Applied Mathematics & Information Sciences
An International Journal

http://dx.doi.org/10.12785/amis/090532

Optimal Control of a Bioeconomic Model for a Two-Stage Species with Recruitment

Lilia M. Ladino[1], *Edison I. Sabogal*[1] *and Jose C. Valverde*[2,*]

[1] Department of Mathematics and Physics, University of Los Llanos, 2621 Villavicencio, Colombia
[2] Department of Mathematics, University of Castilla-La Mancha, 02071 Albacete, Spain

Received: 29 Nov. 2014, Revised: 23 Dec. 2014, Accepted: 24 Dec. 2014
Published online: 1 Sep. 2015

Abstract: In this work, the capture effort exerted on the adult population (or exploitable population) of a two-stage species with recruitment is considered as a control parameter to investigate the optimal exploitation of the resource, sustainability properties of the population and rent earned from the resource. Pontryagin's maximum principle is used to characterize the optimal control. The optimal control is derived and then solved numerically using an iterative method with Euler scheme. Simulation results show that the optimal control scheme allows us to get the sustainability of the ecosystem and to validate the model.

Keywords: Bioeconomic modeling, Optimal control, Pontryagin's maximum principle, Computer simulations.

1 Introduction

The capture and its impact on species conservation has been a matter of interest not only to biologists and ecologists, but also to mathematicians and those devoted to the modeling and simulation of dynamical systems. Mathematical models based on ordinary differential equations of population dynamics with harvesting or capture have been studied for decades and remains being an issue in full swing [2, 18, 19, 17, 10, 9, 15, 26, 29, 30, 31].

Currently, there is great interest about harvesting policies and bioeconomic modeling of different biological populations [6, 8, 9, 10, 16, 23, 31, 26]. Biological resources are renewable resources. Among all the renewable resources, an important one is fishery. Different species of fishes are decreased due to exploitation on a nonsustainable basis, high growth rate of world population and lack of knowledge of the characteristics of exploited species [23]. To ensure the conservation of resources in the future and provide a sustainable flow of benefits to human society, it is a compelling need to control the actions associated to capture. The adequate management of the resources will provide protection from overexploitation. Therefore, further scientific investigation is required in the field of biology and the bioeconomy. Bioeconomic models assist natural resource managers in controlling the appropriate level of stocks and catches [23].

In this sense, nowadays, there is great interest about harvesting policies and bioeconomic modeling of different biological populations. Populations models with optimal harvesting policy have been studied for single populations [20], populations with stage-structured [23, 31, 26], prey-predator systems [9, 10, 23, 22, 31], and models with time delay [32]. However, most of these models remain theoretical and only a few of them have been applied to real case studies (for example, the models proposed by Fresard and Ropars-Collet [12] or Ladino and Valverde [2, 18]).

Following the ideas in [15], where the authors study a two-stage population with a linear recruitment function, but where both stages are harvested, the main purpose of this paper is to analyze the optimal control problem in a model, previously proposed by us in [18], for the population dynamics of a two-stage species with recruitment given by the nonlinear Beverton-Holt function, but where only the post-recruit individuals are harvested, since they constitute the part of the total population that is commonly admitted as visible to fishing.

More concretely, in such a model, it is considered a migratory population with a two-stage structure:

* Corresponding author e-mail: jose.valverde@uclm.es

pre-recruit (eggs, larvae and juvenile ones) and post-recruit or exploitable (adults). It shows how the dynamics is determined by a basic threshold parameter $\mathscr{R}$. As the most important result for this model, it is proved that when $\mathscr{R} \leq 1$, then the extinction equilibrium point is globally asymptotically stable, assuring that the species would disappear under this condition. Now, in this paper, we achieve to maximize the net economic revenue earned from the capture. The capture effort exerted on the adult population (or exploitable population) is considered as a control parameter to investigate the optimal exploitation of the resource, sustainability properties of the population and rent earned from the resource. Pontryagin's maximum principle is used to characterize the optimal control. The optimal control is derived and then solved numerically using an iterative method with Euler scheme. Simulation results show that the optimal control scheme allows us to get the sustainability of the ecosystem and to validate the model.

The numerical simulations, performed by means of our own software [1], have been executed by using statistical data of some fish stocks of *Prochilodus*, which are abundant in the most important river basins of South America and are the most famous long-distance migrants. The fish population migrates, searching temporary shelters, feeding habitats and spawning grounds [3].

The *Prochilodus mariae*, a specific species of *Prochilodus*, has a dual role in aquatic ecosystems of the Orinoco basin because: on one hand, due to its detritivore characteristics, it is within the first link in the food chain; on the other hand, it is captured to be commercialized, becoming a source of products of consumption for coastal populations (indigenous and settler), and being the most important protein base of these communities [24]. Thus, the exploitation of *Prochilodus* should be controlled optimally, because any decline of this species could have an important impact [18].

The organization of this paper is as follows. In Section 2, we review the mathematical model of population dynamics of a two-stage species with recruitment stated in [18]. In Section 3, we obtain an optimal harvesting policy for the model. In Section 4, we provide specific examples and, by means of numerical simulations, we corroborate our analytical results. Finally, in Section 5, we present some interesting conclusions and future research directions which arise with this work.

2 Mathematical Model

We will base our study on a populations dynamics model developed by us in [18] that we review bellow. The assumptions made are coherent with the ideas, concepts and parameters of mathematical modeling of biological populations made by other authors [6,7,21,11,4] and, in this sense, some of them have the same meaning.

– The *stock* is defined as a subset of a given species, in which population parameters remain constant [27]. The stock is represented in the model by the size of the population or population density.
– The total population is divided into two subpopulations: *pre-recruit population* $x(t)$ (eggs, larvae and juvenile ones) and *exploitable population* $y(t)$ (adults).
– The *reproduction rate* of the adults (exploitable) population is δ.
– The *natural death rate* of both the pre-recruit population and exploitable population is μ.
– The *recruitment rate* γ is proportional to the *total number of recruits* R, i.e., $\gamma = cR$; where $c = 1$ for simplicity of the model.
– The total number of recruits depends only on the size of the pre-recruit population. The form of the stock-recruitment relationship of Beverton-Holt [4] is taken in order to establish the number of recruits R in terms of pre-recruit population so:

$$R = \frac{\alpha \cdot x(t)}{\beta + x(t)}$$

– The *maximum recruitment* is α [13].
– The parameter β is the stock needed to produce (on average) a recruitment equal to $\alpha/2$ [13].
– The parameter q is called *catchability coefficient*, which is a measure of the ability to catch individuals. The more efficient the art of capture is, the higher value of q is [27].
– The parameter E is the *capture effort* (in the fishing, for example, is number of boats per day, boat days, etc.) [14,27].
– The parameter F is the *capture death rate*, which is proportional to capture effort (E) and is given by the relationship $F = q \cdot E$ [14,27].
– An homogeneous distribution of both the pre-recruit population and the exploitable population is assumed. That is, every pre-recruit individual has the same probability of being recruited, and every adult individual has the same probability of being caught.

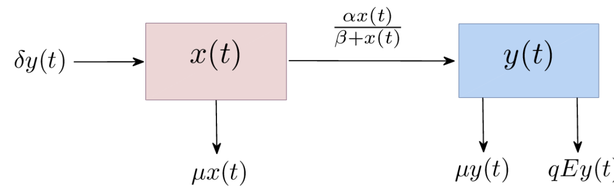


Fig. 1: Diagram of the dynamics of a two-stage population with recruitment.

Given the above assumptions, the Figure 1 shows the dynamics of the population modeled.

Moreover, the system representing the dynamics of a two-stage species with recruitment and capture can be stated by

$$\begin{cases} \dot{x}(t) = \delta y(t) - \dfrac{\alpha x(t)}{\beta + x(t)} - \mu x(t) \\ \dot{y}(t) = \dfrac{\alpha x(t)}{\beta + x(t)} - (\mu + qE)y(t) \end{cases} \quad (1)$$

The parameters in this model are considered non-negative. The region $\Omega = \{(x,y)/x \geq 0, y \geq 0\}$ is a positive invariant set for the system (1). Therefore, it is considered the phase space of this system.

3 The optimal control problem

Nowadays bio-economic models are used as a tool to better understand the impact of policies on natural resources and human welfare [9]. In the context of commercial exploitation of renewable resources, the most important economic problem is to find the optimal trade-off between present and future harvests [10]. For this reason, it is necessary to take into account the studies on the development, conservation and exploitation of resources [9].

Usually, it is assumed that capture per unit effort is proportional to the population size. So, $\frac{H}{E} = \alpha y$ implies $H = qEy$, where H and E are the capture and effort, respectively, applied to the harvested population in the exploited area, and q is the constant called catchability coefficient, as said before.

In fisheries management, it is generally considered that the *total cost (TC)* is proportional to fishing effort [8]. Thus in this model the total cost of fishing effort is defined by

$$TC(E) = cE, \quad (2)$$

where c is the *unit cost of fishing* or the *marginal cost of effort*, and it is constant.

On the other hand, it is usually assumed that the *total revenue (TR)* is proportional to capture [8], that is

$$TR(E) = \rho H(E), \quad (3)$$

where ρ represents the *price per unit biomass* of population captured. Actually, it is evident that the price is a function which decreases with increasing biomass. Therefore, following the usual economic hypotheses, we assume that

$$TR(E) = (p - vqEy)qEy, \quad (4)$$

where $(p - vqEy)$ is the *price per unit biomass* and the constant v represents the variability of the quantity demanded with respect to the variability of the price. This expression can be used to consider monopolistic or oligopolistic concurrency, which is a more general case and includes the one in which the price per unit biomass is constant, what means perfect concurrence ($v = 0$) [28].

The *sustainable economic rent* (profit) at a given level of fishing effort E is the difference between the total revenue of the fishery and the total fishing costs. Therefore sustainable economic rent can be expressed as

$$\pi(E) = TR(E) - TC(E). \quad (5)$$

Thus, the optimal control problem for the model (1), allowing us to maximize the total discounted resource rent earned from the fishery, can be formulated by

$$J(E) = \int_{t_0}^{t_f} e^{-\omega t} \left[(p - vqEy)qEy - cE\right] dt, \quad (6)$$

where ω is the *instantaneous annual discount rate*.

We consider that the present valuation of capital flow over time depends on the discount rate, ω. The discount rate would therefore determine the stock level, maximizing the present value of the flow of the resource rent over time. This is known as the *optimal economic yield biomass*.

To solve the problem of maximizing $J(E)$ in (6), subject to equations of system (1) and the control restriction $0 \leq E \leq E_{max}$ (because a fishery always has a maximum harvesting capability [6], implying a maximum fishing effort), it is possible to apply the Pontryagin's maximum principle ([5],[16],[25]). The convexity of the objective function with respect to E, the linearity of the differential equations in the control and the compactness of the range values of the state variables allows us to give the existence of the optimal control.

Suppose E_ω is an optimal control with respective states x_ω and y_ω. We suppose $A_\omega = (x_\omega, y_\omega)$ as optimal equilibrium point. Now, it is intended to derive optimal control E_ω such that

$$J(E_\omega) = max\{J(E) \mid E \in U\},$$

where U is the control set given by

$$U = \{E : [t_0, t_f] \to [0, E_{max}] \mid E \text{ is Lebesgue measurable}\}.$$

Thus, the *Hamiltonian* of this optimal control problem is

$$L = (p - vqEy)qEy - cE + \lambda_1 \left[\delta y - \tfrac{\alpha x}{\beta + x} - \mu x\right] + \\ + \lambda_2 \left[\tfrac{\alpha x}{\beta + x} - (\mu + qE)y\right]$$

where λ_1 and λ_2 are adjoint variables. Here, the transversality conditions give $\lambda_i(t_f) = 0$, $i = 1, 2$.

Now, it is possible to find the characterization of the optimal control E_ω as follows. On the set $\{t \mid 0 < E_\omega(t) < E_{max}\}$, we have

$$\frac{\partial L}{\partial E} = pqy - 2vq^2Ey^2 - c - \lambda_2 qy.$$

Thus, at $A_\omega = (x_\omega, y_\omega)$, $E = E_\omega(t)$, we have

$$\frac{\partial L}{\partial E} = pqy_\omega - 2vq^2E_\omega {y_\omega}^2 - c - \lambda_2 qy_\omega = 0.$$

This implies that

$$E_\omega = \frac{pqy_\omega - c - \lambda_2 q y_\omega}{2vq^2{y_\omega}^2}. \tag{7}$$

On the other hand, the adjoint equations at the point $A_\omega = (x_\omega, y_\omega)$ are

$$\frac{d\lambda_1}{dt} = \omega\lambda_1 - \frac{\partial L}{\partial x}\,|_{A_\omega} = \lambda_1 \left[\omega + \frac{\alpha\beta}{(\beta + x_\omega)^2} + \mu\right] - \lambda_2 \frac{\alpha\beta}{(\beta + x_\omega)^2} \tag{8}$$

$$\frac{d\lambda_2}{dt} = \omega\lambda_2 - \frac{\partial L}{\partial y}\,|_{A_\omega} = \lambda_1\delta + \lambda_2(\omega + \mu + qE) - (pqE - 2vq^2E^2y_\omega) \tag{9}$$

Equations (8) and (9) constitute a first order system of simultaneous differential equations and it is easy to get the analytical solution of the equations with the help of initial conditions $\lambda_i(t_f) = 0$, $i = 1,2$. In this regard, it has to be noticed that we have formulated the optimal control problem through considering fishing effort as control parameter. Hence, the optimal control problem will be numerically solved using the forward-backward sweep technique of Euler method to pursue numerical simulations in the next section.

The following Theorem summarize the above analysis.

Theorem 3.1. There exist corresponding solutions to the system (1), x_ω and y_ω, and an optimal control E_ω that maximizes $J(E)$ over U. Furthermore, there exist adjoint functions λ_1 and λ_2 satisfying equations (8) and (9) with transversality conditions $\lambda_i(t_f) = 0$, $i = 1,2$. In such a context, the optimal control for the problem is given by

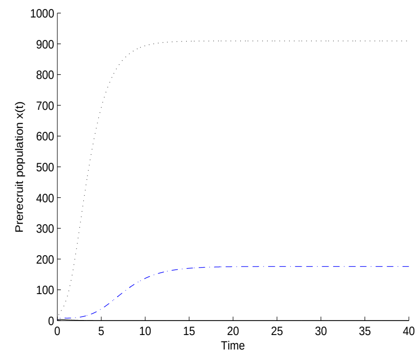
$$E_\omega = \frac{pqy_\omega - c - \lambda_2 q y_\omega}{2vq^2{y_\omega}^2}$$

.

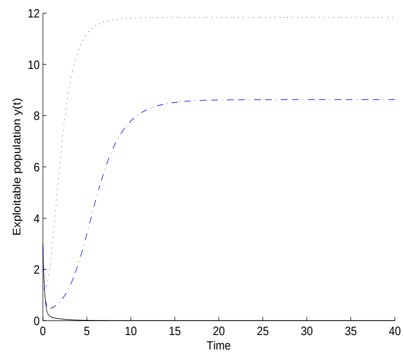
4 Numerical simulation

Below, different scenarios are simulated in order to show different dynamics of the system according to some relevant parameters. From a biological and fisheries point of view, this is important since it is possible to design strategies for the control and sustainability of this population.

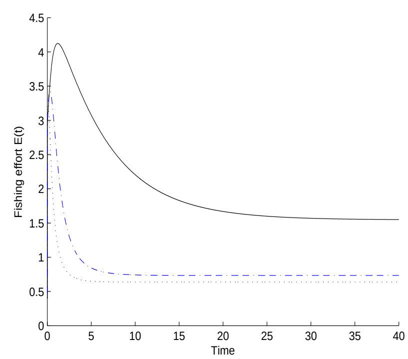
Due to the importance in the fishing trade of the species *Prochilodus mariae*, actual statistics about some population parameters of this species were used for performing the numerical simulations by means of our software [1]. These parameters were obtained from research results on the state of fishing for *Prochilodus mariae* in the Orinoco river of Colombia for the time period from 2005 to 2008. Thus, we have that the total death rate of the species Z is 1.38 and the fishing death F is 0.75 [24]. However, the other parameters were estimated theoretically because of the difficulty of getting serious research publications about them. Although using real data of model parameters would be of great interest, numerical analysis presented has the advantage of being able to simulate and analyze different scenarios of the feasible parametric biological space. In this sense, the simulations of this work should be considered from a qualitative rather than a quantitative approach.



(a)



(b)



(c)

Fig. 2: Variation of optimal pre-recruit and exploitable population and capture effort with (increasing) time. The solid line corresponds to $\delta = 5$, the blue dashed line to $\delta = 14,6$ and the dotted line to $\delta = 50$.

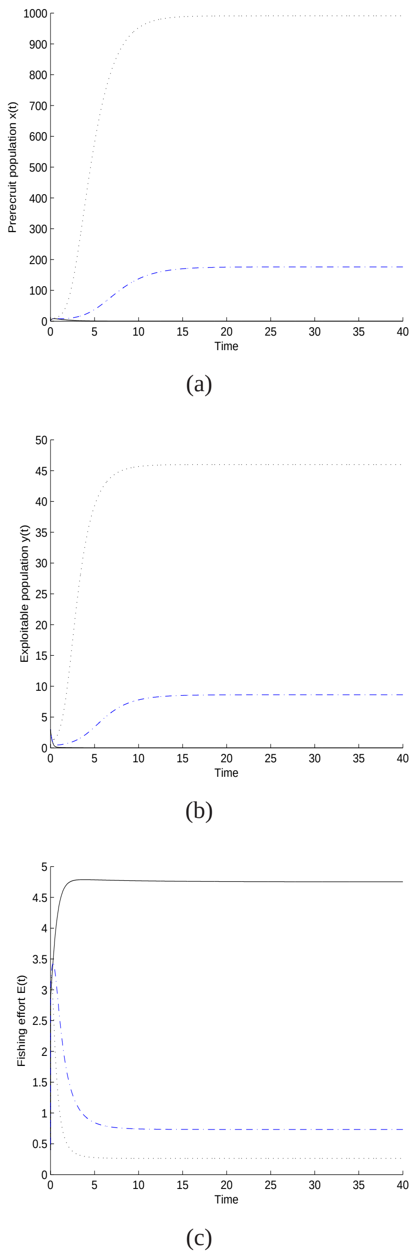


Fig. 3: Variation of optimal pre-recruit and exploitable population and capture effort with the increasing time. The solid line corresponds to $\alpha = 5$, the blue dashed line to $\alpha = 20$ and the dotted line to $\alpha = 50$.

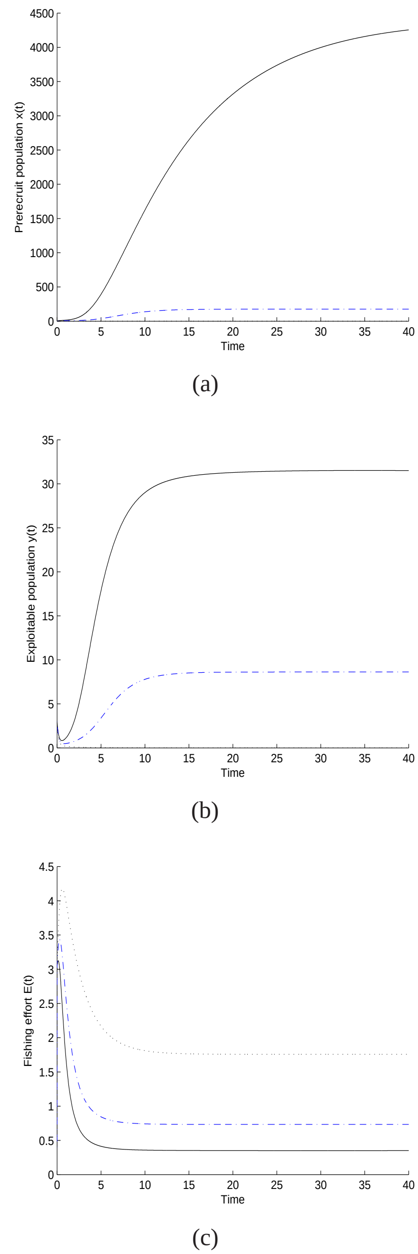
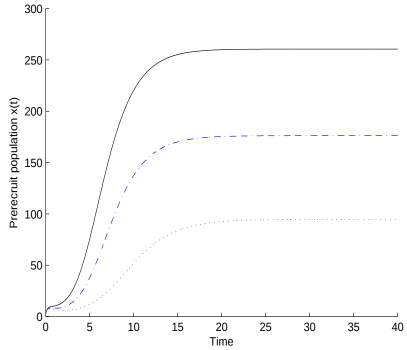


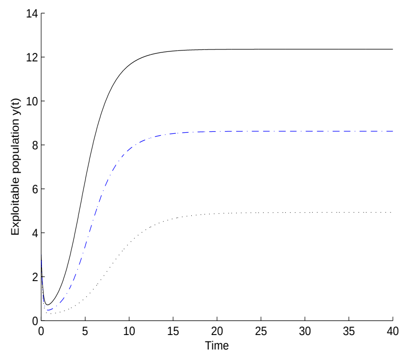
Fig. 4: Variation of optimal pre-recruit and exploitable population and capture effort with the increasing time. The solid line corresponds to $\mu = 0.1$, the blue dashed line to $\mu = 0.63$ and the dotted line to $\mu = 1.2$.

The numerical simulation of optimal control with different sets of parameters is possible with a forward-backward sweep technique of Euler method to solve the system (1) and their corresponding adjoint equations (8) and (9).

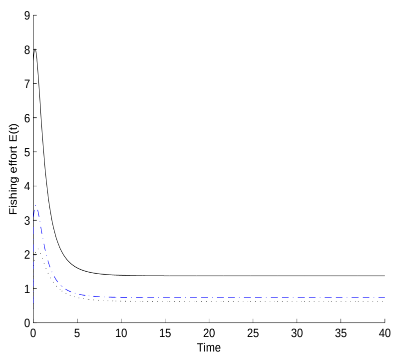
From Figure 2(a) and 2(b) one can deduce that, in presence of capture, the density of the populations is proportional to the reproduction rate of exploitable population δ. From the Figure 2(c), it can be seen that the capture effort used to harvest the populations varies inversely with δ. Note that, at the initial level, capture effort decreases with time. But, after a specific time of span, it is constant and the populations converge to a finite quantity.



(a)

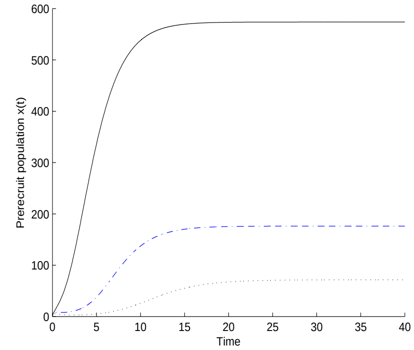


(b)

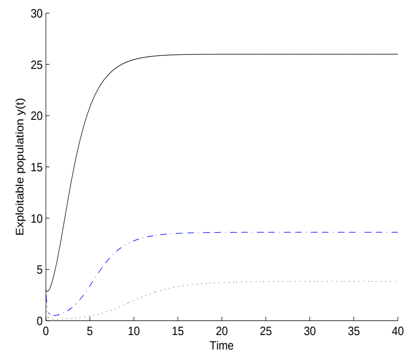


(c)

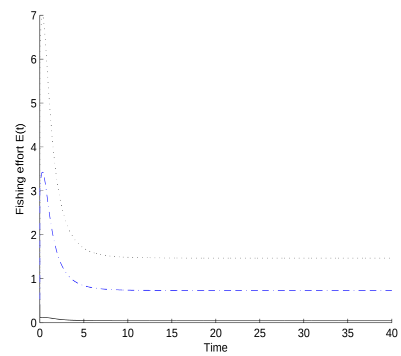
Fig. 5: Variation of optimal pre-recruit and exploitable population and capture effort with (increasing) time. The solid line corresponds to $q = 0.5$, the blue dashed line to $q = 1.5$ and the dotted line to $q = 3$.



(a)



(b)



(c)

Fig. 6: Variation of optimal pre-recruit and exploitable population and capture effort with (increasing) time. The solid line corresponds to $c = 1$, the blue dashed line to $c = 2$ and the dotted line to $c = 4$.

From Figure 3(a) and 3(b), it can be inferred that, in presence of capture, the density of the populations is proportional to the maximum recruitment α. This also happens in the Figure 3(c), in which the capture effort used to harvest the populations varies inversely with α. Observe that, from the initial level, capture effort decreases with time. But, after a precise time of span, the capture is constant and the populations approximate a finite quantity and remain so over the time.

From Figure 4 it can be deduced that the natural death rate μ has an important role in the dynamics of the system. It is clearly observed from Figure 4(a) and 4(b) that, in presence of capture, the densities of the populations vary inversely with μ. On the other hand, from Figure 4(c), it can be infered that the capture effort used to harvest the populations is proportional to μ. Notice again that, at the initial level, capture effort decreases with time. But, after a specific time of span, it remains constant and, as before, the population is set to a finite size.

From Figure 5(a) and 5(b), it can be deduced that, in presence of capture, the densities of the populations vary inversely with the catchability coefficient q. It is also evident from the Figure 5(c) that the capture effort used to harvest the populations varies inversely with q. Again, note that, at the initial level, capture effort decreases with time. However, after a particular time of span, the capture is constant and the populations approximate a finite quantity and remain so over time.

The stock level maximizing the present value of the flow of resource rent over time is determined by the economic parameters (such as price per unit biomass of catch, fishing cost per unit effort and discount rate). From Figure 6, it can be inferred that the fishing effort increases with the increasing of the unit cost of fishing c and consequently both populations decreases.

5 Conclusions and future research directions

The optimal control problem is formulated and solved both analytically and numerically in a bioeconomic context. The sensitivity analysis was performed with respect to some relevant parameters of the system. Since this work is not a particular case, the analytical results may be helpful for researchers in this topic, mainly for those engaged in fisheries research.

The results obtained are biologically consistent and hence they allow us to validate the model. On one hand, these results show that, as the reproduction rate and the recruitment increase, the population increases and therefore the optimal capture effort should be lower. On the other hand, the density of the populations varies inversely with the natural death rate and the catchability coefficient, and in such cases the optimal capture effort used to harvest the populations is proportional to the natural death rate, although varying inversely with the catchability coefficient. Moreover, note that after a specific time the populations converge to a finite quantity when the effort reaches its optimum.

From the work developed here, new research directions raise in this area. For example, generally the reproduction of eggs, natural death and fishing death rates are depending on environmental conditions, fishing seasons and techniques used and thus it may vary. Therefore, it would be interesting to analyze the model with non-constant parameters. Moreover, it is possible to consider other more complex scenarios ([2] and [17]) with general forms of recruitment, depending on special features in the system (such as cannibalism of young fishes by adults), interaction with other species (competition, predation or mutualism), transmission disease, etc.

On the other hand, it is possible that the integration of the pre-recruit population to the exploitable population is not immediately, that is, after a certain period of time, which may be the corresponding recruitment age. This aspect is another interesting research direction that would be appropriate to analyze with the proposed model, but including delay.

Acknowledgement

Jose C. Valverde thanks Ministerio de Ciencia e Innovación of Spain for their support for this work through the Spanish national grant MTM2011-23221.

Lilia M. Ladino thanks University of Los Llanos for support in the project Bioeconomic Analysis of Population Dynamics Models to Capture.

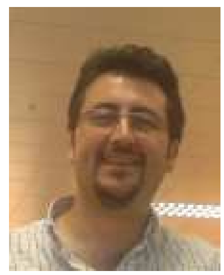
References

[1] A.M. Atehortua, L.M. Ladino, J.C. Valverde, Ing. Invest. **32**, 51–57 (2012).
[2] A.M. Atehortua, L.M. Ladino, J.C. Valverde, Nonlinear Anal.-Real World Appl. **16**, 27–39 (2014).
[3] A. Barbarino, D. Taphorn, K. Winemiller, Env. Biol. Fishes **21**, 33–46 (1998).
[4] R.J. Beverton, S.J. Holt, On the dynamics of exploited fish populations, Chapman and Hall, London, 1993.
[5] V.G. Boltyanskii, R.V. Gamkrelidze, L.S. Pontryagin, Reports Acad. Sci. USSR, **110 (1)**, (1956).
[6] F. Brauer, C. Castillo-Chávez, Mathematical models in population biology and epidemiology, Texts in Applied Mathematics, 40, Springer, New York, 2001.
[7] N.F. Britton, Essential mathematical biology, Springer Undergraduate Mathematics Series, Springer-Verlag, New York, 2003.
[8] C.W. Clark, Mathematical Bioeconomics: The optimal management of renewable resources, Second ed., John Wiley and Sons, New York, 1990.
[9] K. Chakraborty, T.K. Kar, Math. Biosci. **240**, 212–222 (2012).
[10] K. Chakraborty, S. Jana, T.K. Kar, Appl. Math. Comput. **218**, 9271–9290 (2012).
[11] J. Csirke, Introducción a la dinámica de poblaciones de peces, Food and Agriculture Organization of the United Nations (FAO), Roma, 1989.
[12] M. Fresard, C. Ropars-Collet, Ecol. Econ. **106**, 1–11 (2014).
[13] R. Hilborn, C.J. Walters, Quantitative fisheries stock assessment: choice, dynamics, and uncertainty, Chapman and Hall, New York, 1992.
[14] J.F. Isaza, D. Campos, Ecología: una mirada desde los sistemas dinámicos, Editorial Pontificia Universidad Javeriana, Bogotà, 2006.
[15] W. Jing, W. Ke, Appl. Math. Comput. **148(1)**, 235–247 (2004).
[16] M. Kot, Elements of mathematical ecology, Cambridge University Press, New York, 2001.
[17] L.M. Ladino, E.I. Sabogal, J.C. Valverde, Math. Methods. Appl. Sci. (2014), http://dx.doi.org/10.1002/mma.3271.

[18] L.M. Ladino, J.C. Valverde, Math. Methods Appl. Sci. **36**, 722–729 (2013).
[19] E. Liz, F. Hilker, Springer Proceedings in Mathematics and Statistics **102**, 61–73 (2014).
[20] D. Manna, G. Samanta, Electron. J. Appl. Stat. Anal. **7**, 117–131 (2014).
[21] J.D. Murray, Mathematical biology I. An Introduction, Springer, Berlin, 2002.
[22] D. Pal, G.S. Mahapatra, Appl. Math. Comput. **242**, 748–763 (2014).
[23] D. Pal, G.S. Mahaptra, G.P. Samanta, Math. Biosci. **241**, 181–187 (2013).
[24] H. Ramírez, R. Ajiaco, C. Barreto, Estado actual de la pesca del bocachico (Prochilodus mariae) en la Orinoquia colombiana, Proyecto de investigación interinstitucional: Universidad de los Llanos (UNILLANOS), Corporación Colombia Internacional (CCI), Instituto Colombiano Agropecuario (ICA) e Instituto Colombiano de Desarrollo Rural (INCODER), Bogotá, 2010.
[25] S.P. Sethi, G.L. Thompson, Optimal Control Theory: Applications to Management Science and Economics, 2nd edition, Springer, 2000.
[26] X. Song, L. Chen, Math. Biosci. **170**, 173–186 (2001).
[27] P. Sparre, S.C. Venema, Introducción a la evaluación de recursos pesqueros tropicales. Parte 1. Manual, Food and Agriculture Organization of the United Nations (FAO), Roma, 1997.
[28] H.R. Varian, Microeconomic Analysis, Norton and Company, 1992.
[29] L. Yang, S. Zhong, Ecol. Complex. **19**, 111–123 (2014).
[30] R. Yuan, W. Jiang, Y. Wang, J. Math. Anal. Appl. **422**, 1072–1090 (2015) .
[31] X. Zhang, L. Chen, A.U. Neumann, Math. Biosci. **168**, 201–210 (2000).
[32] Y. Zhang, Q. Zhang, X.G. Yan, Physica A **404**, 180–191 (2014).

Lilia M. Ladino graduated from the University of Los Llanos (Colombia) and earned a PhD degree from the doctoral programme of Physics and Mathematics at the University of Castilla-La Mancha (Spain). She is a Professor of the Department Mathematics and Physics at the University of Los Llanos (Colombia). Her research interests include dynamical systems and biomathematics. She has published articles in JCR-journals such as ”Nonlinear Analysis: Real World Applications” and ”Mathematical Methods in the Applied Sciences”. She has been visiting Professor at University of Macerata (Italia). She has also participated in some international congresses.

Edison I. Sabogal graduated from the University National of Colombia (Colombia). He has earned a MSc in Applied Mathematics from EAFIT University (Colombia). He is a Professor of the Department Mathematics and Physics at the University of Los Llanos (Colombia). His research interests include dynamical systems and biomathematics. He has published articles in JCR-journals. He has also participated in some congresses.



Jose C. Valverde graduated from the University of Murcia (Spain) and earned a PhD degree from the doctoral programme of Mathematics at the same University. He is a member of the Institute of Mathematical Research Applied to Science and Engineering of Castilla-La Mancha and a Professor of the Department Mathematics at the University of Castilla-La Mancha (Spain). His research interests include dynamical systems, biomathematics and discrete mathematics in relation to computer science. He is the author/coauthor of 5 books and has published more than 30 journal articles. He has served as editor-in-chief, chief editor, guest editor and editorial board member of several indexed international journals and has referred papers for several JCR-journals. He has been visiting Professor at some Universities and main speaker in some international conferences. He has also participated as a scientific committee member or as a technical expert committee member of some international congresses.