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On Metric Dimension of Two Constructed Families from Antiprism Graph

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Abstract: In this paper we compute the metric dimension of two families of graphs constructed from antiprism graph.

Keywords: Metrics dimension, basis, resolving set, antiprism

1 Introduction

For a connected graph G, the *distance* $d(u,v)$ between two vertices $u,v \in V(G)$ is the length of a shortest path between them in G. Let $W = \{w_1, w_2, ..., w_k\}$ be an ordered set of vertices of G and let v be a vertex of G, the *representation* of the vertex v with respect to W, denoted by $r(v|W)$ is the k-tuple $(d(v,w_1), d(v,w_2), ..., d(v,w_k))$. If distinct vertices of G have distinct representations with respect to W, then W is called a *resolving set* or *locating set* for G [2]. A resolving set of minimum cardinality is called a *metric basis* for G and this cardinality is the *metric dimension* of G $dim(G)$,

For a given ordered set of vertices $W = \{w_1, w_2, ..., w_k\}$ of a graph G, the ith component of $r(v|W)$ is 0 if and only if $v = w_i$. Thus, to show that W is a resolving set it suffices to verify that $r(x|W) \neq r(y|W)$ for each pair of distinct vertices $x, y \in V(G) \setminus W$.

Motivated by the problem of uniquely determining the location of an intruder in a network, the concept of metric dimension was introduced by Slater in [10,11] and studied independently by Harary and Melter in [3]. Applications of this invariant to the navigation of robots in networks are discussed in [8] and applications to chemistry in [2] while applications to problems of pattern recognition and image processing, some of which involve the use of hierarchical data structures are given in [9]. In [4,5,6] Imran et al. proved the metric dimension of some families of convex polytopes.

In [2] Chartrand et al. proved that a graph has metric dimension *1* if and only if it is a *path*, hence paths on n vertices constitute a family of graphs with constant metric dimension. Similarly, *cycle* with $n \geq 3$ vertices also constitute such a family of graphs as their metric dimension is *2*. In [1] J. Caceres et al. proved that:

$$dim(p_m \times C_n) = \begin{cases} 2, & if \quad n, \ is \ odd; \\ 3, & other \ wise. \end{cases}$$

Since *prisms* D_n are the trivalent plane graphs obtained by the cartesian product of the path P_2 with a cycle C_n; they also constitute a family of 3-*regular graphs* with constant metric dimension. Javaid et al. proved in [7] that the graph of *antiprism* A_n constitutes a family of regular graphs with constant metric dimension and $dim(A_n) = 3$ for every $n \geq 5$.

In this paper, we extend this study by considering two families of graph which are constructed from antiprism.

The antiprism A_n, $n \geq 3$, consists of an outer n-cycle $a_1 a_2 ... a_n$, an inner n-cycle $b_1 b_2 ... b_n$, and a set of n spokes $b_i a_i$ and $b_{i+1} a_i$, $i = 1,2,3,...,n$ where $n+i$ is taken modulo n.

The graph H_n is constructed from the graph A_n as follows: We delete the edges $a_i a_{i+1}$ from A_n. For each $i = 1,2,...,n$, we introduce new vertices c_i and d_i for a_i and b_i respectively. For each $i = 1,2,...,n$, introduce new edges $b_i c_i$, $a_i d_i$, $c_i d_i$ and $b_i c_i$, where $n+i$ is taken modulo n.

The graph R_n is constructed from the graph A_n as follows: We delete the edges $a_i a_{i+1}$ from A_n. For each $i = 1,2,...,n$, we introduce new vertices c_i and d_i for a_i and b_i respectively. For each $i = 1,2,...,n$ introduce new edges $b_i c_i$, $a_i d_i$, $c_i d_i$, $d_i d_{i+1}$ and $b_i c_i$ where $n+i$ is taken modulo n.

2 Main Results

Theorem: Let $n \geq 6$ be an integer then *dim*(H_n)=3.
Proof. We distinguish two cases.
Case (i): $n = 2k$, $k \geq 3$, $k \in IN$. We consider $W = \{b_1, b_2, b_{k+1}\} \subset V(H_n)$. We show that W is a resolving set for $V(H_n)$. For this we find the representations of the vertices *of* $V(H_n) \backslash W$ with respect to *W*. The representations of the vertices are as follows;

$$r(b_i \mid W) = \begin{cases} (i-1, i-2, 1+k-i), & \text{for } 3 \leq i \leq k; \\ (2k-i+1, 2k+2-i, i-1-k), & k+2 \leq i \leq n. \end{cases}$$

$$r(c_i \mid W) = \begin{cases} (1,2,k+1), & \text{for } i = 1; \\ (i, i-1, k+2-i), & \text{for } 2 \leq i \leq k+1; \\ (2k+2-i, 3+2k-i, i-k), & \text{for } k+2 \leq i \leq n. \end{cases}$$

$$r(a_i \mid W) = \begin{cases} (1,1,k), & i = 1; \\ (i, i-1, k+1-i), & 2 \leq i \leq k; \\ (k,k,1), & i = k+1; \\ (2k+1-i, 2k+2-i, i-k), & k+2 \leq i \leq n. \end{cases}$$

$$r(d_i \mid W) = \begin{cases} (2,2,k+1), & i = 1; \\ (i+1, i, k+2-i), & 2 \leq i \leq k; \\ (k+1, k+1, 2), & i = k+1; \\ (2k+2-i, 2k+3-i, i+1-k), & k+2 \leq i \leq n. \end{cases}$$

Note that there are no two vertices having the same representations implying that $dim(H_n) \leq 3$.

Now we show that $dim(H_n)\geq 3$, by proving that there is no resolving set W' with $|W'|=2$. We have the following possibilities;

(1). Both vertices belong to $\{b_i : i=1,2,...,n\}\subset V(H_n)$. Without loss of generality we suppose that one resolving vertex is b_1 and the other is b_t, $(2\leq t\leq k+1)$. For $2\leq t\leq k$ we have $r(c_1\,|\,\{b_1,b_t\})=r(a_n\,|\,\{b_1,b_t\})=(1,t)$ and for *t=k+1*, we have $r(a_k\,|\,\{b_1,b_t\})=r(a_{k+1}\,|\,\{b_1,b_t\})=(k,1)$, a contradiction.

(2). Both vertices belong to $\{c_i : i=1,2,...,n\}\subset V(H_n)$. Without loss of generality we suppose that one resolving vertex is c_1 , and the other is c_t , $(2\leq t\leq k+1)$. Then for $2\leq t\leq k$ we have $r(b_n\,|\,\{c_1,c_t\})=r(a_n\,|\,\{c_1,c_t\})=(2,t+1)$ for $t=k+1$ is $r(a_k\,|\,\{c_1,c_t\})=r(a_{k+1}\,|\,\{c_1,c_t\})=(k+1,1)$, a contradiction.

(3). Both vertices belong to $\{a_i : i=1,2,...,n\}\subset V(H_n)$. We suppose that one resolving vertex is a_1 and the other is a_t, $(2\leq t\leq k+1)$. Then for $2\leq t\leq k$ we have $r(c_1\,|\,\{a_1,a_t\})=r(b_n\,|\,\{a_1,a_t\})=(2,t+1)$ and for $t=k+1$, is $r(b_{k+1}\,|\,\{a_1,a_t\})=r(b_k\,|\,\{a_1,a_t\})=(k,1)$, a contradiction.

(4). Both vertices belong to $\{d_i : i=1,2,...,n\}\subset V(H_n)$. We suppose that one resolving vertex is d_1 and the other is d_t, $(2\leq t\leq k+1)$. For $2\leq t\leq k$ we have $r(a_n\,|\,\{d_1,d_t\})=r(b_n\,|\,\{d_1,d_t\})=(3,t+2)$ and for $t=k+1$, we have $r(c_{k+1}\,|\,\{d_1,d_t\})=r(c_{k+1}\,|\,\{d_1,d_t\})=(k+2,1)$, a contradiction.

(5). One vertex belong to $\{b_i : i=1,2,...,n\}\subset V(H_n)$ and another vertex belong to $\{c_i : i=1,2,...,n\}\subset V(H_n)$. Without loss of generality we suppose that one resolving vertex is b_1 and the other is c_t, $(1\leq t\leq k+1)$. For $1\leq t\leq k$ we have $r(a_n\,|\,\{b_1,c_t\})=r(b_n\,|\,\{b_1,c_t\})=(1,t+1)$ and for $t=k+1,$ is $r(a_k\,|\,\{b_1,c_t\})=r(a_{k+1}\,|\,\{b_1,c_t\})=(k,2)$, a contradiction.

(6). One vertex belong to $\{b_i : i=1,2,...,n\}\subset V(H_n)$ and another vertex belong to $\{a_i : i=1,2,...,n\}\subset V(H_n)$. Without loss of generality we suppose that one resolving vertex is b_1 and the other is a_t, $(1\leq t\leq k+1)$. Then for $1\leq t\leq k$ we have $r(a_n\,|\,\{b_1,a_t\})=r(c_1\,|\,\{b_1,a_t\})=(1,t+1)$ and for $t=k+1,$ $r(b_k\,|\,\{b_1,a_t\})=r(a_{k+2}\,|\,\{b_1,a_t\})=(k-1,2)$, a contradiction.

(7). One vertex belong to $\{c_i : i=1,2,...,n\}\subset V(H_n)$ and another vertex belong to $\{a_i : i=1,2,...,n\}\subset V(H_n)$. Without loss of generality we suppose that one resolving vertex is c_1 and the other is a_t, $(1\leq t\leq k+1)$. For $1\leq t\leq k-1$ we have $r(a_n\,|\,\{c_1,a_t\})=r(b_n\,|\,\{c_1,a_t\})=(2,t+1)$ and for *t=k*, $r(a_{k+1}\,|\,\{c_1,a_t\})=r(c_k\,|\,\{c_1,a_t\})=(k,2),$ similarly for *t=k+1*, we have $r(a_k\,|\,\{c_1,a_t\})=r(c_{k+2}\,|\,\{c_1,a_t\})=(k+1,2)$, a contradiction.

(8). One vertex belong to $\{a_i : i=1,2,...,n\}\subset V(H_n)$ and another vertex belong to $\{d_i : i=1,2,...,n\}\subset V(H_n)$. Without loss of generality we suppose that one resolving vertex is a_1 and the other is d_t, $(1\leq t\leq k+1)$. For $1\leq t\leq k$ we have $r(a_n\,|\,\{a_1,d_t\})=r(b_n\,|\,\{a_1,d_t\})=(2,t+2)$ and for $t=k+1$ the representation is $r(c_{k+1}\,|\,\{a_1,d_t\})=r(c_{k+2}\,|\,\{a_1,d_t\})=(k+2,1)$, a contradiction.

(9). One vertex belong to $\{b_i : i=1,2,...,n\}\subset V(H_n)$ and another vertex belong to $\{d_i : i=1,2,...,n\}\subset V(H_n)$. Without loss of generality we suppose that one resolving vertex is b_1 and the other is d_t, $(1\leq t\leq k+1)$. For $1\leq t\leq k-1$ we have $r(a_n\,|\,\{b_1,d_t\})=r(b_n\,|\,\{b_1,d_t\})=(1,t+2)$ and for $t=k$ the representation is

$r(a_k \mid \{b_1, d_t\}) = r(c_k \mid \{b_1, d_t\}) = (k,1),$ similarly for $t = k+1,$ we have $r(a_{k+1} \mid \{b_1, d_t\}) = r(c_{k+2} \mid \{b_1, d_t\}) = (k,1),$ a contradiction.

(10). One vertex belong to $\{c_i : i = 1,2,...,n\} \subset V(H_n)$ and another vertex belong to $\{d_i : i = 1,2,...,n\} \subset V(H_n)$. Without loss of generality we suppose that one resolving vertex is c_1 and the other is d_t, $(1 \le t \le k+1).$ For $1 \le t \le k-1$ we have $r(a_n \mid \{c_1, d_t\}) = r(b_n \mid \{c_1, d_t\}) = (2, t+2)$ and for $t = k,$ $r(a_k \mid \{c_1, d_t\}) = r(c_k \mid \{c_1, d_t\}) = (k+1,1)$, similarly for $t = k+1,$ we have $r(a_{k+1} \mid \{c_1, d_t\}) = r(c_{k+2} \mid \{c_1, d_t\}) = (k+1,1)$, a contradiction.

Hence, from above it follows that there is no resolving set with two vertices for $V(H_n)$ implying that $dim(H_n) = 3$.

Case(ii): $n = 2k+1$, $k \ge 3$, $k \in IN$. Let $W = \{b_1, b_2, b_{k+2}\} \subset V(H_n)$. We show that W is a resolving set for $V(H_n)$. For this we find the representations of vertices of $V(H_n) \setminus W$ with respect to W.

The representations of the vertices are as follow;

$$r(a_i \mid W) = \begin{cases} (1,1,k+1), & for\ i = 1; \\ (i, i-1, k+2-i), & for\ 2 \le i \le k+1; \\ (2k+2-i, 3+2k-i, i-k-1), & for\ k+2 \le i \le n. \end{cases}$$

$$r(b_i \mid W) = \begin{cases} (i-1, i-2, 2+k-i), & for\ 3 \le i \le k+1; \\ (2k-i+2, 2k+3-i, i-2-k), & k+3 \le i \le n. \end{cases}$$

$$r(c_i \mid W) = \begin{cases} (1,2,k+1), & i = 1; \\ (i, i-1, k+3-i), & 2 \le i \le k+1; \\ (k+1, k+1, 1), & i = k+2; \\ (2k+3-i, 2k+4-i, i-1-k), & k+3 \le i \le n. \end{cases}$$

$$r(d_i \mid W) = \begin{cases} (2,2,k+2), & for\ i = 1; \\ (i+1, i, k+3-i), & for\ 2 \le i \le k+1; \\ (2k+3-i, 4+2k-i, i-k), & for\ k+2 \le i \le n. \end{cases}$$

Proceeding on same line as in case(i) we note that there are no two vertices having the same representations, implying that $dim(H_n) \le 3$.

Also as in case(1), it can be shown that there is no set W' with $|W' = 2|$, such that W' is a resolving set for $V(H_n)$ for $n \ge 6$ and n is odd. Thus, $dim(H_n) \ge 3$. Hence $dim(H_n) = 3$ From case(i) and case(ii) we get $dim(H_n) = 3$.

Theorem: Let $n \ge 6$ be an integer then $dim(R_n) = 3$.

Proof. We distinguish two cases:

Case(i) $n = 2k$, $k \ge 3$, $k \in IN$. Suppose $W = \{b_1, b_2, b_{k+1}\} \subset V(R_n)$. We show that W is a resolving set for $V(R_n)$. For this we find the representations of the vertices of $V(R_n) \setminus W$ with respect to W.

The representations of the vertices are as follows;

$$r(b_i \mid W) = \begin{cases} (i-1, i-2, 1+k-i), & for\ 3 \le i \le k; \\ (2k-i+1, 2k+2-i, i-1-k), & k+2 \le i \le n. \end{cases}$$

$$r(c_i \mid W) = \begin{cases} (1,2,k+1), & for\ i=1; \\ (i, i-1, k+2-i), & for\ 2 \le i \le k+1; \\ (2k+2-i, 3+2k-i, i-k), & for\ k+2 \le i \le n. \end{cases}$$

$$r(a_i \mid W) = \begin{cases} (1,1,k), & i=1; \\ (i, i-1, k+1-i), & 2 \le i \le k; \\ (k,k,1), & i=k+1; \\ (2k+1-i, 2k+2-i, i-k), & k+2 \le i \le n. \end{cases}$$

$$r(d_i \mid W) = \begin{cases} (2,2,k+1), & i=1; \\ (i+1, i, k+2-i), & 2 \le i \le k; \\ (k+1, k+1, 2), & i=k+1; \\ (2k+2-i, 2k+3-i, i+1-k), & k+2 \le i \le n. \end{cases}$$

We note that there are no two vertices having the same representations implying that $dim(R_n) \le 3$.

Now we show that $dim(R_n) \ge 3$, by proving that there is no resolving set W' with $|W'| = 2$. We have the following possibilities,

(1). Both vertices belong to $\{b_i : i = 1,2,...,n\} \subset V(R_n)$. Without loss of generality we suppose the resolving vertices b_1 and b_t , $(2 \le t \le k+1)$. For $2 \le t \le k$ we have $r(c_1 \mid \{b_1, b_t\}) = r(a_n \mid \{b_1, b_t\}) = (1,t)$ and for $t = k+1,$ $r(a_k \mid \{b_1, b_t\}) = r(a_{k+1} \mid \{b_1, b_t\}) = (k,1),$ a contradiction.

(2). Both vertices belong to $\{c_i : i = 1,2,...,n\} \subset V(R_n)$. We suppose that one resolving vertex is c_1 and the other is c_t, $(2 \le t \le k+1)$. For $2 \le t \le k$ we have $r(b_n \mid \{c_1, c_t\}) = r(a_n \mid \{c_1, c_t\}) = (2, t+1)$ and for $t = k+1,$ $r(b_n \mid \{c_1, c_t\}) = r(a_n \mid \{c_1, c_t\}) = (2, t+1)$, a contradiction.

(3). Both vertices belong to $\{a_i : i = 1,2,...,n\} \subset V(R_n)$. We suppose that one resolving vertex is a_t and the other is a_t, $(2 \le t \le k+1)$. For $2 \le t \le k$ we have $r(c_1 \mid \{a_1, a_t\}) = r(b_n \mid \{a_1, a_t\}) = (2, t+1)$ and for $t = k+1$ we have $r(b_{k+1} \mid \{a_1, a_t\}) = r(b_k \mid \{a_1, a_t\}) = (k,1)$, a contradiction.

(4). Both vertices belong to $\{d_i : i = 1,2,...,n\} \subset V(R_n)$. We suppose that one resolving vertex is d_1 and the other is d_t, $(2 \le t \le k+1)$. For $2 \le t \le k$ we have $r(a_n \mid \{d_1, d_t\}) = r(b_n \mid \{d_1, d_t\}) = (3, t+2)$ and for $t = k+1,$ $r(c_{k+1} \mid \{d_1, d_t\}) = r(c_{k+1} \mid \{d_1, d_t\}) = (k+2, 1)$, a contradiction.

(5). One vertex belong to $\{b_i : i = 1,2,...,n\} \subset V(R_n)$ and another belong to $\{c_i : i = 1,2,...,n\} \subset V(R_n)$. Without loss of generality we suppose that one resolving vertex is b_1 and the other is c_t, $(1 \le t \le k+1)$. For $1 \le t \le k$ we have $r(a_n \mid \{b_1, c_t\}) = r(b_n \mid \{b_1, c_t\}) = (1, t+1)$ and for *t=k+1*, $r(a_k \mid \{b_1, c_t\}) = r(a_{k+1} \mid \{b_1, c_t\}) = (k,2)$, a contradiction.

(6). One vertex belong to $\{b_i : i = 1,2,...,n\} \subset V(R_n)$ and another belong to $\{a_i : i = 1,2,...,n\} \subset V(R_n)$. Without loss of generality we suppose that one resolving vertex is b_1 and the other is a_t, $(1 \le t \le k+1)$.

For $1 \le t \le k$ we have $r(a_n|\{b_1,a_t\}) = r(c_1|\{b_1,a_t\}) = (1,t+1)$ and for $t=k+1$ is $r(b_k|\{b_1,a_t\}) = r(a_{k+2}|\{b_1,a_t\}) = (k-1,2)$, a contradiction.

(7). One vertex belong to $\{c_i : i=1,2,...,n\} \subset V(R_n)$ and another belong to $\{a_i : i=1,2,...,n\} \subset V(R_n)$. Without loss of generality we suppose that one resolving vertex is c_1 and the other is a_t, $(1 \le t \le k+1)$. For $1 \le t \le k-1$ we have $r(a_n|\{c_1,a_t\}) = r(b_n|\{c_1,a_t\}) = (2,t+1)$ and for *t=k*, $r(a_{k+1}|\{c_1,a_t\}) = r(c_k|\{c_1,a_t\}) = (k,2)$, similarly for $t=k+1$ we have $r(a_k|\{c_1,a_t\}) = r(c_{k+2}|\{c_1,a_t\}) = (k+1,2)$, a contradiction.

(8). One vertex belong to $\{a_i : i=1,2,...,n\} \subset V(R_n)$ and another belong to $\{d_i : i=1,2,...,n\} \subset V(R_n)$. Without loss of generality we suppose that one resolving vertex a_1 and the other is d_t, $(1 \le t \le k+1)$. For $1 \le t \le k$ we have $r(a_n|\{a_1,d_t\}) = r(b_n|\{a_1,d_t\}) = (2,t+2)$ and for $t=k+1$, is $r(c_{k+1}|\{a_1,d_t\}) = r(c_{k+2}|\{a_1,d_t\}) = (k+2,1)$, a contradiction.

(9). One vertex belong to $\{b_i : i=1,2,...,n\} \subset V(R_n)$ and another belong to $\{d_i : i=1,2,...,n\} \subset V(R_n)$. Without loss of generality we suppose that one resolving vertex is b_1 and the other is d_t, $(1 \le t \le k+1)$. For $1 \le t \le k-1$ we have $r(a_n|\{b_1,d_t\}) = r(b_n|\{b_1,d_t\}) = (1,t+2)$ and for $t=k$ is $r(a_k|\{b_1,d_t\}) = r(c_k|\{b_1,d_t\}) = (k,1)$, similarly for $t=k+1$ than we have $r(a_{k+1}|\{b_1,d_t\}) = r(c_{k+2}|\{b_1,d_t\}) = (k,1)$, a contradiction.

(10). One vertex belong to $\{c_i : i=1,2,...,n\} \subset V(R_n)$ and another belong to $\{d_i : i=1,2,...,n\} \subset V(R_n)$. Without loss of generality we suppose that one resolving vertex is c_1 and the other is d_t, $(1 \le t \le k+1)$. For $1 \le t \le k-1$ we have $r(a_n|\{c_1,d_t\}) = r(b_n|\{c_1,d_t\}) = (2,t+2)$ and for $t=k$ is $r(a_k|\{c_1,d_t\}) = r(c_k|\{c_1,d_t\}) = (k+1,1)$, similarly for $t=k+1$ than we have $r(a_{k+1}|\{c_1,d_t\}) = r(c_{k+2}|\{c_1,d_t\}) = (k+1,1)$, a contradiction.

Hence, from above it follows that there is no resolving set with two vertices for $V(R_n)$ implying that $dim(R_n) \ge 3$.

Case (ii): $n = 2k+1$, $k \ge 3$, $k \in IN$. Consider $W = \{b_1, b_2, b_{k+2}\} \subset V(R_n)$. We show that W is a resolving set for $V(R_n)$. For this we find the representations of vertices of $V(R_n) \setminus W$ with respect to *W*. The representations of the vertices are as follow;

$$r(a_i|W) = \begin{cases} (1,1,k+1), & \text{for } i=1; \\ (i,i-1,k+2-i), & \text{for } 2 \le i \le k+1; \\ (2k+2-i, 3+2k-i, i-k-1), & \text{for } k+2 \le i \le n. \end{cases}$$

$$r(b_i|W) = \begin{cases} (i-1,i-2,2+k-i), & \text{for } 3 \le i \le k+1; \\ (2k-i+2, 2k+3-i, i-2-k), & k+3 \le i \le n. \end{cases}$$

$$r(c_i|W) = \begin{cases} (1,2,k+1), & i=1; \\ (i,i-1,k+3-i), & 2 \le i \le k+1; \\ (k+1,k+1,1), & i=k+2; \\ (2k+3-i, 2k+4-i, i-1-k), & k+3 \le i \le n. \end{cases}$$

$$r(d_i \mid W) = \begin{cases} (2,2,k+2), & \text{for } i=1; \\ (i+1,i,k+3-i), & \text{for } 2 \le i \le k+1; \\ (2k+3-i,4+2k-i,i-k), & \text{for } k+2 \le i \le n. \end{cases}$$

Proceeding on same line as in case(i) we observe that there are no two vertices having the same representations, implying that $dim(R_n) \le 3$.

Also as in case(1), it can be shown that there is no set W' with $|W'=2|$, such that W' is a resolving set for $V(R_n)$ for $n \ge 6$ and n is odd. Thus, $dim(R_n) \ge 3$. Hence $dim(R_n) = 3$. From case(i) and case(ii) we get $dim(R_n) = 3$.

3 Conclusion

In this paper we have studied the metric dimension of two families of graphs which are the extension of the antiprism graph. We have seen that the metric dimension of these graphs is finite and does not depend on the order of the graph and only three vertices appropriately chosen suffice to resolve all the vertices of these graphs.

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