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Spatial Vector Autoregressive Model with Calendar Variation for East Java Inflation and Money Supply

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Abstract: Inflation is a macroeconomic variable that can indicate the economy of a country, so it needs to be managed to a stability point. One way to control inflation is to regulate the money supply that was made by the Central Bank. This shows the relationship between inflation and money supply. In addition, inflation and money supply have space-time relationship. One character of inflation and money supply in Indonesia is the effect of Eid al Fitr on these two variables. The purpose of this study is to model the inflation and the money supply simultaneously with the effect of calendar variation Eid al Fitr in four cities in East Java, Indonesia namely Surabaya, Malang, Kediri and Jember. The model used is spatial vector autoregressive model with calendar variation. The result shows that there is a relationship between inflation and money supply although not in all cities, there is a space-time relationship on inflation as well as on the money supply. In addition, there is an effect of Eid al Fitr on inflation and the money supply in the four cities.

Keywords: Inflation, Money Supply, Spatial Vector Autoregressive, Calendar Variation

1 Introduction

Inflation and money supply are macroeconomic indicators of concern to a country. In Indonesia, these two indicators are controlled by Bank Indonesia as the Central Bank. Every country is always trying to keep inflation in the country stable. One of the policies of Bank Indonesia to maintain inflation stability is by controlling the money supply. Based on the phenomenon, it appears there is a relationship between inflation and the money supply, so to model it must be done simultaneously.

Research on inflation and the money supply has been done by several researchers in various regions, among which were [1] who studied the dynamics of money supply, exchange rate and inflation in Nigeria using the vector error correction model and concluded that the increase in the money supply increases inflation in the short term but in the long run does not have a significant effect. This conclusion is supported by the results of the study by [2] which gave the result that the money supply has a positive effect on inflation in Malawi. Furthermore [3] using vector autoregressive concluded that there is a unidirectional causality between the money supply and the interest rate on inflation in Nigeria. In the study, the money supply also has a positive effect on inflation. Study from [4] which was conducted in Pakistan showed the result that the money supply negatively affects inflation. Meanwhile [5], using monthly data from January 2000 to June 2015, the relationship between inflation and money supply was examined. The results of the research using ARIMAX-Neural network hybrid method is the money supply having a positive effect on inflation. While research of [6] suggested that the inflation rate in Colonial Massachusetts is more affected by fiscal policy than growth of money. Completing previous studies, [7] which examined the relationship between inflation rate, interest rate and money supply in Indonesia using VAR concluded that there is a negative relationship between the inflation rate and the interest rate. While the relationship between the inflation rate and the money supply is not proven in Indonesia.

Inflation and the money supply are related to inflation and money supply in the previous period. On the other hand, inflation and money supply in a region are related to inflation and money supply in other regions, especially those close in both geographically and economically.

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Based on these two phenomena, then inflation and money supply have space-time relationship (see [8], [9] and [10]). So, to model them we used space-time model.

In Indonesia, inflation and money supply are also influenced by Eid al Fitr because the majority of the Indonesian population are Mosleem (see [11], [12] and [13]). In the month of Shawwal (the month of Eid al Fitr) and Ramadhan (one month before Shawwal), generally the prices increase because in that month, people consume more goods and services from other month. This trigger an increase in inflation at these months. Noteworthy is that Eid al Fitr occurs on different dates and / or months each year because Eid al-Fitr is based on lunar calendar. This case is known as calendar variation.

Based on the previous explanation, then there are several aspects that need to be considered in modeling inflation and money supply. First, there is a relationship between inflation and the money supply. Second, there is a space-time relationship to inflation and the money supply. Third, there is effect of calendar variations caused by Eid al Fitr on inflation and money supply. However, there has been no research that models inflation and money supply that accommodates the all three aspects in one model. Because of the importance of controlling inflation, the aims of this study is to model inflation and money supply simultaneously (as a case study it was conducted in four cities in East Java, Indonesia i.e Surabaya, Malang, Kediri and Jember). To model inflation and money supply with these three aspects, we propose the spatial vector autoregressive model with calendar variation.

The rest of the paper is organized as follow. Section two presents literature review on space time and calendar variation modeling. On the next section we explore the data and spatial vector autoregressive model with calendar variation that we use in this study. Section four presents the result and discussion. The last section is Conclusion.

2 Literature Review

One space-time model in statistics is Space Time Autoregressive (STAR) which was first developed by [14]. STAR has a disadvantage, i.e. this model assumes space-time phenomena to have homogeneous characteristics in all location. This results that the parameters being constant or equal for all locations, so the model is less flexible because it can not be applied to locations with heterogeneous characteristics. Further, [15] developed the STAR model into a Generalized Space Time Autoregressive model (GSTAR) assuming the parameters in each location to have different values. Furthermore, [16] also studied the method of the parameters estimation of Ordinary Least Square (OLS) along with the characteristic of the estimator. Meanwhile, the Generalized Space Time Autoregressive-Seemingly Unrelated Regression (GSTAR-SUR) model developed by [17] is used when the GSTAR model has error correlation between locations. Hereafter, [18] developed a GSTAR model for seasonal data with errors correlated between locations and named Seasonal-Generalized Space Time Autoregressive-Seemingly Unrelated Regression (S-GSTAR-SUR). While the Generalized Space Time Autoregressive Exogen-Generalized Least Square (GSTARX-GLS) model is the development of GSTAR model that has predictor variables and there is error correlation between location, developed by [11]. Furthermore, [19] extended the GSTAR-Integrated model with an Autoregressive Conditional Heteroscedasticity (ARCH) error, it is called GSTARI ARCH model and it is applied to predict a consumer price index phenomenon at several cities in North Sumatera province using Generalized Least Square (GLS) method.

In addition to STAR and GSTAR, Vector Autoregressive (VAR) can also be used for space-time modeling. Initially VAR (p) developed by [20], and subsequently [21], were used to examine the interrelationships of several interacting variables in time series data. According to [22], the VAR (p) model can also be used to analyze spatio temporal data. Data used in the study is one time series data observed in several locations. The weakness of GSTAR and VAR for space-time analysis is that both models can only use one variable only. For space-time analysis with more than one variable, Spatial VAR (SpVAR) is used. This model was developed by [23] and [24]. In the model [23] the coefficients at each location are assumed to be the same while on the model [24] the coefficients can be different. To accommodate the effects of calendar variations, we developed SpVAR model with calendar variations.

3 Methodology

In this section we discuss the data and models that we use in this paper.

Data

The data used were monthly inflation data for four cities in East Java namely Surabaya, Malang, Kediri and Jember obtained from website of Badan Pusat Statistik (bps.go.id) for 12 years starting from January 2003 to December 2014. While monthly data of money supply for the same city and period is obtained from Bank Indonesia. The data used as an in-sample were the first 11 years while the last one year data are used as an out-sample.

In this paper we propose the method as a procedure to use the SpVAR model with calendar variation.

Spatial Vector Autoregressive (1,p) Model with Calendar Variation

To model the inflation and money supply in four cities in East Java, Indonesia, spatial vector autoregressive $(1,p)$ model with calendar variation is used. In addition we also used SpVAR model. Next, the best model is selected based on the smallest RMSE of out-sample. Spatial vector autoregressive $(1,p)$ model with calendar variation can be expressed as 1

$$y_t = \mathbf{X}_t \gamma + \mathbf{B}_1 y_{t-1} + \cdots + \mathbf{B}_p y_{t-p} + \xi_t. \quad (1)$$

While according [24], spatial vector autoregressive $(1,p)$ model can be expressed as

$$y_t = \mathbf{B}_1 y_{t-1} + \cdots + \mathbf{B}_p y_{t-p} + \eta_t, \quad (2)$$

where $\underset{NK\times 1}{y_t} = [y_{11t}, \ldots, y_{N1t}, \ldots, y_{1Kt}, \ldots, y_{NKt}]'$, y_{nkt} the value of $k-th$ variable observed at $n-th$ location at time t and
$\underset{NKm\times 1}{\gamma} = \left[\gamma_{11} \ \ldots \ \gamma_{N1} \ \ldots \ \gamma_{1K} \ \ldots \ \gamma_{NK}\right]'$,
$\underset{m\times 1}{\gamma_{nk}} = \left[\gamma_{nk1} \ \gamma_{nk2} \ \ldots \ \gamma_{nkm}\right]'$, m is the number of dummy variable for calendar variation,
$\mathbf{X}_t = diag\left(x_{11t}, \ldots, x_{N1t}, \ldots, x_{1Kt}, \ldots, x_{NKt}\right)$,
$\underset{1\times m}{x_{nkt}} = \left[x_{nk1t} \ x_{nk2t} \ \ldots \ x_{nkmt}\right]$.
Following [24], $\mathbf{B}_h$ is defined as

$$\underset{NK\times NK}{\mathbf{B}_h} = \begin{bmatrix} \mathbf{A}_{11}^{(h)} & \mathbf{A}_{12}^{(h)} & \ldots & \mathbf{A}_{1K}^{(h)} \\ \mathbf{A}_{21}^{(h)} & \mathbf{A}_{22}^{(h)} & \ldots & \mathbf{A}_{2K}^{(h)} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{A}_{K1}^{(h)} & \mathbf{A}_{K2}^{(h)} & \ldots & \mathbf{A}_{KK}^{(h)} \end{bmatrix},$$

where $\underset{N\times N}{\mathbf{A}_{kr}^{(h)}} = \Phi_{kr}^{(h0)} + \Phi_{kr}^{(h1)} \mathbf{W}_{kr}$,

$$\underset{N\times N}{\Phi_{kr}^{(hl)}} = diag\left\{\left[\phi_{1kr}^{(hl)}, \ldots, \phi_{Nkr}^{(hl)}\right]'\right\}, k,r = 1,\ldots,K; h = 1,\ldots,p \quad (3)$$

where $\mathbf{W}_{kr}$ is $N\times N$ spatial weight matrix, the element of $\mathbf{W}_{kr}$ are $w_{kr}(i,j)$ which are known apriori and assumed to have non-negative value and positive if location i and j are neighbour and zero if spatial weights are assumed to be fixed all the time. Autoregressive coefficient is assumed to vary between locations.
$\underset{NK\times 1}{\xi_t} = [\xi_{11t}, \ldots, \xi_{N1t}, \ldots, \xi_{1Kt}, \ldots, \xi_{NKt}]'$,
ξ_{nkt} : error from the model of the $k-th$ variable on the $n-th$ location at time t and it is assumed that $\xi_t \sim N(0,\Sigma)$.

$$\underset{NK\times NK}{\Sigma} = \begin{bmatrix} E\left(\xi_{1t}\xi'_{1t}\right) & E\left(\xi_{1t}\xi'_{2t}\right) & \ldots & E\left(\xi_{1t}\xi'_{Kt}\right) \\ E\left(\xi_{2t}\xi'_{1t}\right) & E\left(\xi_{2t}\xi'_{2t}\right) & \ldots & E\left(\xi_{2t}\xi'_{Kt}\right) \\ \vdots & \vdots & \ddots & \vdots \\ E\left(\xi_{Kt}\xi'_{1t}\right) & E\left(\xi_{Kt}\xi'_{2t}\right) & \ldots & E\left(\xi_{Kt}\xi'_{Kt}\right) \end{bmatrix},$$

where $\underset{N\times 1}{\xi_{kt}} = \begin{pmatrix} \xi_{1kt} \\ \xi_{2kt} \\ \vdots \\ \xi_{Nkt} \end{pmatrix}$.

With the restriction of equation (3) then equation (1) can be written as

$$y_t = \mathbf{X}_t \gamma + \mathbf{V}_t \beta + \xi_t, \quad (4)$$

where $\mathbf{V}_t = \left[\mathbf{V}_{1t} \ \mathbf{V}_{2t} \ \ldots \ \mathbf{V}_{pt}\right]$, $\beta = \left[\beta^{(1)} \ \beta^{(2)} \ \ldots \ \beta^{(p)}\right]'$,
$\mathbf{V}_{it} = diag\left(v_{11,t-i}, \ldots, v_{N1,t-i}, \ldots, v_{1K,t-i}, \ldots, v_{NK,t-i}\right)$
$v_{nk,t-i} = \left[y_{n1,t-i} \ y_{n1,t-i}^{(k1)} \ \ldots \ y_{nK,t-i} \ y_{nK,t-i}^{(kK)}\right]$,
$y_{nk,t-i}^{(kj)} = \sum_{u=1}^{N} w_{kj}(n,u) y_{uk,t-i}$ $i = 1,2,\ldots,p$ and
$\beta_{nk}^{(i)} = \left[\beta_{nk1}^{(i0)} \ \beta_{nk1}^{(i1)} \ \ldots \ \beta_{nkK}^{(i0)} \ \beta_{nkK}^{(i1)}\right]'$.

Equation (4) can also be written as

$$y_t = \mathbf{Z}_t \theta + \xi_t \quad (5)$$

where $\mathbf{Z}_t = \left[\mathbf{X}_t \ \mathbf{V}_{1t} \ \mathbf{V}_{2t} \ \ldots \ \mathbf{V}_{pt}\right]$ and

$$\theta = \begin{bmatrix} \gamma \\ \beta^{(1)} \\ \beta^{(2)} \\ \vdots \\ \beta^{(p)} \end{bmatrix}.$$

Equation (5) can be elaborated to become

$$\begin{bmatrix} y_{11t} \\ y_{21t} \\ \vdots \\ y_{N1t} \\ y_{12t} \\ y_{22t} \\ \vdots \\ y_{N2t} \\ \vdots \\ y_{1Kt} \\ y_{2Kt} \\ \vdots \\ y_{NKt} \end{bmatrix} = \left[\mathbf{X}_t \ \mathbf{V}_{1t} \ \mathbf{V}_{2t} \ \ldots \ \mathbf{V}_{pt}\right] \begin{bmatrix} \gamma \\ \beta^{(1)} \\ \beta^{(2)} \\ \vdots \\ \beta^{(p)} \end{bmatrix} + \begin{bmatrix} \xi_{11t} \\ \xi_{21t} \\ \vdots \\ \xi_{N1t} \\ \xi_{12t} \\ \xi_{22t} \\ \vdots \\ \xi_{N2t} \\ \vdots \\ \xi_{1Kt} \\ \xi_{2Kt} \\ \vdots \\ \xi_{NKt} \end{bmatrix} \quad (6)$$

Parameter Estimation of the Spatial Vector Autoregressive model with calendar variation is done using the Full Information Maximum Likelihood (FIML) method.

Modeling procedure

The modeling procedure of SpVAR $(1,p)$ with calendar variation is done by following the steps

Step 1. Specifies the dummy variable for calendar variations. This study used 8 dummy variables, namely

$$x_{bi,t}=\begin{cases}1, & \text{if } t \text{ is a month before Eid al Fitr with Eid al Fitr in week}-i\\ 0, & \text{for other month}\end{cases}$$

$$x_{i,t}=\begin{cases}1, & \text{if } t \text{ is Eid al Fitr month with Eid al Fitr in week}-i\\ 0, & \text{for other month}\end{cases}$$

for $i=1,2,3,4$.

Next, the variable that significantly affects endogenous variables is chosen using backward procedure in modeling

$$y_t=\mathbf{X}_t\gamma+w_t \tag{7}$$

Generalized Least Square (GLS) can be used to estimate the parameter of model (7).

Step 2. Specify spatial weight. This study uses uniform weight.

Step 3. Obtain error from equation (7), w_t, and model the error using SpVAR model to obtain the order. The determination of order is done by testing the null hypothesis and alternative hypothesis of (8) using likelihood ratio test sequentially

$$H_0^1:\beta^{(M)}=\mathbf{0} \text{ vs } H_1^1:\beta^{(M)}\neq\mathbf{0}, \tag{8}$$

where M is a positive integer and it is assumed that M is known as the upper limit of order of the SpVAR with calendar variation. If the null hypothesis is not rejected, then the hypothesis test for the order $(M-1)$ is conducted. Each null hypothesis is tested conditional on the previous null hypothesis is true. This procedure stops if one null hypothesis is rejected. If H_0^i is rejected, then $\hat{p}=M-i+1$ is chosen as order for SpVAR model with calendar variation. The test statistics for this hypothesis is

$$G=\ln\left(\frac{|\hat{\Sigma}_0|}{|\hat{\Sigma}|}\right)^T$$

where $\hat{\Sigma}_0$ is the estimator of the variance covariance matrix under H_0 and $\hat{\Sigma}$ is the estimator of the variance covariance matrix under population. The distribution of G is chi square with degrees of freedom of $2NK^2$, where N is the number of locations and K is the number of variables.

Step 4. Estimate the parameter of SpVAR model with calendar variation and SpVAR model using Full Information Maximum Likelihood (FIML).

Step 5. Testing the significance of parameters

4 Results and Discussion

Determination on Used Calendar Variation Variable

From 8 dummy variables that is used in this study, we select significant variables on endogenous variables using model (7). Based on these analyzes, significant variables of calendar variations and used in the model are summarized in Table 1

Table 1: Variables of Calendar Variations Used in This Study

Endogenous Variable	Calendar Variation Variable
Inflation in Surabaya	$x_{b1,t}, x_{b2,t}$
Inflation in Malang	$x_{b1,t}, x_{b2,t}$
Inflation in Kediri	$x_{b1,t}, x_{2,t}$
Inflation in Jember	$x_{b1,t}, x_{2,t}$
Money Supply in Surabaya	$x_{b1,t}, x_{b2,t}, x_{1,t}, x_{3t}, x_{4t}$
Money Supply in Malang	$x_{b1,t}, x_{b2,t}, x_{1t}, x_{3,t}, x_{4,t}$
Money Supply in Kediri	$x_{b1,t}, x_{b2,t}, x_{1,t}, x_{3,t}, x_{4,t}$
Money Supply in Jember	$x_{b1,t}, x_{b2,t}, x_{1,t}, x_{3,t}, x_{4t}$

The variables of the calendar variations used in this model vary for each equation. However, there is one variable used in all equations, namely $x_{b1,t}$. The value of dummy variable $x_{b1,t}$ is 1 if month t is the month of Ramadhan if Eid al Fitr occurs in the first week. This shows that inflation and money supply in all four cities are influenced by Ramadhan month when Eid fitri occurs on the first week. That is, inflation and money supply are going higher during Ramadhan than any other month.

Furthermore, we determine the SpVAR($1,p$) order through hypothesis testing on (8)

Determination of Order

Based on the number of data, for the parameters can still be estimated, then the maximum order limit specified is 4. The value of the test statistical G for the fourth order obtained is 4.3806, while the value of $\chi^2_{(0.10;32)}$ is 42.5847, so the null hypothesis could not be rejected. Next, the G value for the third order is calculated and the G value is 32.75188, so the null hypothesis could not be rejected. The next step is to calculate the G value for the second order and obtain the value of G equal to 342.7024, so that the null hypothesis could be rejected. Based on this procedure, the chosen order is two.

Parameter Estimation

Based on the result of determination variables of calendar variation and order determination, the model used is SpVAR (1,2) with calendar variation and SpVAR (1,2) model. The dummy variable used in accordance with Table 1. The RMSE of outsample for SpVAR (1,2) and SpVAR(1,2) with calendar variation are presented in Table 2.

Table 2 shows that RMSE for SpVAR(1,2) model with calendar variation is smaller than for SpVAR (1,2) model for five out of eight equation, so we choose SpVAR(1,2)

Table 2: RMSE of SpVAR (1,2) and SpVAR (1,2) with calendar variation

Variable	RMSE	
	SpVAR (1,2)	SpVAR (1,2) with Calendar Variation
Inflation in Surabaya	0.4147	0.4708
Inflation in Malang	0.3874	0.2607
Inflation in Kediri	0.5572	0.6637
Inflation in Jember	0.5186	0.5009
Money Supply in Surabaya	1.7507	1.7375
Money Supply in Malang	0.8656	0.8577
Money Supply in Kediri	0.8514	0.8491
Money Supply in Jember	0.4905	0.4921

with calendar variation to model inflation and money supply in four cities in East Java, Indonesia. From the parameter estimation result, the model is

a. SpVAR(1,2) model with calendar variation for inflation in Surabaya is

$$\begin{aligned}\hat{y}_{11t} &= 3.69x_{b1,t} + 0.49x_{b2,t} - 0.04y_{11,t-1}\\ &+ 0.05y_{12,t-1} - 0.13y_{11,t-2} - 0.12y_{12,t-2}\\ &+ 0.14\left[0.33y_{21,t-1} + 0.33y_{31,t-1} + 0.33y_{41,t-1}\right]\\ &- 0.01\left[0.33y_{22,t-1} + 0.33y_{32,t-1} + 0.33y_{42,t-1}\right]\\ &+ 0.04\left[0.33y_{21,t-2} + 0.33y_{31,t-2} + 0.33y_{41,t-2}\right]\\ &+ 0.37\left[0.33y_{22,t-2} + 0.33y_{32,t-2} + 0.33y_{42,t-2}\right].\end{aligned} \tag{9}$$

b. SpVAR(1,2) model with calendar variation for inflation in Malang is

$$\begin{aligned}\hat{y}_{21t} &= 3.79x_{b1,t} + 0.994x_{b2,t} - 0.11y_{21,t-1}\\ &- 0.38y_{22,t-1} - 0.14y_{21,t-2} - 0.06y_{22,t-2}\\ &+ 0.26\left[0.33y_{11,t-1} + 0.33y_{31,t-1} + 0.33y_{41,t-1}\right]\\ &+ 0.29\left[0.33y_{12,t-1} + 0.33y_{32,t-1} + 0.33y_{42,t-1}\right]\\ &+ 0.04\left[0.33y_{11,t-2} + 0.33y_{31,t-2} + 0.33y_{41,t-2}\right]\\ &+ 0.02\left[0.33y_{12,t-2} + 0.33y_{32,t-2} + 0.33y_{42,t-2}\right].\end{aligned} \tag{10}$$

c. SpVAR(1,2) model with calendar variation for inflation in Kediri is

$$\begin{aligned}\hat{y}_{31t} &= 5.42x_{b1,t} + 0.42x_{2,t} - 0.06y_{31,t-1}\\ &+ 0.23y_{32,t-1} - 0.16y_{31,t-2} + 0.49y_{32,t-2}\\ &+ 0.17\left[0.33y_{11,t-1} + 0.33y_{21,t-1} + 0.33y_{41,t-1}\right]\\ &- 0.07\left[0.33y_{12,t-1} + 0.33y_{22,t-1} + 0.33y_{42,t-1}\right]\\ &+ 0.05\left[0.33y_{11,t-2} + 0.33y_{21,t-2} + 0.33y_{41,t-2}\right]\\ &- 0.42\left[0.33y_{12,t-2} + 0.33y_{22,t-2} + 0.33y_{42,t-2}\right].\end{aligned} \tag{11}$$

d. SpVAR(1,2) model with calendar variation for inflation in Jember is

$$\begin{aligned}\hat{y}_{41t} &= 4.00x_{b1,t} + 0.34x_{2,t} + 0.27y_{41,t-1}\\ &- 0.004y_{42,t-1} - 0.20y_{41,t-2} + 0.03y_{42,t-2}\\ &- 0.13\left[0.33y_{11,t-1} + 0.33y_{21,t-1} + 0.33y_{31,t-1}\right]\\ &+ 0.10\left[0.33y_{12,t-1} + 0.33y_{22,t-1} + 0.33y_{32,t-1}\right]\\ &+ 0.09\left[0.33y_{11,t-2} + 0.33y_{21,t-2} + 0.33y_{31,t-2}\right]\\ &- 0.01\left[0.33y_{12,t-2} + 0.33y_{22,t-2} + 0.33y_{32,t-2}\right].\end{aligned} \tag{12}$$

e. SpVAR(1,2) model with calendar variation for money supply in Surabaya is

$$\begin{aligned}\hat{y}_{12t} &= 2.15x_{b1,t} + 1.31x_{b2,t} - 0.86x_{1,t} + 0.93x_{3,t} + 2.03x_{4,t}\\ &- 0.17y_{11,t-1} - 0.58y_{12,t-1} + 0.12y_{11,t-2} - 0.39y_{12,t-2}\\ &+ 0.07\left[0.33y_{21,t-1} + 0.33y_{31,t-1} + 0.33y_{41,t-1}\right]\\ &- 0.22\left[0.33y_{22,t-1} + 0.33y_{32,t-1} + 0.33y_{42,t-1}\right]\\ &- 0.21\left[0.33y_{21,t-2} + 0.33y_{31,t-2} + 0.33y_{41,t-2}\right]\\ &+ 0.29\left[0.33y_{22,t-2} + 0.33y_{32,t-2} + 0.33y_{42,t-2}\right].\end{aligned} \tag{13}$$

f. SpVAR (1,2) model with calendar variation for money supply in Malang is

$$\begin{aligned}\hat{y}_{22t} &= 0.66x_{b1,t} + 0.39x_{b2,t} - 0.25x_{1,t} + 0.47x_{3,t} + 0.57x_{4,t}\\ &- 0.005y_{21,t-1} - 0.68y_{22,t-1} + 0.01y_{21,t-2} - 0.33y_{22,t-2}\\ &- 0.02\left[0.33y_{11,t-1} + 0.33y_{31,t-1} + 0.33y_{41,t-1}\right]\\ &- 0.04\left[0.33y_{12,t-1} + 0.33y_{32,t-1} + 0.33y_{42,t-1}\right]\\ &- 0.05\left[0.33y_{11,t-2} + 0.33y_{31,t-2} + 0.33y_{41,t-2}\right]\\ &+ 0.03\left[0.33y_{12,t-2} + 0.33y_{32,t-2} + 0.33y_{42,t-2}\right].\end{aligned} \tag{14}$$

g. SpVAR(1,2) model with calendar variation for money supply in Kediri is

$$\begin{aligned}\hat{y}_{32t} &= 0.96x_{b1,t} + 0.52x_{b2,t} - 0.36x_{1,t} + 0.36x_{3,t} + 0.74x_{4,t}\\ &+ 0.003y_{31,t-1} - 0.53y_{32,t-1} - 0.03y_{31,t-2} - 0.30y_{32,t-2}\\ &- 0.05\left[0.33y_{11,t-1} + 0.33y_{21,t-1} + 0.33y_{41,t-1}\right]\\ &- 0.14\left[0.33y_{12,t-1} + 0.33y_{22,t-1} + 0.33y_{42,t-1}\right]\\ &- 0.05\left[0.33y_{11,t-2} + 0.33y_{21,t-2} + 0.33y_{41,t-2}\right]\\ &+ 0.01\left[0.33y_{12,t-2} + 0.33y_{22,t-2} + 0.33y_{42,t-2}\right].\end{aligned} \tag{15}$$

h. SpVAR(1,2) model with calendar variation for money supply in Jember is

$$\begin{aligned}\hat{y}_{42t} &= 0.44x_{b1,t} + 0.27x_{b2,t} - 0.31x_{1,t} + 0.25x_{3,t} + 0.33x_{4,t}\\ &+ 0.03y_{41,t-1} - 0.54y_{42,t-1} - 0.002y_{41,t-2} - 0.23y_{42,t-2}\\ &+ 0.03\left[0.33y_{11,t-1} + 0.33y_{21,t-1} + 0.33y_{31,t-1}\right]\\ &- 0.03\left[0.33y_{12,t-1} + 0.33y_{22,t-1} + 0.33y_{32,t-1}\right]\\ &- 0.03\left[0.33y_{11,t-2} + 0.33y_{21,t-2} + 0.33y_{31,t-2}\right]\\ &- 0.02\left[0.33y_{12,t-2} + 0.33y_{22,t-2} + 0.33y_{32,t-2}\right].\end{aligned} \tag{16}$$

where y_{nkt} is the value of k-th variable observed at n-th location at time t. In this research $k = 1$ is inflation and $k = 2$ is money supply, while $n = 1$ is Surabaya, $n = 2$ is Malang, $n = 3$ is Kediri and $n = 4$ is Jember. The significant variables in the model 9 - 16 are summarized in Table 3.

Table 3: Summary of Significant Variables

Endogenous Variable	Significant Variable
Inflation in Surabaya	$x_{b1,t}$; $x_{b2,t}$
Inflation in Malang	$y_{21,t-2}$; $y^*_{21,t-1}$; $x_{b1,t}$; $x_{b2,t}$
Inflation in Kediri	$y_{31,t-2}$; $y_{32,t-2}$; x_{b1t}; x_{2t}
Inflation in Jember	$y_{41,t-1}$; $y_{41,t-2}$; x_{b1t}; x_{2t}
Money Supply in Surabaya	$y_{11,t-1}$; $y_{12,t-1}$; $y_{12,t-2}$; $y^*_{11,t-2}$; $x_{b1,t}$; $x_{b2,t}$; $x_{3,t}$; $x_{4,t}$
Money Supply in Malang	$y_{22,t-1}$; $y_{22,t-2}$; $y^*_{21,t-2}$; $x_{b1,t}$; $x_{b2,t}$; $x_{3,t}$; $x_{4,t}$
Money Supply in Kediri	$y_{32,t-1}$; $y_{32,t-2}$; $y^*_{32,t-1}$; $x_{b1,t}$; $x_{b2,t}$; $x_{3,t}$; $x_{4,t}$
Money Supply in Jember	$y_{41,t-1}$; $y_{42,t-1}$; $y_{42,t-2}$; x_{b1t}; x_{b2t}; $x_{1,t}$; $x_{3,t}$; $x_{4,t}$

where $y^*_{nkt} = \sum_{i \neq n} 0.33 y_{ikt}$

Based on Table 3, it can be seen that inflation in Kediri and Jember are not related to inflation in other cities. It is also not influenced by the money supply in the city itself or other cities. While inflation in Malang is associated to inflation in Malang and in other cities in the previous period. This finding is in line with finding of [10]. Based on Table 3 it can also be seen that the money supply in Surabaya and Jember are influenced by inflation and related to the money supply in the city itself in the past. While the money supply in Surabaya is also related to inflation in other cities. Meanwhile, money supply in Malang is related to the money supply in Malang in the past and influenced by inflation in other cities. This finding does not inline with the findings of [7] which states that there is no relationship between inflation and money supply. This is because the data used in this research are inflation and money supply in four cities in East Java while in [7] the data used was the inflation and the money supply in Indonesia nationally. Table 3 shows that both inflation and money supply in all four cities are influenced by calendar variation of Eid al Fitr. From equation 9-16, it can be seen that the coefficient of the significant dummy variable is positive. This indicates that inflation and money supply in the four cities increase in Ramadhan (one month before Eid al Fitr) if Eid al Fitr occurs in the first week. Money supply in the four cities also increases during the month of Shawwal (the month of Eid al Fitr), especially if Eid al Fitr occurs in the third or fourth week.

5 Conclusion

Based on the modeling results, it can be concluded that there is a relationship between inflation and the money supply although not in all cities. In some cities, inflation and money supply also indicate the space-time relationship. It also can be concluded that there is effect of calendar variation of Eid al Fitr on inflation and money supply in all cities.

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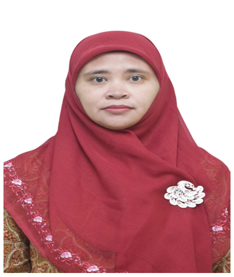

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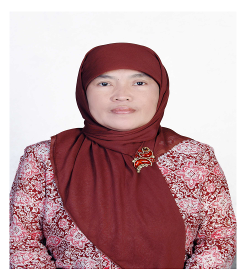

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