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Improved DFT-based Channel Estimation in OFDM Systems Based on Phase Compensation

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Abstract: Based on frequency domain pilot channel estimation is a typical technology of channel estimation in OFDM system. In most cases, the pilot configuration of the system can't start from the first subcarrier, which will have an effect on channel estimation values at the pilot, especially in the algorithm of channel estimation based on DFT interpolation which will largely degrade the performance of the algorithm. The study analyses and deduces in detail phase offsets and the compensation technology under the condition of non-integer multiple of the multipath delay and non-integer multiple of the number of pilot. The simulation results show that the proposed phase compensation technology can effectively improve the performance of the channel estimation.

Keywords: OFDM, Channel estimation, Phase compensation.

1. Introduction

Orthogonal frequency division multiplexing (OFDM), due to its high data rate transmission capability with high bandwidth efficiency has recently been widely used in wireless communication [1].

Channel estimation is important for coherent demodulation, and some methods have been proposed.

Channel estimation based on training sequence of frequency domain, usually is also called based on pilot symbol assisted (Pilot-Symbol-Aided, PSA) of channel estimation [2–5]. Because channel estimation based on training sequence of frequency domain can track the changes in frequency selective channels (it tracks fast fading channels), which has been shown easy to implement in the OFDM system, it is widely adopted [6, 7].

OFDM systems adopting frequency-domain training sequence, usually before the IDFT of the transmitter, insert the pilot sequence with a certain way corresponding to the pilot subcarriers position.

Pilot signal is extracted and received at the receiving end of DFT, and is computed with local pilot sequence through the channel estimation to obtain channel frequency response of the location of the pilot carrier. And then it gets through a variety of different interpolation methods: the linear interpolation method, the cubic interpolation, DFT interpolation and so on to achieve the channel's frequency response at all of the other OFDM subcarriers [8].

DFT-based interpolation techniques received extensive attention because of being able to used noise suppression techniques to further improve the performance of channel estimation. However at this time, the pilot's position and the number can impact on its estimated channel domain impulse response, and finally they have an influence on channel estimation accuracy, accordingly need to analyze its impact.

2. System Model

In the real application, OFDM system modulation divides into continuous modulation and Difference modulation. Using difference modulation, because channel information had included in the difference of adjacent symbols, receiver can finish the coherent demodulation difference without channel estimation. Difference modulation simplifies the receiver's complexity. But compared with coherent modulation system, performance will drop to 3dB. In addition,

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adopting the coherent modulation' OFDM system can use high order modulation technology of higher spectrum efficiency. So channel estimation algorithm has become current research's hotspot and focus.

2.1. The summarize of OFDM channel estimation

Mobile communication system's Wireless channel usually has Time selective and Frequency selective, and in almost situations it's time-varying. So in More carrier system, in each sub-carrier, Decline situation of signal is difference. A higher SNR carrier demodulation Judgment signal has higher reliability, can offer Status information for Channel decoding.

On the one hand Channel estimation Provide reference for equilibrium. On the other hand provide reference for Synchronous algorithm. Channel estimation usually can Divided into three categories:

The first kind: Pilot-symbol-aided channel estimation [2–5]:

1). Channel estimation based on Time domain training sequence

In Time domain, use BPSK Modulation mode mapping PN sequence to a fixed Discrete time series and form vector p. Add vector p to each time domain OFDM signal pieces $s[n]$ comprehensive.

At the same time the period of time domain waveform still acted as protection of signals between eliminate IBI component. In the receiver, the channel information will be obtained by p. This kind of training sequence based on the time domain channel estimation can also use a radio channel relevance, for M OFDM symbol of channel estimation in for moving average processing, improve the CSI estimate.

2). Based on the frequency domain training sequence estimate

At the transmitter, will pilot sequences certain principle installation is inserted into the each frequency domain OFDM symbol information. The definition of frequency than for the system of PR insert the frequency number and system Fourier transform the ratio of the total points. Pilot sub-carrier place channel a frequency response by receiving data and the local store reference guide frequency sequence to get. OFDM symbol information and data in the subcarrier in frequency response can through to the Pilot sub-carrier place frequency response interpolation gain. Common interpolation algorithm including: piecewise linear interpolation, high order interpolation, based on the DFT interpolation, etc.

3). Based on the two dimensional frequency domain frequency when training sequence estimate

Single carrier system can only in time domain insert training sequence, and multiplexing (OFDM) system has the time-frequency 2 d structure, can in time domain and frequency domain insert training sequence. Thus can by using 2-d interpolation method of Winner filter estimates sub-carrier the channel in the frequency domain properties, get under mean-square optimal estimation channel. But two dimensional processing, high complexity, prevents the realization. Usually a radio channel meet WSSUS hypothesis, can the 2 d filter breakdown to time and frequency of the direction of two dimensional filter cascades. Its defect is, because the data information into the guide frequency, will allow the data rate to decrease. Especially in some channel conditions, must insert enough guide to have complete frequency channel state information. The second type is based on the decision feedback of channel estimation [9].

Principle is, and that before and after two OFDM information symbol interval of time, channel is constant. Before sending data, system first hair at the receiving end of a known training sequence, so as to obtain the initial value of CSI. In the process of data transmission, using OFDM symbol information after decoding as training sequence get CSI estimate. The algorithm use ECC to reduce the error, so as to reduce the error of feedback and to improve the receiver of channel estimation accuracy. Because of its don't need guide frequency, effectively improve spectrum efficiency. Defect is when channel were also goes wrong, easy to produce error diffusion, reduce the system performance. Only new training sequence can work to achieve normal. As for channel coding error correction ability request is higher.

The third is blind channel estimation [10].

It does not need to send any auxiliary data, so the system of the spectrum of high utilization rate. But its decision depended on the data, if the verdict is incorrect, easy to cause the error diffusion. At the same time, the blind to the use of a lot of the verdict, extract the channel's statistical properties. This request on the analysis in the process of received signal, channel characteristic can't change. In the fast channel of real-time communication under time-varying system used to realize.

The pattern of the design is about frequency, in what way in the information data insert lead frequency, insert what kind of guide frequency, insert how many guide frequency. On the one hand, it must to ensure that the frequency of auxiliary channel estimation accuracy, on the other hand, to reduce the cost of the pilot tones, ensure that the utilization rate of the system.

For OFDM systems, the frequency of general principles of pattern design is determined according to the channel conditions of channel bandwidth and coherent time coherent. This as the constraint condition, and insert some guide frequency. The shorter than that frequently domain coherent bandwidth, the frequency interval time less than coherent time.

Common guide frequency patterns are as follows:

A). The design is the frequency. In an OFDM symbol in all sub-carrier have put on the frequency, the structure suitable for frequency selective fading channel.

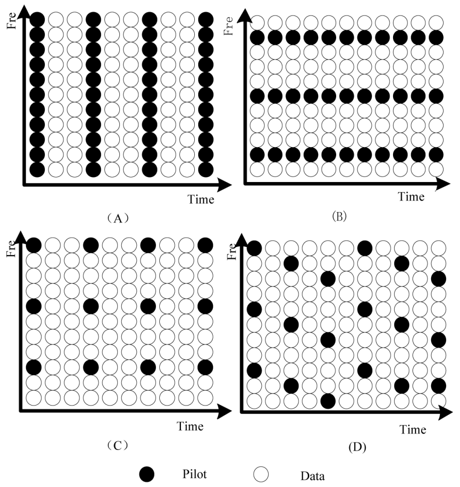

Figure 1 The common frequency pattern design.

B). Comb guide frequency patterns. In the continuous OFDM symbol on the same carrier insert lead frequency, its structure suitable for time selective fading channel.

C). Rectangular guide frequency patterns. The guide in time and frequency of the frequency interval insert, and meet the coherent bandwidth and coherence of time requirements. When compared to the massive and comb guide frequency design, has the high frequency spectrum efficiency.

D). A rotation frequency patterns of the guide. The guide frequency in frequency position, as time and change, relative to the rectangular frequency pattern, with the characteristics of interference.

Considering the specific characteristics and mobile channel spectrum efficiency is adopted in this paper, similar to the rectangular (C) the frequency patterns.

2.2. *Based on interpolation of DFT filter channel estimates*[11]

$s(n) = [s(nN), s(nN+1), \cdots, s(nN+N-1)]^T$ represents the data vector at time n. N is the length of the vector, $N > l$. After IFFT modulation in the sender, get the $x(n)$:

$$x(n) = F_N^H s(n) \tag{1}$$

where, F_N^H are the inverse Fourier transform matrix and its transpose conjugate Fourier transform matrix is F_N.

Due to the inter symbol interference from the last L of the previous symbol $x(n-1)$, thus joining the cyclic prefix in the current symbol $x(n)$. In practice, the length from the tail $x(n)$ copy N_{cp} ($N_{cp} > L$) piece of data to the front $x(n)$, the formation of a new data $\bar{x}(n)$. Mathematical representation as:

$$\bar{x}(n) = T_{cp} x(n) \tag{2}$$

where, $T_{cp} = \left[I^T_{N_{cp} \times N} \; I^T_N \right]^T$, $I^T_{N_{cp} \times N}$ is the last N_{cp} row of the unit matrix I_N. $\bar{x}(n)$ is called a complete OFDM symbol block. The length is $N_s = N + N_{cp}$.

After sending the data over Multipath Fading channels, receiving signals to signals are represented as:

$$r(n) = H_0 \bar{x}(n) + H_1 \bar{x}(n-1) + \eta(n) \tag{3}$$

where $\eta(n)$ the vector of Gaussian is white noise, H_0 and H_1 is the matrix of $N_s \times N_s$. H_0 Reflects the interaction of the signal in the current paragraph, H_1 response signal in the preceding paragraph for the signal with in the current segment. Specifically expressed as:

$$H_0 = \begin{bmatrix} h(0) & 0 & 0 & \ldots & 0 \\ \vdots & h(0) & 0 & \ldots & 0 \\ h(L) & \ldots & \ddots & \ldots & 0 \\ \vdots & \ddots & \ldots & \ddots & 0 \\ 0 & \ldots & h(L) & \ldots & h(0) \end{bmatrix}$$

$$H_1 = \begin{bmatrix} 0 \ldots h(L) \ldots h(1) \\ \vdots \; \ddots \quad 0 \quad \ldots \quad 0 \\ 0 \ldots \quad \ddots \quad \ldots h(L) \\ \vdots \; \ddots \quad \ldots \quad \ddots \quad 0 \\ 0 \ldots \quad 0 \quad \ldots \quad 0 \end{bmatrix} \tag{4}$$

The receiver to the length N_s of the OFDM blocks of data to be processed, after you remove the length N_{cp} of CP, The ISI of the data block are from within the $x(n)$, there is no relationship with $x(n-1)$, that is commonly referred to as OFDM eliminates interference between the symbols. Derived as follows:

$$\begin{aligned} r(n) &= R_{cp}\left(H_0 \bar{x}(n) + H_1 \bar{x}(n-1)\right) + \eta(n) \\ &= R_{cp} H_0 T_{cp} x(n) + \underbrace{R_{cp} H_1 T_{cp}}_{=0} x(n-1) + \eta(n) \\ &= R_{cp} H_0 T_{cp} x(n) + \eta(n) \\ &= \tilde{H} x(n) + \eta(n) \end{aligned} \tag{5}$$

Similar to the T_{cp}, R_{cp} is called CP delete matrix, $R_{cp} = \left[0_{N \times N_{cp}} \; I_N \right]$. According to the nature of the matrix, we can see that $\tilde{H}$ is $N \times N$ loop array:

$$\tilde{H} = \begin{bmatrix} h(0) & 0 & \ldots & 0 & h(L) & \ldots & h(1) \\ \vdots & h(0) & \vdots & \vdots & \vdots & \vdots & \vdots \\ h(L) & \ldots & h(0) & \ldots & \ldots & \ldots & \ldots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \ldots & 0 & h(L) & \ldots & \ldots & h(0) \end{bmatrix} \tag{6}$$

Join the CP is designed according to the characteristics of OFDM system using IFFT/FFT modulation, using cyclic matrix properties make OFDM system channel equalization simplified. The nature of circular matrix $\tilde{H}$ respectively by F_N and then by F_N^H, will be a diagonal matrix. Its diagonal elements are N-point DFT transform of discrete channel impulse response. Mathematical representation for:

$$\begin{aligned} y(n) &= F_N r(n) \\ &= F_N \left(\tilde{H} x(n) + \eta(n) \right) \\ &= \underbrace{F_N \tilde{H} F_N^H}_{D} s(n) + \bar{\eta}(n) \end{aligned} \tag{7}$$

where $\bar{\eta}(n)$ is the noise vector after transformation. D is a diagonal matrix whose diagonal elements:

$$D(k,k) = \sum_{l=0}^{L} h(l) \exp(-j2\pi lk/N) \tag{8}$$

Finishing the above formula can be

$$y(n) = Ds(n) + \bar{\eta}(n) \tag{9}$$

In this way, OFDM system in parallel with different gain can be equivalent to N independent Gaussian channels. Send signals $s(n)$ by inserting pilot sequence, as well as the extract location data at the receiving end $y(n)$, you can get to each sub-carrier channel gain.

The DFT interpolation algorithm is mainly used in traditional training in the frequency domain channel estimation sequence. First of all, first of all, in the transmitter designs into the corresponding pilot sequence $X_p[k]$, in accordance with the pilot. Extracted in the receiver FFT transform to guide the pilot subcarriers at the $Y_p[k]$. The $Y_p[k]$ for transmitting a pilot $X_p[k]$ through the channel, to be received by the receiver. Because the receiving end local entities pilot sequence $X_p[k]$, can be a pilot subcarrier channel frequency response:

$$H_p[k] = \frac{Y_p[k]}{X_p[k]} + \frac{N_p[k]}{X_p[k]} \approx \frac{Y_p[k]}{X_p[k]} \tag{10}$$

where, $k = 1, 2, \ldots, N_p$, N_p is the number of pilots. By (2-9) we can see OFDM system time domain multipath channel impulse response and the carrier frequency domain response into a Fourier transform on the relationship. Therefore, the $H_p[k]$ FFT transform, you can get the time domain multipath channel impulse response $h[k]$ [11]:

$$H_p[k] \longrightarrow h[k] \tag{11}$$

where, $\longrightarrow$ represents IFFT. IFFT transform length to the length of $H_p[k]$, that is N_p.

For ideal multi-path channel impulse response of the h_{ideal}, only in the corresponding delay department will have the impulse, at the rest position is 0. While the actual demand of $h[k]$, generally do not exist non-zero elements, the reason is because of noise and interference. According to this principle, the $h[k]$ filter processing, retention time delay position on impulse, will not delay position on the value zero, i.e., filter out noise and interference to the impulse response of the effect, known as the selection of effective path. Selection of effective path in practice:

1) Set the threshold T. T can be a fixed value, can also be used for statistical time-domain impulse response according to the number of OFDM symbols received, T is a dynamic value.

2) Multi-path time-domain impulse response $h[k]$ of elements in the threshold T for comparison. If the elements in $h[k]$ is greater than the threshold, the retention; otherwise $h[k]$ in the corresponding element is set to 0. So to retain effective multi-path fading channel impulse response, and inhibits some of the noise impulse interference.

That is, according to channel physical model, effective path exists in a single sampling point. However, the actual system, the multi-path delay is often not necessarily an integer multiple of the sampling time, noise and interference will affect the judgment of the threshold, so select a valid path, usually retain greater than the threshold $h[k]$, or the number of elements around $h[k]$. According to the simulation, this deal with the choice of a valid path is very accurate.

Obtained multi-path channel impulse response of the $h^{'}[k]$ through time-domain filter, $h^{'}[k]$ for FFT transform, get all sub-carrier frequency channel frequency response:

$$h^{'}[k] \longrightarrow H \tag{12}$$

where, $\longrightarrow$ represents FFT. The FFT transform length is the total number of OFDM system carrier N.

In practical OFDM Systems, pilot in frequency on the Insert locations will vary depending on the system requirements. Generally speaking, not from the first sub-carrier interval insertion begins. This caused the pilot sequence frequency position offset. Channel estimation for DFT filtering, can lead to channel estimation accuracy decreased.

Because the OFDM system in multi-path channel time domain impulse response and the carrier frequency domain response is the Fourier transform relationship. Although the pilot of the starting position may be relatively the first sub-carrier offset, but if you can get on the carrier in the entire frequency domain response, or can be obtained through the IFFT time-domain impulse response of multipath channel.

Pilot in response to get the whole channel frequency response can be accomplished by interpolation algorithm; Pilot-interpolation algorithm is, within the coherence bandwidth frequency carrier and data carrier frequency domain response of relevance. Through interpolation methods, you can approximate estimate data by frequency domain response of carrier frequency domain response. Common interpolation algorithm includes linear interpolation, second-order interpolation, such as higher-order interpolation. Estimated higher accuracy, the higher complexity. In addition, sometimes before the interpolation calculation is made, according to the frequency of the system design, adding some number in advance, when the operation is complete; remove some inconsistent number of design and structure.

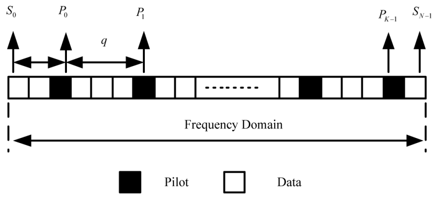

Figure 2 Pilot Sequence's Frequency Domain.

3. Improved DFT-Based Channel Estimation Based On Phase Compensation

Assume that in the OFDM system, the total number of subcarriers is N and the serial number of the sub-carriers is $0 \sim N-1$.

The starting position of the first pilot subcarriers is $\varDelta$, that is to say the offset of the first sub-carrier is $\varDelta$. The spacing of the adjacent pilot subcarriers is q, and the number of pilot subcarriers in the OFDM symbols is K (Fig.2).

So the pilot subcarrier location in the OFDM symbol is for

$$\varphi = \{\varDelta, \varDelta + q, \ldots, \varDelta + (K-1)\,q\} \tag{13}$$

If $\varDelta$ is not 0, because of the FFT's characteristics, it will bring the impact on the phase offsets in channel domain impulse response which is estimated to reach. In the references [12] and [13], they analyzed the phase compensation model when the multipath delay is integer and the number of pilot and subcarriers number with an integer multiple relationship, but its conclusions cannot be extended to non-integer multipath delay and the scene in which the number of pilot and the number of subcarriers is not an integer multiple relationship.

Assumption that the system time is perfectly synchronized, multipath environment is sparse, and in an OFDM symbol period the channel is not changed. To better simulate the impact of the channel, it usually adopts the band-limited discrete multipath channel model [14].

$$g(n) = \sum_{l=1}^{L} h(l) \sin c\left(\frac{\tau_l}{T_s} - n\right) \tag{14}$$

where h_l and τ_l, respectively, are the l path's fading and delay, and L is the number of multipath. Each path is independent, and also meets the Jakes spectrum's complex Gaussian process.

3.1. The Integral Path Delays

Assume that the channel multipath delay is exactly an integer multiple of the sampling's time

$$\tau_l = \tilde{\tau}_l T_s \tag{15}$$

Equivalent the frequency domain channel response can be simplified as [16]

$$H(m) = \sum_{l=0}^{L-1} h(l) \exp\left(-j\frac{2\pi m \tilde{\tau}_l}{N}\right),\ 0 \le m \le N-1 \tag{16}$$

By Eq. (1), the channel frequency response at the pilot subcarriers is

$$H(k) = \sum_{l=0}^{L-1} h(l) e^{-j\frac{2\pi\tau_l}{N}(qk+\varDelta)},\ 0 \le k \le K-1 \tag{17}$$

Without considering the impact of noise, the Eq. (17) as K point's IFFT to get channel time-domain impulse response's estimated value.

$$\hat{g}(n) = \frac{1}{K}\sum_{k=0}^{K-1} H(k) e^{j\frac{2\pi kn}{K}},0 \le n \le K-1 \tag{18}$$

Insert (17) into (18), and may be organized

$$\begin{aligned}\hat{g}(n) = &\frac{1}{K}\sum_{l=0}^{L-1} h(l) e^{-j\frac{2\pi}{N}\tau_l\varDelta} \sum_{k=0}^{K-1} e^{-j\frac{2\pi}{N}\tau_l qk} e^{j\frac{2\pi}{K}kn}\\ &\frac{1}{K}\sum_{l=0}^{L-1} h(l) e^{-j\frac{2\pi}{N}\tau_l\varDelta} \sum_{k=0}^{K-1} e^{j\frac{2\pi}{K}\left(n-\frac{qK}{N}\tau_l\right)k}\end{aligned} \tag{19}$$

1) When $qK = N$ holds, make further organization for (19) and get

$$\hat{g}(n) = \begin{cases} h(l)e^{-j\frac{2\pi}{N}\tau_l\varDelta}, & n = \tau_l \\ 0, & n \ne \tau_l \end{cases} \tag{20}$$

At this point, only the need phase compensation for $\hat{g}(n)$, and that is

$$\tilde{g}(n) = \exp\left(j\frac{2\pi}{N}n\varDelta\right)\hat{g}(n), 0 \le n \le K-1 \tag{21}$$

That will be able to compensate for the effect of the pilot position offset. This is consistent with the literature conclusion [13].

2) When $qK \ne N$ holds, make further organization for Eq.(7) and get

$$\begin{aligned}\hat{g}(n) = &\frac{1}{K}h(0)\sum_{k=0}^{K-1}\exp\left(j\frac{2\pi n}{\mathrm{K}}k\right)\\ &+\frac{1}{K}\sum_{l=1}^{L-1} h(l)\exp\left(-j\frac{2\pi}{N}\tau_l\varDelta\right)\\ &\frac{1-\exp\left(-j2\pi qK\tau_l/N\right)}{1-\exp\left(j2\pi\left(n/K - q\tau_l/N\right)\right)}\\ &= \begin{cases} h(0)+f(0), & n = 0 \\ f(n), & 1 \le n \le K-1 \end{cases}\end{aligned} \tag{22}$$

where:

$$f(n) = \frac{1}{K}\sum_{l=1}^{L-1} h(l)\exp\left(-j\frac{2\pi}{N}\tau_l\Delta\right) \frac{1-\exp\left(-j2\pi qK\tau_l/N\right)}{1-\exp\left(j2\pi\left(n/K-q\tau_l/N\right)\right)} \quad (23)$$

Simplify (11) and get

$$\begin{aligned} f(n) &= \frac{1}{K}\sum_{l=1}^{L-1} h(l) \\ &\frac{2j\sin\left(\pi qK\tau_l/N\right)\exp\left(-j\pi qK\tau_l/N\right)\exp\left(-j2\pi\tau_l\Delta/N\right)}{2j\sin\left(\pi\left(q\tau_l/N-n/K\right)\right)\exp\left(j\pi\left(n/K-q\tau_l/N\right)\right)} \\ &= \frac{1}{K}\sum_{l=1}^{L-1} h(l)\frac{\sin\left(\pi qK\tau_l/N\right)}{\sin\left(\pi\left(q\tau_l/N-n/K\right)\right)} \\ &\exp\left(-j\left(\frac{\pi q(K-1)}{N}\tau_l+\frac{2\pi\Delta}{N}\tau_l+\frac{\pi}{K}n\right)\right) \\ &= \sum_{l=1}^{L-1} A(n,\tau_l)\exp(j\Theta)h(l) \end{aligned} \quad (24)$$

where

$$\begin{cases} A(n,\tau_l) = \frac{1}{K}\frac{\sin(\pi qK\tau_l/N)}{\sin(\pi(q\tau_l/N-n/K))} \\ \Theta(n,\tau_l) = -\left(\frac{\pi q(K-1)}{N}\tau_l+\frac{2\pi\Delta}{N}\tau_l+\frac{\pi}{K}n\right) \end{cases} \quad (25)$$

As Eq.(10) shows, estimate value $\hat{g}(n)$ of each channel time domain impulse response, contains all real time-domain channel impulse response's $h(l)$, $0 \le l \le L-1$ part of the energy, and that is to say after IFFT's K point, $h(l)$'s energy leaks, meanwhile the phase deviation also occurs, the weighted coefficient is (n,τ_l) and the phase offset is Θ.

Sparse channel usually meets $|\tau_k-\tau_l| \gg 1$, $k \ne l$, therefore, the various diameters' interference caused by the IFFT energy leakage is less and in the case of unknown multipath delay there is no need to eliminate the interference.

At this point, the channel estimation errors introduced by the phase offsets play a decisive role in the performance of the DFT channel interpolation algorithm [15].

$$\hat{g}'(n) = \begin{cases} \hat{g}(n), & if\ |\hat{g}(n)| > T \\ 0\quad , & others \end{cases} \quad (26)$$

Because channel multipath delay is unknown, the phase offsets Θ in (13) can make the following approximate:

$$\bar{\Theta} = -\left(\frac{\pi q(K-1)}{N}+\frac{2\pi\Delta}{N}+\frac{\pi}{K}\right)n \quad (27)$$

Thus make corresponding to the phase compensation for Eq.(14)

$$\tilde{g}(n) = \hat{g}'(n)\exp(-j\bar{\Theta}) \quad (28)$$

3.2. The Nonintgral Path Delays

It is worth noting that when the τ_l is decimal, Eq.(5) does not hold. The channel frequency response at the pilot is

$$H(k) = \sum_{m=0}^{N_1+N_2} g(m)\exp\left(-j\frac{2\pi m(qk+\Delta)}{N}\right), \quad 0 \le k \le K-1 \quad (29)$$

where N_1+N_2+1 is the equivalent diameter of the number according to Eq.(2), and transform Eq.(17) into the time domain and get

$$\begin{aligned} \hat{g}(n) &= \frac{1}{K}\sum_{k=0}^{K-1} H(k)\exp\left(j\frac{2\pi kn}{K}\right) \\ &= \frac{1}{K}\sum_{m=0}^{N_1+N_2} g(m)\exp\left(-j\frac{2\pi}{N}m\Delta\right) \\ &\sum_{k=0}^{K-1}\exp\left(j\frac{2\pi}{K}\left(n-\frac{qKm}{N}\right)k\right) \end{aligned} \quad (30)$$

1) When $N = qK$ holds, as the Eq.(9) shows, make some compensation by the following formula:

$$\bar{g}(n) = \exp\left(j\frac{2\pi}{N}n\Delta\right)\hat{g}(n) \quad (31)$$

2) When $N \ne qK$ holds, Simplify Eq.(18) and get:

$$\begin{aligned} \hat{g}(n) &= \frac{1}{K}g(0)\sum_{k=0}^{K-1}\exp\left(j\frac{2\pi n}{K}k\right) + \frac{1}{K}\sum_{m=1}^{N_1+N_2} g(m) \\ &\exp\left(-j\frac{2\pi}{N}m\Delta\right)\frac{1-\exp\left(-j2\pi qKm/N\right)}{1-\exp\left(j2\pi\left(n/K-qm/N\right)\right)} \\ &= \begin{cases} g(0)+f(0), & n=0 \\ f(n), & 1 \le n \le K-1 \end{cases} \end{aligned} \quad (32)$$

where,

$$\begin{aligned} f(n) &= \frac{1}{K}\sum_{m=1}^{N_1+N_2} g(m)\exp\left(-j\frac{2\pi}{N}m\Delta\right) \\ &\frac{1-\exp\left(-j2\pi qKm/N\right)}{1-\exp\left(j2\pi\left(n/K-qm/N\right)\right)} \\ &= \frac{1}{K}\sum_{m=1}^{N_1+N_2} g(m) \\ &\frac{2j\sin\left(\pi qKm/N\right)\exp\left(-j\pi qKm/N\right)\exp\left(-j2\pi m\Delta/N\right)}{2j\sin\left(\pi\left(qm/N-n/K\right)\right)\exp\left(j\pi\left(n/K-qm/N\right)\right)} \\ &= \frac{1}{K}\sum_{m=1}^{N_1+N_2} g(m)\frac{\sin\left(\pi qKm/N\right)}{\sin\left(\pi\left(qm/N-n/K\right)\right)} \\ &\exp\left(-j\pi\left(\frac{qm(K-1)}{N}+\frac{2m\Delta}{N}+\frac{n}{K}\right)\right) \\ &= \sum_{m=1}^{N_1+N_2}(-1)^n A(n,m)\exp\left(j\Theta\left(n,m\right)\right)g(m) \end{aligned} \quad (33)$$

where,

$$\begin{cases} A(n,m) = \frac{1}{K}\left|\frac{\sin(\pi qKm/N)}{\sin(\pi(qm/N-n/K))}\right| \\ \Theta\left(n,m\right) = -\pi\left(\frac{qm(K-1)}{N} + \frac{2m\Delta}{N} + \frac{n}{K}\right) \end{cases} \quad (34)$$

For Eq.(22), when m is fixed, the function (n,m) is a non-monotonic function and its maximum is at $n = m$. This shows that $g(n)$ play a major impact on $\hat{g}(n)$, but the effects on which $g(m)$, $m \neq n$ plays $\hat{g}(n)$ is temporarily unable to eliminate, and therefore, Eq. (20) can be made the following approximation.

$$\hat{g}(n) \approx (-1)^n \exp\left(j\Theta\left(n,n\right)\right) g(n) \quad (35)$$

4. Simulation Results and Analysis

4.1. Typical channel estimation algorithm performance

This section in the single-antenna OFDM system simulates and analyses linear interpolation and DFT-based channel estimation. The simulation parameters such as shown in Table 1:

Table 1 Single antenna OFDM system channel estimation simulation parameters

OFDM number of carriers	1024
CP Length	216
OFDM Symbol period (ms)	31
Modulation mode	QPSK
Guided frequency pattern	comb
Pilot carrier offset	$\Delta = 1$
Pilot interval	4
Channel conditions	COST207 Typical urban standard channel
$f_d T$	0.015&0.1
Channel estimation	1. linear interpolation channel estimation 2. DFT interpolation channel estimation
Signal detection	Linear MMSE detector

Fig. 3 is the comparison several channel estimation algorithms MSE performance. As shown. MSE performance of the optimal estimation algorithm based on the optimization of the phase compensation channel is the best. It also compares the DFT interpolation channel estimation's performance whether it exists offset in the pilot position ($\Delta = 1$) or offset ($\Delta = 0$). When it's $\Delta = 1$ DFT interpolation channel estimation MSE performance is poor. This also proves that the pilot the position of the offset will seriously affect the DFT channel estimation accuracy.

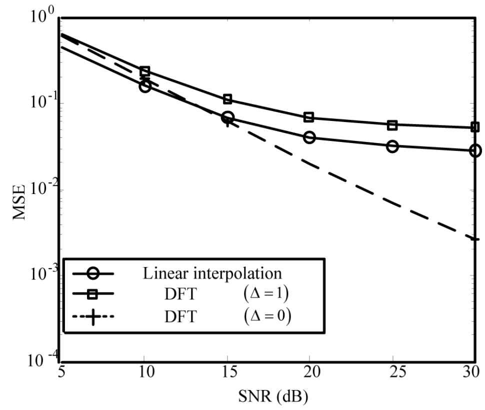

Figure 3 The MSE performance comparison of several channel estimation in the OFDM system.

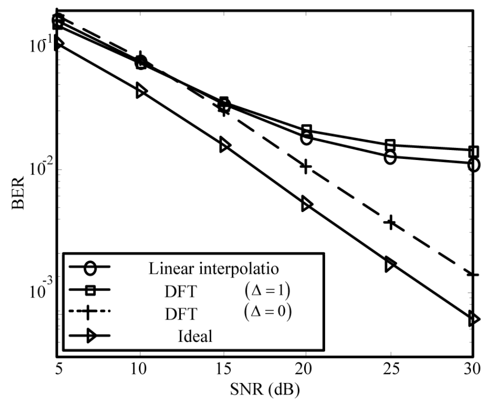

Figure 4 The BER performance comparison under several channel estimation in OFDM system ($f_d T = 0.015$).

Fig.4 is several channel estimation algorithm OFDM system bit error rate performance comparison. As shown there's big gap in DFT interpolation channel estimation BER performance when it is ($\Delta = 1$) or ($\Delta = 0$).

Fig. 5 is when increasing the Doppler frequency makes owned by one of the Doppler shift $f_d T = 0.1$, the comparison of several channel estimation algorithms for OFDM system BER performance. We can get the same conclusion from the figure. However, Doppler shift the ICI can not be ignored because of the rapid changes in channel ($f_d T \gg 0.01$). Therefore, when compared to $f_d T = 0.015$, the overall system performance is worse.

4.2. The phase compensation

The simulation parameters as follows: Rayleigh fading channel is shown in Table 2. OFDM system parameters are shown in Table3.

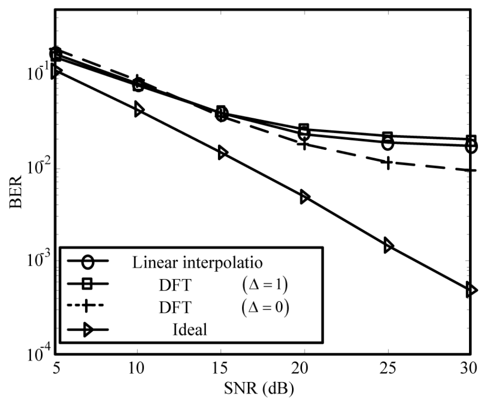

Figure 5 The BER performance comparison under several channel estimation in OFDM system ($f_d T = 0.015$).

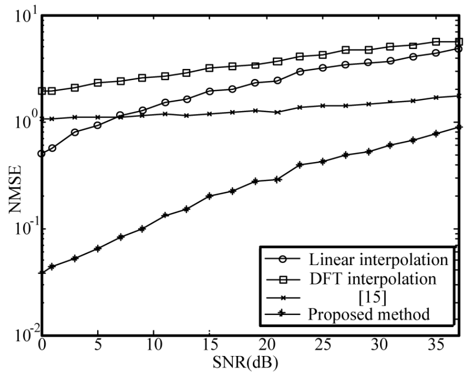

Figure 6 Performance of the channel estimation.

Fig. 6 compares the performance of linear interpolation and DFT interpolation algorithm, because the product of the adjacent pilot interval and the pilot number is not equal to the total number of subcarriers, ie $mk \neq N$, which makes DFT interpolation performance dramatically deteriorate compared to the case of $mk = N$.

In DFT interpolation algorithm with noise suppression, since most of the noise diameter was inhibited that makes its performance better than the DFT interpolation, and phase compensation operation makes the performance of the algorithm to be further improved.

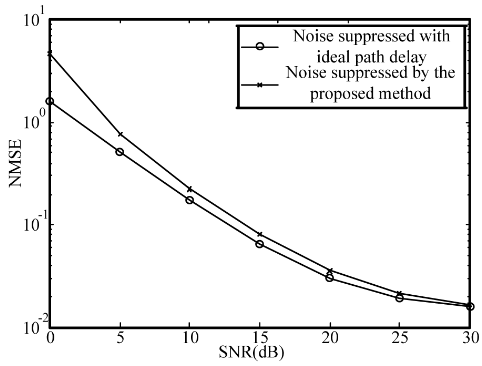

Figure 7 NMSE of the channel estimation with the phase compensation.

Table 2 Channel parameters

Multipath Numbers	Multipath normalized delay τ_l	Multipath fading(dB)
1	0	0
2	4.2	0
3	10.3	0

Table 3 OFDM system parameters

Parameter name	Parameter values
The number of subcarriers N	1024
Constellation modulation mode	QPSK
Offset Δ	4
Adjacent the pilot interval m	6
Number of pilot subcarriers K	170

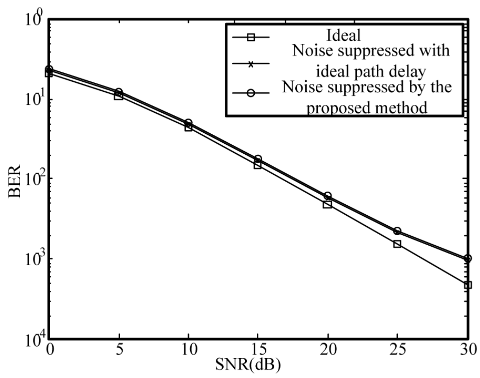

Figure 8 BER of system with the phase compensation.

Fig. 7 shows the channel NMSE by phase compensation, and retained the number is 12 before and after. The curves labeled "noise suppressed with ideal path delay" refers to retain only the effective diameter of the channel time-domain impulse response, while the value of the noise path is forced to zero. Fig. 8 is a corresponding system BER performance map.

5. Conclusions

Based on frequency domain training sequence channel estimation in OFDM systems are widely used. The pilot configuration of the system often cannot start from the first subcarrier, which will impact on the channel estimation at the pilot. The study analyses and deduces in detail phase offsets and the compensation technology under the condition of non-integer multiple of the multipath delay and non-integer multiple of the number of pilot. The simulation results show that the proposed phase compensation technology can effectively improve the performance of the channel estimation.

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