

# Numerical Study of Nonlinear Micro Circular Plate Analysis using Hybrid Method (H.M.)

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Received: 3 Dec. 2013, Revised: 3 Apr. 2014, Accepted: 4 Apr. 2014

Published online: 1 Feb. 2015

**Abstract:** Micro-electro-mechanical systems is that the size of component or the movement range is within micro grade, also called the Micromechanical or Microsystems. It is a cross-curricular research field, covering topics such as physics, optics, mechanics, electricity, biology and chemistry. The micro fluid system is an important branch of micro-electro-mechanical systems, it can be widely applied to fields such as medical, chemical analysis and lubricating, etc.. The micro pump is the actuating component of the micro fluid system. The analytical modeling of the micro circular plate devices by electrostatic is problematic due to the complexity of the interactions between the electrostatic coupling effect, residual stress and the nonlinear electrostatic force. Therefore, the dynamic behavior of the micro circular plates is not easily analyzed using traditional analytic methods. Accordingly, this study develops an efficient computational scheme in which the nonlinear governing equation of the coupled electrostatic force acting, residual stress and hydrostatic pressure acting on the micro circular plates system is solved using a hybrid method (H.M.) which differential transformation with finite difference approximation method. In addition, this study shows the dynamic behavior of the micro circular plates by a DC actuating load.

**Keywords:** Pull-in voltage; Micro circular plate; Electrostatic actuator; Hybrid Method; Differential Transformation.

## 1 Introduction

In recent years, micro-electro-mechanical systems (MEMS) devices have been widely applied for a diverse range of applications, ranging from accelerometer and pressure sensors in automotive security systems [1], to micro-scale actuators in the aerospace and medical fields [2], electrostatic rotary comb actuators for micromirrors [3], actuators for Digital Micromirror Devices (DMDs) [4], and so forth. Therefore, MEM is highly worth to research for industry. The actuation systems used in today's MEMS devices can be broadly classified as either electrostatic [5], thermal, piezoelectric [6] or electromagnetic [7]. Of these various techniques, electrostatic actuation schemes are commonly preferred since they are easily fabricated using established surface micromachining techniques, have a rapid response and a low power consumption. In practice, the actuation effect in such schemes is created by generating an electrostatic force between the stationary and the moving parts of the actuator through the application of an external voltage.

However, in implementing such a system, great care must be taken to specify appropriate values of the device parameters and operating conditions in order to prevent the so-called "pull-in phenomenon", in which the attractive electrostatic force induced by the external voltage exceeds the restoring force developed within the deflected membrane and causes it to collapse and make a momentary contact with the lower electrode. The voltage at which this phenomenon takes place is referred to as the "pull-in voltage" and is of critical importance in the design of many MEMs-based devices [8]. Generally speaking, the dynamic behavior of the micro circular plates used in MEMS electrostatic actuators is not easily analyzed using traditional methods such as Galerkin method due to the complexity of the interactions between the electrostatic coupling effects.

Differential transformation theory was originally proposed by Zhao in 1986 as a means of solving linear and nonlinear initial value problems in the circuit analysis domain. However, in more recent years, researchers have extended its use to the analysis of a variety of initial value

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problems in the mechanical engineering field [9,10]. Chen *et al.* [11] demonstrated that the hybrid method (H.M.) which differential transformation and finite difference method provides a precise and computationally-efficient means of analyzing the nonlinear dynamic behavior of fixed-fixed micro-beams. The same group also used the hybrid method to analyze the nonlinear dynamic response of an electrostatically-actuated micro circular plate subject to the effects of residual stress and a uniform hydrostatic pressure acting on the upper surface [12]. Finally, Kuo and Chen [13] employed the hybrid method (H.M.) which differential transformation and finite difference method to solve the nonlinear Burgers' equation for various values of the Reynolds number, including high values.

In the current study, the hybrid method (H.M.) is employed to analyze the electrostatic gap, membrane thickness, membrane radius of the dynamic behavior of the micro circular plates by a DC actuating load. In formulating the nonlinear governing equation of the circular plate, explicit account is taken of both the hydrostatic pressure and the residual stress. The hybrid method can be applied easily and efficiently for nonlinear electrostatic behavior to obtain numerical results of the micro circular plate.

## 2 Differential Transformation Theory

This section reviews the basic principles of differential transformation theory. Assume that  $y(t)$  is an analytic function in the time domain  $T$ . The differential transformation of  $y$  at time  $t = t_0$  in the  $K$  domain is given by

$$Y(k; t_0) = W(k) \left( \frac{d^k}{dt^k} (s(t)y(t)) \right)_{t=t_0}, \quad k \in K \quad (1)$$

where  $k$  belongs to a set of non-negative integers which collectively define the  $K$  domain;  $W(k)$  is a weighting factor; and  $s(t)$  is a kernel function corresponding to  $y(t)$ . Note that  $W(k)$  and  $s(t)$  are both non-zero and  $s(t)$  is analytic in the time domain. The inverse differential transformation of  $Y(k; t_0)$  is formulated as

$$y(t) = \frac{1}{s(t)} \sum_{k=0}^{\infty} \frac{(t-t_0)^k}{k!} \frac{Y(k; t_0)}{W(k)}, \quad t \in T, \quad (2)$$

in which  $W(k) = H^k/k!$  and  $s(t) = 1$ . Note that  $H$  is the time interval.

$$Y(k) = \frac{H^k}{k!} \left[ \frac{d^k y(t)}{dt^k} \right]_{t=0}, \quad k \in K. \quad (3)$$

From Eq. (2), the inverse differential transformation of  $Y(k)$  is obtained as

$$y(t) = \sum_{k=0}^{\infty} \left( \frac{t}{H} \right)^k Y(k), \quad t \in T. \quad (4)$$

Substituting Eq. (3) into Eq. (4) gives

$$y(t) = \sum_{k=0}^{\infty} \frac{t^k}{k!} \left[ \frac{d^k y(t)}{dt^k} \right]_{t=0}, \quad t \in T. \quad (5)$$

Eq. (5) has the form of a Taylor series expansion. Therefore, the basic operational properties of the differential transformation method (D.T.M) can be summarized as follows:

(a) Linearity operation

$$T[\alpha p(t) + \beta q(t)] = \alpha P(k) + \beta Q(k), \quad (6)$$

where  $T$  denotes differential transformation and  $\alpha$  and  $\beta$  are any real number.

(b) Convolution operation

$$T[p(t)q(t)] = P(k) \otimes Q(k) = \sum_{\ell=0}^k P(\ell)Q(k-\ell),$$

$$T[p^m(t)] = kP(0)P^m(k) = \sum_{\ell=1}^k [(m+1)\ell] P(\ell)P^m(k-\ell), \quad m \in N \quad (7)$$

where  $\otimes$  denotes convolution.

(c) Differential operation

$$T \left[ \frac{d^n p(t)}{dt^n} \right] = \frac{(k+n)!}{k!H^n} P(k+n), \quad (8)$$

where  $n$  is the order of differentiation

(d) Differential transformation of  $\sin(t)$  and  $\cos(t)$  functions

$$T[\sin(\alpha t + \beta)] = \frac{(\alpha H)^k}{k!} \sin\left(\frac{\pi k}{2} + \beta\right),$$

$$T[\cos(\alpha t + \beta)] = \frac{(\alpha H)^k}{k!} \cos\left(\frac{\pi k}{2} + \beta\right), \quad (9)$$

where  $\alpha$  and  $\beta$  are any real number. [8,11-12].

## 3 Modeling of Micro Circular Plate

In deriving the nonlinear governing equation of motion of the micro circular plate shown in Fig. 1, an assumption is made that the plate is subject to displacements and small strains and undergoes an axi-symmetric bending effect. The dynamic governing equation is normalized for analytical convenience and is then solved using the hybrid method (H.M.).

### 3.1 Nonlinear governing equation of micro circular plate

As shown in Fig. 1, the micro-actuator system considered in this study comprises a movable circular plate with a

thickness  $b$  attached at its perimeter to a fixed rigid substrate. The gap between the two plates is filled with air and has an initial height of  $u$ . The application of an external voltage across the two electrodes creates an electrostatic attractive force which causes the micro circular plate to deflect in the downward direction toward the lower substrate. The displacement  $w$  of the circular plate is assumed to vary as a function of both the radial position  $r$  and the time  $t$ , i.e.  $w = w(r,t)$ . Due to the small physical size of the MEMS actuator system shown in Fig.1, it is essential to take the effects of hydrostatic pressure and residual stress into account when modeling the pull-in phenomenon. It is supposed that the symmetry deflection of micro circular plate is irrelevant to polar coordinate  $\theta$ . Thus, the dynamic governing equation should be rewritten as follows [12]:

$$\rho A \frac{\partial^2 w}{\partial t^2} + D \left( \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} \right) \left( \frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} \right) - F_r \left( \frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} \right) = \frac{\epsilon_0 V^2}{2(u-w)^2} + h_0 \quad (10)$$

where  $\rho$  is the density of the micro circular plate,  $F_r$  is the residual force,  $\epsilon_0$  is the permittivity of free space,  $V$  is the voltage between the upper and lower electrodes and  $h_0$  is the hydrostatic pressure which acts on the upper surface of the plate. Furthermore,  $D$  is the flexural rigidity of the plate can be expressed as

$$D = \frac{Eb^3}{12(1-\nu^2)}, \quad (11)$$

where  $E$ ,  $b$  and  $\nu$  are the Young's modulus, thickness and Poisson ratio of the micro circular plate, respectively. The boundary conditions for Eq. (10) are as follows:

$$w(r,t) = \frac{\partial w(r,t)}{\partial r} = 0, \text{ at } r = 0 \quad (12)$$

$$w(r,t) = \frac{\partial w(r,t)}{\partial r} = 0, \text{ at } r = \pm R \quad (13)$$

where  $R$  denotes the radius of the circular plate. Meanwhile, the initial conditions are given by:

$$w(r,0) = \frac{\partial w(r,0)}{\partial t} = 0. \quad (14)$$

### 3.2 Normalized nonlinear governing equation of micro circular plate

For analytical convenience, the displacement term  $w$  in the governing equation is normalized with respect to the initial gap height between the plates, the radial position term  $r$  is normalized with respect to the plate radius, and the time term  $t$  is normalized with respect to the constant  $T_1$ , i.e.

$$w^* = \frac{w}{u}, \quad r^* = \frac{r}{R}, \text{ and } t^* = \frac{t}{T_1}, \quad (15)$$

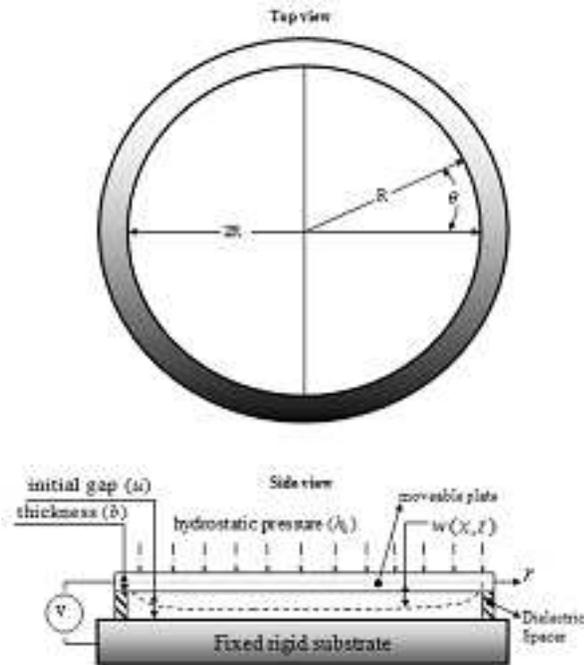


Fig. 1: Schematic illustration of micro circular plate actuator system.

where  $T_1 = \sqrt{\rho b R^4 / D}$ .

In addition, the voltage, residual stress and hydrostatic pressure terms are normalized as follows:

$$V^* = \sqrt{\frac{\epsilon_0 R^4 V^2}{2Du^3}}, \quad F_r^* = \frac{F_r R^2}{D}, \quad h_0^* = \frac{h_0 R^4}{Du}, \quad (16)$$

Substituting Eqs. (15) and (16) into Eqs. (10) and (12)-(14), the normalized governing equation can be expressed as follows:

$$\frac{\partial^2 w^*}{\partial t^{*2}} + \frac{\partial^4 w^*}{\partial r^{*4}} + \frac{2}{r^*} \frac{\partial^3 w^*}{\partial r^{*3}} - \frac{1}{r^{*2}} \frac{\partial^2 w^*}{\partial r^{*2}} + \frac{1}{r^{*3}} \frac{\partial w^*}{\partial r^*} - F_r^* \frac{\partial^2 w^*}{\partial r^{*2}} - F_r^* \frac{1}{r^*} \frac{\partial w^*}{\partial r^*} = \frac{V^{*2}}{(1-w^*)^2} + h_0^* \quad (17)$$

The corresponding boundary conditions are given as

$$w^*(r^*, t^*) = \frac{\partial w^*(r^*, t^*)}{\partial r^*} = 0, \text{ at } r^* = 0$$

$$w^*(r^*, t^*) = \frac{\partial w^*(r^*, t^*)}{\partial r^*} = 0, \text{ at } r^* = 1 \quad (18)$$

The initial condition is given by

$$w^*(r^*, 0) = \frac{\partial w^*(r^*, 0)}{\partial t^*} = 0. \quad (19)$$

The nonlinear electrostatic force term  $V^{*2}/(1-w^*)^2$  in Eq. (17) can be approximated via the following Taylor

expansion series:

$$\frac{V^{*,2}}{(1-w^*)^2} = V^{*,2} (1 + 2w^* + 3w^{*,2} + 4w^{*,3} + 5w^{*,4} \dots \dots \dots), \tag{20}$$

Neglecting the higher-order terms, and substituting Eq. (20) into Eq. (17), the nonlinear governing equation of the micro circular plate subject to the combined effects of electrostatic force, hydrostatic pressure and residual stress can be expressed as

$$\begin{aligned} & \frac{\partial^2 w^*}{\partial r^{*,2}} + \frac{\partial^4 w^*}{\partial r^{*,4}} + \frac{2}{r^*} \frac{\partial^3 w^*}{\partial r^{*,3}} - \frac{1}{r^{*,2}} \frac{\partial^2 w^*}{\partial r^{*,2}} \\ & + \frac{1}{r^{*,3}} \frac{\partial w^*}{\partial r^*} - F_r^* \frac{\partial^2 w^*}{\partial r^{*,2}} - F_r^* \frac{1}{r^*} \frac{\partial w^*}{\partial r^*} \\ & = V^{*,2} (1 + 2w^* + 3w^{*,2} + 4w^{*,3} + 5w^{*,4}) + h_0^* \end{aligned} \tag{21}$$

### 3.3 Application of H.M. to solution of nonlinear governing equation

In this section, the normalized governing equation given in Eq. (21), and the corresponding boundary conditions and initial condition given in Eqs. (18) and (19), respectively, are solved using the hybrid method (H.M.). The solution procedure commences by applying the differential transformation process with respect to the time domain  $t$  to each term in the governing equation, i.e.

$$T \left[ \frac{\partial^2 w^*}{\partial t^{*2}} \right] = \frac{(k+1)(k+2)}{H^2} W(r^*, k+2),$$

$$T \left[ \frac{\partial^4 w^*}{\partial r^{*,4}} \right] = \frac{d^4 W(r^*, k)}{dr^{*,4}},$$

$$T \left[ \frac{2}{r^*} \frac{\partial^3 w^*}{\partial r^{*,3}} \right] = \frac{2}{r^*} \frac{d^3 W(r^*, k)}{dr^{*,3}},$$

$$T \left[ \frac{1}{r^{*,2}} \frac{\partial^2 w^*}{\partial r^{*,2}} \right] = \frac{1}{r^{*,2}} \frac{d^2 W(r^*, k)}{dr^{*,2}},$$

$$T \left[ \frac{1}{r^{*,3}} \frac{\partial w^*}{\partial r^*} \right] = \frac{1}{r^{*,3}} \frac{dW(r^*, k)}{dr^*},$$

$$T \left[ F_r^* \frac{\partial^2 w^*}{\partial r^{*,2}} \right] = F_r^* \frac{d^2 W(r^*, k)}{dr^{*,2}},$$

$$T \left[ F_r^* \frac{1}{r^*} \frac{\partial w^*}{\partial r^*} \right] = F_r^* \frac{1}{r^*} \frac{dW(r^*, k)}{dr^*},$$

$$T [2V^{*2}w^{*2}] = 2V^{*2}W(r^*, k),$$

$$\begin{aligned} T [3V^{*2}w^{*2}] &= 3V^{*2} (W(r^*, k) \otimes W(r^*, k)) \\ &= 3V^{*2} (\sum_{\ell=0}^k W(r^*, \ell)W(r^*, k-\ell)) \end{aligned} ,$$

$$\begin{aligned} T [4V^{*2}w^{*3}] &= \\ 4V^{*2} (\sum_{\ell=1}^k [(3+1)\ell - k] W(r^*, \ell)W^3(r^*, k-\ell)) \end{aligned} ,$$

$$\begin{aligned} T [5V^{*2}w^{*4}] &= \\ 5V^{*2} (\sum_{\ell=1}^k [(4+1)\ell - k] W(r^*, \ell)W^4(r^*, k-\ell)) \end{aligned} ,$$

$$T [V^{*2}] = V^{*2}\delta(k),$$

$$T [h_0^*] = h_0^*\delta(k). \tag{22}$$

Note that  $\delta(k)$  is specified as

$$\delta(k) = \begin{cases} 1 & \text{for } k=0 \\ 0 & \text{otherwise} \end{cases} .$$

In the second stage of the solution procedure, the finite difference approximation method is applied with respect to  $r^*$  to the transformed versions of the equation of motion, boundary conditions and initial conditions, respectively. Applying the fourth-order accurate central difference scheme, which the equation can be expressed as follows:

$$\begin{aligned} & \frac{(k+1)(k+2)}{H^2} W_i(k+2) \\ & + \frac{W_{i+2}(k) - 4W_{i+1}(k) + 6W_i(k) - 4W_{i-1}(k) + W_{i-2}(k)}{\Delta r^{*4}} \\ & + \frac{1}{r_i^{*3}} \frac{W_{i+1}(k) - W_{i-1}(k)}{2\Delta r^*} - \frac{1}{r_i^{*2}} \frac{W_{i+1}(k) - 2W_i(k) + W_{i-1}(k)}{\Delta r^{*2}} \\ & + \frac{2}{r_i^*} \frac{W_{i+2}(k) - 2W_{i+1}(k) + 2W_{i-1}(k) - W_{i-2}(k)}{2\Delta r^{*3}} \\ & - F_r^* \frac{W_{i+1}(k) - 2W_i(k) + W_{i-1}(k)}{\Delta r^{*2}} \\ & - F_r^* \frac{1}{r_i^*} \frac{W_{i+1}(k) - W_{i-1}(k)}{2\Delta r^*} \\ & = V^{*2}\delta(k) + 2V^{*2}W_i(k) + 3V^{*2} (\sum_{\ell=0}^k W_i(\ell)W_i(k-\ell)) \\ & + 4V^{*2} (\sum_{\ell=1}^k [(3+1)\ell - k] W_i(\ell)W_i^3(k-\ell)) \\ & + 5V^{*2} (\sum_{\ell=1}^k [(4+1)\ell - k] W_i(\ell)W_i^4(k-\ell)) + h_0^*\delta(k) \end{aligned} \tag{23}$$

where  $\Delta r^*$  is the radius interval and  $i$  is a position index in the  $r$  direction. In Eq. (23),  $W_i(k+2)$  is the only unknown parameter.

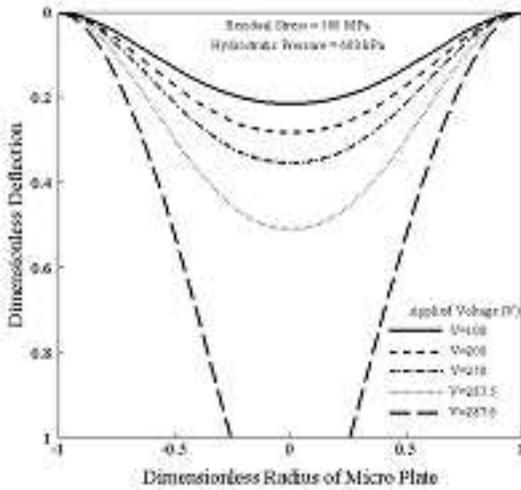
## 4 Numerical Results and Discussion

In this study, the material and geometry parameters considered in the present analyses are summarized in Table 1. Figure 2 illustrates the variation of the micro plate deflection in the radial direction as a function of the applied voltage. The results show that at voltages lower than the pull-in voltage, the micro plate deflects symmetrically about its center point. As expected, the deflection of the micro plate increases with an increasing voltage. Furthermore, it is observed that the pull-in voltage has a theoretical value of 287.6 V for the micro plate parameters considered in Table 1.

Figure 3 shows the variation of the dimensionless center-point deflection with the applied voltage as a function of the initial gap between the two plates. The results indicate that the pull-in voltage increases as the initial gap is increased from 0.5  $\mu\text{m}$  to 1.5  $\mu\text{m}$ .

**Table 1:** Material and geometry parameters of micro fixed-fixed beam model

Parameters	Value
Young's modulus ( $E$ ) (GPa)	169
Poisson's Ratio ( $\nu$ )	0.3
Density ( $\rho$ ) ( $\text{Kg/m}^3$ )	$2.33 \times 10^4$
Permittivity of free space ( $\epsilon_0$ ) (F/m)	$8.8541878 \times 10^{-12}$
Thickness of the micro circular plate ( $b$ ) ( $\mu\text{m}$ )	20
Initial gap ( $u$ ) ( $\mu\text{m}$ )	1
Radius of the plate ( $R$ ) ( $\mu\text{m}$ )	250

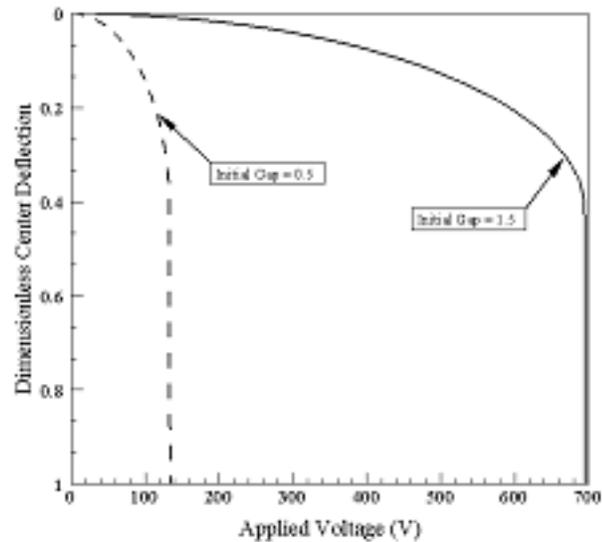


**Fig. 2:** Variation of dimensionless deflection with dimensionless radius of circular plate as function of applied voltage.

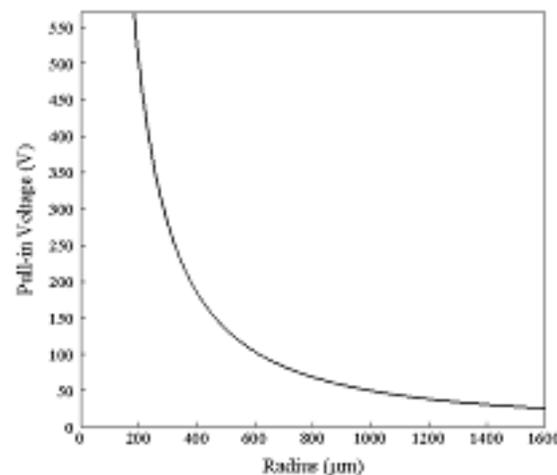
Figure 4 illustrates the variation of the pull-in voltage with the circular plate radius. In performing the analysis, the pull-in voltage reduces as the circular plate radius increases as the result of a loss in rigidity as the circular plate radius is increased. Finally, Figure 5 depicts the dependence of the pull-in voltage on the circular plate thickness. It is seen from figure 5 that the the circular plate thickness increases, pull-in voltage are increased due to an increase in the circular plate flexural rigidity.

## 5 Conclusion

This study has applied a hybrid method (H.M.) scheme comprising the differential transformation method and the finite difference approximation technique to analyze the nonlinear dynamic response of the circular plate. In contrast to previous studies, the governing equation developed in this study takes into account the effects of both the hydrostatic force acting on the upper surface of the circular plate and the residual stress within the plate itself. In general, the present results have confirmed that the micro circular plate becomes structurally unstable at



**Fig. 3:** Variation of dimensionless center-point displacement with applied voltage



**Fig. 4:** Variation of pull-in voltage with radius of the micro circular plate.

voltages equal to or greater than the pull-in voltage and collapses and makes

contact with the lower electrode as a result. Furthermore, the results have shown that the magnitude of the pull-in voltage reduces as the circular plate radius increases due to a loss in structural rigidity. Overall, the results have confirmed that the proposed hybrid method scheme represents a computationally efficient of predicting the dynamic response of the micro circular plate commonly employed in electrostatically actuated MEMS-based devices.

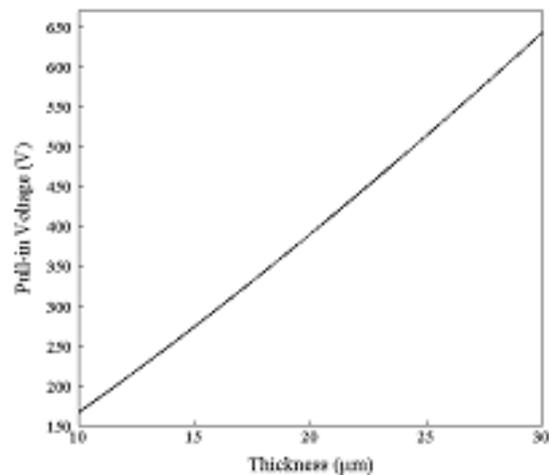


Fig. 5: Pull-in voltage versus micro-beam thickness

## Acknowledgements

The authors gratefully acknowledge the financial support provided to this study by the National Science Council of Taiwan under Grant Number NSC-99-2218-E-167-002.

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