_1^{*2}\psi_0}{c_1^2}\phi,$$

$$t' = \omega_1^* t, \quad \tau_o' = \omega_1^*\tau_o, \quad \tau_1' = \omega_1^*\tau_1, \quad a' = \frac{\omega_1^*}{c_1}a,$$

$$T' = \frac{T}{T_0}, \quad P_1' = \frac{P_1}{\mu_0}, \quad P_2' = \frac{P_2}{T_0}, \tag{3.3}$$

where

$$c_1 = (\frac{\lambda_0 + 2\mu_0}{\rho})^{\frac{1}{2}} \quad and \quad \omega_1^* = \frac{c_1^2}{\kappa}$$

Equations (2.1)-(2.3), with the help of equations (3.1)-(3.3) may be recast into the dimensionless form after suppressing the primes as:

$$\nabla^2 u + a_1\frac{\partial e}{\partial x} + a_2\frac{\partial \phi}{\partial x} - a_3\frac{\partial T}{\partial x} = a_4\frac{\partial^2 u}{\partial t^2}, \tag{3.4}$$

$$\nabla^2 w + a_1\frac{\partial e}{\partial z} + a_2\frac{\partial \phi}{\partial z} - a_3\frac{\partial T}{\partial z} = a_4\frac{\partial^2 w}{\partial t^2}, \tag{3.5}$$

$$\nabla^2 \phi - a_5 e - a_6\phi - a_7\frac{\partial \phi}{\partial t} + a_8 T = a_9\frac{\partial^2 \phi}{\partial t^2}, \tag{3.6}$$

$$\nabla^2 T - \epsilon_1(\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2})e - \epsilon_2(\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2})\phi = (\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2})T, \tag{3.7}$$

where

$$a_1 = \frac{\lambda_0 + \mu_0}{\mu_0}, \quad a_2 = \frac{b_0 c_1^2}{\omega_1^{*2}\mu_0\psi_0}, \quad a_3 = \frac{\beta_0 T_0}{\mu_0}, \quad a_4 = \frac{\rho c_1^2 A^*}{\mu_0},$$

$$a_5 = \frac{b_0\psi_0}{\alpha_0}, \quad a_6 = \frac{\xi_{10} c_1^2}{\omega_1^{*2}\alpha_0}, \quad a_7 = \frac{\omega_{10}\kappa}{\alpha_0}, \quad a_8 = \frac{m_0 T_0 \psi_0}{\alpha_0},$$

$$a_9 = \frac{\rho c_1^2 \psi_0}{\alpha_0}, \quad \epsilon_1 = \frac{\beta_0 c_1^2}{K_0 \omega_1^*}, \quad \epsilon_2 = \frac{m_0 c_1^4}{K_0 \psi_0 \omega_1^{*3}},$$

with

$$A^* = \frac{1}{(1-\alpha^* T_0)}, \quad \rho C_e = \frac{K}{\kappa},$$

also ϵ_1, ϵ_2 are the coupling constants and κ is the diffusivity. Using the expression relating displacement components$u(x,z,t)$ and $w(x,z,t)$ to the scalar potential functions $\psi_1(x,z,t)$ and $\psi_2(x,z,t)$ in dimensionless form

$$u = \frac{\partial \psi_1}{\partial x} - \frac{\partial \psi_2}{\partial z}, \quad w = \frac{\partial \psi_1}{\partial z} + \frac{\partial \psi_2}{\partial x}, \tag{3.8}$$

in equations (3.4)-(3.7) and applying the Laplace and Fourier transforms defined by

$$\hat{f}(x,z,s) = \int_o^\infty e^{-st} f(x,z,t)dt,$$

and

$$\tilde{f}(\xi,z,s) = \int_{-\infty}^\infty e^{\imath\xi x}\hat{f}(x,z,s)dx, \tag{3.9}$$

on resulting equations,and eliminating$\tilde{\psi}_1$,$\tilde{\phi}$,$\tilde{T}$ and $\tilde{\psi}_2$,we obtain

$$\left(\frac{d^6}{dz^6} + A\frac{d^4}{dz^4} + B\frac{d^2}{dz^2} + C\right)(\tilde{\psi}_1, \tilde{\phi}, \tilde{T}) = 0, \tag{3.10}$$

$$\left(\frac{d^2}{dz^2} - \lambda_4^2\right)\tilde{\psi}_2 = 0, \tag{3.11}$$

where

$$A = \frac{1}{b_1}[b_1(-3\xi^2 - (f_1+f_2)) - a_4 s^2 + a_2 a_5 - a_3\epsilon_1 f_1],$$

$$B = \frac{1}{b_1}[b_1(3\xi^4 + 2\xi^2(f_1+f_2) + f_1 f_2 - a_8\epsilon_2 f_1) + s^2(2a_4\xi^2$$
$$+a_4(f_1+f_2)) - a_2(2\xi^2 a_5 + a_5 f_1 - a_8\epsilon_1 f_1)$$
$$+2\xi^2 a_3\epsilon_1 f_1 - a_3 a_5 \epsilon_2 f_1 + a_3\epsilon_1 f_1 f_2],$$

$$C = \frac{1}{b_1}[-b_1\xi^6 + \xi^4(-b_1(f_1+f_2) - a_4s^2 + a_2a_5 - a_3\epsilon_1 f_1)$$
$$+\xi^2(-b_1f_2f_1 + a_8\epsilon_2 f_1 b_1 - a_4s^2(f_1+f_2) + a_2a_5f_1 - \epsilon_1a_2a_8f_1$$
$$+a_3a_5\epsilon_2f_1 - a_3\epsilon_1f_1f_2) + s^2(-a_4f_1f_2 + a_4a_8\epsilon_2f_1)],$$

with

$$b_1 = 1 + a_1, \quad f_1 = s + \tau_0 s^2, \quad f_2 = a_6 + a_7 s + a_9 s^2, \quad \lambda_4^2 = \xi^2 + a_4 s^2.$$

The roots of the equations (3.10)and(3.11)are λ_ℓ,$(\ell = 1,2,3,4)$. Assuming the regularity conditions, the solutions of equations may be written as

$$(\tilde{\psi_1}, \tilde{\phi}, \tilde{T}) = (\sum_{\ell=1}^{3} A_\ell \exp^{-\lambda_\ell z}, \sum_{\ell=1}^{3} d_\ell A_\ell \exp^{-\lambda_\ell z}, \sum_{\ell=1}^{3} e_\ell A_\ell \exp^{-\lambda_\ell z}), \tag{3.12}$$

$$\tilde{\psi_2} = A_4 \exp^{-\lambda_4 z}, \tag{3.13}$$

where
$$d_\ell = \frac{a_{13}c_{\ell 1} - a_{\ell 1}c_{\ell 3}}{a_{12}c_{\ell 3} - a_{13}c_{12}}, e_\ell = \frac{a_{\ell 1}c_{12} - a_{12}c_{\ell 1}}{a_{12}c_{\ell 3} - a_{13}c_{12}},$$

and

$$a_{\ell 1} = b_1(\lambda_\ell^2 - \xi^2) - a_4s^2 \quad , a_{12} = a_2 \quad , a_{13} = -a_3 \quad , c_{\ell 1} = \epsilon_1 f_1(\lambda_\ell^2 - \xi^2)$$
$$c_{12} = a_{10}f_1 \quad , c_{\ell 3} = (\xi^2 - \lambda_\ell^2) + f_1 \quad ; (\ell = 1,2,3).$$

4 Boundary Conditions

The appropriate boundary conditions are

$$(i) \quad t_{33} = -P_1 f_1(x,t), \tag{4.1}$$

$$(ii) \quad t_{31} = 0, \tag{4.2}$$

$$(iii) \quad \frac{\partial \phi}{\partial z} = 0, \tag{4.3}$$

$$(iv) \quad T = P_2 f_2(x,t), \tag{4.4}$$

where P_1 is the magnitude of force,P_2 is the constant temperature applied on the boundary and $f_1(x,t)$ and $f_2(x,t)$ are known functions. Making use of equations (1.4),(3.2)-(3.3)and (3.8)and applying the Laplace and Fourier transforms defined by(3.9) and substituting the value of $\tilde{\psi}_1$,$\tilde{\phi}$,$\tilde{T}$,and $\tilde{\psi}_2$, from equations (3.12)and(3.13)in the boundary conditions (4.1)-(4.4),we obtain the components of displacement, stress, change in volume fraction field and temperature distribution as

$$\tilde{u} = \frac{1}{\triangle}[P_1\tilde{f}_1(\xi,s)\sum_{\ell=1}^{3}\{(-\imath\xi)(\triangle_\ell \exp^{-\lambda_\ell z}) + \lambda_4\triangle_{\ell+6}\exp^{-\lambda_4 z}\}$$

$$+P_2\tilde{f}_2(\xi,s)\sum_{\ell=1}^{3}\{(-\imath\xi)(\triangle_{\ell+3}\exp^{-\lambda_\ell z}) + \lambda_4\triangle_{\ell+9}\exp^{-\lambda_4 z}\}]$$

$$\tilde{w} = \frac{1}{\triangle}[P_1\tilde{f}_1(\xi,s)\sum_{\ell=1}^{3}\{-\lambda_\ell(\triangle_\ell \exp^{-\lambda_\ell z}) + (-\imath\xi)\triangle_{\ell+6}\exp^{-\lambda_4 z}\}$$

$$+P_2\tilde{f}_2(\xi,s)\sum_{\ell=1}^{3}\{-\lambda_\ell(\triangle_{\ell+3}\exp^{-\lambda_\ell z}) + (-\imath\xi)\triangle_{\ell+9}\exp^{-\lambda_4 z}\}]$$

$$\tilde{t_{33}} = \frac{1}{\triangle}[P_1\tilde{f}_1(\xi,s)\sum_{\ell=1}^{3}\{R_\ell(\triangle_\ell \exp^{-\lambda_\ell z}) + R_4\triangle_{\ell+6}\exp^{-\lambda_4 z}\}$$

$$+P_2\tilde{f}_2(\xi,s)\sum_{\ell=1}^{3}\{R_\ell(\triangle_{\ell+3}\exp^{-\lambda_\ell z}) + R_4\triangle_{\ell+9}\exp^{-\lambda_4 z}\}]$$

$$\tilde{t_{31}} = \frac{1}{\triangle}[P_1\tilde{f}_1(\xi,s)\sum_{\ell=1}^{3}\{q_\ell(\triangle_\ell \exp^{-\lambda_\ell z}) + q_4\triangle_{\ell+6}\exp^{-\lambda_4 z}\}$$

$$+P_2\tilde{f}_2(\xi,s)\sum_{\ell=1}^{3}\{q_\ell(\triangle_{\ell+3}\exp^{-\lambda_\ell z}) + q_4\triangle_{\ell+9}\exp^{-\lambda_4 z}\}]$$

$$\tilde{\phi} = \frac{1}{\triangle}[P_1\tilde{f}_1(\xi,s)\sum_{\ell=1}^{3}d_\ell\triangle_\ell \exp^{-\lambda_\ell z} + P_2\tilde{f}_2(\xi,s)\sum_{\ell=1}^{3}d_\ell\triangle_{\ell+3}\exp^{-\lambda_\ell z}]$$

$$\tilde{T} = \frac{1}{\triangle}[P_1\tilde{f}_1(\xi,s)\sum_{\ell=1}^{3}e_\ell\triangle_\ell \exp^{-\lambda_\ell z} + P_2\tilde{f}_2(\xi,s)\sum_{\ell=1}^{3}e_\ell\triangle_{\ell+3}\exp^{-\lambda_\ell z}] \qquad (4.5)$$

where

$$\triangle = (\lambda_2 d_2 e_3 - \lambda_3 d_3 e_2)(R_1 q_4 - R_4 q_1) + (\lambda_3 d_3 e_1 - \lambda_1 d_1 e_3)(R_2 q_4 - R_4 q_2)$$

$$+(\lambda_1 d_1 e_2 - \lambda_2 d_2 e_1)(R_3 q_4 - R_4 q_3),$$

$$\triangle_1 = q_4(\lambda_3 d_3 e_2 - \lambda_2 d_2 e_3),$$

$$\triangle_2 = q_4(\lambda_1 d_1 e_3 - \lambda_3 d_3 e_1),$$

$$\triangle_3 = q_4(\lambda_2 d_2 e_1 - \lambda_1 d_1 e_2),$$

$$\triangle_4 = q_4(\lambda_3 d_3 R_2 - \lambda_2 d_2 R_3) + R_4(\lambda_2 d_2 q_3 - \lambda_3 d_3 q_2),$$

$$\triangle_5 = (R_4 q_1 - R_1 q_4)\lambda_3 d_3 + (R_3 q_4 - R_4 q_3)\lambda_1 d_1,$$

$$\triangle_6 = (R_1 q_4 - R_4 q_1)\lambda_2 d_2 + (R_4 q_2 - R_2 q_4)\lambda_1 d_1,$$

$$\triangle_7 = -q_1(\lambda_2 d_2 e_3 - \lambda_3 d_3 e_2),$$

$$\triangle_8 = -q_2(\lambda_3 d_3 e_1 - \lambda_1 d_1 e_3),$$

$$\triangle_9 = -q_3(\lambda_1 d_1 e_2 - \lambda_2 d_2 e_1),$$

$$\triangle_{10} = (R_1 q_2 - R_2 q_1)\lambda_3 d_3,$$

$$\triangle_{11} = (R_3 q_1 - R_1 q_3)\lambda_2 d_2,$$

$$\triangle_{12} = (R_2 q_3 - R_3 q_2)\lambda_1 d_1,$$

and

$$R_\ell = \frac{((b_2+2)\lambda_\ell^2 - b_2\xi^2 + a_2 d_\ell - a_3 e_\ell)}{A^*}, \quad R_4 = \frac{2(\imath\xi)\lambda_4}{A^*},$$

$$q_\ell = \frac{2(\imath\xi)\lambda_\ell}{A^*}, \quad q_4 = \frac{-(\xi^2+\lambda_4^2)}{A^*}, \quad b_2 = \frac{\lambda_0}{\mu_0}; (\ell = 1, 2, 3.).$$

The corresponding expressions are obtained for mechanical source by taking $P_2 = 0$ and for thermal source by taking $P_1 = 0$ in equation(4.5),respectively.

5 Applications

We take $f_1(x,t)$ and $f_2(x,t)$ as

$$(f_1(x,t), f_2(x,t)) = \begin{cases} (g_1(x), g_2(x))\delta(t) & \text{for instantaneous source,} \\ (g_1(x), g_2(x))H(t) & \text{for continuous source} \end{cases} \tag{5.1}$$

where $\delta()$ is the Dirac delta function and $H()$ is the Haviside distribution function, $g_1(x)$ and $g_2(x)$ are the known function.

Applying the Laplace and Fourier transforms defined by equation(3.9) on equation (5.1) we get

$$(\tilde{f_1}(\xi,s), \tilde{f_2}(\xi,s)) = \begin{cases} (\tilde{g_1}(\xi), \tilde{g_2}(\xi)) & \text{for instantaneous source,} \\ (\tilde{g_1}(\xi), \tilde{g_2}(\xi))\frac{1}{s} & \text{for continuous source} \end{cases} \tag{5.2}$$

(I)Uniformly distributed source: In this case, the solution is obtained by using

$$(g_1(x), g_2(x)) = \begin{cases} 1 & \text{if } |\mathrm{x}| \leq \mathrm{a} \\ 0 & \text{if } |\mathrm{x}| > \mathrm{a} \end{cases} \tag{5.3}$$

Fourier transform of $g_1(x)$,$g_2(x)$ with respect to the pair $(x;\xi)$ for the case of uniform strip load of non-dimensional width 2a applied at z = 0 is given by

$$(\tilde{g}_1(\xi), \tilde{g}_2(\xi)) = \left\{ \begin{array}{ll} \frac{2\sin\xi\, a}{\xi}, & \xi \neq 0, \end{array} \right. \tag{5.4}$$

(II)Linearly Distributed Source: In this case, the solution due to linearly distributed source is obtained by using

$$(g_1(x), g_2(x)) = \left\{ \begin{array}{ll} 1 - \frac{|x|}{a} & \text{if} \quad |\mathrm{x}| \leq \mathrm{a} \\ 0 & \text{if} \quad |\mathrm{x}| > \mathrm{a} \end{array} \right. \tag{5.5}$$

Fourier transform of $g_1(x)$,$g_2(x)$ are given by

$$(\tilde{g}_1(\xi, s), \tilde{g}_2(\xi, s)) = \left\{ \begin{array}{ll} \frac{2[1-\cos(\xi\, a)]}{\xi^2 a} & \xi \neq 0 \end{array} \right. \tag{5.6}$$

The corresponding solutions are obtained for uniformly or linearly distributed source by substituting the value of $\tilde{f}_1(\xi, s)$ and $\tilde{f}_2(\xi, s)$ from equations (5.2) in equation (4.5)with the help of equations (5.4)and(5.6),respectively.

6 Particular Cases

(i)Taking $A^* = 1$ in equation (4.5) and with the help of equation(5.2), we obtained the corresponding expressions in generalized porous thermoelastic half-space without dependence of modulus of elasticity for uniformly or linearly distributed source,respectively. These results tally with those obtained by Kumar and Rani(2005) after some modification.
(ii)By putting $\tau_0 = 0$ in equation (4.5)and with the help of equations (5.2), we obtained the corresponding expressions of the porous thermoelastic half-space with and without dependence of modulus of elasticity,for CT theory due to uniformly or linearly distributed source, respectively. These results are in aggrement with those if we solve the problem directly in coupled thermoelasticity.
(iii) If we neglect the voids effect ($\alpha = b = \xi_1 = m = \psi = \omega_0 = 0$), in equations (4.5),and with the help of equations (5.2), we obtained the components of displacement,stress, temperature distribution in generalized thermoelastic half-space with dependence of modulus of elasticity by replacing values of $\triangle$ with $\triangle^*$, $\triangle_\ell$ with $\triangle_\ell^*(\ell = 1, 2, 4, 5, 7, 8, 10, 11)$, q_ℓ, e_ℓ, R_ℓ, λ_ℓ, E_ℓ with q_ℓ^*, e_ℓ^*, R_ℓ^*, λ_ℓ^*, E_ℓ^* $(\ell = 1, 2.)$ and $d_\ell = q_3 = e_3 = R_3 = \lambda_3 = E_3 = \triangle_3 = \triangle_6 = \triangle_9 = \triangle_{11} = \triangle_{12} = 0$ $(\ell = 1, 2, 3.)$ respectively, where

$$\triangle^* = q_4(R_2^* e_1^* - R_1^* e_2^*) + R_4(e_2^* q_1^* - e_1^* q_2^*), \quad \triangle_1^* = q_4 e_2^*, \quad \triangle_2^* = -q_4 e_1^*,$$

$$\triangle_4^* = q_4 R_2^* - q_2^* R_4, \quad \triangle_5^* = R_4 q_1^* - R_1^* q_4, \quad \triangle_7^* = -q_1^* e_2^*, \quad \triangle_8^* = q_2^* e_1^*,$$

$$\triangle_{10}^* = R_1^* q_2^* - R_2^* q_1^*, \quad R_\ell^* = \frac{((b_2 + 2)\lambda_\ell^{*2} - b_2 \xi^2 - a_3 e_\ell^*)}{A^*}, \quad q_\ell^* = \frac{2(\imath\xi)\lambda_\ell^*}{A^*},$$

$$e_\ell^* = \frac{\epsilon_1 f_1(\lambda_\ell^{*2} - \xi^2)}{(\lambda_\ell^{*2} - \xi^2) - f_1}, \quad E_\ell^* = \exp^{-\lambda_\ell^* z}, \quad \lambda_\ell^{*2} = \frac{-A + (-1)^{\ell+1}\sqrt{A^2 - 4B}}{2},$$

with

$$A = \frac{-1}{b_1}[b_1(2\xi^2 + f_1) + a_4 s^2 + \epsilon_1 a_3 f_1],$$

$$B = \frac{1}{b_1}[(b_1\xi^2 + a_4 s^2)(\xi^2 + f_1) + \xi^2 \epsilon_1 a_3 f_1].$$

The above results are similar as those obtained by Kumar and Rani(2005)

7 Inversion of the transforms

The transformed expressions in equation(4.5) are inverted by using numerical inversion technique given by Kumar and Ailawalia (2003)

8 Numerical Results and Discussion

Following Dhaliwal and Singh (1980) magnesium material was chosen for purposes of numerical evaluations. The constants of the problem were taken as

$$\lambda = 2.17 \times 10^{10} N/m^2, \quad \mu = 3.278 \times 10^{10} N/m^2, K = 1.7 \times 10^2 W/mdeg,$$

$$\rho = 1.74 \times 10^3 Kg/m^3, \quad \beta = 2.68 \times 10^6 N/m^2 deg, \quad C_e = 1.04 \times 10^3 J/Kgdeg,$$

$$T_0 = 298K, \quad \omega_1^* = 3.58 \times 10^{11}/s$$

and the voids parameters are

$$\psi = 1.753 \times 10^{-15} m^2, \quad \alpha = 3.688 \times 10^{-5} N, \quad \xi_1 = 1.475 \times 10^{10} N/m^2,$$

$$b = 1.13849 \times 10^{10} N/m^2, \quad m = 2 \times 10^6 N/m^2 deg, \quad \omega_0 = .0787 \times 10^{-3} N/m^2 s.$$

The comparison were carried out for

$$\alpha^* = .00051/K, \quad P_1 = P_2 = 1, \quad \tau_0 = 0.05.$$

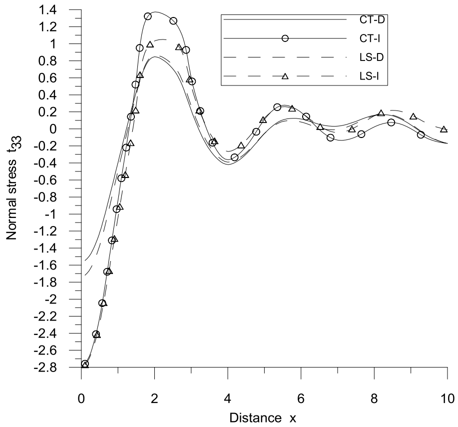

Fig.1. Variations of normal stressb t_{33} for uniformly distributed force (instantaneous).

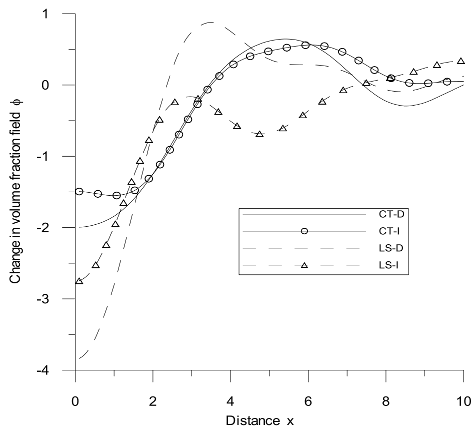

Fig.2. Variation of change in volume fraction field ϕ for uniformly distributed force (instantaneous).

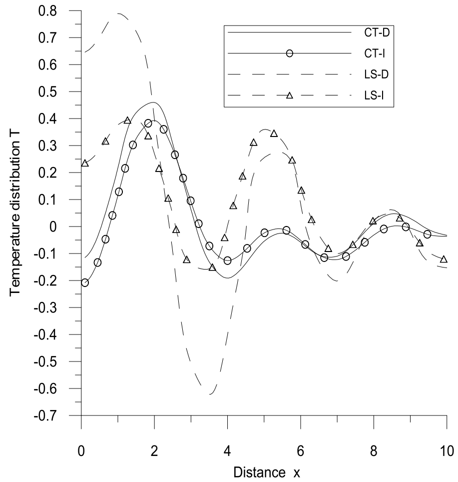

Fig.3. Variation of temperature distribution T for uniformly distributed force (instantaneous).

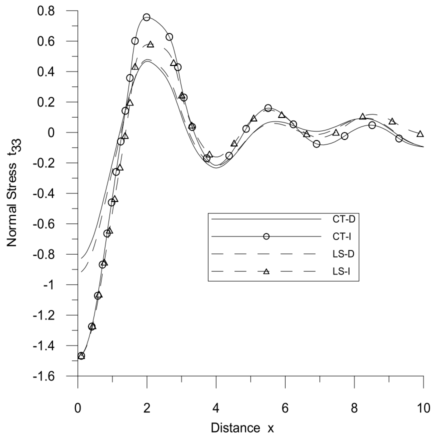

Fig.4. Variation of normal stress t_{33} for linearly distributed force (instantaneous).

The comparison of values of normal stress t_{33}, change in volume fraction field ϕ and temperature distribution T with distance x for uniformly distributed source(UDS) and linearly distributed source(LDS) with and without dependence of modulus of elasticity are shown graphically in figures 1-12, for coupled theory CT and Lord and Shulman theory (L-S). The solid line,the small dashed lines, in graphs represents the variations of normal stress t_{33}, change in volume fraction field ϕ and temperature distribution T for coupled theory with dependence of modulus of elasticity (CT-D) and Lord and Shulman theory with dependence of modulus of elasticity (LS-D)due to uniformly distributed source(UDS) and linearly distributed source(LDS).The solid line with center symbol circle,the small dashed lines with center symbol triangle, in graphs represents the variations of normal stress t_{33}, change in volume fraction field ϕ and temperature distribution T for coupled theory without dependence of modulus of elasticity (CT-I) and Lord and Shulman theory without dependence of modulus of elasticity (LS-I)due to uniformly distributed source(UDS) and linearly distributed source(LDS). The results for a distributed sources(mechanical and thermal)are presented for dimensionless width $a = 0.6$ for instantaneous source. The computations are carried out for value of non-dimensional time $t = 0.5$. in the range $0 \leq x \leq 10$.

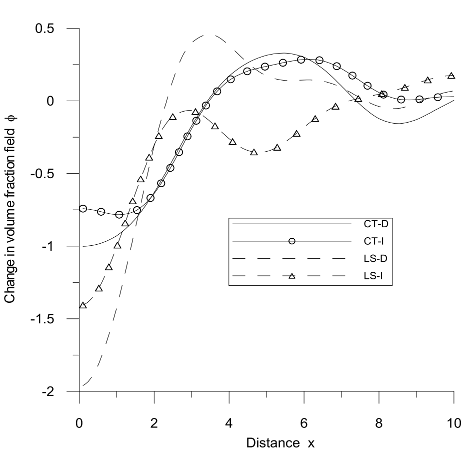

Fig.5. Variation of change in volume fraction field ϕ for linearly distributed force (instantaneous).

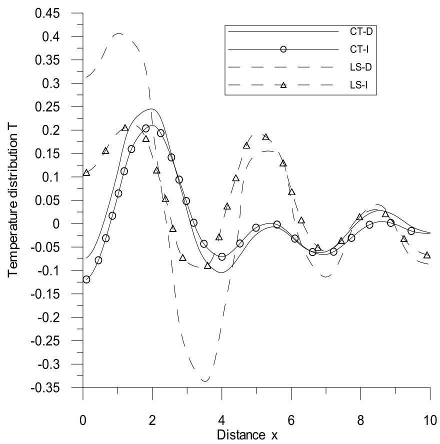

Fig.6. Variation of temperature distribution T for linearly distributed source (instantaneous).

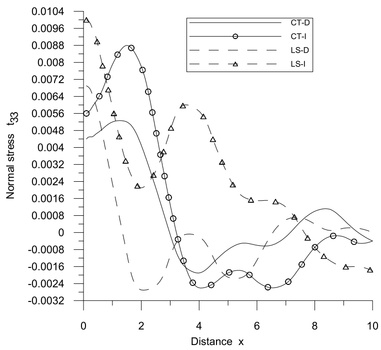

Fig.7. Variation of normal stress t_{33} for uniformly distributed source (instantaneous).

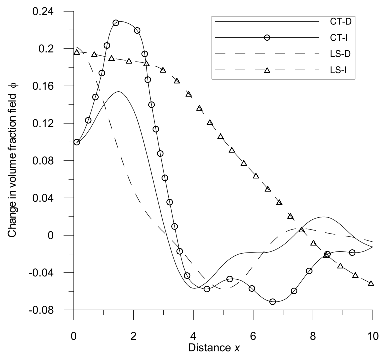

Fig.8. Variation of Change in volume fraction field ϕ for uniformly distributed source(instantaneous).

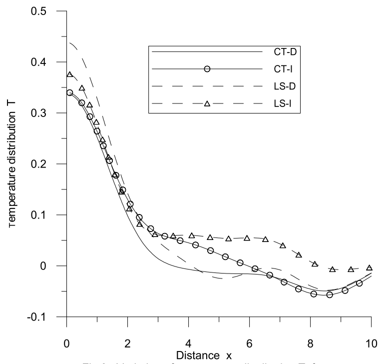

Fig.9. Variation of temperature distribution T for uniformly distributed source(instantaneous).

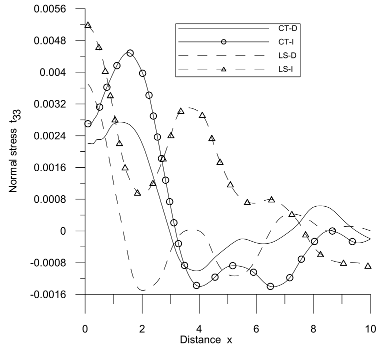

Fig.10. Variation of Normal stress t_{33} for linearly distributed source (instantaneous).

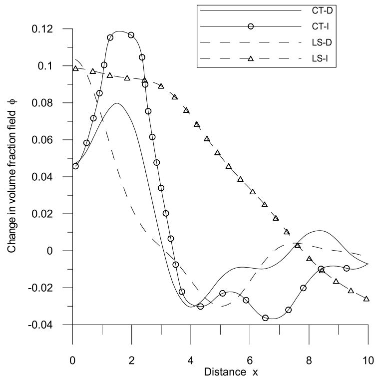

Fig.11. Variation of Change in volume fraction field ϕ for linearly distributed source (instantaneous).

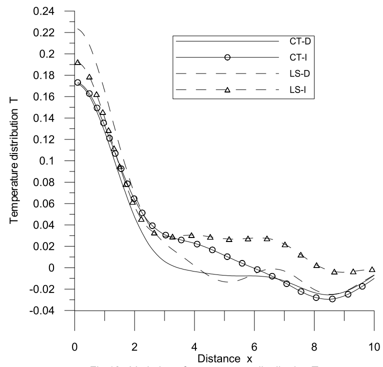

Fig.12. Variation of temperature distribution T for linearly distributed source (instantaneous).

8.1 Mechanical distributed force

Fig.1. shows the variations of normal stress t_{33} for UDS with distance x.From this figure we find that the t_{33} increases steadily between the boundary and the location $x = 2$ and decreases thereafter up to the location $x = 4$ and oscillatory around zero beyond this location. The normal stress is negative at $x = 0$ where its magnitude is maximum.

Fig.2. shows the variations of change in volume fraction field ϕ for UDS with distance x. Initially the magnitude of ϕ for (CT-I,LS-I) is found to be greater than that for(CT-D,LS-

D), respectively.

Fig.3. shows the variations of temperature distribution T for UDS with distance x. It is noticed that the variations of T in the context of (LS-D,LS-I)attain more values as compared to (CT-D,CT-I)just in the vicinity of the load. But the behavior of variations of T for (CT-D,CT-I,LS-D,LS-I) is oscillatory in the whole domain. Also it is observed that, near the point of application of source the values of T for (CT-D,,LS-D) are more in comparison to the (CT-I,LS-I), respectively.

Fig.4. shows the variations of normal stress t_{33} for LDS with distance x. The values of t_{33} for (LS-D,CT-D) is large in comparison to the values of t_{33} for (LS-I,CT-I),respectively.Initially the values of t_{33} start with sharp increase in the range $0 \leq x \leq 2.2$ and then follows an oscillatory pattern with reference to x.

Fig.5. depicts the variations of change in volume fraction field ϕ for LDS with distance x. For (CT-I,LS-I), the values of ϕ are greater than those for (CT-D,LS-D), at the initial value i.e. $x = 0$. The values of ϕ for (LS-D,LS-I) start with sharp increase in the range $0 \leq x \leq 3.2$ whereas for (CT-D,CT-I) increase in the range $0 \leq x \leq 5.6$ and then oscillatory as x increase further.

Fig.6.depicts the variations of temperature distribution T for LDS with distance x. The values of T for(CT-I,LS-I) are less than those for (CT-D,LS-D) in the range $0 \leq x \leq 2.2$. The behavior of variations of T with reference to x is same i.e. oscillatory for (CT-D,CT-I,LS-D,LS-I)with difference in their magnitudes.

8.2 Thermal distributed source

Fig.7. shows the variations of normal stress t_{33} for UDS with distance x At the bounding surface $x = 0$, t_{33} attain its maximum value$\approx 0.0100, \approx 0.0073$ for (LS-I,LS-D), respectively. The normal stress curves are not continuous suffering jumps at different locations. Fig.8. shows the variations of change in volume fraction field ϕ for UDS with distance x. It is interesting to note that the peak value attain by ϕ is much higher for CT-D as compared to(CT-I,LS-D,LS-I), but the value gets reduced when x is increased.

Fig 9. shows the variations of temperatre distribution T for UDS with distance x. The trend of variations of T for (CT-D,CT-I,LS-D, LS-I) is same whereas their corresponding values are different in magnitudes. It decreases rapidly in the domain $0 \leq x \leq 2.5$ and ultimately it vanishes, as x increases further.

Fig.10. depicts the variations of normal stress t_{33} for LDS with distance x. The t_{33} for (LS-D,LS-I) has a sharp fall in values for $0 \leq x \leq 2$ and then the values start oscillating and approach to constant value. Also the values of t_{33} for (CT-D,CT-I)increase slowly in the range $0 \leq x \leq 1.4$,decrease sharply in the range $1.4 \leq x \leq 4$ and as x increases further its behavior is oscillatory.

Fig.11. shows the variations of change in volume fraction field ϕ for LDS with distance

x. The trend of variations of ϕ for(CT-D,CT-I)is same i.e. oscillatory in the whole range of x whereas their corresponding values are different in magnitudes. The value of ϕ for LS-D decreases in the range $0 \leq x \leq 5$ and oscillatory in the remaining range of x. But the value of ϕ for LS-I decreases in the whole range of x.

Fig.12. depicts the variations of temperatre distribution T for LDS with distance x. The behavior of variations of T for (CT-D,CT-I,LS-D,LS-I) is similar but the corresponding values are different. The values of T decrease sharply in the range $0 \leq x \leq 2.6$, gradually decrease in the range $2.6 \leq x \leq 8.5$ and then increase as x increases further.

9 Conclusion

The comparison of the Lord and Shulman(L-S) theory and coupled theory of thermoelasticity with and without dependence of modulus of elasticity is carried out. It is noticed that the results obtained by using either the coupled or the Lord and Shulman theories are different,near the point of application of source and quite similar far from the source. The behavior of normal stress, change in volume fraction field and temperature distribution for uniformly distributed source(instantaneous) are similar to those of the linearly distributed source(instantaneous), respectively with difference in their magnitudes, respectively. It is observed that the magnitude of normal stress, change in volume fraction field and temperature distribution follow an oscillatory pattern as x diverges from the point of application of source.

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R. Kumar received his M.Sc. (1980) from Guru Nanak Dev University (G.N.D.U.), Amritsar (Punjab), M Phill (1982) from Kurukshetra University Kurukshetra (K.U.K.) and Ph. D. (1986) in Applied Mathematics from Guru Nanak Dev University (G.N.D.U.), Amritsar. Guided 52 Mphill students, 9 students awarded Ph.D. degree and 8 students are doing Ph.D. under his supervision. He has 181 papers published in Journal of international repute. His area of research work is Continuum Mechanics(Micropolar elasticity, thermoelasticity, poroelasticity, magnetoelasticity, micropolar porous couple stress theory, viscoelasticity, mechanics of fluid.)

S. Devi is a Ph. D. student and received her M.Sc. (2001), B.Ed. (2002) from Kurukshetra University Kurukshetra (K.U.K.). Presently working as a Lecturer in Mathematics in Dayanand College Hisar-125001(Haryana)-India since Jan, 2007.