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Combined Optimized Domain Decomposition Method and a Modified Fixed Point Method for Non Linear Diffusion Equation

M. R. Amattouch [*] *and H. Belhadj*

Department of mathematics, Faculty of sciences and techniques, University of Abdelmalek Essaadi, BP.416, Tangier, Morocco

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Abstract: This work is devoted to an optimized domain decomposition method applied to a non linear anisotropic reaction equation. The proposed method is based on the idea of the optimized of two order (OO2) method developed this last two decades. We first use a modified fixed point technique to linearize the problem in a way to converge fately, then we generalize the OO2 method and modify it to obtain a new more optimized rate of convergence of the Schwarz algorithm. To compute the new rate of convergence we have used Fourier analysis. For the numerical computation we minimize this rate of convergence using a global optimization algorithm. Several test-cases of analytical problems illustrate this approach and show the efficiency of the proposed new method.

Keywords: Non linear diffusion equation, Schauder fixed point theorem, Domain decomposition method, Optimized interface conditions.

1 Introduction

The aim goal of this paper is to propose an optimized domain decomposition method (DDM) to solve a non linear diffusion equation on a bounded domain such that:

$$\begin{cases} \frac{\partial u}{\partial t} + cu - div(g(u)\nabla u) = f(x,y) \;\; on\; \Omega \\ u = h \;\; on\; \partial\Omega \\ u = h_o \;\; on\; t = 0 \end{cases} \tag{1}$$

By an implicit Scheme of discretization, its stationary equation is:

$$\begin{cases} cu - div\, g(u)\nabla u = f(x,y) \;\; on\; \Omega \\ u = h \;\; on\; \partial\Omega \end{cases} \tag{2}$$

g is a nonlinear function which I name the viscosity depending on the speed or the concentration u to find. The function g is depending on the physical model (Thermo-elasticity, Plasma physics, Porous media model, Beltrami model, heat transfer, turbulence modeling...).There is so money kind of approach (eventually analytical and variational methods)to solve this kind of equation ([1]...[7]). Here in this paper, we study generally the existence of a solution to the problem and new robust, accurate, fast numerical methods to solve this equation.
We first describe the fixed point to linearize the Equation 2, We prove the existence of a solution of the variational problem resulting from it, then we propose a new modified fixed point to converge faster to the solution.
Secondly, We use and generalize an optimized domain decomposition to reduce the cost of time of our algorithm. The domain decomposition method (OO2) is a tool we use for large domain sizes and thin meshes, it consists of solving in parallel our equations on sub domains with new interface condition in order to reduce the size and the complexity of the problem, the parallelism in new devices of computer science and architecture make this method easy to handle. There is so many kind of methods in the domain decomposition method (see [17] and [18] for some classical ones), but in this paper we consider an optimized domain decomposition method of two order (OO2). Severally used this last decades ([8],...[13]), this method is powerful and optimized. It's optimized because ([8,9]) we can compute explicitly the rate of convergence between two iteration using the Fourier transform and make this rate optimal so the method will converge quickly. But, the rate of convergence of this method can be computed only for

[*] Corresponding author e-mail: amattouch36@yahoo.fr

linear partial differential equation, and it's difficult to have and explicit rate for nonlinear partial differential equation, Thus come the idea of the modified fixed method combined to this optimized fixed point to solve this problem of nonlinearity. In addition, we can prove that the rate of convergence of the OO2 method increases if the viscosity increases, so we have generalized the method to treat this challenge.
At the end of this work you can find some numerical experiment compared to analytical solution that effectively illustrate the efficiency of our proposed methods.

2 Method of Modified Fixed point

In this section we present the Fixed point algorithm and the modified one to solve the stationary problem 2
The Fixed point method involves given one initial function u_0, we construct iteratively a function sequence u_n as follows:

$$\begin{cases} cu_{n+1} - div\, g(u_n)\nabla u_{n+1} = f(x,y) \;\; on\; \Omega \\ u_{n+1} = h \;\; on\; \partial\Omega \end{cases} \quad (3)$$

So we give below a convergence result of this algorithm:

Theorem 1. *Suppose that:*
- $\exists \nu > 0$ *and* μ; $c \geq 0$ *and* ; $g(x) > \nu$
- *g is K-lipshitzian.*
- $\frac{K}{\nu} < 1$

Then the sequence $u_{n+1} = \phi(u_n)$ *converge to the unique solution of the nonlinear problem (3)*

Proof:
By an adequate change, it is sufficient to consider a proof with the Dirichlet condition $u = 0$ (The same thing could be done with a Neumann condition).
Let $V = \{u \in H_0^1(\Omega) / \, g(u) \in L^1(\Omega) \; and \; \|\nabla u\| < M\}$. Consider the following application:

$$\begin{aligned} \phi : V &\mapsto V \\ v &\mapsto u \end{aligned}$$

such that u is the unique solution of the variational formulation:

$$\int_\Omega cuw + \int_\Omega g(v)\nabla u \nabla w = \int_\Omega f(x,y)w \;\; \forall w \in H_0^1(\Omega) \quad (4)$$

♣ ϕ bellow is well defined, Indeed: Consider the bilinear form :

$$a(u,w) = \int_\Omega cuw + \int_\Omega g(u)\nabla u \nabla w$$

by the Holder and the Poincarré Inequality a is continuous:

$$\begin{aligned} a(u,w) &\leq \sup_\Omega |c| \; \|u\|\|w\| + M\|\nabla u\|\|\nabla w\| \\ &\leq (\sup_\Omega |c| C^2_{Poincarre}) + M)\|\nabla u\|\|\nabla w\| \end{aligned}$$

a `is coercive`:

$$\begin{aligned} a(u,u) = \int_\Omega cuu + \int_\Omega g(u)\nabla u \nabla u \\ \geq \nu\|u\|^2 \end{aligned}$$

The Lax Milgram theorem involves that there exist one solution of (3) named $\phi(v)$
♣ we have by substraction:

$$a(\phi(u) - \phi(v), w) = 0 \;\; \forall w \in H_0^1(\Omega)$$

in another way

$$\int_\Omega c(\phi(u) - \phi(v))w + \int_\Omega g(u)\nabla(\phi(u) - \phi(v))\nabla w = \int_\Omega \nabla\phi(v)(g(v) - g(u))$$

We take $w = \phi(u) - \phi(v)$, so (Thinks to holder and Poincarre Inequalities)

$$\nu\|\phi(u) - \phi(v)\|^2 \leq K \times M \times C_{Poincarre}\|\phi(u) - \phi(v)\|\|u - v\|$$

M could be chosen as $M \times C_{Poincarre} < 1$, Thus

$$\|\phi(u) - \phi(v)\| \leq \frac{K}{\nu}\|u - v\|$$

The theorem of fixed point applied to the application ϕ show thats the equation $\phi(u) = u)$ have one solution and the suite $u_{n+1} = \phi(u_n)$ converge to this solution which is the solution of problem (4)

Remark. In the proof of the theorem we have:

$$\nu\|\phi(u) - \phi(v)\| \leq K\|u - v\|$$

If K is smaller, the Fixed point suite converge quickly. So we introduce a new modified fixed point by adding a new function r such that:

$$\begin{cases} cu_{n+1} - div((g(u_n) + \mathbf{r}(\mathbf{u_n}, \nabla\mathbf{u_n}))\nabla u_{n+1}) = f(x,y) - div(\mathbf{r}(\mathbf{u_n}, \nabla\mathbf{u_n})\nabla u_n) \;\; on\; \Omega \\ u_{n+1} = o \;\; on\; \partial\Omega \end{cases} \quad (5)$$

r is a function we choose to have the Coefficient K_r small.
It's necessary to have $r : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ that assume
$|g'(x) + r'(x, \overrightarrow{y})| < \varepsilon_1 << 1$ and $\| \nabla_y(r(x, \| \overrightarrow{y} \|))\overrightarrow{y}) \| < \varepsilon_2 << 1$
Using r, the condition $g(x) > \nu$ also we don't have to look for a solution with small gradient shown in theorem 1.1 proof.

Theorem 2. *Suppose that, there exist a function r such that:*
a) $\nu \prec g(x) + r(x, \overrightarrow{y}) \succ \mu$ *b)* $|g'(x) + r'(x, \overrightarrow{y})| < \varepsilon_1 << \frac{1}{2}$.
c) $\|\nabla_y(r(x, \overrightarrow{y}))\overrightarrow{y})\| < \varepsilon_2 << \frac{1}{2}$
then the sequence $u_{n+1} = \phi(u_n)$ *of the modified fixed point converge to a solution of the nonlinear problem (3)*

Proof: Let $V = \{u \in H_0^1(\Omega) / \, g(u) \; and \; r(u) \in L^1(\Omega) \; and \; \|\nabla u\| < M\}$.
Consider the following application:

$$\begin{aligned} \phi : V &\mapsto V \\ v &\mapsto u \end{aligned}$$

such that u is the unique solution of the variational formulation:

$$\int_\Omega cuw + \int_\Omega (g(v)+r(v))\nabla u \nabla w = \int_\Omega fw + \int_\Omega r(v)\nabla v \nabla w \ \ \forall w \in H_0^1(\Omega) \tag{6}$$

♣ ϕ is well defined, Indeed: Consider the bilinear form :

$$a(u,w) = \int_\Omega cuw + \int_\Omega (g(v)+r(v))\nabla u \nabla w$$

by the Holder and the Poincaré Inequality a is continuous:

$$\begin{aligned} a(u,w) &\leq \sup_\Omega |c| \, \|u\|\|w\| + \mu\|\nabla u\|\|\nabla w\| \\ &\leq (\sup_\Omega |c| C^2_{Poincarre}) + \mu)\|\nabla u\|\|\nabla w\| \end{aligned}$$

a is coercive:

$$\begin{aligned} a(u,u) = \int_\Omega cuu + \int_\Omega g(u)\nabla u \nabla u \\ \geq \nu\|u\|^2 \end{aligned}$$

The Lax Milgram theorem involves that there exist one solution of (6) named $\phi(v)$

♣ It's easy to make:

$$\begin{aligned} &\int_\Omega c|\phi(u)-\phi(v)|^2 + \int_\Omega (g(u)+r(u)) \mid \nabla(\phi(u)-\phi(v)) \mid^2 = \\ &= \int_\Omega (g(v)+r(v)-g(u)-r(u))\nabla\phi(v)\nabla(\phi(u)-\phi(v)) + \\ &+ \int_\Omega (r(u)\nabla u - r(v)\nabla v)\nabla(\phi(u)-\phi(v)) \end{aligned}$$

so

$$\begin{aligned} \nu \int_\Omega \| \nabla(\phi(u)-\phi(v)) \|^2 &\leq \varepsilon_1 M \| u-v \|\| \nabla(\phi(u)-\phi(v)) \| + \varepsilon_2 M \| u-v \|\| \nabla(\phi(u)-\phi(v)) \| \\ &\leq C_{Poincarre}(M+1)(\varepsilon_1+\varepsilon_2) \| u-v \|\| \nabla(\phi(u)-\phi(v)) \| \end{aligned}$$

We choose M such that $\frac{C_{Poincarre}(M+1)}{\nu} < 1$ The theorem of fixed point applied to the application ϕ show thats the equation $\phi(u) = u$) have one solution and the suite $u_{n+1} = \phi(u_n)$ converge to this solution which is the solution of the variational problem of (2)

Notice that in practice, we can found a function r for more general conditions even if g and g' are not bounded. And the choice of r is optimal when the physical model of g is known. Also the convergence is optimal for small and bounded function g.

3 Domain decomposition with optimized interface of second order(DDM OO2)

The use of finite volumes , finite differences or finite elements solvers on high order meshes requires a high cost of computation.

Domain decomposition methods can reduce this cost by splitting initial problem into two or more sub-problems with smaller dimensions. Many authors have studied domain decomposition methods these last decades [17,18]. Among these methods we consider in this work the method called second order optimized method OO2. This method was developed by different authors [8,9]. Our stationary equation (2) could be treated in what follow as a reaction advection diffusion equation (the same equation indeed). The main idea of the OO2 technique is described briefly as follows:

We split the domain Ω for example in two sub-domains Ω_1 and Ω_2 with an interface Γ (see figure 1) then we built

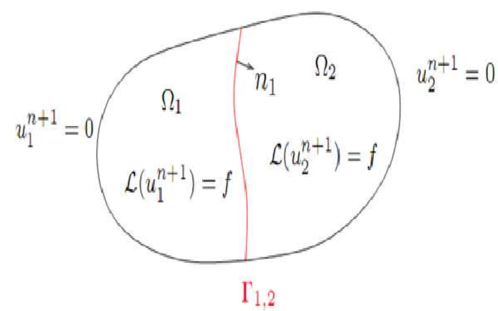

Fig. 1: splitting of the domain in two sub-domain

two sequences $u^n_{1,p}$ and $u^n_{2,p}$ respectively solutions of two sub-problems as described bellow:

we choose two initials functions u_1^0 defined on Ω_1 and u_2^0 defined on Ω_2 then we consider the two problems:

$$\begin{cases} L(u_1^{p+1}) = f(x,y) \ \ on\ \Omega_1 \\ u_1^{p+1} = 0 \ \ on\ \partial\Omega \\ B_1(u_1^{p+1}) = B_1(u_2^p) \ \ on\ \Gamma \end{cases} \tag{7}$$

and

$$\begin{cases} L(u_2^{p+1}) = f(x,y) \ \ on\ \Omega_2 \\ u_2^{p+1} = 0 \ \ on\ \partial\Omega \\ B_2(u_2^{p+1}) = B_2(u_1^p) \ \ on\ \Gamma \end{cases} \tag{8}$$

Where

$$L(u) = cu + a\frac{\partial u}{\partial x} + b\frac{\partial u}{\partial y} - \mu\Delta u \tag{9}$$

$$\begin{aligned} B_1(u) &= \tfrac{\partial u}{\partial n} - C_1 u + C_2\tfrac{\partial u}{\partial \tau} - C_3\tfrac{\partial^2 u}{\partial \tau^2} \\ B_2(u) &= -\tfrac{\partial u}{\partial n} - (C_1 - \tfrac{a}{\mu})u + C_2\tfrac{\partial u}{\partial \tau} - C_3\tfrac{\partial^2 u}{\partial \tau^2} \end{aligned} \tag{10}$$

n and τ are the normal and the tangent on Ω_1

Because of the Fourier analysis we show that the rate of convergence in the fourier way is (see [6] for proof)

$$\rho(C_1,C_2,C_3,k) = \left(\frac{\lambda^-(k) - C_1 + ikC_2 + C_3k^2}{\lambda^+(k) - C_1 + ikC_2 + C_3k^2}\right)^2 \tag{11}$$

where

$$\lambda^{\mp}(k) = \frac{a \mp \sqrt{a^2 + 4c\mu - 4i\mu bk + 4k^2\mu^2}}{2\mu}$$

The next theorem gives a condition of convergence of the OO2 method

Theorem 3. *Suppose that $c > 0$ and $sign(b) = sign(C_2)$ and $C_3 \geq 0$ then,*

$$\max_{|k|<\frac{\pi}{h}} \|\rho(C_1, C_2, C_3, k)\| < 1$$

Proof. see [9]

Remark. In order to optimize the method we need to optimize the rate of convergence so we look for:

$$\min_{C_1,C_2,C_3} \max_{|k|<\frac{\pi}{h}} \|\rho(C_1, C_2, C_3, k)\|$$

For optimizing this rate we have implemented the global optimization method [14]. Notice that this last problem have at least two optimums.

In our work, the viscosity is high and for the OO2 method proposed the convergence take more time because the rate of convergence is not small enough and that can be seen experimentally on so much example of high viscosity, also we can prove that theoretically. Thus, to have a convergence near to two iterations, we take the generalized artificial coefficients:

$$\begin{aligned} B_1(u) = \frac{\partial u}{\partial n} - C_1 u + C_1' u(0, y-a) + C_2 \frac{\partial u}{\partial \tau} + C_2' \frac{\partial u(0,y-a)}{\partial \tau} \\ -C_3 \frac{\partial^2 u}{\partial \tau^2} + C_3' \frac{\partial^2 u(0,y-a)}{\partial \tau^2} + C_4 \frac{\partial^2 u}{\partial n \partial \tau} \\ B_2(u) = -\frac{\partial u}{\partial n} - C_1 u + C_1' u(0, y-a) + C_2 \frac{\partial u}{\partial \tau} + C_2' \frac{\partial u(0,y-a)}{\partial \tau} \\ -C_3 \frac{\partial^2 u}{\partial \tau^2} + C_3' \frac{\partial u(0,y-a)}{\partial \tau^2} + C_4 \frac{\partial^2 u}{\partial n \partial \tau} \end{aligned}$$

The fourier transform of the the rate of convergence is calculated using:

$$\begin{aligned} \mathfrak{F}(f(x-a)) = e^{-ika} \mathfrak{F}(f(x)) \\ \mathfrak{F}(\frac{\partial^2 f}{\partial x \partial y})) = ik \mathfrak{F}(\frac{\partial f}{\partial x}) \end{aligned}$$

We also optimize the step of our algorithm by Coupling OO2 and Fixed point next subproblems:

$$\begin{cases} L(u_1^{p+1}) = f(x,y) + div(r(u_1^p) \nabla u_1^p) \text{ on } \Omega_1 \\ u_1^{p+1} = 0 \text{ on } \partial\Omega \\ B_1(u_1^{p+1}) = B_1(u_2^p) \text{ on } \Gamma \end{cases} \tag{12}$$

and

$$\begin{cases} L(u_2^{p+1}) = f(x,y) + div(r(u_1^p) \nabla u_2^p) \text{ on } \Omega_2 \\ u_2^{p+1} = 0 \text{ on } \partial\Omega \\ B_2(u_2^{p+1}) = B_2(u_1^p) \text{ on } \Gamma \end{cases} \tag{13}$$

where:

$$L(u) = cu + div((r(u, \| \nabla u \|) + g(u)) \nabla u) \tag{14}$$

4 Numerical simulation

Numerically, we have implemented the finite volumes method to approach the sub-problems obtained after applying the fixed point algorithm. We obtain a good accuracy, the error is in the order of 10^{-6} forward the third iteration on some usual test-functions .We take for g respectively the expressions
$g(u) = u^p, (p = -1, p = 1, p = \frac{1}{2})$
$g(u) = e^u$
$g(u) = \frac{a}{b+u}$
$g(u) = \frac{1}{1+u^2}$
$g(u) = \frac{1}{1-u^2}$
First the figure (2) shows the theoretical error of the OO2 method while changing the constant viscosity μ. And we aknowlege that the error rate rises when the viscosity rises to become big and then without interest for higher viscosity.

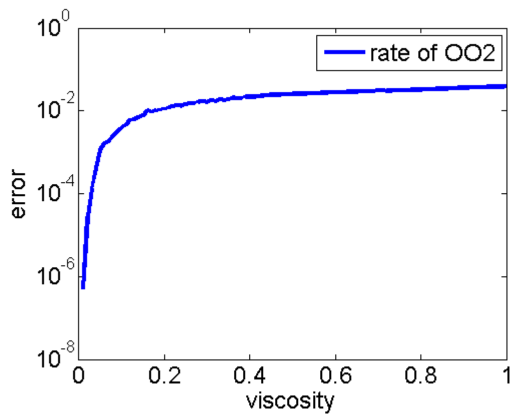

Fig. 2: error in function of viscosity

The following results show the error between the approximate solution using the modified fixed point method and the exact solution $uexact = xy(1-x)(1-y)$ of the problem (1).$Error = \|u - u_{exact}\|_2$

h is the mesh grid.

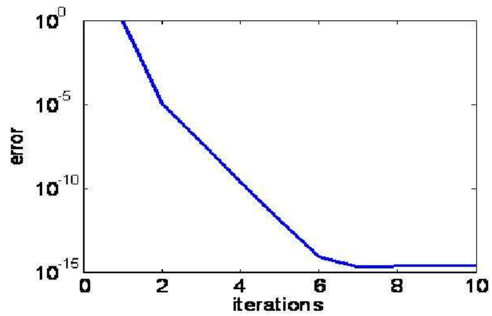

Fig. 3: The error: c=10, h=0.025; g(u)=u*u

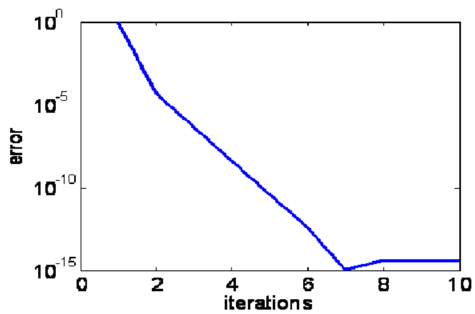

Fig. 4: The error: c=1, h=0.001 and F(u)=exp(u)

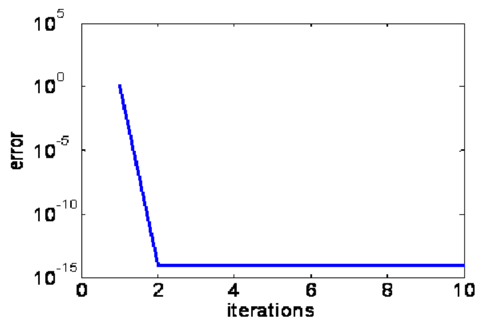

Fig. 5: The error: c=20, a=-2, b=1, h=0.001 and F(u)=a/(b+u)

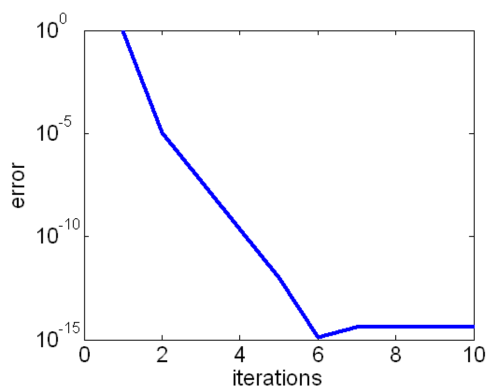

Fig. 6: c=1, h=0.001 and g(u)=10 sin($\frac{\pi}{3}u$)

Now, we show the results of combining modified fixed point and OO2 algorithm.

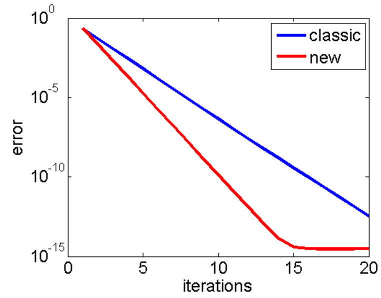

Fig. 7: case1

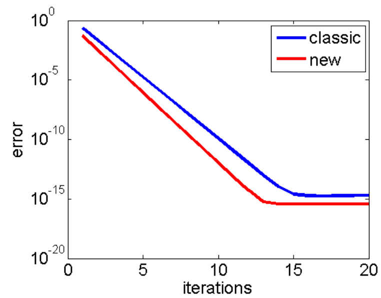

Fig. 8: case2

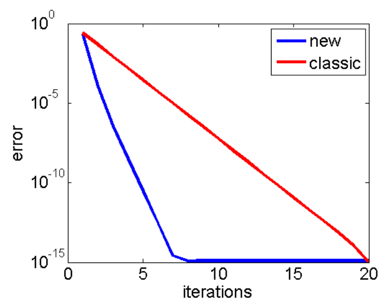

Fig. 9: case3

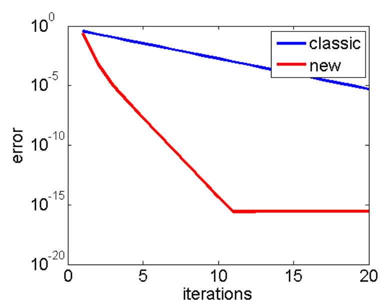

Fig. 10: case4

The figures 7,8,9,10 showing the L^2 error between the combining modified fixed point solution and the OO2 algorithm and the exact solution.

Test case1: c=1, , h=0.0001, $g(u) = log(u)$and $u_{exact} = (x+y)(1-x)(1-y)$.
Test case2: c=10, h=0.001, g(u)=$\frac{u}{1+u^2}$ and $uexact = exp(xy(1-x)(1-y)) - 1$.
Test case3: c=1, h=0,001, $g(u) = u^{\frac{1}{2}}$ and $u_{exact} = sin(\pi(x+y))exp(xy(1-x)(1-y)) - 1$.
Test case4: c=10, h=0,001, $g(u) = exp(u) - 1$ and $u_{exact} = xy(1-x)(1-y)$. h is the mesh grid

5 Conclusion

In this work, we have developed an optimized domain decomposition algorithm applied to a non linear PDE. We firstly, have proposed a proof of the convergence of the fixed point technique applied to the non linear equation. We have proposed a new approach for computing the convergence rate using the Fourier analysis and global optimization. Secondly we have presented several test-cases to show the efficiency of this approach. The fundamental result is that we obtained high accuracy and a fast method with a well optimized rate of convergence of the proposed algorithm in comparison with global calculation using classical solvers. As perspective of the present work, we can study the following ideas:
- Obtaining a rate of convergence of two iterations to this PDE
- Generalize the approach to multi dimensional nonlinear PDE such Navier stokes and Compressible Euler equation.
- Apply the method to real problems in fluid dynamics, environmental sciences or the image processing.

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AMATTOUCH MOHAMED RIDOUAN is a Ph. D student in the University of Abdelmalek Essaadi. FST, Department of Mathematics, Tangier, Morocco. He received his Master degree in 2005 in applied mathematics and application in the Jean-Monnet University, France. His Research Interests are domain decomposition methods for PDEs, Scientific computing, Image analysis, Computational fluid dynamics, stability and accelerating the convergence of numerical methods.

Hassan Belhadj was born 1960, Taza, Morocco. He is a Professor in the University of Abdelmalek Essaadi, FST, Department of Mathematics,Tangier, Morocco. He received a Ph.D degree in Civil Engineering, Mohammadia Schoul of Engineers, University of Mohammed V, Rabat, Morocco, 2007 and a Ph.D degree in SPI, University of Franche Comt, Besanon, France, 1992. He get a master degree in Applied Mathematics, University of Franche Comt, Besanon, France, 1987. His research interests are Domain decomposition methods for PDEs, Scientific computing,Time-splitting methods for multiscale and multimodel simulation, Computational fluid dynamics and Acceleration of the convergence and extrapolation methods.