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Some fixed point theorems in G-metric spaces

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Abstract: In this paper we introduced the (E.A.)-property and weak compatibility of mappings in G-metric spaces. We have utilized these concepts to deduce certain common fixed point theorems in G-metric space

1. Introduction and Preliminaries

Mustafa and Sims [9] introduced the concept of G-metric spaces in the year 2004 as a generalization of the metric spaces. In this type of spaces a non-negative real number is assigned to every triplet of elements. In [11] Banach contraction mapping principle was established and a fixed point results have been proved. After that several fixed point results have been proved in these spaces. Some of these works may be noted in [2–4, 10–13] and [14]. Several other studies relevant to metric spaces are being extended to G-metric spaces. For instances we may note that a best approximation result in these type of spaces established by Nezhad and Mazaheri in [15], the concept of w-distance, which is relevant to minimization problem in metric spaces [8], has been extended to G-metric spaces by Saadati et al. [23]. Also one can note that fixed point results in G-metric spaces have been applied to proving the existence of solutions for a class of integral equations [25].

Now we give some preliminaries and basic definitions which are used throughout the paper.

Definition 1.1. G-metric Space [9, ?]
Let X be a nonepmty set and let $G : X \times X \times X \to \mathbf{R}^+$ be a function satisfying the following :

1.$G(x, y, z) = 0$ if $x = y = z$,
2.$0 < G(x, x, y)$; for all $x, y \in X$, with $x \neq y$,
3.$G(x, x, y) \leq G(x, y, z)$, for all $x, y, z \in X$ with $z \neq y$,
4.$G(x, y, z) = G(x, z, y) = G(y, z, x) = ...$ (symmetry in all three variables),
5.$G(x, y, z) \leq G(x, a, a) + G(a, y, z)$, for all $x, y, z, a \in X$ (rectangle inequality),

then the function is called a generalized metric, or, more specifically a G-metric on X and the pair (X, G) is a G-metric space.

Definition 1.2. [10] Let (X, G) be a G-metric space and $\{x_n\}$ be a sequence of points in X. We say that $\{x_n\}$ is G-convergent to x if $\lim_{n\to\infty} G(x, x_n, x_m) = 0$, that is, for each $\epsilon > 0$ there exists a positive integer N such that $G(x, x_n, x_m) < \epsilon$ for all $m, n \geq N$. We call that x is the limit of the sequence and we write $x_n \to x$ or $\lim_{n\to\infty} x_n = x$.

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It has been shown in [10] that the G-metric induces a Hausdorff topology and the convergence described in the above definition is relative to this topology. The topology being Hausdorff, a sequence can converge at most to one point.

Proposition 1.1. [10] Let (X, G) be a G-metric space then the following are equivalent:

1. $\{x_n\}$ is G-convergent to x,
2. $G(x_n, x_n, x) \to 0$ as $n \to \infty$,
3. $G(x_n, x, x) \to 0$ as $n \to \infty$,
4. $G(x_m, x_n, x) \to 0$ as $m, n \to \infty$.

Definition 1.3. [10] Let (X, G) be a G-metric space. A sequence $\{x_n\}$ is said to be a G-Cauchy sequence for each $\epsilon > 0$ there exists a positive integer N such that $G(x_n, x_m, x_l) < \epsilon$ for all $l, m, n \geq N$.

Proposition 1.2. [10] Let (X, G) be a G-metric space then the following are equivalent:

1. the sequence $\{x_n\}$ is G-Cauchy,
2. for each $\epsilon > 0$ there exists a positive integer N such that $G(x_n, x_m, x_l) < \epsilon$ for all $l, m, n \geq N$.

Proposition 1.3. [10] Let (X, G) be a G-metric space then the function $G(x, y, z)$ is jointly continuous in all three variable.

Definition 1.4. [10] A G-metric space (X, G) is called a symmetric G-metric space if $G(x, y, y) = G(y, x, x)$ for all $x, y \in X$.

Proposition 1.4. [10] Every G-metric (X, G) defines a metric space (X, d_G) by

1. $d_G(x, y) = G(x, y, y) + G(y, x, x)$ for all $x, y \in X$.

If (X, G) is a symmetric G-metric space, then
2. $d_G(x, y) = 2G(x, y, y)$ for all $x, y \in X$.

However, if (X, G) is not a symmetric G-metric space, then it follows from the G-metric properties that
3. $\frac{3}{2}G(x, y, y) \leq d_G(x, y) = 3G(x, y, y)$ for all $x, y \in X$.

Proposition 1.5. [10] A G-metric space (X, G) is G-complete if and only if (X, d_G) is a complete metric space.

Proposition 1.6. [10] Let (X, G) be a G-metric space. Then, for any $x, y, z, a \in X$ it follows that

1. if $G(x, y, z) = 0$ then $x = y = z$,
2. $G(x, y, z) \leq G(x, x, y) + G(x, x, z)$,
3. $G(x, y, y) \leq 2G(y, x, x)$,
4. $G(x, y, z) \leq G(x, a, z) + G(a, y, z)$,
5. $G(x, y, z) \leq \frac{2}{3}(G(x, a, a) + G(y, a, a) + G(z, a, a))$.

Next we give two examples of non-symmetric G-metric spaces.

Example 1.1. [10] Let $X = \{a, b\}$, let $G(a, a, a) = G(b, b, b) = 0$, $G(a, a, b) = 1$, $G(a, b, b) = 2$ and extend G to all of $X \times X \times X$ by symmetry in the variables. Then G is a G-metric. It is non-symmetric since $G(a, b, b) \neq G(a, a, b)$.

Example 1.2. [4] Let $X = \{0, 1, 2, 3....\}$ and $G : X \times X \times X \to R^+$ be defined as follows:

$$G(x,y,z) = \begin{cases} x+y+z, & \text{if } x,\ y,\ z \text{ are all distinct and different from zero,} \\ x+z, & \text{if } x = y \neq z \text{ and all are different from zero,} \\ y+z+1, & if\ x = 0, y \neq z \text{ and } y, z \text{ are different from zero,} \\ y+2, & if\quad x = 0, y = z \neq 0, \\ z+1, & if\quad x = 0, y = 0, z \neq 0, \\ 0, & \text{if}\quad x = y = z. \end{cases}$$

Then (X, G) is a complete G-metric space. Then G is also non-symmetric, since $G(0, 0, 1) \neq G(1, 1, 0)$.

There has been a considerable interest to study common fixed point for a pair (or family) of mappings satisfying some contractive conditions in metric spaces. Several interesting and elegant results were obtained in this direction by various authors. It was the turning point in the "fixed point arena" when the notion of commutativity was introduced by G. Jungck [5] to obtain common fixed point theorems. This result was further generalized and extended in various ways by many authors. In one direction Jungck [6] introduced the compatibility in 1986. It has also been noted that fixed point problems of non-compatible mappings are also important and have been considered in a number of works, a few may be noted in [7, 18, **?**]. In another direction weaker version of commutativity has been considered in a large number of works. One such concept is R-weakly commutativity. This is an extension of weakly commuting mappings [16, 24]. Some other references may be noted in [17–20] and [21].

Definition 1.5. [6] Let f and g be two self mappings on a metric space (X, d). The mappings f and g are said to be compatible if $\lim_{n\to\infty} d(fgx_n, gfx_n) = 0$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = z$ for some $z \in X$.

In particular, now we look in the context of common fixed point theorem in G-metric spaces. Start with the following contraction conditions:

Definition 1.6. Let (X, G) be a G-metric space and $T : X \to X$ be a self mapping on (X, G). Now T is said to be a contraction if

$$G(Tx, Ty, Tz) \leq \alpha G(x, y, z) \tag{1.1}$$

for all $x, y, z \in X$ where $0 \leq \alpha < 1$.

It is clear that every self mapping $T : X \to X$ satisfying condition (1.1) is continuous. Now we focus to generalize the condition (1.1) for a pair of self mappings S and T on X in the following way:

$$G(Sx, Sy, Sz) \leq \alpha G(Tx, Ty, Tz) \tag{1.2}$$

for all $x, y, z \in X$ where $0 \leq \alpha < 1$.

Let $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = z$ for some $z \in X$. To prove the existence of common fixed points for mappings satisfying inequality (1.2), it is necessary to add additional assumptions of the following type:

1. construction of the sequence $\{x_n\}$,
2. some mechanism to obtain common fixed point and this problem was overcomed by imposing additional hypothesis of commutative pair $\{S, T\}$.

Most of the theorems followed a similar pattern of mappings :

1. contraction,
2. continuity of functions (either one or both) and
3. commuting pair of mappings were given.

In some cases condition (ii) can be relaxed but conditions (i) and (iii) are unavoidable.

Definition 1.7. Let f and g be two self mappings on a G-metric space (X, G). The mappings f and g are said to be compatible if $\lim_{n\to\infty} G(fgx_n, gfx_n, gfx_n) = 0$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = z$ for some $z \in X$.

Definition 1.7. Two maps f and g are said to be weakly compatible if they commute at coincidence points.

Example 1.3. Let $X = [-1, 1]$ and let G be the G-metric on $X \times X \times X$ defined as follows: $G(x, y, z) = |x - y| + |y - z| + |z - x|$ for all $x, y, z \in X$.
Then (X, G) be a G-metric space. Let us define $fx = x$ and $gx = \frac{x}{4}$. Consider the sequence $\{x_n\}$, where $x_n = \frac{1}{n}$ and n is a natural number. It is clearly that the mappings f and g are compatible, since $\lim_{n\to\infty} G(fgx_n, gfx_n, gfx_n) = 0$, here the sequence $\{x_n\}$ in X such that $x_n = \frac{1}{n}$ and $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = 0$ for $0 \in X$.

Example 1.4. Let $X = [0, 3]$ and let G be the G-metric on $X \times X \times X$ defined as follows: $G(x, y, z) = |x - y| + |y - z| + |z - x|$ for all $x, y, z \in X$. If we define f and g as follows:

$$fx = \begin{cases} 0 \text{ if } & x \in [0,1), \\ 3, \text{ if } & x \in [1,3] \end{cases} \quad \text{and} \quad gx = \begin{cases} 3 - x \text{ if } & x \in [0,1), \\ 3, \quad \text{if} & x \in [1,3]. \end{cases}$$

Then for any $x \in [1,3]$, x is a coincidence point and $fgx = gfx$, showing that f, g are weakly compatible maps on $[0,3]$.

2. Main Results

Now we come to our main result for a pair of compatible maps.

Theorem 2.1. Let (X, G) be a complete G-metric space and f, g be two self mappings on (X, G) satisfies the following conditions:

1. $f(X) \subseteq g(X)$, (2.1)
2. f or g is continuous, (2.2)
3. $G(fx, fy, fz) \leq \alpha G(fx, gy, gz) + \beta G(gx, fy, gz) + \gamma G(gx, gy, fz)$, (2.3)

for every $x, y, z \in X$ and $\alpha, \beta, \gamma \geq 0$ with $0 \leq \alpha + 3\beta + 3\gamma < 1$. Then f and g have a unique common fixed point in X provided f and g are compatible maps.

Proof. Let x_0 be an arbitrary point in X. By (2.1), one can choose a point $x_1 \in X$ such that $fx_0 = gx_1$. In general one can choose x_{n+1} such that $y_n = fx_n = gx_{n+1}, n = 0, 1, 2, \ldots \ldots \ldots$

From (2.3), we have

$$\begin{aligned} G(fx_n, fx_{n+1}, fx_{n+1}) &\leq \alpha G(fx_n, gx_{n+1}, gx_{n+1}) + \beta G(gx_n, fx_{n+1}, gx_{n+1}) \\ &\quad + \gamma G(gx_n, gx_{n+1}, fx_{n+1}) \\ &= \alpha G(fx_n, fx_n, fx_n) + \beta G(fx_{n-1}, fx_{n+1}, fx_n) \\ &\quad + \gamma G(fx_{n-1}, fx_n, fx_{n+1}) \\ &= (\beta + \gamma) G(fx_{n-1}, fx_n, fx_{n+1}). \end{aligned} \tag{2.4}$$

By the rectangular inequality of G-metric space, we have

$$\begin{aligned} G(fx_{n-1}, fx_n, fx_{n+1}) &\leq G(fx_{n-1}, fx_n, fx_n) + G(fx_n, fx_n, fx_{n+1}) \\ &\leq G(fx_{n-1}, fx_n, fx_n) + 2G(fx_n, fx_{n+1}, fx_{n+1}). \\ &\qquad \text{(by using Proposition 1.6)} \end{aligned}$$

From (2.3), we have

$$(1 - 2\beta - 2\gamma) G(fx_n, fx_{n+1}, fx_{n+1}) \leq (\beta + \gamma) G(fx_{n-1}, fx_n, fx_n),$$

that is, $$G(fx_n, fx_{n+1}, fx_{n+1}) \leq \frac{(\beta + \gamma)}{(1 - 2\beta - 2\gamma)} G(fx_{n-1}, fx_n, fx_n)$$

that is, $$G(fx_n, fx_{n+1}, fx_{n+1}) \leq q G(fx_{n-1}, fx_n, fx_n) \text{ where } q = \frac{(\beta + \gamma)}{(1 - 2\beta - 2\gamma)} < 1.$$

Continuing in the same way, we have

$$G(fx_n, fx_{n+1}, fx_{n+1}) \leq q^n G(fx_0, fx_1, fx_1).$$

Therefore, for all $n, m \in N, n < m$, we have by rectangle inequality that

$$\begin{aligned} G(y_n, y_m, y_m) &\leq G(y_n, y_{n+1}, y_{n+1}) + G(y_{n+1}, y_{n+2}, y_{n+2}) + \ldots \ldots \ldots + G(y_{m-1}, y_m, y_m) \\ &\leq (q^n + q^{n+1} + \ \ldots \ldots \ + q^{m-1}) G(y_0, y_1, y_1) \\ &\leq \frac{q^n}{1-q} G(y_0, y_1, y_1). \end{aligned}$$

Letting as $n, m \to \infty$, we have $\lim_{n\to\infty} G(y_n, y_m, y_m) = 0$. Thus $\{y_n\}$ is a G-Cauchy sequence in X. Since (X, G) is complete G-metric space, therefore, there exists a point $z \in X$ such that $\lim_{n\to\infty} y_n = \lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_{n+1} = z$. Since the mapping f or g is continuous, for definiteness one can assume that g is continuous, therefore $\lim_{n\to\infty} gfx_n = \lim_{n\to\infty} ggx_n = gz$. Further, f and g are compatible, therefore, $\lim_{n\to\infty} G(fgx_n, gfx_n, gfx_n) = 0$, implies $\lim_{n\to\infty} fgx_n = gz$.

From (2.3), we have

$$G(fgx_n, fx_n, fx_n) \leq \alpha G(fgx_n, gx_n, gx_n) + \beta G(ggx_n, fx_n, gx_n) + \gamma G(ggx_n, gx_n, fx_n).$$

Proceeding limit as $n \to \infty$, we have $gz = z$.

Again from (2.3), we have

$$G(fx_n, fz, fz) \le \alpha G(fx_n, gz, gz) + \beta G(gx_n, fz, gz) + \gamma G(gx_n, gz, fz).$$

Taking limit $n \to \infty$, we have $z = fz$. Therefore, we have $gz = fz = z$. Thus z is a common fixed point of f and g.

For uniqueness, we assume that $z_1(\neq z)$ be another common fixed point of f and g . Then $G(z, z_1, z_1) > 0$ and

$$\begin{aligned} G(z, z_1, z_1) &= G(fz, fz_1, fz_1) \\ &\le \alpha G(fz, gz_1, gz_1) + \beta G(gz, fz_1, gz_1) + \gamma G(gz, gz_1, fz_1) \\ &= (\alpha + \beta + \gamma)G(z, z_1, z_1) \\ &< G(z, z_1, z_1), \text{ a contradiction,} \end{aligned}$$

which demands that $z = z_1$.

This completes the proof of the theorem.

Corollary 2.1. Let (X, G) be a complete G-metric space and f, g be two compatible self mappings on (X, G) satisfies (2.1), (2.2) and the following condition:

$$G(fx, fy, fz) \le qG(x, y, z)$$

for every $x, y, z \in X$ and $0 < q < 1$. Then f and g have a unique common fixed point in X.

Proof. Proof follows easily from above theorem.

Theorem 2.2. Let f and g be weakly compatible self maps of a G-metric space (X, G) satisfying conditions (2.1) and (2.3) and any one of the subspace $f(X)$ or $g(X)$ is complete. Then f and g have a unique common fixed point in X.

Proof. From Theorem 2.1, we conclude that $\{y_n\}$ is a G-Cauchy sequence in X. Since either $f(X)$ or $g(X)$ is complete, for definiteness assume that $g(X)$ is complete subspace of X then the subsequence of $\{y_n\}$ must get a limit in $g(X)$. Call it be z. Let $u \in g^{-1}z$. Then $gu = z$ as $\{y_n\}$ is a G-Cauchy sequence containing a convergent subsequence, therefore the sequence $\{y_n\}$ also convergent implying thereby the convergence of subsequence of the convergent sequence. Now we show that $fu = z$.

On setting $x = u, y = x_n$ and $z = x_n$, in (2.3), we have

$$G(fu, fx_n, fx_n) \le \alpha G(fu, gx_n, gx_n) + \beta G(gu, fx_n, gx_n) + \gamma G(gu, gx_n, fx_n).$$

Letting as $n \to \infty$ in the above inequality, we have

$$G(fu, z, z \le \alpha G(fu, z, z),$$

which implies that, $fu = z$.

Therefore, $fu = gu = z$, i.e., u is a coincident point of f and g. Since f and g are weakly compatible, it follows that $fgu = gfu$, i.e., $fz = gz$.

We now show that $fz = z$. Suppose that $fz \neq z$, therefore $G(fz, z, z) > 0$. From (2.3), on setting $x = z, y = u, z = u$, we have

$$\begin{aligned} G(fz, z, z) &= G(fz, fu, fu) \\ &\le \alpha G(fz, gu, gu) + \beta G(gz, fu, gu) + \gamma G(gz, gu, fu) \\ &= (\alpha + \beta + \gamma)G(fz, z, z) \\ &< G(fz, z, z), \text{ a contradiction,} \end{aligned}$$

which implies that $fz = z$.

Therefore, $fz = gz = z$ i.e., z is common fixed point of f and g. Uniqueness follows easily.

We now give an example to illustrate Theorem 2.1.

Example 2.1. Let $X = [-1, 1]$ and let G be the G-metric on $X \times X \times X$ defined as $G(x, y, z) = |x-y|+|y-z|+|z-x|$ for all $x, y, z \in X$. Then (X, G) be a G-metric space. Let us define $fx = \frac{x}{6}$ and $gx = \frac{x}{2}$. Here we note that, f is continuous and $f(X) \subseteq g(X)$. Also, $G(fx, fy, fz)) \le qG(gx, gu, gz)$, holds for all $x, y, z \in X, \frac{1}{3} \le q < 1$ and 1 is the unique common fixed point of f and g.

3. Property (E.A.) in G-metric spaces

Recently, Amari and Moutawakil [1] introduced a generalization of non compatible maps as property (E.A.) in metric spaces as follows:

Definition 3.1. Let A and S be two self-maps of a metric space (X,d). The pair (A,S) is said to satisfy property (E.A.), if there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z$, for some $z \in X$.

In [22] property (E.A.) in metric spaces has been used to prove a common fixed point result. In similar mode we use property (E.A.) in G-metric spaces. Now we prove a common fixed point theorem for a pair of weakly compatible maps along with property (E.A.).

Theorem 3.1. Let f and g be two self maps on a G-metric space (X,G) satisfying condition (2.3) and the following conditions:

1. f and g satisfy property (E.A.), (3.1)
2. $g(X)$ is a closed subspace of X. (3.1)

Then f and g have a unique common fixed point in X provided f and g are weakly compatible self maps.

Proof : Since f and g satisfy property (E.A.), therefore, there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = u \in X$. Since $g(X)$ is a closed subspace of X, therefore every convergent sequence of points of $g(X)$ has a limit point in $g(X)$ .Therefore, $\lim_{n\to\infty} fx_n = uuga = \lim_{n\to\infty} gx_n$ for some $a \in X$. This implies that $u = ga \in g(X)$.

Now from (2.3), we have
$$G(fa, fx_n, fx_n) \leq \alpha G(fa, gx_n, gx_n) + \beta G(ga, fx_n, gx_n) + \gamma G(ga, gx_n, fx_n).$$

Letting $n \to \infty$ and using $0 \leq \alpha + 3\beta + 3\gamma < 1$, we have $u = fa$. This implies $u = ga = fa$. Thus a is the coincidence point of f and g. Since f and g are weakly compatible, therefore, $fu = fga = gfa = gu$.

Again from (2.3), we have
$$G(fu, fa, fa) \leq \alpha G(fu, ga, ga) + \beta G(gu, fa, ga) + \gamma G(gu, ga, fa),$$
since $0 \leq \alpha+3\beta+3\gamma < 1$, above inequality implies that $fu = u$. Hence u is common fixed point of f and g. Uniqueness follows easily.

Corollary 3.1. Let (X,G) be a complete G-metric space and f, g be two self mappings on (X,G) satisfying (3.1), (3.2) and the following condition:
$$G(fx, fy, fz) \leq qG(gx, gy, gz)$$
for every $x, y, z \in X$ and $0 < q < 1$. Then f and g have a unique common fixed point in X provided f and g are weakly compatible self maps.

Proof. Proof follows easily from above Theorem 3.1.

Conclusions : Our results involve the followings:

1. to relax the continuity requirement of maps completely,
2. to minimize the commutativity requirement of the maps to the point of coincidence,
3. to weaken the completeness requirement of the space,
4. property (E.A.) buys containment of ranges without any continuity requirement to the points of coincidence.

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