Applied Mathematics & Information Sciences
An International Journal

http://dx.doi.org/10.18576/amis/13S143

New Solitary Wave Solutions to the Three Coupled Nonlinear Maccari's-System with a Complex Structure

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Received: 2 May 2019, Revised: 2 Jul. 2019, Accepted: 24 Jul. 2019.
Published online: 1 Aug. 2019.

Abstract: In this article, the three nonlinear Maccari's-system (TNLMS) which describe how isolated waves are propagates in finite region of space is included. New accurate travelling wave solutions of this model are obtained using the balanced modified extended tanh-function method (BMETFM).The obtained results give an accuracy interpretation of the propagation of these isolated waves. We listing comparison between our realized results and that satisfied in the previous work.

Keywords: the TNLMS, the BMETFM, travelling wave solutions.

1 Introduction

In different branches of physics new methods and techniques for dealing the physical phenomena exactly that described by the non-linear partial differential equations are still an open area? Recently different technics have been introduced through several authors to study the behaviors of NLPDEs arising in various branches of physics, mathematics and physical engineering through references [1-27]. It is important to find new accurate solution for this system which illustrates how these waves propagate in a small part of space.

Several tries are introduced through different authors [28-31] to solve to the proposed system (TNLMS) which plays a vital role in different branches of nonlinear physical science.

$$\begin{aligned} &iH_t + H_{xx} + RH = 0,\\ &iM_t + M_{xx} + RM = 0,\\ &iL_t + L_{xx} + RL = 0,\\ &R_t + R_y + \left(\left| H + M + L \right|^2\right)_x = 0. \end{aligned}$$

The aim of this article is applying the proposed method to find a new exact solution for this system [28] given above in terms of some parameters. The solitary wave solution well be realized when one give these parameters finite values.

2 The BMETFM [22-25]

The proposed method admits the solution in the form:

$$u(\zeta) = a_0 + \sum_{i=1}^{m} (a_i \phi_i + \frac{b_i}{\phi_i}), \tag{1}$$

The balance rule [18], is applied to evaluate the integer m mentioned at Eq. (1) such that either $a_m \neq 0$ or $b_m \neq 0$ while a_i, b_i are unknowns to be defined. Also the function ϕ in this equation must satisfy the Riccati equation $\phi' = b + \phi^2$, and according to the value of b three forms of solutions are admitted namely:

$$\varphi = -\sqrt{-B}\tanh\left(\sqrt{-B}\zeta\right), or \quad \varphi = -\sqrt{-B}\coth\left(\sqrt{-B}\zeta\right),$$

when $B < 0$, (2)

$$\varphi = \sqrt{B}\tan\left(\sqrt{B}\zeta\right), or \quad \varphi = -\sqrt{B}\cot\left(\sqrt{B}\zeta\right),$$

when $B > 0$, (3)

$$\varphi = -\frac{1}{\zeta},$$ When $B = 0$, (4)

According to these forms of solutions when the coefficients

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of exponential powers of φ^i are equal to zero an algebraic system of equations will generated which solving by any computer program to get the values of the required constants.

2.1 Application

According to the transformations given in [31],

$$H(x,y,t)=p(x,y,t)\,e^{\theta},\quad M(x,y,t)=q(x,y,t)\,e^{\theta},\quad L(x,y,t)=s(x,y,t)\,e^{\theta}, \tag{5}$$

Such that $\theta=i(kx+\alpha y+\lambda t+l)$, k,α,λ are unknowns to be defined while l is arbitrary constant, the TNLMS [31] mentioned above transfer to,

$$\begin{aligned}&i(p_t+2\alpha\,p_x)+p_{xx}-(\lambda+k^2)\,p+pR=0,\\&i(q_t+2\alpha\,q_x)+q_{xx}-(\lambda+k^2)\,q+qR=0,\\&i(s_t+2\alpha\,s_x)+s_{xx}-(\lambda+k^2)\,s+sR=0,\\&R_t+R_y+(p^2)_x=0.\end{aligned} \tag{6}$$

Now, let us consider $p=P(\zeta),q=Q(\zeta),s=S(\zeta),\zeta=\mu(x+y-2kt)$ the constructed equation became,

$$\begin{aligned}&\mu^2P''-(\lambda+k^2)P+PR=0,\\&\mu^2Q''-(\lambda+k^2)Q+QR=0,\\&\mu^2S''-(\lambda+k^2)S+SR=0,\\&\mu(1-2\alpha)\frac{\partial R}{\partial\zeta}+\frac{\partial(P+Q+S)^2}{\partial\zeta}=0.\end{aligned} \tag{7}$$

Integrating the last term of Eq. (8), we get,

$$R=\frac{1}{\mu(1-2k)}(P+Q+S)^2. \tag{8}$$

Substitute from Eq. (8) at the first three aspects of Eq. (7), we obtain,

$$\begin{aligned}&\mu^2P''-(\lambda+k^2)P-\frac{(P+Q+S)^2}{\mu(1-2k)}P=0,\\&\mu^2Q''-(\lambda+k^2)Q-\frac{(P+Q+S)^2}{\mu(1-2k)}Q=0,\\&\mu^2S''-(\lambda+k^2)S-\frac{(P+Q+S)^2}{\mu(1-2k)}S=0.\end{aligned} \tag{9}$$

Latterly, put $Q=r_1P,\ S=r_2P$, we get,

$$\mu^2P''-(\lambda+k^2)P-\frac{(1+r_1+r_2)^2}{\mu(1-2k)}P^3=0. \tag{10}$$

Apply the balance rule between $P''\ and\ P^3\Rightarrow m=1$, hence the proposed method [22] adimt the solution in the form,

$$P(\zeta)=a_0+a_1\phi(\zeta)+\frac{b_1}{\phi(\zeta)}, \tag{11}$$

Sub., about $P''\ and\ P$, and its higher orders at Eq. (10), and let the coefficients of exponential powers of $\phi(\zeta)$ equal to zero we get,

$$\begin{aligned}&2\mu^2-\frac{(1+r_1+r_2)^2}{\mu(1-2k)}a_1^2=0,\\&-3\frac{(1+r_1+r_2)^2}{\mu(1-2k)}a_0\,a_1^2=0,\\&2\mu^2a_1B-3\frac{(1+r_1+r_2)^2}{\mu(1-2k)}(a_0^2a_1+a_1^2\,b_1)-(\lambda+k^2)\,a_1=0,\\&2B^2\mu^2-\frac{(1+r_1+r_2)^2}{\mu(1-2k)}b_1^2=0,\\&-3\frac{(1+r_1+r_2)^2}{\mu(1-2k)}a_0\,b_1^2=0,\\&2\mu^2b_1B-3\frac{(1+r_1+r_2)^2}{\mu(1-2k)}(a_0^2b_1+a_1^2\,b_1^2)-(\lambda+k^2)\,b_1=0,\\&\frac{(1+r_1+r_2)^2}{\mu(1-2k)}(a_0^2+6a_1\,b_1)+(\lambda+k^2)=0.\end{aligned} \tag{12}$$

When this system of equations is solved we get,

$$\begin{aligned}&(1)\ a_0=0,a_1=\frac{2\mu^3(1-2k)}{1+r_1+r_2},\ b_1=(\lambda+k^2)\sqrt{\frac{2(1-2k)}{\mu(1+r_1+r_2)}},B=-\frac{(\lambda+k^2)}{\mu^2},\\&(2)\ a_0=0,a_1=\frac{2\mu^3(1-2k)}{1+r_1+r_2},\ b_1=-(\lambda+k^2)\sqrt{\frac{2(1-2k)}{\mu(1+r_1+r_2)}},B=-\frac{(\lambda+k^2)}{\mu^2},\\&(3)\ a_0=0,a_1=-\frac{2\mu^3(1-2k)}{1+r_1+r_2},\ b_1=(\lambda+k^2)\sqrt{\frac{2(1-2k)}{\mu(1+r_1+r_2)}},B=-\frac{(\lambda+k^2)}{\mu^2},\\&(4)\ a_0=0,a_1=-\frac{2\mu^3(1-2k)}{1+r_1+r_2},\ b_1=-(\lambda+k^2)\sqrt{\frac{2(1-2k)}{\mu(1+r_1+r_2)}},B=-\frac{(\lambda+k^2)}{\mu^2}.\end{aligned} \tag{13}$$

As the result of these Fourth solutions (10-13), we can obtain twelve solutions according to different probabilities of b while it takes values less than or greater than or equal to zero.
To illustrate this, we choose only one case say (1),

$$a_0=0,\quad a_1=\frac{2\mu^3(1-2k)}{1+r_1+r_2},\quad b_1=(\lambda+k^2)\sqrt{\frac{2(1-2k)}{\mu(1+r_1+r_2)}},\quad B=-\frac{(\lambda+k^2)}{\mu^2}.$$

According to these constants, the solution is,

$$P(x,y,t)=\frac{2\mu^3(1-2k)}{1+r_1+r_2}\varphi(x,y,t)+(\lambda+k^2)\sqrt{\frac{2(1-2k)}{\mu(1+r_1+r_2)}}\,\frac{1}{\varphi(x,y,t)}, \tag{14}$$

From which three forms of solution are generated namely,

$$(1)\;P(x,y,t)=\frac{2\mu^3(1-2k)}{1+r_1+r_2}\sqrt{\frac{(\lambda+k^2)}{\mu^2}}\tanh\sqrt{\frac{(\lambda+k^2)}{\mu^2}}\,\mu(x+y-2kt)+(\lambda+k^2)\sqrt{\frac{2(1-2k)}{\mu(1+r_1+r_2)}}\,\frac{1}{\sqrt{\frac{(\lambda+k^2)}{\mu^2}}\tanh\sqrt{\frac{(\lambda+k^2)}{\mu^2}}\,\mu(x+y-2kt)},$$

$$B<0, \tag{15}$$

$$Q(x,y,t)=r_1P=r_1\left[\begin{array}{l}\frac{2\mu^3(1-2k)}{1+r_1+r_2}\sqrt{\frac{(\lambda+k^2)}{\mu^2}}\tanh\sqrt{\frac{(\lambda+k^2)}{\mu^2}}\,\mu(x+y-2kt)\\+(\lambda+k^2)\sqrt{\frac{2(1-2k)}{\mu(1+r_1+r_2)}}\,\frac{1}{\sqrt{\frac{(\lambda+k^2)}{\mu^2}}\tanh\sqrt{\frac{(\lambda+k^2)}{\mu^2}}\,\mu(x+y-2kt)}\end{array}\right], \tag{16}$$

$$S(x,y,t)=r_2P=r_2\left[\begin{array}{l}\frac{2\mu^3(1-2k)}{1+r_1+r_2}\sqrt{\frac{(\lambda+k^2)}{\mu^2}}\tanh\sqrt{\frac{(\lambda+k^2)}{\mu^2}}\,\mu(x+y-2kt)\\+(\lambda+k^2)\sqrt{\frac{2(1-2k)}{\mu(1+r_1+r_2)}}\,\frac{1}{\sqrt{\frac{(\lambda+k^2)}{\mu^2}}\tanh\sqrt{\frac{(\lambda+k^2)}{\mu^2}}\,\mu(x+y-2kt)}\end{array}\right], \tag{17}$$

$$H(x,y,t)=Pe^{\theta}=e^{\theta}\left[\begin{array}{l}\frac{2\mu^3(1-2k)}{1+r_1+r_2}\sqrt{\frac{(\lambda+k^2)}{\mu^2}}\tanh\sqrt{\frac{(\lambda+k^2)}{\mu^2}}\,\mu(x+y-2kt)\\+(\lambda+k^2)\sqrt{\frac{2(1-2k)}{\mu(1+r_1+r_2)}}\,\frac{1}{\sqrt{\frac{(\lambda+k^2)}{\mu^2}}\tanh\sqrt{\frac{(\lambda+k^2)}{\mu^2}}\,\mu(x+y-2kt)}\end{array}\right], \tag{18}$$

$$M(x,y,t)=Qe^{\theta}=r_1e^{\theta}\left[\begin{array}{l}\frac{2\mu^3(1-2k)}{1+r_1+r_2}\sqrt{\frac{(\lambda+k^2)}{\mu^2}}\tanh\sqrt{\frac{(\lambda+k^2)}{\mu^2}}\,\mu(x+y-2kt)+\\(\lambda+k^2)\sqrt{\frac{2(1-2k)}{\mu(1+r_1+r_2)}}\,\frac{1}{\sqrt{\frac{(\lambda+k^2)}{\mu^2}}\tanh\sqrt{\frac{(\lambda+k^2)}{\mu^2}}\,\mu(x+y-2kt)}\end{array}\right], \tag{19}$$

$$L(x,y,t)=Se^{\theta}=r_2e^{\theta}\left[\begin{array}{l}\frac{2\mu^3(1-2k)}{1+r_1+r_2}\sqrt{\frac{(\lambda+k^2)}{\mu^2}}\tanh\sqrt{\frac{(\lambda+k^2)}{\mu^2}}\,\mu(x+y-2kt)\\+(\lambda+k^2)\sqrt{\frac{2(1-2k)}{\mu(1+r_1+r_2)}}\,\frac{1}{\sqrt{\frac{(\lambda+k^2)}{\mu^2}}\tanh\sqrt{\frac{(\lambda+k^2)}{\mu^2}}\,\mu(x+y-2kt)}\end{array}\right], \tag{20}$$

$$R=\frac{1}{\mu(1-2k)}(P+Q+S)^2, \tag{21}$$

Where $\theta=i\left(kx+\alpha y+\lambda t+l\right).$

Or

$$P(x,y,t)=-\frac{2\mu^3(1-2k)}{1+r_1+r_2}\sqrt{\frac{(\lambda+k^2)}{\mu^2}}\coth\sqrt{\frac{(\lambda+k^2)}{\mu^2}}\,\mu(x+y-2kt)-(\lambda+k^2)\sqrt{\frac{2(1-2k)}{\mu(1+r_1+r_2)}}\,\frac{1}{\sqrt{\frac{(\lambda+k^2)}{\mu^2}}\coth\sqrt{\frac{(\lambda+k^2)}{\mu^2}}\,\mu(x+y-2kt)}, \tag{22}$$

$$Q(x,y,t)=r_1P=r_1\left[\begin{array}{l}\frac{-2\mu^3(1-2k)}{1+r_1+r_2}\sqrt{\frac{(\lambda+k^2)}{\mu^2}}\coth\sqrt{\frac{(\lambda+k^2)}{\mu^2}}\,\mu(x+y-2kt)-\\(\lambda+k^2)\sqrt{\frac{2(1-2k)}{\mu(1+r_1+r_2)}}\,\frac{1}{\sqrt{\frac{(\lambda+k^2)}{\mu^2}}\coth\sqrt{\frac{(\lambda+k^2)}{\mu^2}}\,\mu(x+y-2kt)}\end{array}\right], \tag{23}$$

$$S(x,y,t)=r_2P=r_2\left[\begin{array}{l}-\frac{2\mu^3(1-2k)}{1+r_1+r_2}\sqrt{\frac{(\lambda+k^2)}{\mu^2}}\coth\sqrt{\frac{(\lambda+k^2)}{\mu^2}}\,\mu(x+y-2kt)-\\(\lambda+k^2)\sqrt{\frac{2(1-2k)}{\mu(1+r_1+r_2)}}\,\frac{1}{\sqrt{\frac{(\lambda+k^2)}{\mu^2}}\coth\sqrt{\frac{(\lambda+k^2)}{\mu^2}}\,\mu(x+y-2kt)}\end{array}\right], \tag{24}$$

$$H(x,y,t)=Pe^{\theta}=e^{\theta}\left[\begin{array}{l}\frac{-2\mu^3(1-2k)}{1+r_1+r_2}\sqrt{\frac{(\lambda+k^2)}{\mu^2}}\coth\sqrt{\frac{(\lambda+k^2)}{\mu^2}}\,\mu(x+y-2kt)-\\(\lambda+k^2)\sqrt{\frac{2(1-2k)}{\mu(1+r_1+r_2)}}\,\frac{1}{\sqrt{\frac{(\lambda+k^2)}{\mu^2}}\coth\sqrt{\frac{(\lambda+k^2)}{\mu^2}}\,\mu(x+y-2kt)}\end{array}\right], \tag{25}$$

$$M(x,y,t)=Qe^{\theta}=r_1e^{\theta}\left[\begin{array}{l}\frac{-2\mu^3(1-2k)}{1+r_1+r_2}\sqrt{\frac{(\lambda+k^2)}{\mu^2}}\coth\sqrt{\frac{(\lambda+k^2)}{\mu^2}}\,\mu(x+y-2kt)-\\(\lambda+k^2)\sqrt{\frac{2(1-2k)}{\mu(1+r_1+r_2)}}\,\frac{1}{\sqrt{\frac{(\lambda+k^2)}{\mu^2}}\coth\sqrt{\frac{(\lambda+k^2)}{\mu^2}}\,\mu(x+y-2kt)}\end{array}\right], \tag{26}$$

$$L(x,y,t)=Se^{\theta}=r_2e^{\theta}\left[\begin{array}{l}\frac{-2\mu^3(1-2k)}{1+r_1+r_2}\sqrt{\frac{(\lambda+k^2)}{\mu^2}}\coth\sqrt{\frac{(\lambda+k^2)}{\mu^2}}\,\mu(x+y-2kt)-\\(\lambda+k^2)\sqrt{\frac{2(1-2k)}{\mu(1+r_1+r_2)}}\,\frac{1}{\sqrt{\frac{(\lambda+k^2)}{\mu^2}}\coth\sqrt{\frac{(\lambda+k^2)}{\mu^2}}\,\mu(x+y-2kt)}\end{array}\right], \tag{27}$$

$$R=\frac{1}{\mu(1-2k)}(P+Q+S)^2. \tag{28}$$

$$(2)\,P(x,y,t)=\frac{2\mu^3(1-2k)}{1+r_1+r_2}\sqrt{\frac{-(\lambda+k^2)}{\mu^2}}\tan\sqrt{\frac{-(\lambda+k^2)}{\mu^2}}\;\mu(x+y-2kt)+(\lambda+k^2)\sqrt{\frac{2(1-2k)}{\mu(1+r_1+r_2)}}\,\frac{1}{\sqrt{\frac{-(\lambda+k^2)}{\mu^2}}\tan\sqrt{\frac{-(\lambda+k^2)}{\mu^2}}\;\mu(x+y-2k}$$

$$B<0, \tag{29}$$

$$Q(x,y,t)=r_1P=r_1\left[\begin{array}{l}\frac{2\mu^3(1-2k)}{1+r_1+r_2}\sqrt{\frac{-(\lambda+k^2)}{\mu^2}}\tan\sqrt{\frac{-(\lambda+k^2)}{\mu^2}}\;\mu(x+y-2kt)+\\(\lambda+k^2)\sqrt{\frac{2(1-2k)}{\mu(1+r_1+r_2)}}\,\frac{1}{\sqrt{\frac{-(\lambda+k^2)}{\mu^2}}\tan\sqrt{\frac{-(\lambda+k^2)}{\mu^2}}\;\mu(x+y-2kt}\end{array}\right. \tag{30}$$

$$S(x,y,t)=r_2P=r_2\left[\begin{array}{l}\frac{2\mu^3(1-2k)}{1+r_1+r_2}\sqrt{\frac{-(\lambda+k^2)}{\mu^2}}\tan\sqrt{\frac{-(\lambda+k^2)}{\mu^2}}\;\mu(x+y-2kt)+\\(\lambda+k^2)\sqrt{\frac{2(1-2k)}{\mu(1+r_1+r_2)}}\,\frac{1}{\sqrt{\frac{-(\lambda+k^2)}{\mu^2}}\tan\sqrt{\frac{-(\lambda+k^2)}{\mu^2}}\;\mu(x+y-2kt}\end{array}\right. \tag{31}$$

$$H(x,y,t)=Pe^{\theta}=e^{\theta}\left[\begin{array}{l}\frac{2\mu^3(1-2k)}{1+r_1+r_2}\sqrt{\frac{-(\lambda+k^2)}{\mu^2}}\tan\sqrt{\frac{-(\lambda+k^2)}{\mu^2}}\;\mu(x+y-2kt)+\\(\lambda+k^2)\sqrt{\frac{2(1-2k)}{\mu(1+r_1+r_2)}}\,\frac{1}{\sqrt{\frac{-(\lambda+k^2)}{\mu^2}}\tan\sqrt{\frac{-(\lambda+k^2)}{\mu^2}}\;\mu(x+y-2kt}\end{array}\right. \tag{32}$$

$$M(x,y,t)=Qe^{\theta}=r_1e^{\theta}\left[\begin{array}{l}\frac{2\mu^3(1-2k)}{1+r_1+r_2}\sqrt{\frac{-(\lambda+k^2)}{\mu^2}}\tan\sqrt{\frac{-(\lambda+k^2)}{\mu^2}}\;\mu(x+y-2kt)+\\(\lambda+k^2)\sqrt{\frac{2(1-2k)}{\mu(1+r_1+r_2)}}\,\frac{1}{\sqrt{\frac{-(\lambda+k^2)}{\mu^2}}\tan\sqrt{\frac{-(\lambda+k^2)}{\mu^2}}\;\mu(x+y-2kt)}\end{array}\right] \tag{33}$$

$$L(x,y,t)=Se^{\theta}=r_2e^{\theta}\left[\begin{array}{l}\frac{2\mu^3(1-2k)}{1+r_1+r_2}\sqrt{\frac{-(\lambda+k^2)}{\mu^2}}\tan\sqrt{\frac{-(\lambda+k^2)}{\mu^2}}\;\mu(x+y-2kt)+\\(\lambda+k^2)\sqrt{\frac{2(1-2k)}{\mu(1+r_1+r_2)}}\,\frac{1}{\sqrt{\frac{-(\lambda+k^2)}{\mu^2}}\tan\sqrt{\frac{-(\lambda+k^2)}{\mu^2}}\;\mu(x+}\end{array}\right. \tag{34}$$

$$R=\frac{1}{\mu(1-2k)}(P+Q+S)^2. \tag{35}$$

Or

$$P(x,y,t)=\frac{-2\mu^3(1-2k)}{1+r_1+r_2}\sqrt{\frac{-(\lambda+k^2)}{\mu^2}}\cot\sqrt{\frac{-(\lambda+k^2)}{\mu^2}}\;\mu(x+y-2kt)-(\lambda+k^2)\sqrt{\frac{2(1-2k)}{\mu(1+r_1+r_2)}}\,\frac{1}{\sqrt{\frac{-(\lambda+k^2)}{\mu^2}}\cot\sqrt{\frac{-(\lambda+k^2)}{\mu^2}}\;\mu(x+y-2kt)}, \tag{36}$$

$$Q(x,y,t)=r_1P=r_1\left[\begin{array}{l}-\frac{2\mu^3(1-2k)}{1+r_1+r_2}\sqrt{\frac{-(\lambda+k^2)}{\mu^2}}\cot\sqrt{\frac{-(\lambda+k^2)}{\mu^2}}\;\mu(x+y-2kt)-\\(\lambda+k^2)\sqrt{\frac{2(1-2k)}{\mu(1+r_1+r_2)}}\,\frac{1}{\sqrt{\frac{-(\lambda+k^2)}{\mu^2}}\cot\sqrt{\frac{-(\lambda+k^2)}{\mu^2}}\;\mu(x+y-2kt)}\end{array}\right], \tag{37}$$

$$S(x,y,t)=r_2P=r_2\left[\begin{array}{l}\frac{-2\mu^3(1-2k)}{1+r_1+r_2}\sqrt{\frac{-(\lambda+k^2)}{\mu^2}}\cot\sqrt{\frac{-(\lambda+k^2)}{\mu^2}}\;\mu(x+y-2kt)-\\(\lambda+k^2)\sqrt{\frac{2(1-2k)}{\mu(1+r_1+r_2)}}\,\frac{1}{\sqrt{\frac{-(\lambda+k^2)}{\mu^2}}\cot\sqrt{\frac{-(\lambda+k^2)}{\mu^2}}\;\mu(x+y-2kt)}\end{array}\right], \tag{38}$$

$$H(x,y,t)=Pe^{\theta}=e^{\theta}\left[\begin{array}{l}\frac{-2\mu^3(1-2k)}{1+r_1+r_2}\sqrt{\frac{-(\lambda+k^2)}{\mu^2}}\cot\sqrt{\frac{-(\lambda+k^2)}{\mu^2}}\;\mu(x+y-2kt)-\\(\lambda+k^2)\sqrt{\frac{2(1-2k)}{\mu(1+r_1+r_2)}}\,\frac{1}{\sqrt{\frac{-(\lambda+k^2)}{\mu^2}}\cot\sqrt{\frac{-(\lambda+k^2)}{\mu^2}}\;\mu(x+y-2kt)}\end{array}\right], \tag{39}$$

$$M(x,y,t)=Qe^{\theta}=r_1e^{\theta}\left[\begin{array}{l}\frac{-2\mu^3(1-2k)}{1+r_1+r_2}\sqrt{\frac{-(\lambda+k^2)}{\mu^2}}\cot\sqrt{\frac{-(\lambda+k^2)}{\mu^2}}\;\mu(x+y-2kt)-\\(\lambda+k^2)\sqrt{\frac{2(1-2k)}{\mu(1+r_1+r_2)}}\,\frac{1}{\sqrt{\frac{-(\lambda+k^2)}{\mu^2}}\cot\sqrt{\frac{-(\lambda+k^2)}{\mu^2}}\;\mu(x+y-2kt)}\end{array}\right], \tag{40}$$

$$L(x,y,t)=Se^{\theta}=r_2e^{\theta}\left[\begin{array}{l}-\frac{2\mu^3(1-2k)}{1+r_1+r_2}\sqrt{\frac{-(\lambda+k^2)}{\mu^2}}\cot\sqrt{\frac{-(\lambda+k^2)}{\mu^2}}\;\mu(x+y-2kt)-\\(\lambda+k^2)\sqrt{\frac{2(1-2k)}{\mu(1+r_1+r_2)}}\,\frac{1}{\sqrt{\frac{-(\lambda+k^2)}{\mu^2}}\cot\sqrt{\frac{-(\lambda+k^2)}{\mu^2}}\;\mu(x+y-2kt)}\end{array}\right], \tag{41}$$

$$R=\frac{1}{\mu(1-2k)}(P+Q+S)^2. \tag{42}$$

(1)

$$P(x,y,t)=\frac{-2\mu^3(1-2k)}{1+r_1+r_2}\frac{1}{\mu(x+y-2kt)}-(\lambda+k^2)\sqrt{\frac{2(1-2k)}{\mu(1+r_1+r_2)}}\ \mu(x+y-2kt),$$

$$B=0, \tag{43}$$

$$Q(x,y,t)=r_1P=r_1\left[\frac{-2\mu^3(1-2k)}{1+r_1+r_2}\frac{1}{\mu(x+y-2kt)}-(\lambda+k^2)\sqrt{\frac{2(1-2k)}{\mu(1+r_1+r_2)}}\ \mu(x+y-2kt)\right] \tag{44}$$

$$S(x,y,t)=r_2P=r_2\left[\frac{-2\mu^3(1-2k)}{1+r_1+r_2}\frac{1}{\mu(x+y-2kt)}-(\lambda+k^2)\sqrt{\frac{2(1-2k)}{\mu(1+r_1+r_2)}}\ \mu(x+y-2kt)\right] \tag{45}$$

$$H(x,y,t)=Pe^{\theta}=e^{\theta}\left[\frac{-2\mu^3(1-2k)}{1+r_1+r_2}\frac{1}{\mu(x+y-2kt)}-(\lambda+k^2)\sqrt{\frac{2(1-2k)}{\mu(1+r_1+r_2)}}\ \mu(x+y-2kt)\right], \tag{46}$$

$$L(x,y,t)=Se^{\theta}=r_2e^{\theta}\left[\frac{-2\mu^3(1-2k)}{1+r_1+r_2}\frac{1}{\mu(x+y-2kt)}-(\lambda+k^2)\sqrt{\frac{2(1-2k)}{\mu(1+r_1+r_2)}}\ \mu(x+y-2kt),\right], \tag{48}$$

$$R=\frac{1}{\mu(1-2k)}(P+Q+S)^2. \tag{49}$$

In the same manner we can easily obtain the other solutions.

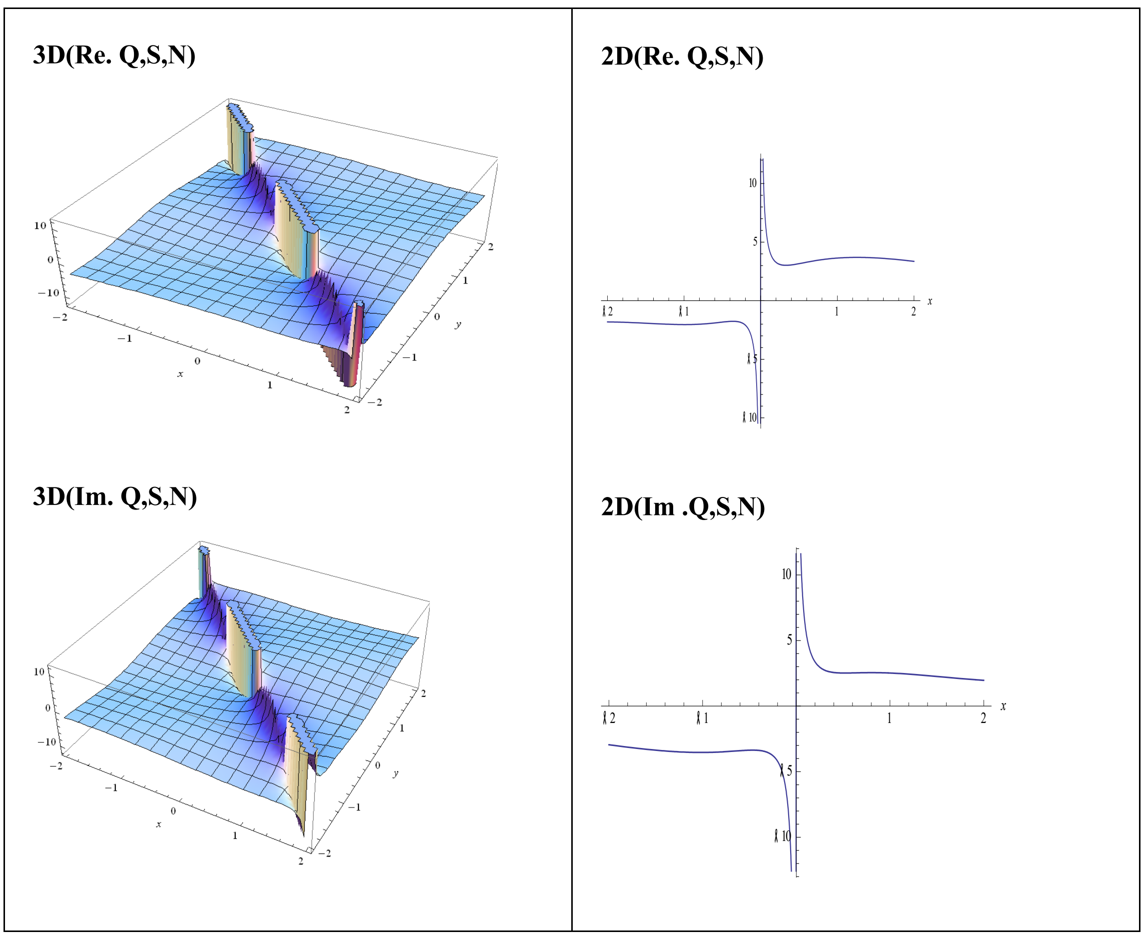

Fig.1: Plot of Eqs. (18, 19, 20) in three and two dimensions of when k =-1, α = 2, μ = 1, l = 1, $r_1 = r_2 = 1$, t = 0.002, −2 < x < 2, −2 < t < 2 and y = 0.001 for the two dimension graphics.

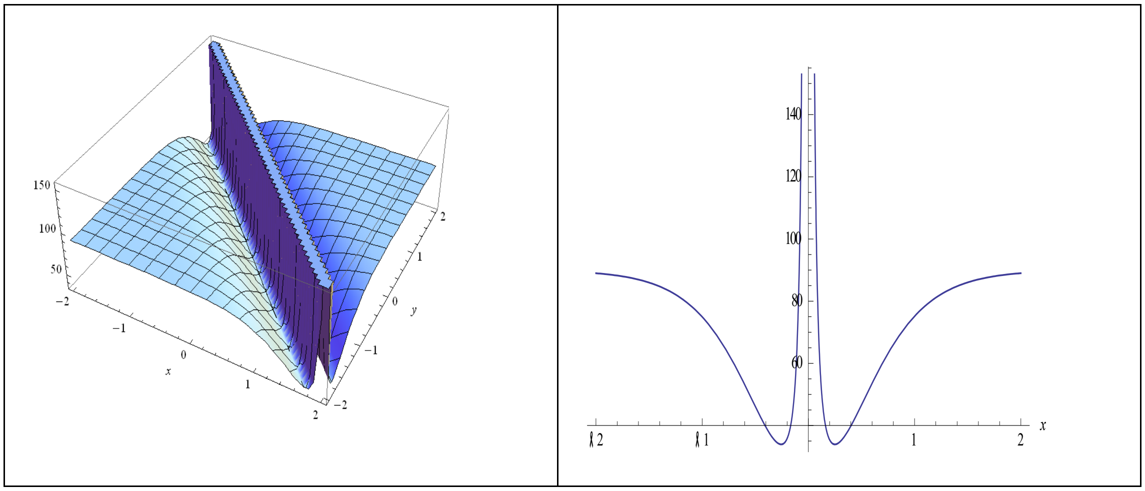

Fig.2: Plot of Eqs. (21) in three and two dimensions of when k = -1, α = 2, μ = 1, l = 1, $r_1 = r_2 = 1$, t = 0.002, $-2 < x < 2$, $-2 < t < 2$ and y = 0.001 for the two dimension graphics.

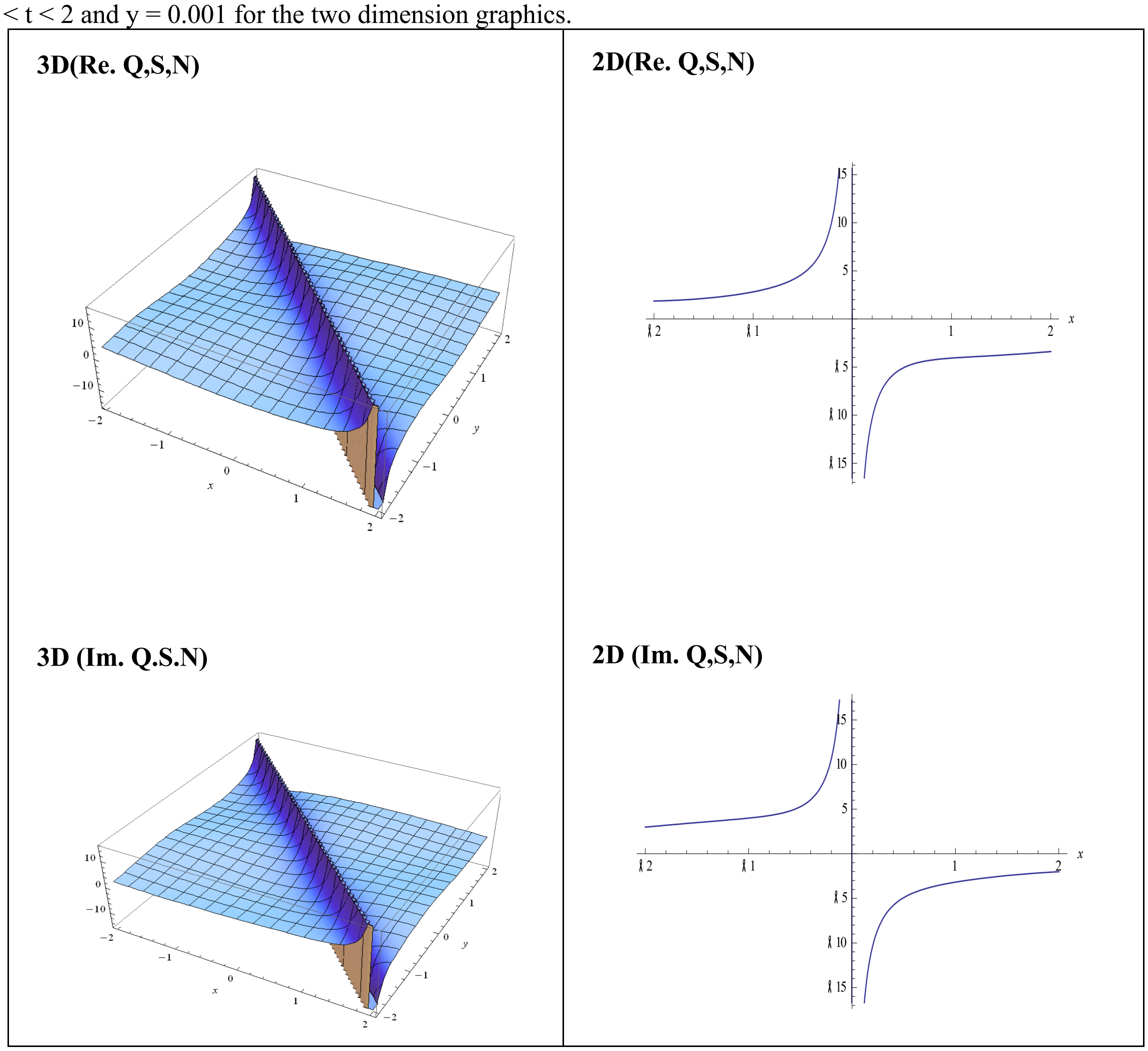

Fig.3: Plot of Eqs. (25, 26, 27) in three and two dimensions of when k = -1, α = 2, μ = 1, l = 1, $r_1 = r_2 = 1$, t = 0.002, $-2 < x < 2$, $-2 < t < 2$ and y = 0.001 for the two dimension graphics.

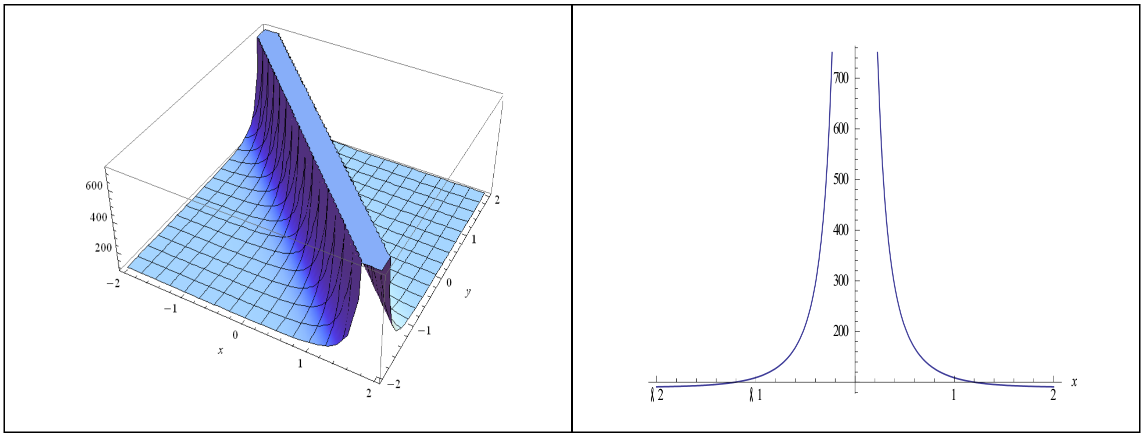

Fig.4: Plot of Eqs. (28) in three and two dimensions of when k = -1, $\alpha = 2$, $\mu = 1$, l = 1, $r_1 = r_2 = 1$, t = 0.002, $-2 < x < 2$, $-2 < t < 2$ and y = 0.001 for the two dimension graphics.

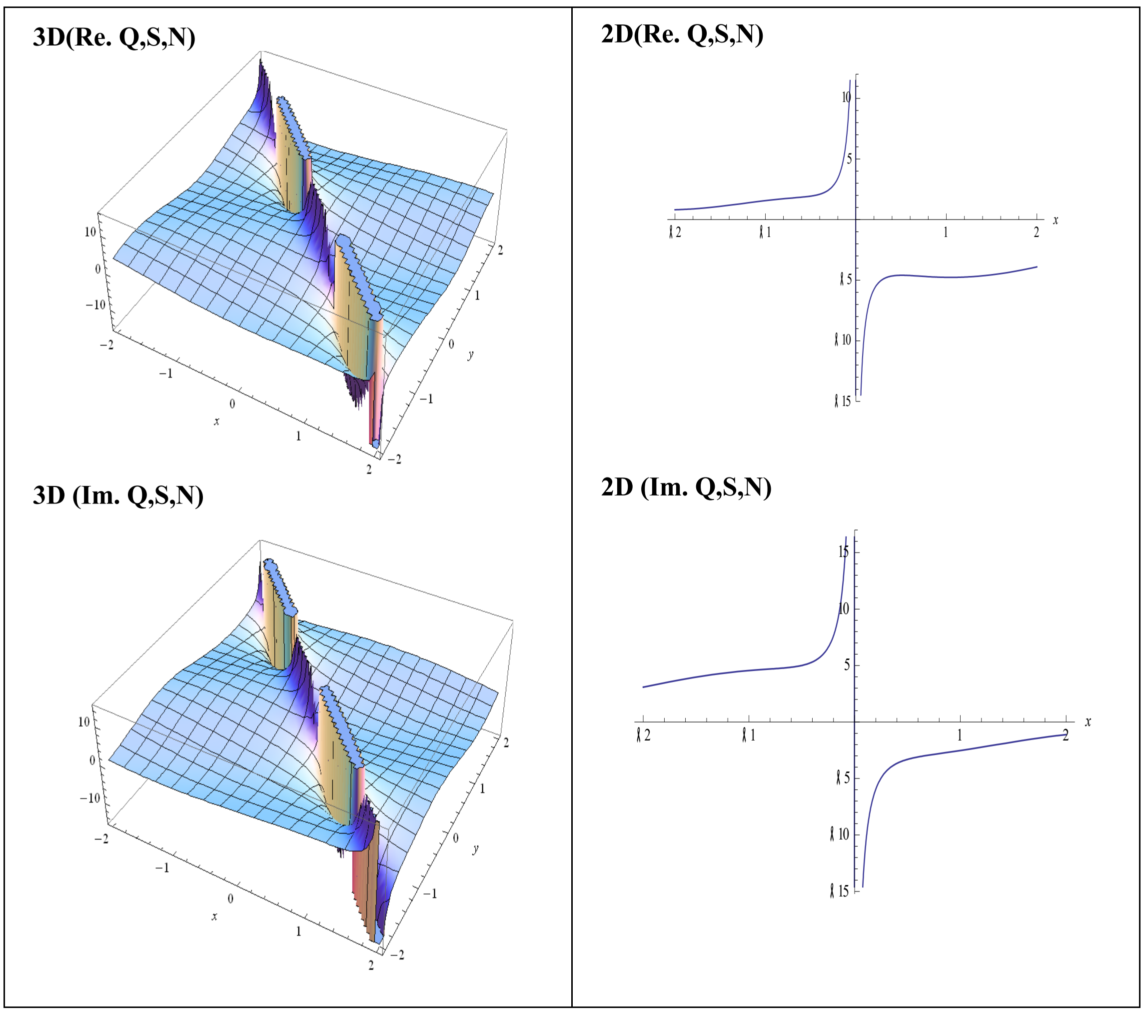

Fig.5: Plot of Eqs. (32, 33, 34) in three and two dimensions of when k = -1, $\alpha = 2$, $\mu = 1$, l = 1, $r_1 = r_2 = 1$, t = 0.002, $-2 < x < 2$, $-2 < t < 2$ and y= 0.001 for the two dimension graphics.

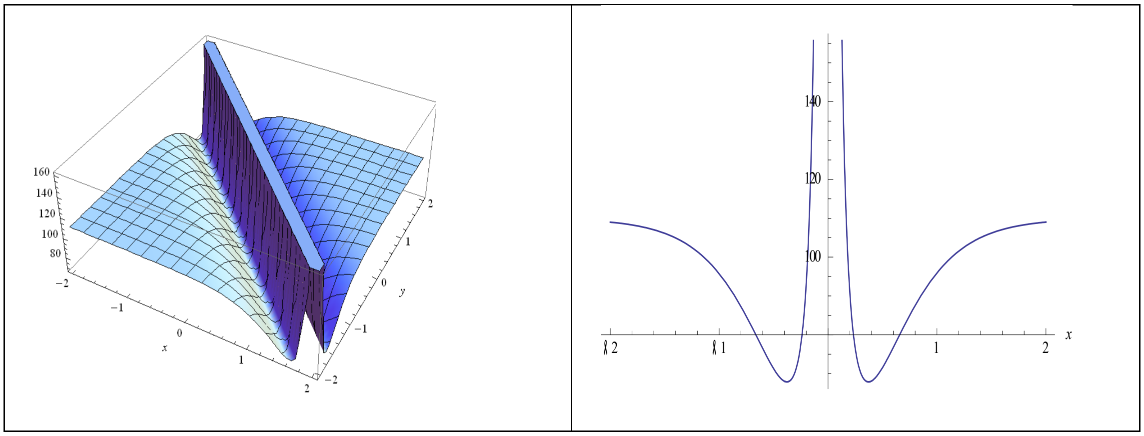

Fig.6: Plot of Eqs. (35) in three and two dimensions of when k = -1, α = 2, μ = 1, l =1, $r_1 = r_2 = 1$, t = 0.002, −2 < x < 2, −2 < t < 2 and y = 0.001 for the two dimension graphics.

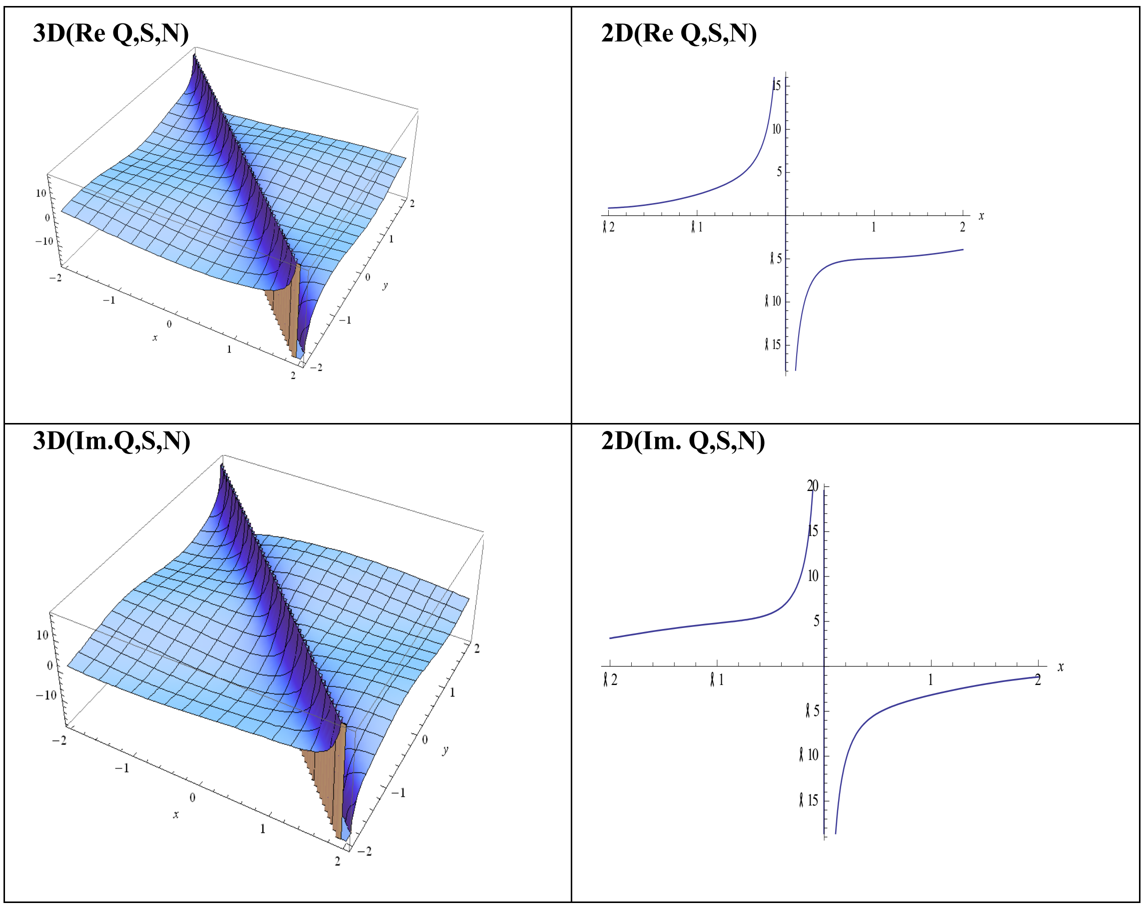

Fig.7: Plot of Eqs. (39, 40, 41) in three and two dimensions of when k = -1, α = 2, μ = 1, l = 1, $r_1 = r_2 = 1$, t = 0.002, −2 < x < 2, −2 < t < 2 and y = 0.001For the two dimension graphics.

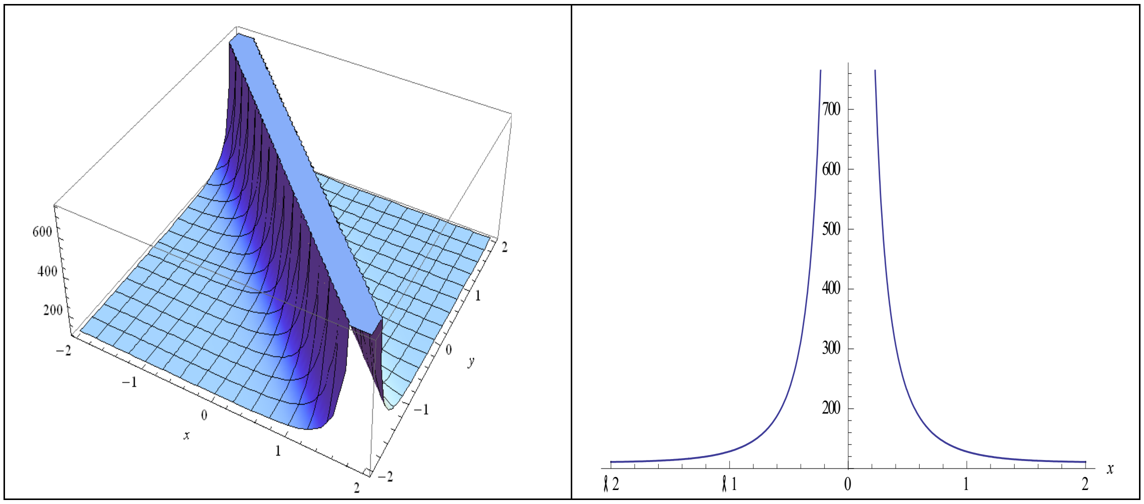

Fig.8: Plot of Eqs. (42) In three and two dimensions of when = -1, α = 2, μ = 1, l = 1, $r_1 = r_2 = 1$, t = 0.002, −2 < x < 2, −2 < t < 2 and y = 0.001 for the two dimension graphics.

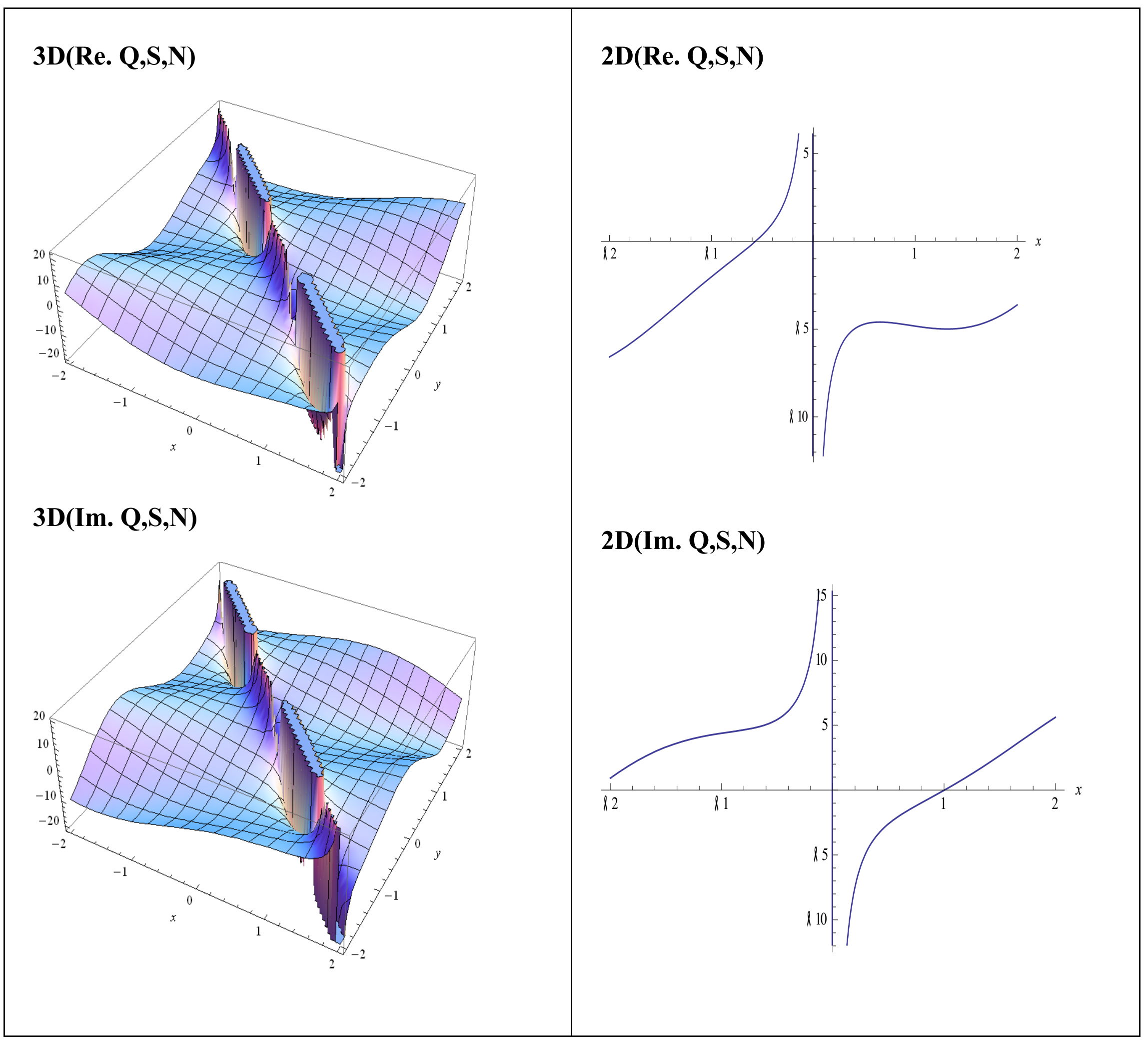

Fig.9: Plot of Eqs. (46, 47, 48) in three and two dimensions of when k = -1, α = 2, μ = 1, l = 1, $r_1 = r_2 = 1$, t = 0.002, −2 < x < 2, −2 < t < 2 and y = 0.001 for the two dimension graphics.

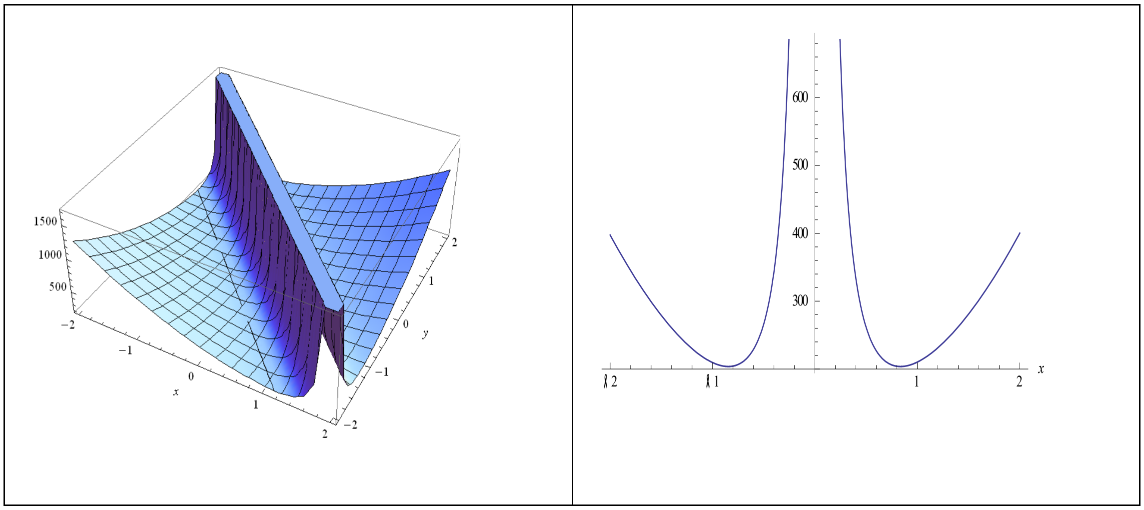

Fig.10: Plot of Eqs. (49) In three and two dimensions of when $k = -1$, $\alpha = 2$, $\mu = 1$, $l = 1$, $r_1 = r_2 = 1$, $t = 0.002$, $-2 < x < 2$, $-2 < t < 2$ and $y = 0.001$ for the two dimension graphics.

4 Conclusions

Accurate traveling wave solutions of an important model TNLMS [31] have been established using important technique BMETFM [22]. Comparing our solutions with that obtained by other authors indicate that this method give many new of solitary solutions which give a large area to physical interpretation for this model experimentally. The located solutions via the proposed methods are equivalence in some cases with that established by [33-36] –but not in the other cases. Also this pushes in the forward direction on the application of this model in the different branches of physics. The results obtained by these methods are reliable, effective and can be considered as benchmark against the numerical results.

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