

**Fuzzy Soft α-Connectedness in Fuzzy Soft Topological Spaces**

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Received: 31 Mar. 2015, Revised: 16 Oct. 2015, Accepted: 30 Oct. 2015
Published online: 1 Jan. 2016

Abstract: In the present paper, we study and investigate the properties of fuzzy soft α-connected sets, fuzzy soft α-separated sets and fuzzy soft α-s-connected sets and establish several interesting properties supported by examples. Moreover, we show that, a fuzzy soft α-disconnectedness is not an hereditary property in general. Finally, we show that the fuzzy α-irresolute surjective soft image of fuzzy soft α-connected (resp. fuzzy soft α-s-connected) is also fuzzy soft α-connected (resp. fuzzy soft α-s-connected).

Keywords: Fuzzy soft topological space, Fuzzy α-open soft, Fuzzy α-continuous soft functions, Fuzzy soft connected, Fuzzy soft α-connected.

1 Introduction

In real life situation, the problems in economics, engineering, social sciences, medical science etc. do not always involve crisp data. So, we cannot successfully use the traditional classical methods because of various types of uncertainties presented in these problems. To exceed these uncertainties, some kinds of theories were given like theory of fuzzy set, intuitionistic fuzzy set, rough set, bipolar fuzzy set, i.e. which we can use as mathematical tools for dealings with uncertainties. But, all these theories have their inherent difficulties. The reason for these difficulties Molodtsov [34] initiated the concept of soft set theory as a new mathematical tool for dealing with uncertainties which is free from the above difficulties. In [34,35], Molodtsov successfully applied the soft theory in several directions, such as smoothness of functions, game theory, operations research, Riemann integration, Perron integration, probability, theory of measurement, and so on. After presentation of the operations of soft sets [32], the properties and applications of soft set theory have been studied increasingly [7,27,35]. Xiao et al.[44] and Pei and Miao [38] discussed the relationship between soft sets and information systems. They showed that soft sets are a class of special information systems. In recent years, many interesting applications of soft set theory have been expanded by embedding the ideas of fuzzy sets [5,6,10,16,25,30,31,32,33,35,36,47]. To develop soft set theory, the operations of the soft sets are redefined and a uni-int decision making method was constructed by using these new operations [11].

Recently, in 2011, Shabir and Naz [41] initiated the study of soft topological spaces. They defined soft topology on the collection τ of soft sets over X. Consequently, they defined basic notions of soft topological spaces such as open soft and closed soft sets, soft subspace, soft closure, soft nbd of a point, soft separation axioms, soft regular spaces and soft normal spaces and established their several properties. Min in [43] investigate some properties of these soft separation axioms. In [17], Kandil et. al. introduced some soft operations such as semi open soft, pre open soft, α-open soft and β-open soft and investigated their properties in detail. Kandil et al. [24] introduced the notion of soft semi separation axioms. In particular they study the properties of the soft semi regular spaces and soft semi normal spaces. The notion of soft ideal was initiated for the first time by Kandil et al.[20]. They also introduced the concept of soft local function. These concepts are discussed with a view to find new soft topologies from the original one, called soft topological spaces with soft ideal (X,τ,E,I). Applications to various fields were further investigated by Kandil et al. [18,19,21,22,23,26]. The notion of b-open soft sets was initiated for the first time by El-sheikh and Abd El-latif [13]. Maji et. al. [30] initiated the study

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involving both fuzzy sets and soft sets. In [8] the notion of fuzzy soft set was introduced as a fuzzy generalization of soft sets and some basic properties of fuzzy soft sets are discussed in detail. Then, many scientists such as X. Yang et. al. [45], improved the concept of fuzziness of soft sets. In [4], Karal et al. defined the notion of a mapping on classes of fuzzy soft sets, which is fundamental important in fuzzy soft set theory, to improve this work and they studied properties of fuzzy soft images and fuzzy soft inverse images of fuzzy soft sets. Chang [12] introduced the concept of fuzzy topology on a set X by axiomatizing a collection 𝖲 of fuzzy subsets of X. Tanay et.al. [42] introduced the definition of fuzzy soft topology over a subset of the initial universe set while Roy and Samanta [40] gave the definition of fuzzy soft topology over the initial universe set. Some fuzzy soft topological properties based on fuzzy pre (resp. semi, β-) open soft sets, were introduced in [1, 2, 3, 15, 16, 25].

In the present paper, we generalize the notion of fuzzy soft connectedness [29], by using the notions of fuzzy α-open soft sets.

2 Preliminaries

In this section, we present the basic definitions and results of fuzzy soft set theory which will be needed in the paper.

Definition 2.1. [46] A fuzzy set A of a non-empty set X is characterized by a membership function μA : X → [0, 1] = I whose value μA(x) represents the "degree of membership" of x in A for x ∈ X. We denote family of all fuzzy sets by IX.

Definition 2.2. [30] Let A ⊆ E. A pair (f, A), denoted by fA, is called fuzzy soft set over X, where f is a mapping given by f : A → IY defined by fA(e) = μfA,e where μfA,e = 0 if e ∉ A and μfA,e = 1 if e ∈ A, where Y(x) = 0 ∀ x ∈ X. The family of all these fuzzy soft sets over X denoted by FSS(X).

Definition 2.3. [39] Let 𝖲 be a collection of fuzzy soft sets over a universe X with a fixed set of parameters E, then 𝖲 ⊆ FSS(X) is called fuzzy soft topology on X if

(1) ̄I E, 0 E ∈ 𝖲, where 0 E(e) = 0 and ̄I E(e) = 1, ∀ e ∈ E,
(2) the union of any members of 𝖲 belongs to 𝖲,
(3) the intersection of any two members of 𝖲 belongs to 𝖲.

The triplet (X, 𝖲, E) is called fuzzy soft topological space over X. Also, each member of 𝖲 is called fuzzy open soft in (X, 𝖲, E). We denote the set of all open soft sets by FOS(X, 𝖲, E), or FOS(X).

Definition 2.4. [39] Let (X, 𝖲, E) be a fuzzy soft topological space. A fuzzy soft set fA over X is said to be fuzzy closed soft set in X, if its relative complement fA c is fuzzy open soft set. We denote the set of all fuzzy closed soft sets by FCS(X, 𝖲, E), or FCS(X).

Definition 2.5. [37] Let (X, 𝖲, E) be a fuzzy soft topological space and fA ∈ FSS(X)E. The fuzzy soft closure of fA, denoted by Fcl(fA), is the intersection of all fuzzy soft closed super sets of fA, i.e.,

Fcl(fA) = ∩{hD : hD is fuzzy soft set and fA ⊆ hD}.

The fuzzy soft interior of gB, denoted by Fin(gB) is the fuzzy soft union of all fuzzy soft open super sets of gB, i.e.,

Fin(gB) = ∪{hD : hD is fuzzy open soft set and hD ⊆ gB}.

Definition 2.6. [29] The fuzzy soft set fA ∈ FSS(X)E is called fuzzy soft point if there exist x ∈ X and e ∈ E such that μfA,e(x) = α (0 < α ≤ 1) and μfA,e(y) = 0 for each y ∈ X − {x}, and this fuzzy soft point is denoted by x̄A or fA.

Definition 2.7. [29] The fuzzy soft point x̄A is said to be belonging to the fuzzy soft set (g, A), denoted by x̄A ∈ (g, A), if the element e ∈ A, α ≤ μfA,e(x).

Definition 2.8. [29] A fuzzy soft set gB in a fuzzy soft topological space (X, 𝖲, E) is called fuzzy soft neighborhood of the fuzzy soft point x̄A if there exists a fuzzy open soft set hC such that x̄A ⊆ hC ⊆ gB. A fuzzy soft set gB in a fuzzy soft topological space (X, 𝖲, E) is called fuzzy soft neighborhood of the fuzzy soft set fA if there exists a fuzzy open soft set hC such that fA ⊆ hC ⊆ gB. The fuzzy soft neighborhood system of the fuzzy soft point x̄A, denoted by N̄(x̄A), is the family of all its fuzzy soft neighborhoods.

Definition 2.9. [29] Let (X, 𝖲, E) be a fuzzy soft topological space and Y ⊆ X. Let hY be a fuzzy soft set over (Y, E) such that hY : E → IY such that hY(e) = μhY,e,

μhY,e(x) = \begin{cases} 1 & x ∈ Y, \\ 0 & x ∉ Y. \end{cases}

Let 𝖲Y = {hY ∩ gB : gB ∈ 𝖲}, then the fuzzy soft topology 𝖲Y on (Y, E) is called fuzzy soft subspace topology for (Y, E) and (Y, 𝖲Y, E) is called fuzzy soft subspace of (X, 𝖲, E). If hY ∈ 𝖲 (resp. hY ∈ 𝖲), then (Y, 𝖲Y, E) is called fuzzy open (resp. closed) soft subspace of (X, 𝖲, E).

Definition 2.10. [37] Let FSS(X)E and FSS(Y)K be families of fuzzy soft sets over X and Y, respectively. Let u : X → Y and p : E → K be mappings. Then the map fpu is called fuzzy soft mapping from X to Y and denoted by fpu : FSS(X)E → FSS(Y)K such that,

(1) If fA ∈ FSS(X)E. Then the image of fA under the fuzzy soft mapping fpu is the fuzzy soft set Y defined by fpu(fA), where ∀ k ∈ p(E), ∀ y ∈ Y,

fpu(fA)(k)(y) = \begin{cases} \bigvee_{e \in u^{-1}(y)} | \bigvee_{e \in p^{-1}(k)} fA(e)(x) | & if x ∈ u^{-1}(y), \\ 0 & otherwise. \end{cases}

(2) If gB ∈ FSS(Y)K, then the pre-image of gB under the fuzzy soft mapping fpu is the fuzzy soft set X defined by fpu⁻¹(gB), where ∀ e ∈ p⁻¹(K), ∀ x ∈ X,

fpu⁻¹(gB)(e)(x) = \begin{cases} gB(p(e)(u(x))) & for p(e) ∈ B, \\ 0 & otherwise. \end{cases}
The fuzzy soft mapping \( f_{pu} \) is called surjective (resp. injective) if \( p \) and \( u \) are surjective (resp. injective), also it is said to be constant if \( p \) and \( u \) are constant.

**Definition 2.11.** [37] Let \((X, \mathcal{S}_1, E)\) and \((Y, \mathcal{S}_2, K)\) be two fuzzy soft topological spaces and \( f_{pu} : FSS(X)_E \rightarrow FSS(Y)_K \) be a fuzzy soft mapping. Then \( f_{pu} \) is called

1. Fuzzy continuous soft if \( f_{pu}^{-1}(g_B) \subseteq \mathcal{S}_1 \forall (g_B) \subseteq \mathcal{S}_2. \)
2. Fuzzy open soft if \( f_{pu}(g_A) \subseteq \mathcal{S}_2 \forall (g_A) \subseteq \mathcal{S}_1. \)

**Theorem 2.1.** [4] Let \( FSS(X)_E \) and \( FSS(Y)_K \) be two families of fuzzy soft sets. For the fuzzy soft function \( f_{pu} : FSS(X)_E \rightarrow FSS(Y)_K \), the following statements hold,

(a) \( f^{-1}_{pu}(g_B^c) = (f^{-1}_{pu}(g_B))^c \forall (g_B) \subseteq FSS(Y)_K. \)
(b) \( f_{pu}(f^{-1}_{pu}(g_B)) \subseteq (g_B) \subseteq FSS(Y)_K. \) If \( f_{pu} \) is surjective, then the equality holds.
(c) \( (f, A) \subseteq f^{-1}_{pu}(f^{-1}(f, A)) \forall (f, A) \subseteq FSS(X)_E. \) If \( f_{pu} \) is injective, then the equality holds.
(d) \( f_{pu}(\mathcal{O}_E) = \mathcal{O}_K, f_{pu}(\mathcal{I}_E) \subseteq \mathcal{I}_K. \) If \( f_{pu} \) is surjective, then the equality holds.
(e) \( f^{-1}_{pu}(1_K) = 1_E \) and \( f^{-1}_{pu}(\mathcal{O}_E) = \mathcal{O}_E. \)
(f) If \( (f, A) \subseteq (g, A) \), then \( f_{pu}(f, A) \subseteq f_{pu}(g, A). \)

If \( f_{pu} \) is surjective, then the equality holds.

\( f^{-1}_{pu}(f_{pu}(f, A)) = (f_{pu}(f, A))^c \subseteq (g, A) \subseteq FSS(Y)_K. \) If \( f_{pu} \) is injective, then the equality holds.

**Definition 2.12.** [29] Let \((X, \mathcal{S}, E)\) be a fuzzy soft topological space. A fuzzy soft separation of \( \mathcal{I}_E \) is a pair of non null proper fuzzy soft open sets \( g_B, h_C \subseteq X \) such that \( g_B \cap h_C = \emptyset_E \) and \( \mathcal{I}_E = g_B \cup h_C. \)

**Definition 2.13.** [29] A fuzzy soft topological space \((X, \mathcal{S}, E)\) is said to be fuzzy soft connected if and only if there is no fuzzy soft separations of \( \mathcal{X}. \) Otherwise, \((X, \mathcal{S}, E)\) is said to be fuzzy soft disconnected space.

**Definition 2.14.** [17] Let \((X, \mathcal{S}, E)\) be a soft topological space and \( F_A \subseteq SS(X)_E. \) If \( F_A \subseteq int(\mathcal{cl}(int(F_A))) \), then \( F_A \) is called \( \mathcal{S} \)-open soft set. We denote the set of all \( \mathcal{S} \)-open soft sets by \( \mathcal{AOS}(X, \mathcal{S}, E) \), or \( \mathcal{AOS}(X) \) and the set of all \( \mathcal{S} \)-closed soft sets by \( \mathcal{ACS}(X, \mathcal{S}, E) \), or \( \mathcal{ACS}(X) \).

**Definition 2.15.** [25] Two fuzzy soft sets \( f_A \) and \( g_B \) are said to be disjoint, denoted by \( f_A \cap g_B = \emptyset_E \), if \( A \cap B = \emptyset \) and \( \mu^c_{f_A} \cap \mu^c_{g_B} = \emptyset \forall e \in E. \)

### 3 Fuzzy soft \( \alpha \)-connected spaces

In this section, we introduce the notions of fuzzy soft \( \alpha \)-connectedness in fuzzy soft topological space and examine its basic properties.
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μ_C^*_y = \{a_0.2, b_0.5, c_0.8\}, μ_C^2_y = \{a_0.1, b_0.6, c_0.7\},
μ_C^1_y = \{a_1, b_1, c_1\}, μ_C^0_y = \{a_1, b_1, c_1\},

Theorem 3.4. Let (X1, X2, E) and (X2, X3, E) be fuzzy soft topological spaces and fpu_1: (X1, X2, E) → (X2, X3, E) be a fuzzy α-irresolute surjective soft function. If (X1, X2, E) is fuzzy soft α-connected, then (X2, X3, E) is also fuzzy soft α-connected.

Proof. Let (X2, X3, E, K) be a fuzzy soft α-connected space. Then, there exist f_u, g_b pair of non null proper fuzzy α-open soft subsets of \( X = \{0, 1\} \) such that \( f_u \cap g_b = \emptyset \) and \( X \) is a contradiction with the fuzzy soft α-connectedness of (X1, X2, E). Therefore, (X2, X3, E) is fuzzy soft α-connected.

4 Fuzzy soft α-s-connectedness

In this section, we introduce the notions of fuzzy soft α-separated sets and use it to introduce the notions of fuzzy α-s-connectedness in fuzzy soft topological spaces and study its basic properties.

Definition 4.1. A non null fuzzy soft subsets \( f_a, g_b \) of fuzzy soft topological space (X, Y, E) are said to be fuzzy soft α-separated sets if \( F\alpha cl(f_a) \cap g_b = F\alpha cl(g_b) \cap f_a = \emptyset \).

Theorem 4.1. Let \( f_a \subset g_b \), \( h_c \subset k_d \) and \( g_b, k_d \) are soft fuzzy soft α-separated subsets of fuzzy soft topological space (X, Y, E). Then, \( f_a, h_c \) are fuzzy soft α-separated sets.

Proof. Let \( f_a \subset g_b \), then \( F\alpha cl(f_a) \subset F\alpha cl(g_b) \). It follows that, \( F\alpha cl(f_a) \subset F\alpha cl(f_a) \cup k_d \subset F\alpha cl(g_b) \cap k_d = \emptyset \). Also, since \( h_c \subset k_d \). Then, \( F\alpha cl(h_c) \subset F\alpha cl(k_d) \). Hence, \( f_a \cap F\alpha cl(h_c) \subset F\alpha cl(k_d) \cap g_b = \emptyset \). Thus, \( f_a, h_c \) are fuzzy soft α-separated sets.

Theorem 4.2. Two fuzzy α-connected soft subsets of fuzzy soft topological space (X, Y, E) are fuzzy soft α-connected sets if and only if they are disjoint.

Proof. Let \( f_a, g_b \) are fuzzy soft α-separated sets. Then, \( F\alpha cl(g_b) \cap f_a = g_b \cap F\alpha cl(f_a) = \emptyset \). Since \( f_a, g_b \) are fuzzy α-connected soft sets. Then, \( f_a \cap g_b = \emptyset \). Conversely, let \( f_a \cap g_b = \emptyset \). It follows that, \( f_a, g_b \) are fuzzy soft α-separated sets.

Definition 4.2. A fuzzy soft topological space (X, Y, E) is said to be fuzzy soft α-connected if and only if \( X \) can not be expressed as the fuzzy soft union of two fuzzy soft α-separated sets in (X, Y, E).
Theorem 4.3. Let \((Z, \mathcal{S}_Z, E)\) be a fuzzy soft subspace of \(Z\) such that \(\mathcal{S}_Z\) is fuzzy soft \(\alpha\)-separated on \(Z\) if and only if \(f_A\) and \(g_B\) are fuzzy soft \(\alpha\)-separated on \(Z\).

**Proof.** Assume that \(f_A\) and \(g_B\) are fuzzy soft \(\alpha\)-separated on \(Z\) if and only if \(f_A\) and \(g_B\) are fuzzy soft \(\alpha\)-separated on \(Z\), where \(Z\) is fuzzy soft subspace for \(Z\).

Theorem 4.4. Let \(Z\) be a fuzzy soft subset of fuzzy soft topology space \((X, \mathcal{S}_X, E)\). Then, \(Z\) is fuzzy soft \(\alpha\)-separated w.r.t \((Z, \mathcal{S}_Z, E)\) if and only if \(Z\) is fuzzy soft \(\alpha\)-connected w.r.t \((Z, \mathcal{S}_Z, E)\).

**Proof.** Suppose that \(\alpha\) is not fuzzy soft \(\alpha\)-connected w.r.t \((Z, \mathcal{S}_Z, E)\). Then, \(Z\) is fuzzy soft \(\alpha\)-connected w.r.t \((Z, \mathcal{S}_Z, E)\), where \(f_A\) and \(g_B\) are fuzzy soft \(\alpha\)-separated on \(Z\) if and only if \(f_A\) and \(g_B\) are fuzzy soft \(\alpha\)-separated on \(Z\) if and only if \(f_A\) and \(g_B\) are fuzzy soft \(\alpha\)-separated on \(Z\).

Theorem 4.5. Let \((Z, \mathcal{S}_Z, E)\) be a fuzzy soft \(\alpha\)-separated subspace of fuzzy soft topology space \((X, \mathcal{S}_X, E)\) and \(f_A, g_B\) be fuzzy soft \(\alpha\)-separated of \(\mathcal{S}_X\) with \(Z\) \(\subseteq f_A \cup g_B\), then \(Z\) \(\subseteq f_A\) or \(Z\) \(\subseteq g_B\).

**Proof.** Let \(Z\) \(\subseteq f_A \cup g_B\) for some fuzzy soft \(\alpha\)-separated subsets \(f_A, g_B\) of \(\mathcal{S}_X\). Then, \(Z\) \(\subseteq (f_A \cap g_B)\). Since \(Z\) \(\subseteq f_A \cup g_B\), then \(Z\) \(\subseteq f_A\) or \(Z\) \(\subseteq g_B\).

Theorem 4.6. Let \((Z, \mathcal{S}_Z, N)\) and \((Y, \mathcal{S}_Y, M)\) be fuzzy soft \(\alpha\)-connected subspaces of fuzzy soft topology space \((X, \mathcal{S}_X, E)\) such that none of them is fuzzy soft \(\alpha\)-separated. Then, \(Z\) \(\subseteq N\) and \(Y\) \(\subseteq M\) fuzzy soft \(\alpha\)-connected.

**Proof.** Let \((Z, \mathcal{S}_Z, N)\) and \((Y, \mathcal{S}_Y, M)\) be fuzzy soft \(\alpha\)-connected subspaces of fuzzy soft topology space \((X, \mathcal{S}_X, E)\) such that \(Z\) \(\subseteq N\) and \(Y\) \(\subseteq M\) fuzzy soft \(\alpha\)-connected. Then, there exist two non null fuzzy soft \(\alpha\)-connected sets \(f_A\) and \(g_B\) of \(\mathcal{S}_X\) such that \(Z\) \(\subseteq N\) is fuzzy soft \(\alpha\)-separated on \(Z\), and \(Y\) \(\subseteq M\) is fuzzy soft \(\alpha\)-separated on \(Z\).

Theorem 4.7. Let \((Z, \mathcal{S}_Z, E)\) be a fuzzy soft \(\alpha\)-separated subspace of fuzzy soft topology space \((X, \mathcal{S}_X, E)\) and \(S_M \subseteq F\mathcal{S}(X)_A\). Then, \(\mathcal{S}_Z\) is fuzzy soft \(\alpha\)-separated subspace of \(\mathcal{S}_X\).

**Proof.** Assume that \((S, \mathcal{S}_S, M)\) is not fuzzy soft \(\alpha\)-separated subspace of \((X, \mathcal{S}_X, E)\). Then, there exist fuzzy soft \(\alpha\)-separated sets \(f_A\) and \(g_B\) on \(\mathcal{S}_X\). If \(\mathcal{S}_Z\) \(\subseteq f_A \cup g_B\) such that \(f_A \subseteq g_B\). So, we have \(Z\) is fuzzy soft \(\alpha\)-separated subset of fuzzy soft \(\alpha\)-disconnected space. By Theorem 4.5, either \(Z\) \(\subseteq f_A\) or \(Z\) \(\subseteq g_B\). If \(Z\) \(\subseteq f_A\), then, \(F\mathcal{S}(f_A) \subseteq F\mathcal{S}(f_A)\). It follows \(F\mathcal{S}(f_A) \subseteq F\mathcal{S}(f_A)\). Hence, \(F\mathcal{S}(f_A) \subseteq F\mathcal{S}(f_A)\) which is a contradiction. If \(Z\) \(\subseteq g_B\), by a similar way, we can get \(F\mathcal{S}(f_A) \subseteq F\mathcal{S}(f_A)\) which is a contradiction. Hence, \((S, \mathcal{S}_S, M)\) is fuzzy soft \(\alpha\)-connected subspace of \((X, \mathcal{S}_X, E)\).

Corollary 4.1. If \((Z, \mathcal{S}_Z, N)\) is fuzzy soft \(\alpha\)-connected subspace of fuzzy soft topology space \((X, \mathcal{S}_X, E)\), then, \(F\mathcal{S}(f_A)\) is fuzzy soft \(\alpha\)-connected.

**Proof.** It obvious from Theorem 4.7.

Theorem 4.8. If for all pair of distinct fuzzy soft point \(f_e, g_e\), there exists a fuzzy soft \(\alpha\)-connected set \(Z\) \(\subseteq f_\alpha\) with \(f_e, g_e \in \mathcal{S}_N\), then \(f_\alpha\) is fuzzy soft \(\alpha\)-connected.

**Proof.** Suppose that \(f_e, g_e\) are distinct fuzzy soft \(\alpha\)-separated. Then, \(f_\alpha\) \(\subseteq f_\alpha\) and \(g_\alpha\) \(\subseteq g_\alpha\). If \(f_\alpha\) \(\subseteq f_\alpha\) and \(g_\alpha\) \(\subseteq g_\alpha\), then \(f_\alpha, g_\alpha \in \mathcal{S}_N\). So, \(f_\alpha\) is fuzzy soft \(\alpha\)-connected set \(\alpha\)-connected. This implies that \(f_\alpha\) \(\subseteq f_\alpha\) or \(g_\alpha\) \(\subseteq g_\alpha\).

Theorem 4.9. Let \(\{Z_j: j \in J\}\) be a non null family of fuzzy soft \(\alpha\)-connected subspace of fuzzy soft topology space \((X, \mathcal{S}_X, E)\). If \(\bigcup_{j \in J} Z_j \neq \emptyset\), then \(\bigcup_{j \in J} Z_j\) is also fuzzy soft \(\alpha\)-connected fuzzy subspace of \((X, \mathcal{S}_X, E)\).

**Proof.** Assume that \((Z, \mathcal{S}_Z, N)\) \(\subseteq f_A \cup g_B\) of \(\mathcal{S}_X\). Since \(\bigcup_{j \in J} Z_j \neq \emptyset\), then \(\bigcup_{j \in J} Z_j\) is fuzzy soft \(\alpha\)-connected. Then, \(Z\) \(\subseteq f_A \cup g_B\) for some fuzzy soft \(\alpha\)-separated sets \(f_A, g_B\) of \(\mathcal{S}_X\). Since \(\bigcup_{j \in J} Z_j \neq \emptyset\), then \(\bigcup_{j \in J} Z_j\) is fuzzy soft \(\alpha\)-connected. So, \(f_A \cup g_B\) is fuzzy soft \(\alpha\)-connected. Therefore, \(f_A \cup g_B\) is fuzzy soft \(\alpha\)-connected.
(Z, τZ, N) = (∪j∈J Zj, τ∪j∈J Zj, N) is fuzzy soft α-s-connected.

**Theorem 4.10.** Let \{Zj, τZj, N : j ∈ J\} be a family of fuzzy soft α-s-connected subspaces of fuzzy soft topological space (X, τ, E) such that one of the members of the family intersects every other members, then (∪j∈J Zj, τ∪j∈J Zj, N) is fuzzy subspace of (X, τ, E).

**Proof.** Let \( (Z, \tau Z, N) = (∪j∈J Zj, \tau∪j∈J Zj, N) \) and \( (z, N)_{jo} \in \{ (z_j, N) : j \in J \} \) such that \( (z_j, N)_{jo} \cap (z_j, N) \neq \emptyset \) ∀j ∈ J. Then, \( (z, N)_{jo} \cup (z_j, N) \) is fuzzy soft α-s-connected ∀j ∈ J by Theorem 4.6. Hence, the collection \{ (z, N)_{jo} \cup (z_j, N) : j ∈ J \} is a collection of fuzzy soft α-s-connected subsets of \( \emptyset \), which having a non null fuzzy soft intersection. Therefore, \( (Z, \tau Z, N) = (∪j∈J Zj, \tau∪j∈J Zj, N) \) is fuzzy soft α-s-connected subspace of \( (X, \tau, E) \) by Theorem 4.6.

**Theorem 4.11.** Let \( (X_1, \tau_1, E) \) and \( (X_2, \tau_2, K) \) be fuzzy soft α-topological spaces and \( f_{pu} : (X_1, \tau_1, E) \to (X_2, \tau_2, K) \) be a fuzzy α-irresolute surjective soft function. If \( (X_1, \tau_1, E) \) is fuzzy soft α-s-connected, then \( (X_2, \tau_2, K) \) is also a fuzzy soft α-s-connected.

**Proof.** Let \( (X_2, \tau_2, K) \) be fuzzy soft α-disconnected space. Then, there exist \( f_{A}, g_{B} \) pair of non null proper fuzzy soft α-separated sets such that \( I_{K} = f_{A} \cup g_{B} \). Since \( f_{pu} \) is fuzzy α-irresolute soft function, then \( f_{pu}^{-1}(f_{A}), f_{pu}^{-1}(g_{B}) \) are pair of non null proper fuzzy α-open soft subsets of \( I_{E} \) such that
\[
F_{\alpha cl}(f_{pu}^{-1}(f_{A})) \cap f_{pu}^{-1}(g_{B}) \subseteq f_{pu}^{-1}(F_{\alpha cl}(f_{A})) \cap f_{pu}^{-1}(g_{B}) = f_{pu}^{-1}(f_{A} \cap g_{B}) = f_{pu}^{-1}(\emptyset_{K}) = \emptyset_{E},
\]
\[
f_{pu}^{-1}(f_{A}) \cap F_{\alpha cl}(f_{pu}^{-1}(g_{B})) \subseteq f_{pu}^{-1}(f_{A}) \cap f_{pu}^{-1}(F_{\alpha cl}(g_{B})) = f_{pu}^{-1}(f_{A}) \cap g_{B} = f_{pu}^{-1}(\emptyset_{K}) = \emptyset_{E}
\]
and
\[
f_{pu}^{-1}(f_{A}) \cup f_{pu}^{-1}(g_{B}) = f_{pu}^{-1}(f_{A} \cup g_{B}) = f_{pu}^{-1}(1_{K}) = I_{E}
\]
from Theorem 2.1 and [25], Theorem 4.2. This means that \( f_{pu}^{-1}(f_{A}), f_{pu}^{-1}(g_{B}) \) are pair of non null proper fuzzy soft α-separated sets of \( I_{E} \), which is a contradiction of the fuzzy soft α-s-connectedness of \( (X_1, \tau_1, E) \). Therefore, \( (X_2, \tau_2, K) \) is fuzzy soft α-s-connected.

5 Conclusion

Since the authors introduced topological structures on fuzzy soft sets [8, 14, 42], so the fuzzy soft topological properties, which introduced by Mahanta et al. [29], is generalized here to the fuzzy α-soft sets which will be useful in the fuzzy systems. Because there exists compact connections between soft sets and information systems [38, 44], we may use the results deducted from the studies on fuzzy soft topological space to improve these kinds of connections. We hope that the findings in this paper will help researcher enhance and promote the further study on fuzzy soft topology to carry out a general framework for their applications in practical life.

**Acknowledgements**

The authors express their sincere thanks to the reviewers for their careful checking of the details and for helpful comments that improved this paper. The authors are also thankful to the editors-in-chief and managing editors for their important comments which helped to improve the presentation of the paper.

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