Application of PSO with Different Typical Neighbor Structure to Complex Job Shop Scheduling Problem

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Abstract: Job shop scheduling is to schedule a set of jobs on a set of machines, which is subject to the constraint that each machine can process at most one job at a given time and the fact that each job has a specified processing order through the machines. It is not only a NP-hard problem, it also has the well-earned reputation of being one of the strong combinatorial problems in manufacturing systems. In this paper, the job-shop scheduling problem (JSSP), with the optimization goal of the scheduling problem is minimum of total process time \( C_{\text{max}} \), was modeled. An improved particle swarm optimization with acceleration factor (AFPSO) is proposed to improve the ability of particles to explore the global and local optimization solutions, and to reduce the probability of being trapped into the local optima. The neighbor structure of different particle candidate was studied to improve the information exchange speed in optimizing process. Simulation results show that the proposed model and algorithm are effective to task evaluation and implementation.

Keywords: Job Shop Scheduling Problem, Particle Swarm Optimization, Topology Structure

1 Introduction

Job shop scheduling [1,2] is to schedule a set of jobs on a set of machines, which is subject to the constraint that each machine can process at most one job at a given time and the fact that each job has a specified processing order through the machines. It is not only a NP-hard problem, it also has the well-earned reputation of being one of the strong combinatorial problems in manufacturing systems.

In the previous studies, JSSP has been primarily treated by mathematics methods [3], branch and bound methods [4] and heuristics based on priority rules [5]. Over the past two decades, meta-heuristics have gained wide research attention, including such topics as simulated annealing (SA) [6], tabu search [7], genetic algorithm (GA) [8], particle swarm optimization (PSO) [9], and scatter search (SS) [10].

Owing to the good performance of convergence speed in continuous and discrete problems, PSO has been adopted as an optimization method for JSSP for decades. By modifying the particle position representation, particle movement, and particle velocity, Sha [11] constructed a particle swarm optimization (PSO) for an elaborate multi-objective job-shop scheduling problem. A General PSO (GPSO) model, which utilized different typical topology, is adopted by particles to exchange information and search randomly in candidate space. Optimization of the JSP, with a search space division scheme and the meta-heuristic method of PSO, by assigning each machine in a JSP with an independent swarm of particles, was obtained by multiple independent particle swarms [12]. Ivers et al. examines the optimization of the Job Shop Scheduling Problem (JSP) by a search space division scheme and use of the meta-heuristic method of Particle Swarm Optimization (PSO) to solve it [13]. Besides the velocity representation and the guide equations used by the simple PSO, eight extensions focused on the relationship between the particles and the injection of knowledge of the problem was developed to JSP [14]. Many studies have been consequently carried out to prevent premature convergence and to balance the exploration and exploitation abilities [15].

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2 Job Shop Scheduling Problem

There are \( m \) distinct machines to process \( n \) jobs that have their specific processing routines. Each job’s operation has its precedence and takes up a deterministic time period at a specific machine. At one time, there is only one operation at a machine and the job does not leave this machine until the operation is completed. Scheduling model is the corresponding evolution process to the initial scheduling model, so the initial problem modeling is as follows.

\[
\begin{align*}
\min & \quad \text{max}\{ C_i \mid i \in I \} \\
\text{s.t.:} & \quad s_{ij+1} \geq s_{ij} + p_{ij}, i \in I, J \in \{1, \ldots, s-1\} \\
& \quad (m_{ij} \neq m_{ij'}) \lor (s_{ij} \geq c_{ij} \lor s_{ij'} \geq c_{ij'}) \\
& \quad i_1, i_2 \in I, i_1 \neq i_2, j \in J \\
& \quad c_{ij} = s_{ij} + p_{ij}, i \in I, j \in J \\
& \quad s_i \geq 0, s_j \geq \sum_{k=1}^{j-1} p_{ik}, i \in I, j \in \{2, \ldots, s\} \\
& \quad m_{ij} \in R_j = \{ r_{jj}, \ldots, r_{jj'} \mid j \in J \}, i \in I 
\end{align*}
\]

Where, \( i \) is the workpiece number and \( i \in I, I = \{1, 2, \ldots, s\} \), \( j \) is the level number and \( j \in J, J = \{1, 2, \ldots, s\} \), \( r_{jj} \) is the machine number, \( s_{ij} \) is the start time of initial scheduling, \( m_{ij} \) is the start machine, \( p_{ij} \) is the processing time of the workpiece.

3 The Improved PSO with an Acceleration Factor and Its Performance with Different Topology Structure

In this paper, an improved PSO algorithm, based on standard PSO algorithm, improves the velocity updating formula by adding an acceleration factor. The modified algorithm can be described as follows:

\[
\begin{align*}
\dot{x}_{id}^{t+1} &= \omega \dot{x}_{id}^t + c_1 r_1 (p_{id} - x_{id}^t) + c_2 r_2 (p_{gd} - x_{id}^t) + l r_3 \\
\dot{x}_{id}^{t+1} &= x_{id}^t + \dot{x}_{id}^{t+1}
\end{align*}
\]

Where, \( l = -d_1 (x - d_2) \), which is a linear decline function controlled by parameters \( d_1 \) and \( d_2 \). The variable \( r_3 \) is a random positive number, drawing from the uniform distribution \([0, 1]\). The mathematical model of the dynamic evolutionary equations can be simply described as follows:

\[
\begin{align*}
\dot{v}(t+1) &= \omega v(t) + c_1 r_1 (p(t) - x(t)) + c_2 r_2 (p_g(t) - x(t)) + l r_3 \\
v(t+1) &= x(t) + v(t+1)
\end{align*}
\]

The typical particle topology structure of neighbor domain of particle candidate in PSO has following performance.

Star structure: every particle has direct connection with other members. In this structure, the information exchange rate between particles is very fast, so the convergence speed is fast as well. However, it has the property of the tendency to trap into local optimization in the later evolutions. The topology structure of star structure is shown as Figure 1(a).

Ring structure: all candidates form a ring structure. Each candidate interacts with the neighbor candidate directly to guarantee diversity of the population. The information transfer is slower than that of other structures. The topology structure of ring structure is shown as Figure 1(b).

Rotate structure: each neighbor domain is not formed by 3 candidates. It is formed by a rotate structure with one in center and the rest connected with the one in center. The topology structure of rotate structure is shown as Figure 1(c).

Von Neumann structure: each candidate directly connect with the four candidate circled with it to form a net structure. The topology structure of rotate structure is shown as Figure 1(d).

Fig. 1 The neighbor structure of candidate

The following 5 test functions were selected as the test case to the PSO algorithm. In the experiment setup, the
population size, dimension and iteration number are set 20, 30 and 1000, respectively.

\[ f_1(X) = \sum_{i=1}^{n}[x_i^2 - 10\cos(2\pi x_i) + 10], -10 < x_i < 10 \]

\[ f_2(X) = \frac{1}{4000} \sum_{i=1}^{n} x_i^2 - \prod_{i=1}^{n} \cos\left(\frac{x_i}{\sqrt{i}}\right) + 1, -600 < x_i < 600 \]

\[ f_3(X) = \sum_{i=1}^{n} [100(x_{i+1}^2 - x_i)^2 + (1 - x_i)^2], -100 < x_i < 100 \]

\[ f_4(X) = \sum_{i=1}^{n} x_i^2, -100 < x_i < 100 \]

\[ f_5(X) = \sum_{i=1}^{n} \sum_{j=1}^{n} x_i^2, -100 < x_i < 100 \]

Table 1 The Optimization Result

<table>
<thead>
<tr>
<th>Test</th>
<th>Topology</th>
<th>Mean value</th>
<th>Variance</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>4*f1</td>
<td>Star</td>
<td>42.1271612</td>
<td>7.6479676</td>
<td>1.489</td>
</tr>
<tr>
<td></td>
<td>Ring</td>
<td>31.397248</td>
<td>15.0846012</td>
<td>0.912</td>
</tr>
<tr>
<td></td>
<td>Rotate</td>
<td>50.8840914</td>
<td>10.2164443</td>
<td>0.796</td>
</tr>
<tr>
<td></td>
<td>Von Neumann</td>
<td>69.0705582</td>
<td>10.7142657</td>
<td>1.914</td>
</tr>
<tr>
<td>4*f2</td>
<td>Star</td>
<td>1.3653234</td>
<td>4.4009111</td>
<td>0.626</td>
</tr>
<tr>
<td></td>
<td>Ring</td>
<td>3.0643189</td>
<td>2.429919</td>
<td>0.541</td>
</tr>
<tr>
<td></td>
<td>Rotate</td>
<td>19.2374623</td>
<td>17.2051948</td>
<td>0.492</td>
</tr>
<tr>
<td></td>
<td>Von Neumann</td>
<td>0.0016375</td>
<td>0.0043772</td>
<td>0.644</td>
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<tr>
<td>4*f3</td>
<td>Star</td>
<td>30.2673792</td>
<td>21.0242057</td>
<td>0.593</td>
</tr>
<tr>
<td></td>
<td>Ring</td>
<td>27.3382716</td>
<td>0.6353043</td>
<td>0.594</td>
</tr>
<tr>
<td></td>
<td>Rotate</td>
<td>57.1663611</td>
<td>32.6399529</td>
<td>0.258</td>
</tr>
<tr>
<td></td>
<td>Von Neumann</td>
<td>29.9828141</td>
<td>17.1436943</td>
<td>0.418</td>
</tr>
<tr>
<td>4*f4</td>
<td>Star</td>
<td>3.469510-17</td>
<td>2.324310-16</td>
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<tr>
<td></td>
<td>Ring</td>
<td>0.0000000</td>
<td>2.464210-22</td>
<td>0.698</td>
</tr>
<tr>
<td></td>
<td>Rotate</td>
<td>1.0510-13</td>
<td>7.271910-13</td>
<td>0.549</td>
</tr>
<tr>
<td></td>
<td>Von Neumann</td>
<td>1.27510-15</td>
<td>9.015210-15</td>
<td>0.64</td>
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<td>Star</td>
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<tr>
<td></td>
<td>Ring</td>
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<td></td>
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<td>1.856</td>
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<td></td>
<td>Von Neumann</td>
<td>4.774410-18</td>
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<td>2.386</td>
</tr>
</tbody>
</table>

From the Table 1, f1 function has best optimization effect using star topology, but has best stability with star topology. It needs longer running time than that of obtained by other topology because of the neighbor structure. For function f2, the optimization performance and stability was obtained under Von Neumann structure. For function f3, the best optimization was got by star structure. Under star structure, rotate structure and Von Neumann structure, f3 can obtain good performance as well. So the neighbor structure has important influence to optimization performance. The optimization result and convergence curves of f1 and f2 are shown as Figure 2 (a), (b).

4 Numerical Test and Comparisons

To test the performance of the proposed AFPSO, a computational simulation is carried out with some well studied benchmarks. For the problems used as test case, two regular criteria, the makespan \( C_{max}(s) \) and the maximum tardiness \( T_{max}(s) \), are used as criteria. We set the two objectives as follows \( s \) is the job solution:

\[
\text{Minimize } f_1(s) = C_{max}(s) \\
\text{Minimize } f_2(s) = T_{max}(s)
\]

The performances used in the parameters tuning are "Best", "Mean", "Min", and "Best rate", where the "Best", "Mean" and "Min" stand for the best one, the mean one and the minimum aspects of the objective values achieved in 100 runs. The "Best rate" is the rate to reach the best value. The algorithm was run 100 times with different random seeds for each parameter setting to test the random effect on the solution. From the results shown in Table 2, we can see that all the test cases obtained the best value. In order to testify the effective and efficiency of AFPSO, the results reported in Xia and Wu [16], Sha and Hsu [17] were compared with the results obtained under AFPSO, as shown in Table 2.
Table 2 Result by using AFPSO

<table>
<thead>
<tr>
<th>2*Test</th>
<th>2*nxm</th>
<th>2<em>C</em></th>
<th>AFPSO</th>
</tr>
</thead>
<tbody>
<tr>
<td>LA01</td>
<td>10x5</td>
<td>926</td>
<td>926 926 926 50.9</td>
</tr>
<tr>
<td>LA11</td>
<td>20x5</td>
<td>1222</td>
<td>1222 1222 1222 129.4</td>
</tr>
<tr>
<td>LA16</td>
<td>10x10</td>
<td>945</td>
<td>945 945.5 946 133.3</td>
</tr>
<tr>
<td>LA21</td>
<td>15x10</td>
<td>1046</td>
<td>1046 1052.5 1059 724.5</td>
</tr>
<tr>
<td>LA26</td>
<td>20x10</td>
<td>1218</td>
<td>1218 1218 1218 2389.6</td>
</tr>
<tr>
<td>LA31</td>
<td>30x10</td>
<td>1784</td>
<td>1784 1784 1784 3698.2</td>
</tr>
<tr>
<td>LA36</td>
<td>15x15</td>
<td>1268</td>
<td>1268 1282.4 1290 3356.2</td>
</tr>
</tbody>
</table>

5 Conclusions

A modified PSO with an acceleration factor based on basic particle swarm optimization was presented in this paper. The modified algorithm effectively improved the deficiencies of the prone to local optimization solution and slow convergence by adding an acceleration factor to the velocity updating formula of the evolutionary computing algorithm. Theoretical analysis, which is based on stochastic processes, proves that the trajectory of particle is a Markov processes and AFPSO algorithm converges to the global optimal solution with mean square merit. Experimental simulations show that the AFPSO algorithm has better performance on the convergence, and achieves better solutions in shorter time for typical benchmark functions than the standard PSO.

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References


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