Quantum Physics Letters
An International Journal

http://dx.doi.org/10.18576/qpl/070302

Supersymmetry and the Magnetic Moment

Simon Davis

Research Foundation of Southern California 8861 Villa La Jolla Drive #13595 La Jolla, CA 92039, USA

Received: 2 Sep. 2018, Revised: 12 Nov. 2018, Accepted: 24 Nov. 2018
Published online: 1 Dec. 2018

Abstract: The relation between the presence of supersymmetry and the value of the magnetic moment of the electron will be examined. A method is given for the evaluation of the magnetic moment to arbitrary large orders in a series expansion.

Keywords: Dalgaard-Strulik model, energy, economic growth, time delay, limit cycle

1 Introduction

The magnetic moment of the electron predicted by the coupling of the spin to the magnetic field in the non-relativistic limit of the Dirac equation is $\frac{e}{2m}$. Given the relation $\mu = g\frac{e}{2m}$, the introduction of supersymmetry is sufficient to produce the classical value of $g = 2$. The precise measurement of the magnetic moment of the electron gives a value which is predicted by quantum electrodynamics to twelve decimal places [1].

Since the exact value of g differs from 2, the relation between supersymmetry and the magnetic moment can be clarified. Although the classical supersymmetric point-particle model does not have this value of g, it remains to be determined if supersymmetry can be preserved in the field theory after radiative corrections.

A review of the calculation of the magnetic moment in supersymmetric theories in §3 indicates that perturbative corrections would vanish as a result of spin sum rules. This conclusion then forms the basis of the order estimates of the series expansion of the vertex amplitude with an electromagnetic coupling that exists only in the phase of broken supersymmetry.

The terms in a superstring perturbation series will be sufficient to establish in §4 the higher-order terms in the expansion of the squared absolute value of the summed amplitude. The coefficients that compensate for the division by a factorial are determined by a combinatorial series derived from each of the components of the compactification divisor at genus g. The alternation of the signs will yield cancellations between the terms and reduce the amplitude from an exponential function of the genus to an expression of the form $(-1)^g k_g \frac{\kappa_{e.m}^g}{g!}$. Given the multiplicative factor $q_g = 1 + \frac{1}{g-1}$ for $g \geq 2$ resulting from quantum surfaces in the coefficients of the supermoduli space integrals, and (-1) for fermion loops, evaluation of k_g would yield an estimate of the terms in the series $A_0 + c_0 \sum_{g=1} (-1)^g k_g q_g \frac{\alpha^g}{g!}$ for the magnetic moment, where $\kappa_{e.m.}$ is set equal to α after the renormalization group flow of the coupling and the multiplicative constant c_0 can be calculated from the experimental value.

2 The Superysmmetric Point-Particle Action

Consider the Lagrangian

$$L = \frac{1}{2} m\dot{x}^2 + \frac{i}{2} f_a \dot{f}_a - -eA_a \dot{x}_a - \mu S_a B_a \tag{1}$$

where f_a is a Grassmann variable and $S_a = -\frac{i}{2}\varepsilon_{abc} f_b f_c$ [2]. The equations of motion are

$$m\ddot{x}_a - e\dot{A}_a = -\mu S_j \partial_i B_j \qquad \dot{f}_a + \mu \varepsilon_{abc} f_a B_b = 0 \tag{2}$$

The precession of the spin operator follows from

$$\dot{S}_a = -\frac{i}{2}\varepsilon_{abc}(\dot{f}_b f_c + f_b \dot{f}_c) = -i\varepsilon_{abc}(-\mu\varepsilon_{bde} f_d B_e) f_c = -i\mu f_a f_c B_c = \mu\varepsilon_{abc} B_b S_c.$$

The value of μ will be fixed by invariance under supersymmetry. Under the supersymmetry transformations

$$x_a \to x_a - i\varepsilon \frac{f_a}{\sqrt{m}} \qquad f_a \to f_a + \varepsilon\sqrt{m}\dot{x}_b, \tag{3}$$

* Corresponding author e-mail: sbavis@resfdnsca.org

$$-\mu S_a B_a \to -\mu S_a B_a + i\mu\sqrt{m}\varepsilon\varepsilon_{abc}B_a\dot{x}_a f_c \qquad -eA_a\dot{x}_a \to -eA_a\dot{x}_a + i\frac{e}{\sqrt{m}}\varepsilon A_a\dot{f}_a, \tag{4}$$

and, since

$$i\mu\sqrt{m}\varepsilon\varepsilon_{abc}B_a\dot{x}_b f_c = -i\mu\sqrt{m}\varepsilon(\partial_b A_c - \partial_c A_b)\dot{x}_b f_c,$$

$$\int dt i\mu\sqrt{m}\varepsilon(\partial_b A_c - \partial_c A_b)\dot{x}_b f_c \tag{5}$$
$$= -\int dt\ i\mu\sqrt{m}\varepsilon[\partial_b(A_c x_b \dot{f}_c) - 2A_c\dot{f}_c - \partial_c(A_b x_b \dot{f}_c) + A_b\delta_{bc}\dot{f}_c].$$

Integration of the divergence terms will vanish at spatial infinity, leaving

$$\int d^3x dt\ i\mu\sqrt{m}(-A_c\dot{f}_c) = -\int d^3x dt\ i\mu\sqrt{m}A_c\dot{f}_c, \tag{6}$$

which will cancel the variation from $-eA_a\dot{x}_a$ if $\mu = \frac{e}{m}$ or $g = 2$.

3 Supersymmetry Breaking and the Magnetic Moment

The point-particle limit of the Lorentz model [3] is known to coincide with the non-relativistic limit of the Dirac equation. The self-energy infinities are evident in the equations of motion of the point-particle theory [4] with the action $I = -\frac{1}{2}\int d\tau\dot{x}^2 + \int d^4x\left(A_\mu j^\mu - \frac{1}{16\pi}F_{\mu\nu}F^{\mu\nu}\right)$ and $j^\mu(x) = q\int\delta^4(x - x(\tau))\dot{x}^\mu(\tau)d\tau$,

$$e\ddot{x}_\mu + \dot{e}\dot{x}_\mu = qF_{\mu\nu}\dot{x}^\nu \qquad \partial^\mu F_{\mu\nu} = 4\pi j^\mu \tag{7}$$

where e^a_μ is the world-line density. The reaction of the radiation to the accelerated motion of the charge causes a divergence in the solution for the field. These infinities can be partially removed when $\dot{x}^2 = 0$ and entirely removed if the particle has a mass through the inclusion of counterterms in the effective action [**?**]. The effect of the renormalization of the massive equations of motion is the occurrence of new terms that may be added to Eq.(2), which includes the coupling of the spin to the magnetic field.

The generalization of the Dirac theory to renormalizable theory of quantum electrodynamics allows the systematic removal of all divergences from the perturbative calculation of the magnetic moment of the electron. The radiative corrections alter the value of g from 2 to a slightly larger value.

The self-energy infinities are eliminated in supersymmetric quantum electrodynamics, which has an invariance generated by a superalgebra with an abelian bosonic sector. The radiative corrections of the magnetic moment again can evaluated. Any change from $g = 2$ would render the classical point-particle model to be not invariant under supersymmetry transformations. The removal of infinities and supersymmetry breaking may be considered precisely within the supersymmetric field theory. There are renormalization theorems, for example, which prevent supersymmetry breaking in sQED. The renormalization theorems tend to imply that couplings or relations between parameters in the Lagrangian are unaffected by radiative corrections. The contributions of the photon and the paired spin-$\frac{1}{2}$ field cancel in the evaluation of the perturbative expansion in supersymmetric quantum electrodynamics[**?**]. Therefore, it remains to be determined whether a soft breaking of supersymmetry can accommodate a change in the value of g by a factor of $1 \pm \mathcal{O}(10^{-3})$. The summability of the perturbation series once supersymmetry is broken is affected by a class of bubble diagrams [7]. A study of the electron magnetic dipole moment in a minimal supersymmetic standard model with other vector bosons has yielded conditions on the parameters [8].

Sum rules for magnetic moments of charged particles in supermultiplets receive corrections if supersymmetry-breaking terms are included [**?**]. While the anomalous magnetic moments of the charged particles have the same signs in the standard model and the minimal supersymmetric extension, based on the most general form of the $WW\gamma$ vertex, the coefficient of the term $q_\alpha g_{\beta\mu} - q_\beta g_{\alpha\mu}$ have opposite signs for the two theories, and the condition of the same sign requires a light upper bound for the mass of the Higgs boson [10]. It has been suggested that calculations of the muon magnetic dipole moment, where theoretical and experimental values for $\frac{g_\mu - 2}{2}$ differ by $(4.3 \pm 1.6) \times 10^{-9}$ only, could confirm supersymmetry, and the supersymmetric contributions are found to be $\mathcal{O}(10^{-9})$ [11] Much of the difference may be traced to hadronic polarization effects [**?**]. The expansion of $\frac{g_\mu - 2}{2}$ consists of terms resulting from broken supersymmetry of $\mathcal{O}(2 \times 10^{-10})$ at two loops in the leading logarithm approximation [13], and the viability of the model would be determined by a sum over the higher-loop terms. If it is included within a superstring calculation, the convergence can be ensured.

4 Magnetic Moment in Superstring Theory

Finite ultraviolet and infrared cut-offs for quantum electrodynamics yields a convergent perturbation expansion, even though the number of diagrams increases at a factorial rate [14] [**?**]. This diagrammatic expansion may be derived from a superstring perturbation expansion, since an exponential bound is derived yielding a convergent series for sufficiently low values of the coupling [16].

The electric magnetic moment has been given by the coefficient of $\frac{e}{2m}\hbar s\hat{e}_s$ in the coupling to the magnetic field at zero orbital angular momentum in the renormalized Lagrangian of quantum electrodynamics, where s is the intrinsic spin quantum number and $\hat{e}_s$ is the unit vector in direction of the spin vector. The coefficients of the diagrams in this expansion decrease with the order n of perturbation series as α^n where α is the fine structure constant. The problem of the factorial divergence of the number of diagrams at n^{th} order can be resolved within superstring perturbation theory. Initial calculations indicate that correction to the magnetic moment at one loop vanishes when supersymmetry is preserved [**?**].

The magnetic moment can be calculated through a perturbation series derived from string theory since there may be contributions from higher loops. With a perturbation series of the form

$$A_0 + c_0 \sum_{g=1}^{\infty} (-1)^g \frac{\pi^2}{4} \frac{k_g \kappa_{e.m.}^g}{g!},$$

where A_0 is the leading-order vertex amplitude, the factor of $(-1)^g$ arises from the superstring measure, $\kappa_{e.m}$ is the gauge coupling at electromagnetic scales, set equal to α, k_g represents a compensating factor that is from a combinatorial series related to the components of the compactification divisor [18] at each genus

$$e^g/\sqrt{2\pi g}\Bigg[\frac{1}{1+\frac{1}{g}} - \frac{1}{2}\sum_{k=1}^{\frac{1}{2}\left(\frac{g}{2}+1\right)} \frac{(-1)^{k-1}\cdot 2}{(2\pi)^{2k}}\zeta(2k)\left(1-\frac{1}{g}\right)\ldots\left(1-\frac{2k-2}{g}\right)$$
$$+\sum_{k=1}^{\frac{1}{2}\left(\frac{g}{2}-1\right)} \frac{(-1)^{\frac{g}{2}-k}\cdot 2(2\pi)^{g-2k+2}}{\zeta}(g-2k+2)\left(1-\frac{1}{g}\right)\ldots\left(1-\frac{2g-3}{g}\right)g^{-g+4k-3}\Bigg]$$

and the relative weighting for the terms is $-k_1\alpha : \frac{1}{2!}k_2\alpha^2 : -\frac{1}{3!}k_3\alpha^3 : \frac{1}{4!}k_4\alpha^4 : \ldots$.

Quantum deformations of surfaces in the string path integral have been found to contribute a factor $q_g = 1 + \frac{1}{g-1}$ for $g \geq 2$ [19]. Since the amplitudes then differ from the superstring perturbation expansion, the vanishing of radiative contributions by supersymmetry is no longer valid. There is a factor of -1 for each fermion loop. Setting $q_1 = 1$, the perturbative series is

$$A_0 + c_0 \sum_{g=1}^{\infty} \frac{\pi^2}{4} \frac{k_g q_g \kappa_{e.m.}^g}{g!} \tag{8}$$

after cancellation of the factors of $(-1)^g$. The coefficient c_0 can be selected such that

$$\frac{A_0 + c_0 \sum_{g=1}^{3} (-1)^g \frac{\pi^2}{4} \frac{k_g q_g \alpha^g}{g!}}{A_0} = 1.0011596521 \tag{9}$$

when $\kappa_{e.m.}$ is set equal to the fine structure constant α. The observed amplitudes in Lorentzian space with a $G_2/SU(3)$ compactification differ from the Euclidean amplitudes in $\mathbb{R}^{10}$ by a factor that receives contributions from the space-time instantons resulting from windings around the compact space and the transformation of the integration ranges for the coordinates the light-cone worldsheet [20] and the Riemann surfaces and equal to

$$\sqrt{2}\left(\sum_{\{n_i\}\in\pi_9(S^6)} e^{-n_i\langle I_C\rangle n_i}\right)^{-1} = \sqrt{2}(0.7880624660869638) \simeq 1.1144886275. \tag{10}$$

This factor will be cancelled in the ratio in Eq.(9). The series for the electron magnetic moment then can be evaluated to an arbitrary large order.

5 Perspective

Classical supersymmetry is sufficient to derive a gyromagnetic ratio of 2. It also may be found through the spin coupling term in the Dirac equation. When supersymmetry is present, spin sum rules tend to cause the higher-order terms in the perturbative series for the magnetic moment to vanish. Therefore, the computation of the electron and muon magnetic moments through a perturbation expansion will be valid only when supersymmetry is broken. It follows that order estimates of each term in the scattering matrix would contribute to the magnetic moment.

The superstring perturbation series provides order estimates for each term in a nonvanishing amplitude. There is a factor of $(-1)^{\frac{g}{2}}$ the holomorphic part of the superstring measure that yields alternating signs in the series expansion of the amplitude. When the supersymmetry is present, the value of the effective string coupling is close to the unified gauge coupling of the minimal supersymmetric standard model. The conventional value of the electromagnetic coupling, the fine structure constant, is different from the unified gauge coupling, and therefore, it results from the renormalization group flow after supersymmetry breaking. With the value of α in place of the effective string coupling, a phenomenologically valid estimate of the magnetic moment can be given to arbitrary precision. It therefore can be calculated with a finer resolution than the current theoretical estimates of quantum electrodynamics.

References

[1] G. Gabrielse, D. Hanneke, T. Kinoshita, M. Nio and B. Odom, New Determination of the Fine Structure Constant from the Electron g Value and QED, Phys. Rev. Lett. **9**7 (2006) 030802.

[2] A. P. Balachandran, G. Marmo, B.-S. Skagerstam and A. Stern, Gauge Symmetries and Fibre Bundles, Lect. Notes Phys. **1**88, Springer-Verlag, Berlin, 1983.

[3] H. A. Lorentz, Theory of Electron, Teubner, Leipzig,1906, reprinted, Dover, New York, 1952.

[4] P. A. M. Dirac, Classical Theory of Radiating Elctrons, Proc. R. Soc. **1**67 (1938) 148-169.

[5] P. O. Kazinski and A. A. Sharapov, Radiation Reaction for Massless Charged Particle, Class. Quantum Grav. **2**0 (2003) 2715-2725.
[6] S. Ferrara and E. Remiddi, Absence of the Anomalous Magnetic Moment in a Supersymmetric Abelian Gauge Theory, Phys. Lett. **B**53 (1974) 347-350.
[7] A. Axelrod, Broken Supersymmetry and Divergence of the Perturbation Series in QED, Phys. Lett. **B**196 (1987) 365-368.
[8] J. Scott Burgess, Magnetic Moments in Minimal Supersymmetric Standard Model, Indiana Univ. Ph. D. Dissertation, 1995.
[9] C. L. Bilchak, R. Gastmans and A. van Proeyen, Static Quantities in Supersymmetry, Nucl. Phys. **B**273 (1986) 46-74.
[10] A. Culatti, SUSY Magnetic Moments Sum Rules and Supersymmetry Breaking, Z. Phys. **C**65 (1995) 537-544.
[11] S. P. Martin and J. D. Wells, Muon Anomalous Magnetic Dipole Moment in Supersymmetric Theories, Phys. Rev. **64** (2001) 035003: 1-12.
[12] F. Jegerlehner and A. Nyffeler, The Muon $g-2$, Phys. Rep. **4**77 (2009) 1-110.
[13] D. Stöckinger, The Muon Magnetic Moment and Supersymmetry, J. Phys. **G**34 (2007) R45-R92.
[14] E. Caianello, Number of Feynman Graphs and Convergence, Nuovo Cim. **3** (1956) 223-225.
[15] M. Salmhofer and C. Wieczerkowski, Positivity and Convergence in Fermionic Quantum Field Theory, J. Stat. Phys. **9**9 (2000) 557-586.
[16] S. Davis, On the Absence of Large-Order Divergences in Superstring Theory, Fortschr. Phys. **5**1 (2003) 22-61.
[17] A. Pasquinucci and K. Roland, On the Anomalous Magnetic in Heterotic Superstrings, Mod. Phys. Lett. **A**9 (1994) 3023-3034.
[18] S. Davis, The Resolution of the Integration Region Problem for the Supermoduli Space Integral, Int. J. Mod. Phys. **A**29 (2014) 1450190: 1-60.
[19] S. Davis, Quantum Riemann Surfaces and the Measure of Noncommutativity, RFSC-14-14.
[20] S. Davis, Lorentzian Path Integrals and Moduli Space, RFSC-17-05.