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Some Statistical Properties for a Non-Linear Tavis Cummings Model

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Abstract: A non-linear Tavis Cummings model is studied, especially, two two-level atoms interacting with two modes of radiation field with Stark shift effect. The time-dependent wave function and consequently the density matrix are obtained from which we discussed the effect of Stark shift on different statistical aspects for the present system, for example, atomic inversion, entanglement, variance squeezing and degree of coherence. It is observed that the Stark shift parameter plays an important role on the evolution of these aspects. When Stark shift parameter moves away from 1, the atomic inversion is shifted upwards, entanglement decreases and more squeezing occur.

Keywords: Stark shift, Quantum entropy, Negativity, Variance squeezing.

1 Introduction

Quantum correlation and quantum entanglement are two interesting quantities in quantum world. They play a central role in revealing the quantum features of quantum information and communication theory[1,2,3,4]. Also the inseparable (entangled) states have applications in quantum computing[5], teleportation[6], cryptographic [7] and dense coding [8].

In atomic physics, the Stark effect is the split and shift of a spectral line into several components in the presence of an electric field. The amount of spliting is called the Stark shift [9,10,11]. Its effect has been studied on some phenomena in different quantum systems[12,13,14,15]. It has been shown that an increase in the value of the Stark shift parameter leads to an increase in the degree of entanglement[15] also, Squeezing phenomena have affected by any variation in the Stark parameter[12].
In this paper we study the interaction between an atomic system described by a two two-level atoms and the quantized field in the rotating wave approximation taking into account Stark shift effect. We obtain the wave function of the total system at any time $t > 0$ and we study the effect of Stark shift on some statistical aspects. This paper is arranged as follows: Sec. 2, is devoted to the physical system. Sec. 3, is devoted to the analytical solution of the system, this is followed by a discussion of the atomic inversion in Sec. 4, and entanglement in Sec. 5. Variance squeezing is considered in Sec. 6. Second order coherence for one of the field modes is discussed in Sec. 7. Finally, some conclusions are given in Sec. 8.

2 Description of the model

We consider a system of two three-level atoms in the same cavity, each atom interacting with two modes of radiation field. We obtain the Stark shift contribution to two two-level atoms by using the adiabatic elimination method for the intermediate level. The effective Hamiltonian of a two two-level atoms interacting with two modes in the presence of Stark shift is obtained [15, 16,17,18]. The effective Hamiltonian of this system in the rotating wave approximation (RWA) can be written as $(\hbar = 1)$

$$\hat{H}_{eff} = \sum_{j=1}^{2} \omega_j \hat{a}_j^\dagger \hat{a}_j + \frac{1}{2}\sum_{j=1}^{2} \Omega_j \hat{\sigma}_z^{(j)} + \sum_{j=1}^{2} (\xi_1^{(j)} \hat{n}_1 \hat{\sigma}_+^{(j)} \hat{\sigma}_-^{(j)} + \xi_2^{(j)} \hat{n}_2 \hat{\sigma}_-^{(j)} \hat{\sigma}_+^{(j)}) + \sum_{j=1}^{2} \lambda^{(j)} (\hat{\sigma}_-^{(j)} \hat{a}_1^\dagger \hat{a}_2^\dagger + \hat{\sigma}_+^{(j)} \hat{a}_1 \hat{a}_2), \tag{1}$$

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where ω_j (Ω_j), $(j=1,2)$ is the field (the atomic transition) frequencie, $\hat{a}_j^{\dagger}(\hat{a}_j)$ is the creation (annhilation) operator that satisfy the Boson commutation relation $[\hat{a}_i,\hat{a}_j^{\dagger}]=\delta_{ij}$. The operators $\hat{\sigma}_+^{(j)}(\hat{\sigma}_-^{(j)})$ and $\hat{\sigma}_z^{(j)}$are the usual raising (lowering) and the inversion operators for the j^{th} two-level atomic system, satisfying the commutation relations $[\hat{\sigma}_z^{(i)},\hat{\sigma}_{\pm}^{(j)}]=\pm 2\hat{\sigma}_{\pm}^{(j)}\ \delta_{ij}$ and $[\hat{\sigma}_+^{(i)},\hat{\sigma}_-^{(j)}]=\hat{\sigma}_z^{(j)}\delta_{ij},\ (i,j=1,2)$. $\xi_1^{(j)}$, $\xi_2^{(j)}$ are the parameters of the intensity-dependent Stark shifts to the two levels of the j atom, that are due to the virtual transitions to the intermediate level, and we note that $\lambda^{(j)}=\sqrt{\xi_1^{(j)}\xi_2^{(j)}}$.

3 The analytical solution

In this section we derive the wave function $|\Psi(t)\rangle$, the reduced atomic density matrix and the reduced density operator of the field.

By using the Heisenberg equation of motion $(\frac{d\hat{O}}{dt}=\frac{1}{i\hbar}[\hat{O},\hat{H}]$) for the operators $\hat{n}_j=\hat{a}_j^{\dagger}\hat{a}_j$ and $\hat{\sigma}_z^{(j)}$, $(j=1,2)$, we can deduce the following constants of motion

$$\hat{N}_1=2\hat{n}_1+(\hat{\sigma}_z^{(1)}+\hat{\sigma}_z^{(2)}),\ \hat{N}_2=2\hat{n}_2+(\hat{\sigma}_z^{(1)}+\hat{\sigma}_z^{(2)}). \quad (2)$$

Here, we consider the case in which the atoms and the field are exactly resonant [15], $\xi_1^{(1)}=-\xi_1^{(2)}$, $\xi_2^{(1)}=-\xi_2^{(2)}$ and $\lambda^{(1)}=\lambda^{(2)}=\lambda$. Thus, the Hamiltonian(1) becomes

$$\hat{H}_{eff}=\frac{1}{2}\omega_1\hat{N}_1+\frac{1}{2}\omega_2\hat{N}_2+\hat{\eta}+\hat{C} \quad (3)$$

where

$$\begin{aligned}\hat{C}=&\frac{1}{2}\hat{\delta}_1(\hat{n}_1,\hat{n}_2)(\hat{\sigma}_+^{(1)}\hat{\sigma}_-^{(1)}-\hat{\sigma}_+^{(2)}\hat{\sigma}_-^{(2)})\\&-\frac{1}{2}\hat{\delta}_2(\hat{n}_1,\hat{n}_2)(\hat{\sigma}_-^{(1)}\hat{\sigma}_+^{(1)}-\hat{\sigma}_-^{(2)}\hat{\sigma}_+^{(2)})\\&+\lambda\sum_{j=1}^{2}(\hat{\sigma}_-^{(j)}\hat{a}_1^{\dagger}\hat{a}_2^{\dagger}+\hat{\sigma}_+^{(j)}\hat{a}_1\hat{a}_2)\end{aligned}$$

and
$$\begin{aligned}\hat{\eta}=&\frac{1}{2}\hat{Z}_1(\hat{n}_1,\hat{n}_2)(\hat{\sigma}_+^{(1)}\hat{\sigma}_-^{(1)}-\hat{\sigma}_+^{(2)}\hat{\sigma}_-^{(2)})\\&+\frac{1}{2}\hat{Z}_2(\hat{n}_1,\hat{n}_2)(\hat{\sigma}_-^{(1)}\hat{\sigma}_+^{(1)}-\hat{\sigma}_-^{(2)}\hat{\sigma}_+^{(2)}) \quad (4)\end{aligned}$$

with

$$\begin{aligned}\hat{\delta}_1(\hat{n}_1,\hat{n}_2)&=\xi_1\hat{n}_1-\xi_2(\hat{n}_2+1),\\\hat{\delta}_2(\hat{n}_1,\hat{n}_2)&=\hat{\delta}_1(\hat{n}_1-1,\hat{n}_2-1),\\\hat{Z}_1(\hat{n}_1,\hat{n}_2)&=(\xi_1\hat{n}_1+\xi_2(\hat{n}_2+1))\\\text{and }\hat{Z}_2(\hat{n}_1,\hat{n}_2)&=\hat{Z}_1(\hat{n}_1-1,\hat{n}_2-1).\end{aligned} \quad (5)$$

We assume that the two atoms and the field are initially prepared in excited states and uncorrelated coherent states respectively. In this case the wave function of the system at $t=0$ can be written as

$$|\Psi(0)\rangle=|+,+\rangle\otimes(\sum_{n_1,n_2=0}^{\infty}q_{n_1,n_2}|n_1,n_2\rangle), \quad (6)$$

where

$$q_{n_1,n_2}=\exp(-\frac{1}{2}\sum_{j=1}^{2}|\alpha_j|^2)\frac{\alpha_1^{n_1}\alpha_2^{n_2}}{\sqrt{n_1!n_2!}}.$$

The wave function $|\Psi(t)\rangle$ at $t>0$ for the system takes the following form

$$\begin{aligned}|\Psi(t)\rangle=&\sum_{n_1,n_2=0}^{\infty}e^{-i\chi t}[X_1(n_1,n_2,t)|++,n_1,n_2\rangle\\&+X_2(n_1,n_2,t)|+-,n_1+1,n_2+1\rangle\\&+X_3(n_1,n_2,t)|-+,n_1+1,n_2+1\rangle\\&+X_4(n_1,n_2,t)|--,n_1+2,n_2+2\rangle], \quad (7)\end{aligned}$$

where $\chi=(n_1+1)\omega_1+(n_2+1)\omega_2$. The coefficients $X_j(n_1,n_2,t)$, $(j=1,2,3,4)$ can be obtained by solving the Schrödinger equation $(i\partial/\partial t)|\Psi(t)\rangle=\hat{H}|\Psi(t)\rangle$, where $\hat{H}$ is given by (3).

The explicit expressions for these coefficients are given by

$$\begin{aligned}X_1(n_1,n_2,t)=&\,q_{n_1,n_2}*\\&(\frac{\mu_1^2(n_1,n_2)-2\upsilon_1^2(n_1,n_2)(1-\cos\mu_1(n_1,n_2)t)}{\mu_1^2(n_1,n_2)}),\\X_2(n_1,n_2,t)=&-q_{n_1,n_2}*\\&(\frac{\Lambda(n_1,n_2)\upsilon_1(n_1,n_2)(1-\cos\mu_1(n_1,n_2)t)}{\mu_1^2(n_1,n_2)}\\&+i\frac{\upsilon_1(n_1,n_2)\sin\mu_1(n_1,n_2)t}{\mu_1(n_1,n_2)}),\\X_3(n_1,n_2,t)=&\,q_{n_1,n_2}*\\&(\frac{\Lambda(n_1,n_2)\upsilon_1(n_1,n_2)(1-\cos\mu_1(n_1,n_2)t)}{\mu_1^2(n_1,n_2)}\\&-i\frac{\upsilon_1(n_1,n_2)\sin\mu_1(n_1,n_2)t}{\mu_1(n_1,n_2)}),\\X_4(n_1,n_2,t)=&-q_{n_1,n_2}*\\&(\frac{2\upsilon_1(n_1,n_2)\upsilon_2(n_1,n_2)(1-\cos\mu_1(n_1,n_2)t)}{\mu_1^2(n_1,n_2)}), \quad (8)\end{aligned}$$

where

$$\begin{aligned}\upsilon_1(n_1,n_2)&=\lambda\sqrt{(n_1+1)(n_2+1)},\\\upsilon_2(n_1,n_2)&=\lambda\sqrt{(n_1+2)(n_2+2)},\\\Lambda(n_1,n_2)&=Z(n_1,n_2)+\delta(n_1,n_2),\\\text{and }\mu_1(n_1,n_2)&=\sqrt{\Lambda^2(n_1,n_2)+2(\upsilon_1^2(n_1,n_2)+\upsilon_2^2(n_1,n_2))},\end{aligned}$$

with

$$\begin{aligned}Z(n_1,n_2)&=Z_1(n_1+1,n_2+1)-Z_1(n_1,n_2)\\\text{and }\ \delta(n_1,n_2)&=\frac{\delta_1(n_1+1,n_2+1)+\delta_1(n_1,n_2)}{2}.\end{aligned}$$

These results consider more general than the results which have been obtained earlier, where the cavity field (one mode) has interacted with two coupled atoms and the interaction between the atoms taken in to consideration. Also, the Stark shift effect has been neglected [19].

We can write the reduced atomic density matrix in the following form

$$\hat{\rho}_{atoms}(t) = Tr_{field} |\Psi(t)\rangle \langle \Psi(t)|, \tag{9}$$

where $|\Psi(t)\rangle$ is the time-dependent wave function (7).

The density matrix for a single atom is obtained when we take the trace over one of the atoms, thus we have

$$\hat{\rho}_a^{(j)}(t) = Tr_{a_{(i)}} \hat{\rho}_{atoms}(t), \quad i, j = 1, 2. \tag{10}$$

$$\begin{aligned}\hat{\rho}_a^{(1)}(t) &= \rho_{ee}^{(1)}(t)|+\rangle\langle+| + \rho_{eg}^{(1)}(t)|+\rangle\langle-| \\ &+ \rho_{ge}^{(1)}(t)|-\rangle\langle+| + \rho_{gg}^{(1)}(t)|-\rangle\langle-|,\end{aligned} \tag{11}$$

where

$$\begin{aligned}\rho_{ee}^{(1)}(t) &= \sum_{n_1,n_2=0}^{\infty} (|X_1(n_1,n_2,t)|^2 + |X_2(n_1,n_2,t)|^2), \\ \rho_{gg}^{(1)}(t) &= \sum_{n_1,n_2=0}^{\infty} (|X_3(n_1,n_2,t)|^2 + |X_4(n_1,n_2,t)|^2), \\ \rho_{eg}^{(1)}(t) &= \sum_{n_1,n_2=0}^{\infty} (X_1(n_1+1,n_2+1,t)X_3^*(n_1,n_2,t) \\ &+ X_2(n_1+1,n_2+1,t)X_4^*(n_1,n_2,t)) = \rho_{ge}^{(1)*}(t).\end{aligned} \tag{12}$$

Also the reduced density operator of the field is obtained when we take the trace over the atoms

$$\hat{\rho}_{field}(t) = Tr_{atoms} |\Psi(t)\rangle \langle \Psi(t)|, \tag{13}$$

thus we have

$$\begin{aligned}\hat{\rho}_f(t) = &\sum_{n_1,n_2=0}^{\infty} \sum_{m_1,m_2=0}^{\infty} \\ &[X_1(n_1,n_2,t)X_1^*(m_1,m_2,t)|n_1,n_2\rangle\langle m_1,m_2| \\ &+ X_2(n_1,n_2,t)X_2^*(m_1,m_2,t) \\ &|n_1+1,n_2+1\rangle\langle m_1+1,m_2+1| \\ &+ X_3(n_1,n_2,t)X_3^*(m_1,m_2,t) \\ &|n_1+1,n_2+1\rangle\langle m_1+1,m_2+1| \\ &+ X_4(n_1,n_2,t)X_4^*(m_1,m_2,t) \\ &|n_1+2,n_2+2\rangle\langle m_1+2,m_2+2|].\end{aligned} \tag{14}$$

In what follows we employ the results obtained here, in order to discuss some statistical aspects of the present system. This is performed in the forthcoming sections.

4 The single atomic inversion

The atomic inversion is defined as the difference between the probability of finding the atom in the excited state $|+\rangle$ and in the ground state $|-\rangle$. To discuss the atomic inversion for the first atom we use Eqs.(11) and (12), thus we have

$$W(t) = \rho_{ee}^{(1)}(t) - \rho_{gg}^{(1)}(t). \tag{15}$$

We display the evolution of the atomic inversion for different values of the Stark shift parameter ($r = \xi_1/\xi_2$) in **Fig.** 1, where the values of the intensity of the initial coherent parameters have been fixed as $\alpha_1 = \alpha_2 = 3$. In **Fig. 1(a),** at $r = 1$ (almost absence of the Stark shift effects), it is noted that there are regular fluctuation between excited and ground state as should be expected. The maximal state be clear on a regular basis and the function $W(t)$ is symmetric around $W(t) = 0.1$. To visualize the influence of the Stark shift in the atomic inversion, we set $r = 0.5$, it is noted that the oscillations decreased during the revival period also there is propagation of collapse period and the function $W(t)$ is symmetric around $W(t) = 0.4$, see **Fig.** 1**(b).** By decreasing the Stark shift parameter ($r = 0.25$ or $r = 0.1$), it is noted that the atom oscillates in the excited state and never reaches its ground state and the amplitude of oscillation decreases as observed in **Figs.** 1**(c)** and 1**(d).** In **Fig.** 1**(c),** there is superstructure from $\lambda t = 6\pi$ to 8π. We conclude that the influence of the Stark shift on the atomic inversion is very small when $r = 1$ and becomes strong when r move away from 1.

5 Entanglement

It is well known that a key aspect of quantum information processing is the concept of entanglement, therefore this section is devoted to measure the entanglement.

5.1 Quantum entropy

The quantum dynamics described by the Hamiltonian (1) leads to an entanglement between the field and the atoms. Therefore, a suitable diagnostic tool which is used in this case to measure the degree of entanglement between the field and the atoms is the quantum entropy (or von Neumann entropy). It is defined in quantum mechanics as a generalization of the classical Boltzmann entropy[20, 21, 22],

$$S = -Tr[\hat{\rho} \ln \hat{\rho}] \tag{16}$$

where $\hat{\rho}$ is the density operator for a given quantum system, we set Boltzmann constant $K = 1$. For an initial pure state of the system the entropy of the total system vanishes ($S = 0$), while if $\hat{\rho}$ describes a mixed state, then $S \neq 0$. We can either use the field entropy $S_f(t)$ to measure the amount of entanglement. The field entropy

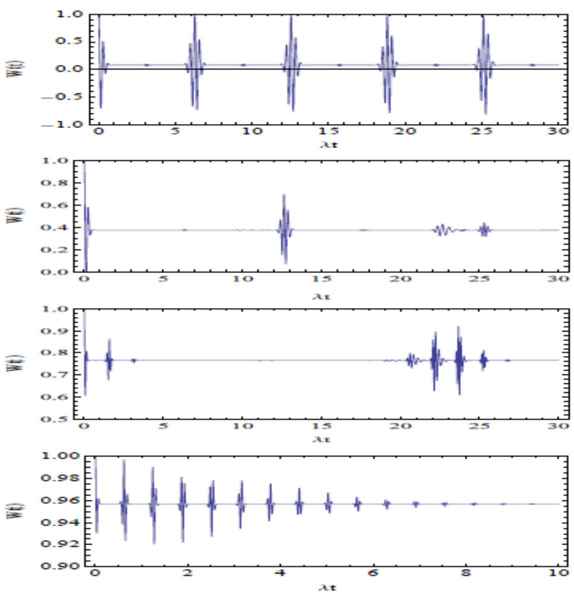

Fig. 1: The time evolution of the atomic inversion as a function of scaled time λt with fixed parameters $\alpha_1 = \alpha_2 = 3$, (a) r=1, (b)r=0.50, (c) r=0.25 and (d) r=0.10.

may be expressed in terms of the eigenvalues $\lambda_i(t)$, $(i = 1,2,3,4)$ for the reduced atomic density matrix as

$$S_f(t) = -\sum_{i=1}^{i=4} \lambda_i(t) \ln \lambda_i(t). \tag{17}$$

For the first atom, the field entropy can be written as

$$S_f(t) = -\lambda_-(t)\ln\lambda_-(t) - \lambda_+(t)\ln\lambda_+(t), \tag{18}$$

where $\lambda_\pm(t)$ are the eigenvalues of the reduced density matrix $\hat{\rho}_a^{(1)}(t)$ which can be easily evaluated from Eqs. (11) and (12) as the following form

$$\lambda_\pm(t) = \frac{1}{2} \pm \frac{1}{2}\sqrt{\left\langle \hat{\sigma}_x^{(1)}(t)\right\rangle^2 + \left\langle \hat{\sigma}_y^{(1)}(t)\right\rangle^2 + \left\langle \hat{\sigma}_z^{(1)}(t)\right\rangle^2}, \tag{19}$$

where

$$\left\langle \hat{\sigma}_x^{(1)}(t)\right\rangle = 2Re\rho_{eg}^{(1)}(t), \; \left\langle \hat{\sigma}_y^{(1)}(t)\right\rangle = 2\rho_{eg}^{(1)}(t)$$

and $$\left\langle \hat{\sigma}_z^{(1)}(t)\right\rangle = \rho_{ee}^{(1)}(t) - \rho_{gg}^{(1)}(t). \tag{20}$$

It is worth mentioning that the relations related to the second atom can be obtained from the relations related to the first atom by using the interchange $X_2(n_1,n_2,t) \leftrightarrow X_3(n_1,n_2,t)$.

Now, we turn our attention to examine numerically the dynamical evolution of the field entropy, we use the same initial parameters of the above figures (for the single atomic inversion). When $r = 1$, the entropy reaches to the maximum value $(\ln 2)$, see **Fig. 2(a).** It reachs to disentanglement at the ponits of revival as show in **Fig. 1(a)**. If we decrease the value of r in order to visualize the influence of the stark shift in the field entropy, we see that when $r = 0.5$ the function $S_f(t)$ oscillates around 0.5, this means that, the maximum value of this function decreases, see **Fig. 2(b),** this effect is also shown clearly in **Figs. 2(c)** and **2(d),** where in the case of $r = 0.25$ the function $S_f(t)$ oscillates around 0.25 and weakly entanglement occurs most of the time (almost disentanglement). At $r = 0.1$, $S_f(t)$ oscillates around 0.06, this means that, very weakly entanglement occurs. Thus we conclude that when r increases the entanglement phenomenon between the field and the atom increases.

In this subsection, we measure the amount of entanglement between the field and the first atom using quantum entropy which is a suitable tool to measure entanglement in a pure state. But in the next subsection we use the negativity to measure the amount of entanglement between the two atoms.

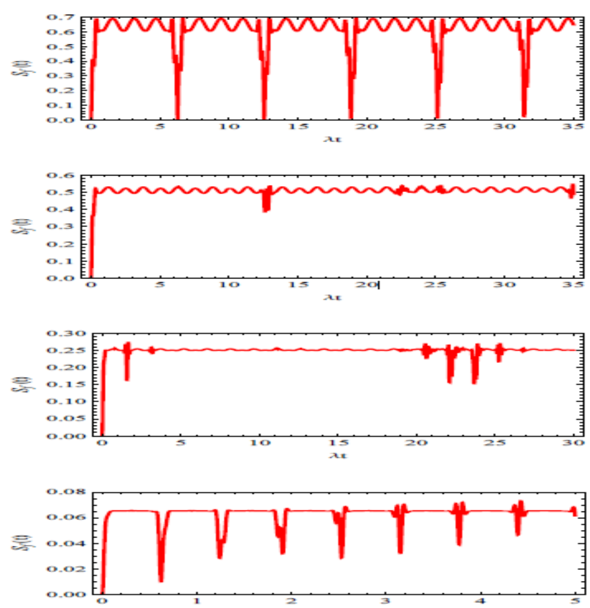

Fig. 2: The time evolution of the field entropy as a function of the scaled time λt and other parameters same as in Fig.(1).

5.2 The negativity

This subsection is devoted to discuss the entanglement measures through the negativity, which is defined by [23]

$$N(\rho) = \frac{\|\rho_{AB}^{T_A}\| - 1}{d-1}, \tag{21}$$

with

$$\|\rho_{AB}^{T_A}\| = (\sum_i |\grave{\mu}_i| - \sum_i \grave{\mu}_i) + 1,$$

where $\rho_{AB}^{T_A}$ is the partial transpose of ρ_{AB} and $\sum_i \grave{\mu}_i$ is sum of the all eigenvalues of $\rho_{AB}^{T_A}$. The value $N(\rho) = 1$ corresponds to maximum entanglement, while $N(\rho) = 0$ indicates that the bipartite system is separable.

Now, we measure the entanglement between the two atoms using the negativity by using the reduced atomic density matrix $\hat{\rho}_{atoms}(t)$ (9), then we take partial transpose of one of the atoms. We display in **Fig.** 3 the evolution of the negativity with the scaled time λt, in order to see the effect of the Stark shift on the degree of the entanglement between the two atoms. We take the values of the intensity of the initial coherent parameters as $|\alpha_1|^2 = |\alpha_2|^2 = 2$. It is noted that at $r = 1$ the maximum degree of the entanglement is 0.1. When Stark shift effect increases the entanglement decreases, also it is noted that when $r = 0.25$, the sudden death and rebirth of entanglement is clearly displayed, see **Fig.** 3**(c).** Thus we conclude that, in this system the degree of the entanglement between the two atoms is weak.

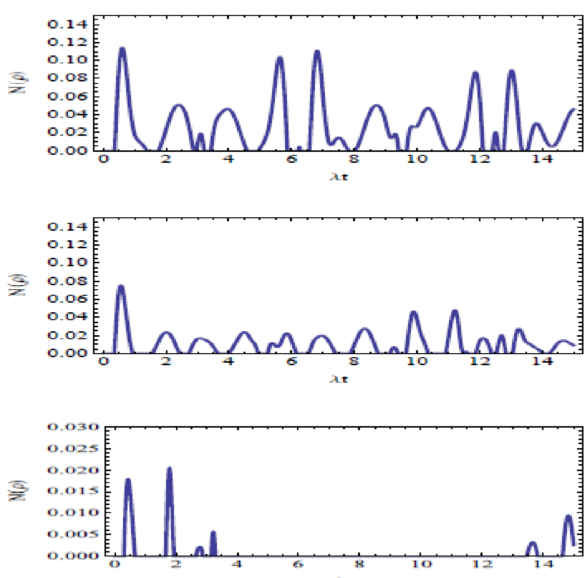

Fig. 3: The time evolution of the negativity as a function of scaled time λt with the atomic initially in excited state, and the field is prepared in a coherent states with fixed parametesr $|\alpha_1|^2 = |\alpha_2|^2 = 2$, (a) r=1,(b) r=0.50 and (c) r=0.25.

6 Variance squeezing

The squeezing phenomenon is one of the nonclassical phenomena in the field of quantum optics. It has been established that squeezing in entropy and variance squeezing are useful tools to measure the phenomenon of squeezing in quantum fluctuations. In fact, both the squeezing in entropy and variance squeezing are built up on the concept of uncertainty relations. There is another kind of squeezing that depends on the field quadratures rather than on the atomic quadratures, which is the usual single-mode squeezing. In this section, we study the variance squeezing.

For a quantum mechanical system with two physical observables represented by the Hermitian operators $\hat{A}$ and $\hat{B}$ satisfying the commutation relation $[\hat{A}, \hat{B}] = i\hat{C}$, one can write the Heisenberg uncertainty relation in the form

$$\langle(\triangle \hat{A})^2\rangle \langle(\triangle \hat{B})^2\rangle \geqslant \frac{1}{4} |\langle\hat{C}\rangle|^2, \tag{22}$$

where
$$\langle(\triangle \hat{A})^2\rangle = \langle\hat{A}^2\rangle - \langle\hat{A}\rangle^2.$$

Consequently, the uncertainty relation for a two-level atom characterized by the Pauli operators $\hat{\sigma}_x$, $\hat{\sigma}_y$ and $\hat{\sigma}_z$ satisfying the commutation relation $[\hat{\sigma}_x, \hat{\sigma}_y] = 2i\hat{\sigma}_z$ can be written as

$$\triangle \hat{\sigma}_x \triangle \hat{\sigma}_y \geqslant |\langle\hat{\sigma}_z\rangle|. \tag{23}$$

Fluctuations in the component $\hat{\sigma}_\gamma^{(i)}$ of the atomic dipole are said to be squeezed if $\triangle \hat{\sigma}_\gamma^{(i)}$ satisfies the following condition[24]

$$V(\hat{\sigma}_\gamma^{(i)}) = \triangle \hat{\sigma}_\gamma^{(i)}(t) - \sqrt{\left|\left\langle \hat{\sigma}_z^{(i)}(t)\right\rangle\right|} < 0, \tag{24}$$
$$\gamma = x \text{ or } y,\ i = 1, 2.$$

Here, we examine the effect of Shark shift on the atomic dipole squeezing of the first atom; so we plot several figures of the variance squeezing against the scaled time λt. First, we measure the variances $V(\hat{\sigma}_x^{(1)})$ and $V(\hat{\sigma}_y^{(1)})$ against $\left\langle \hat{\sigma}_z^{(1)} \right\rangle$. At $r = 1$, it is observed that the squeezing occurs in the quadrature variance $V(\hat{\sigma}_y^{(1)})$ and no squeezing occurs in the quadrature variance $V(\hat{\sigma}_x^{(1)})$. In this case it is found that maximum squeezing equal to 0.2, see **Fig.** 4**(a).** When Stark parameter decreases, it is found that the squeezing occurs in the quadrature variance $V(\hat{\sigma}_x^{(1)})$ and squeezing in the quadrature variance $V(\hat{\sigma}_y^{(1)})$ is almost non-existent, see **Fig.** 4**(b).** Second, we measure the variances $V(\hat{\sigma}_y^{(1)})$ and $V(\hat{\sigma}_z^{(1)})$ against $\left\langle \hat{\sigma}_x^{(1)} \right\rangle$. In **Fig.** 5**(a),** it is observed that no squeezing occurs in the two quadrature variances. By decreasing the value of r to 0.1 it is observed that, there is squeezing all the time in quadrature variance $V(\hat{\sigma}_z^{(1)})$ and no squeezing occurs in the quadrature variance $V(\hat{\sigma}_y^{(1)})$, see **Fig. 5(b).** Third, we measure the variances $V(\hat{\sigma}_x^{(1)})$ and $V(\hat{\sigma}_z^{(1)})$ against $\left\langle \hat{\sigma}_y^{(1)} \right\rangle$. By taking different value of r, it is observed that, there is squeezing most of the time in quadrature variance $V(\hat{\sigma}_z^{(1)})$ and no squeezing occurs in the quadrature variance $V(\hat{\sigma}_x^{(1)})$, see **Fig. 6**. Finally, we conclude that the influence of the Stark shift on the variance squeezing shows clearly. In the case of small values of Stark shift parameter we observe that more squeezing occur and the phenomenon gets more pronounced, also the maximum squeezing occurs after a short of time.

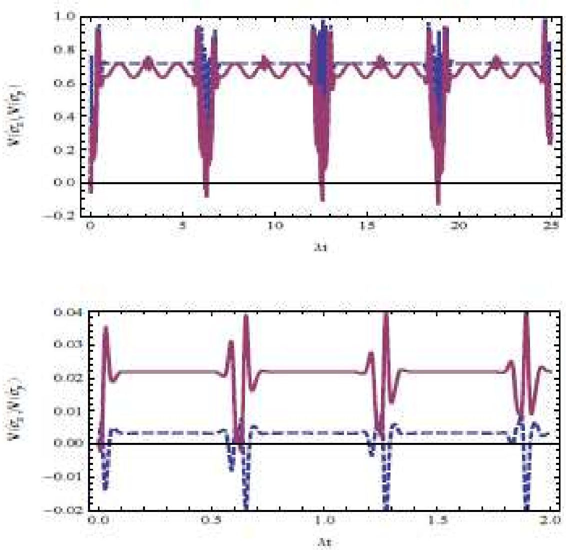

Fig. 4: The time evolution of the variance $V(\hat{\sigma}_x^{(1)})$ and $V(\hat{\sigma}_y^{(1)})$ against $\left\langle \hat{\sigma}_z^{(1)} \right\rangle$ as a function of the scaled time λt, with fixed parameters $\alpha_1 = \alpha_2 = 3$, (a) r=1, and (b) r=0.10. the dashed (blue) curve represent $V(\hat{\sigma}_x^{(1)})$ and the solid (red) curve represent $V(\hat{\sigma}_y^{(1)})$.

7 Second-order coherence for one of the field modes

In fact the correlation function is usually used to discuss the sub-Poissonian and super-Poissonian behaviour of the photon distribution from which we can distinguish between classical and nonclassical behaviour. The normalized second-order correlation function is defined

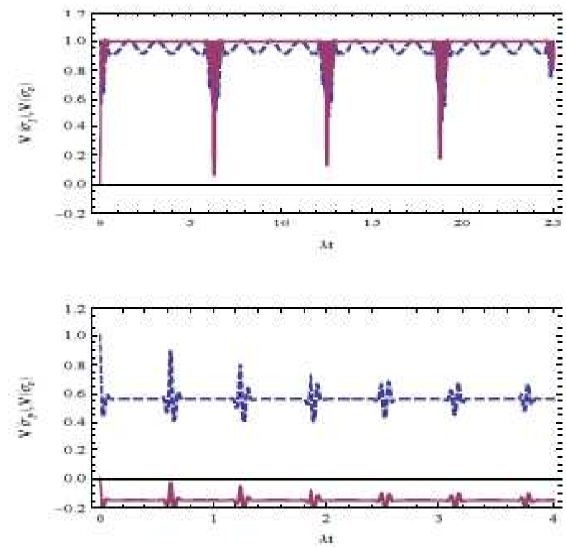

Fig. 5: The time evolution of the variance $V(\hat{\sigma}_y^{(1)})$ and $V(\hat{\sigma}_z^{(1)})$ against $\left\langle \hat{\sigma}_x^{(1)} \right\rangle$ and the same parameters of Fig. (4), the dashed (blue) curve represent $V(\hat{\sigma}_y^{(1)})$ and the solid (red) curve represent $V(\hat{\sigma}_z^{(1)})$.

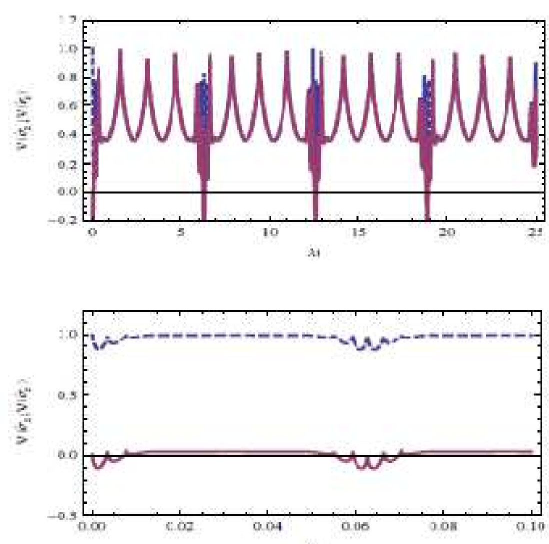

Fig. 6: The time evolution of the variance $V(\hat{\sigma}_x^{(1)})$ and $V(\hat{\sigma}_z^{(1)})$ against $\left\langle \hat{\sigma}_y^{(1)} \right\rangle$ and the same parameters of Fig. (4), (a) r=1 and (b) r=0.01, the dashed (blue) curve represent $V(\hat{\sigma}_x^{(1)})$ and the solid (red) curve represent $V(\hat{\sigma}_z^{(1)})$.

by [25]

$$g_i^{(2)}(t) = \frac{\left\langle \hat{a}_i^{\dagger 2} \hat{a}_i^2 \right\rangle}{\left\langle \hat{a}_i^{\dagger} \hat{a}_i \right\rangle^2}, \tag{25}$$

where the subscript i relates to the i^{th} mode. In order to discuss the behaviour of the correlation function of the first mode, we calculate the expectation values of the quantitys $\left\langle \hat{a}_1^{\dagger 2} \hat{a}_1^2 \right\rangle$ and $\left\langle \hat{a}_1^{\dagger} \hat{a}_1 \right\rangle$

$$\begin{aligned}\left\langle \hat{a}_1^{\dagger} \hat{a}_1 \right\rangle = \sum_{n_1,n_2=0}^{\infty} &[n_1 \left|X_1(n_1,n_2,t)^2\right| + (n_1+1)\left|X_2(n_1,n_2,t)^2\right| \\ &+(n_1+1)\left|X_3(n_1,n_2,t)^2\right| + (n_1+2)\left|X_4(n_1,n_2,t)^2\right|].\end{aligned} \tag{26}$$

$$\begin{aligned}\left\langle \hat{a}_1^{\dagger 2} \hat{a}_1^2 \right\rangle &= \langle \hat{n}_1(\hat{n}_1-1)\rangle \\ &= \sum_{n_1,n_2=0}^{\infty} [n_1(n_1-1)\left|X_1(n_1,n_2,t)\right|^2 \\ &\quad + n_1(n_1+1)\left|X_2(n_1,n_2,t)\right|^2 \\ &\quad + n_1(n_1+1)\left|X_3(n_1,n_2,t)\right|^2 \\ &\quad + (n_1+1)(n_1+2)\left|X4(n_1,n_2,t)\right|^2].\end{aligned} \tag{27}$$

By using Eqs. (25)-(27) we can easily get $g_1^{(2)}(t)$. We display the correlation function against the scaled time λt, to exhibit the effect of the Stark shift on the behavior of the state. When $r = 1$, it is noted that during the short period of time the behavior of the function start Poissanian then changes from super-Poissonian to sub-Poissonian until it reaches to the Poissonian behavior again and this behavior is repeated during the time period, see **Fig.** 7**(a).** In **Figs.** 7**(b)** and 7**(c),** the state has a light super-Poissonian behavior almost all the time. In the end we conclude that by increasing the effect of the Stark shift on the present system the behavior of the state shown super-Poissonian.

8 Conclusion

This paper has been proposed to study the effect of Stark shift parameter on some statistical aspects related to a system of non-linear interaction of a two two-level atoms and two modes of radiation field such as atomic inversion, entanglement, variance squeezing and degree of coherence. The constants of motion and the wave function of the system Hamiltonian are obtained. It is shown that the influence of the Stark shift on the atomic inversion is very small at $r = 1$, (almost disappears) and becomes strong when r decreased, where, we found that the atomic inversion is shifted upwards which means that energy is stored in the atomic system and it does not return to the ground state. The increase in the Stark shift

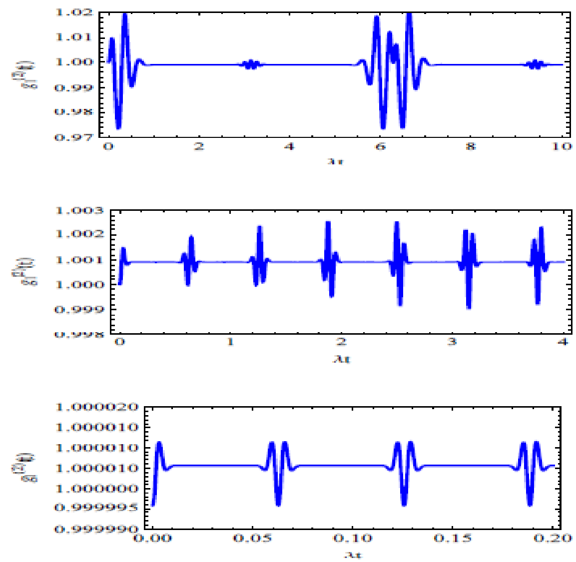

Fig. 7: The time evolution of the correlation function against time λt and the other parameters are the same as in Fig.(1), (a)r=1, (b)r=0.10 and (c)r=0.010.

effect leads to an increase in the degree of entanglement between the field and the atoms. We have observed that more squeezing occurs and the phenomenon is more pronounced in the case of small values of Stark shift parameter. Also maximum squeezing is occurred after a short time. By increasing the effect of the Stark shift, the behavior of the state leads to super-Poissonian behavior. Moreover, in all statistical aspects, when the Stark shift parameter increases the effect of the Stark shift is decreased.

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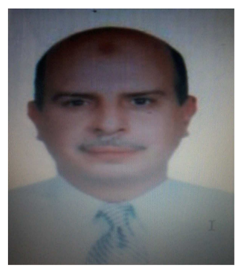

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