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Stochastic Analysis of a System with Cold Standby, General Distribution and Random Change in Units

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Abstract: This paper discusses the stochastic analysis of a two-unit cold standby system, taking into account that the operative and the standby units are exchanged at random time intervals. Failure, repair and exchanging time are following general distributions. Using semi-Markov process and regenerative point technique in Markov renewal process, we develop the explicit expressions for the mean time to system failure, MTSF and steady-state availability, A(∞) for the system. Some special cases have been studied numerically and graphically to explain the effect of the system parameters on system performance. We also compute the sensitivity and relative sensitivity analysis for the MTSF and A(∞) along with changes in specific values of the system parameters.

Keywords: Mean time to system failure, steady-state availability, sensitivity analysis, relative Sensitivity analysis.

1 Introduction

The theory reliability plays an active role in our life because of the high development of devices in many fields. Nowadays, it touches technological, economic, structural, industrial, and other similar subjects. Several authors [1,2,3,4] have studied a two dissimilar unit standby system with constant failure rate.[5] discussed reliability measures of three models with three types of failures and attended by one repairman. The model 1 is under preventive maintenance before failure, while model 2 and model 3 are analyzed without preventive maintenance.

A repairable K-out-of-(M+W) retrial system with M identical primary components, W standby components and one repair facility investigated by [6]. [7] studied the reliability measures of a repairable system with M operating units, W warm standby units, and R repairmen in which there are switching failures and reboot delay. [8] has considered the reliability and sensitivity analysis of a system with M operating machines, S warm standbys, and a repairable service station.

The reliability and sensitivity analysis of a repairable system with imperfect coverage under service pressure condition studied by [9]. [10] provided a two-unit cold standby system with hardware, human error failures. They added time preventive maintenance for the system. [11] analyzed profit analysis of a reliability model for a single-unit system with preventive maintenance subject to maximum operation time. [12] considered reliability and MTTF of complex systems, with different types of failures and one type of repair.

The main contributions of this paper are: study a two-unit cold standby system with random change between the units and all time distribution of the system are arbitrary. Various reliability characteristics of interest are evaluated by using semi-Markov process and regenerative point technique. The effect of the different parameters on mean time to system failure and steady-state availability are shown tabular and graphically. Finally, the sensitivity analysis and the relative sensitivity analysis for the mean time to system failure and steady-state availability are discussed.

The organization of the paper is as follows. In section 2, 3 we given a detailed description for system consist of two non-identical units. Transition probabilities and sojourn times are presented in section 4. Some reliability characteristics of the system are derived in sections 5 and 6 respectively. The results of our numerical simulations and sensitive analysis and relative sensitivity analysis of the reliability characteristics are discussed in section 7. Finally, we make a concluding remark in section 8.

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2 Assumptions

1- A redundant system comprises from two non-identical units. Initially, one unit is operative and the other is cold standby.

2- The switch is perfect and instantaneous.

3- After a random amount of time t, the operative unit becomes standby and the standby unit becomes operative if the standby is available.

4- The distributions of all times are arbitrary.

5- There is a single repair facility is available for repair.

6- Service discipline is FCFS.

7- After the repair, the unit is as good as new.

3 Nomenclature

$F_i(t)$ Cumulative distribution function of failure time from normal mode to complete failure, i = 1,2.

$K(t)$ Cumulative distribution function of repair time of a failed unit.

$G_i(t)$ Cumulative distribution function of times after which operative unit changes, i = 1,2.

E_0 State of the system at t = 0.

E Set of regenerative states.

$q_{ij}(t)$, $Q_{ij}(t)$ Probability density function and cumulative distribution function of transition time from regenerative state S_i to S_j.

$q_{ij}^k(t)$, $Q_{ij}^k(t)$ Probability density function and cumulative distribution function of time for the system transits from regenerative state S_i to S_j via the non-regenerative state $S_k \in$ E.

$(\Pi)_i(t)$ Cumulative distribution function of time to system failure starting from state $S_k \in$ E.

m_{ij} Contribution to mean sojourn time in state S_i, when system transits direct to S_j.

μ_i $\int$ P[system sojourn in state S_i for at least time t]dt.

$M_i(t)$ P[system up initially in state $S_i \in$ E is up at time t without going to any other regenerative state or returning to itself through one or more states $\in$ E].

$Av_i(t)$ P[starting from $S_i \in$ E, the system is up at time t].

s Dummy variable in Lapace transform (LT).

$*$ Symbol for Laplace transform, i.e. $q_{ij}^*(t) = \int$ exp (-st) $q_{ij}(t)$ dt.

© Symbol for ordinary convolution, i.e. $A(t)$ © $B(t)$ = $\int_0^t$ A(t-x) B(x) dx.

3.1 Symbols for states of the system

O : normal unit when it is operative, CS : normal unit when it is cold standby, O_u : old normal operative unit, O_{uc3} : old normal operative unit when it is continued from state S_3 , O_{uc4} : old normal operative unit when it is continued from state S_4 , F_r :failed unit under repair, w_r :failed unit waiting for the repair, F_R :repair of failed unit is continued from earlier state.

With these symbols, the possible states of the system model under study are:

Up states:

S_0 = (O, CS), S_1 = (CS, O), S_2 = (O_u, CS),
S_3 = (F_r, O), S_{c3} = (CS, O_{uc3}),
S_4 = (O_u, F_r), S_{c4} = (O_{uc4}, CS), S_7 = (CS, O_u).

Down states:

S_5 = (F_R, w_r), and S_6 = (w_r, F_R).

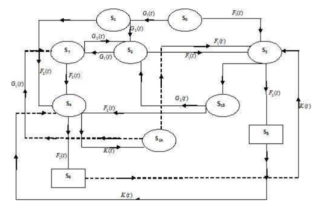

Fig. 1: Possible states and transitions between them

4 Transition probabilities and sojourn times

It is evident that the epochs of entry into any one of the states S_0, S_1, S_2, S_3, S_4 and S_7 are regeneration points and is the set of these states. Let T_0, T_1, T_2,... denote the epochs at which the system enters any state $S_i \in$ E and let X_n denote the state visited at epoch T_n^+, i.e. just after the transition at T_n. Then (X_n, T_n) is a Markov renewal process with state space E and $Q_{ij}(t)$ = P (X_{n+1} = j, T_{n+1} - $T_n \leq$ t | X_n = i) is the semi-Markov kernel over E. The transition probability matrix of embedded Markov-chain is P $\equiv (P_{ij}(t)) \equiv (Q_{ij}(\infty) = Q(\infty))$ with non-zero elements P_{ij} as follows:

$P_{01} = P_{27} = \int \overline{F_1}(t)\, dG_1(t)$, $P_{03} = P_{23} = \int \overline{G_1}(t)\, dF_1(t)$,
$P_{12} = \int \overline{F_2}(t)\, dG_2(t)$, $P_{14} = \int \overline{G_2}(t)\, dF_2(t)$,
$P_{32}^{C_3} = \int \overline{F_2}(t)\, K(t)\, dG_2(t)$, $P_{34}^{5} = \int F_2(t)\, dK(t)$,
$P_{43}^{6} = \int F_1(t)\, dK(t)$, $P_{47}^{C_4} = \int \overline{F_1}(t)\, K(t)\, dG_1(t)$,
$P_{72} = \int \overline{F_2}(t)\, dG_2(t)$, $P_{74} = \int \overline{G_2}(t)\, dF_2(t)$,
$P_{3C_3} = \int \overline{F_2}(t)\, dK(t)$, $P_{4C_4} = \int \overline{F_1}(t)\, dK(t)$,
$P_{35} = \int \overline{K}(t)\, dF_2(t)$, $P_{46} = \int \overline{K}(t)\, dF_1(t)$,
$P_{34}^{C_3} = \int \overline{G_2}(t)\, K(t)\, dF_2(t) + \int \int_0^t G_2(x)\, dK(x)\, dF_2(t)$,
$P_{43}^{C_4} = \int \overline{G_1}(t)\, K(t)\, dF_1(t) + \int \int_0^t G_1(x)\, dK(x)\, dF_1(t)$.

Evidently,

$P_{01} + P_{03} = P_{12} + P_{14} = P_{27} + P_{23} = P_{3C_3} + P_{35} = P_{4C_4} + P_{46} = 1$,
$P_{32}^{C_3} + P_{34}^{C_3} + P_{34}^{5} = P_{43}^{C_4} + P_{47}^{C_4} + P_{43}^{6} = 1$.

Mean sojourn times μ_i in state S_i are:

$\mu_0 = \mu_2 = \int \overline{F_1}(t)\, \overline{G_1}(t)$ dt, $\mu_1 = \mu_7 = \int \overline{F_2}(t)\, \overline{G_2}(t)$ dt,
$\mu_3 = \int \overline{F_2}(t)\, \overline{K}(t)$ dt, $\mu_4 = \int \overline{F_1}(t)\, \overline{K}(t)$ dt.

5 Mean time to system failure

According to the arguments of theory of regenerative processes, we obtain the following equation:

$$\overline{\Pi_0}(t) = \overline{F_1}(t)\overline{G_1}(t) + q_{01}(t)\copyright\overline{\Pi_1}(t) + q_{03}(t)\copyright\overline{\Pi_3}(t), \quad (1)$$

$$\overline{\Pi_1}(t) = \overline{F_2}(t)\overline{G_2}(t) + q_{12}(t)\copyright\overline{\Pi_2}(t) + q_{14}(t)\copyright\overline{\Pi_4}(t), \quad (2)$$

$$\overline{\Pi_2}(t) = \overline{F_1}(t)\overline{G_1}(t) + q_{23}(t)\copyright\overline{\Pi_3}(t) + q_{27}(t)\copyright\overline{\Pi_7}(t), \quad (3)$$

$$\overline{\Pi_3}(t) = \overline{F_2}(t)\overline{K}(t) + q_{32}^{C_3}(t)\copyright\overline{\Pi_2}(t) + q_{34}^{C_3}(t)\copyright\overline{\Pi_4}(t), \quad (4)$$

$$\overline{\Pi_4}(t) = \overline{F_1}(t)\overline{K}(t) + q_{43}^{C_4}(t)\copyright\overline{\Pi_3}(t) + q_{47}^{C_4}(t)\copyright\overline{\Pi_7}(t), \quad (5)$$

$$\overline{\Pi_7}(t) = \overline{F_2}(t)\overline{G_2}(t) + q_{72}(t)\copyright\overline{\Pi_2}(t) + q_{74}(t)\copyright\overline{\Pi_4}(t) \quad (6)$$

Taking the Laplace transform of these relations and solving for $\overline{\Pi_0}(s)$ considering s = 0, we have the time to system failures MTSF as follows:

$$MTSF = E(T) = \frac{N_1}{D_1} \quad (7)$$

where,

D_1 = 1 - P_{23} {$P_{32}^{C_3}$ (1-$P_{47}^{C_4}$ P_{74}) + $P_{34}^{C_3}$ $P_{47}^{C_4}$ P_{72}} - P_{27} {P_{72} (1- $P_{34}^{C_3}$ $P_{43}^{C_4}$) + $P_{32}^{C_3}$ $P_{43}^{C_4}$ P_{74}} - $P_{34}^{C_3}$ $P_{43}^{C_4}$ - $P_{47}^{C_4}$ P_{74},

N_1 = {μ_0 + P_{01} μ_1} {($P_{34}^{C_3}$ $P_{43}^{C_4}$ - 1) (1 - P_{27} P_{72}) - P_{74} (P_{27} $P_{32}^{C_3}$ $P_{43}^{C_4}$ + $P_{47}^{C_4}$) + P_{23} ($P_{32}^{C_3}$ ($P_{47}^{C_4}$ + P_{74} - 1) - $P_{34}^{C_3}$ $P_{47}^{C_4}$ P_{72})} + μ_0 {P_{03} ($P_{34}^{C_3}$ $P_{47}^{C_4}$ P_{72} + $P_{32}^{C_3}$ ($P_{47}^{C_4}$ P_{74} - 1)) (P_{12} P_{74} - P_{14} P_{72}) - P_{03} (P_{27} P_{72} + P_{74} $P_{47}^{C_1}$ - 1)} + {μ_0 + ω_3 + ω_4} {P_{03} ($P_{34}^{C_3}$ (1 - P_{27} P_{72}) + P_{27} $P_{32}^{C_3}$ P_{74}) + P_{01} (P_{14} ($P_{32}^{C_3}$ $P_{43}^{C_4}$ + $P_{47}^{C_4}$ P_{72}) - P_{12} ($P_{34}^{C_3}$ $P_{43}^{C_4}$ + $P_{47}^{C_4}$ P_{74} - 1))} {μ_1 + ω_1 + ω_2} {P_{01} (P_{14} P_{23} + P_{14} $P_{43}^{C_4}$) + P_{01} (P_{27} $P_{43}^{C_4}$ - P_{23} $P_{47}^{C_4}$) + P_{01} (P_{14} (1 - P_{23} $P_{32}^{C_3}$ - P_{27} P_{72}) + P_{12} (P_{23} $P_{34}^{C_3}$ + P_{27} P_{74}))} + μ_1 {P_{27} (P_{03} $P_{32}^{C_3}$ + P_{01} (P_{12} + (P_{14} $P_{32}^{C_3}$ - P_{12} $P_{34}^{C_3}$) $P_{43}^{C_4}$)) - $P_{47}^{C_4}$ (P_{01} P_{14} (P_{23} $P_{32}^{C_3}$ - 1) - (P_{03} + P_{01} P_{12} P_{23}) $P_{34}^{C_3}$)},

$\omega_1 = \int \overline{F_2}(t)\, \overline{G_2}(t)\, K(t)$ dt,

$\omega_2 = \int \overline{F_2}(t) \int_0^t G_2(x)\, dK(x)$ dx,

$\omega_3 = \int \overline{F_1}(t)\, \overline{G_1}(t)\, K(t)$ dt,

$\omega_4 = \int \overline{F_1}(t) \int_0^t G_1(x)\, dK(x)$ dx.

6 System Availability

From the theory of regenerative processes, the pointwise availabilities $Av_i(t)$ of the system starting from a given regenerative point are seen to satisfy the following recursion relations:

$$Av_0(t) = M_0(t) + q_{01}(t)\copyright Av_1(t) + q_{03}(t)\copyright Av_3(t), \quad (8)$$

$$Av_1(t) = M_1(t) + q_{12}(t)\copyright Av_2(t) + q_{14}(t)\copyright Av_4(t), \quad (9)$$

$$Av_2(t) = M_2(t) + q_{23}(t)\copyright Av_3(t) + q_{27}(t)\copyright Av_7(t), \quad (10)$$

$$Av_3(t) = M_3(t) + q_{32}^{C_3}(t)\copyright Av_2(t) + (q_{34}^{C_3}(t) + q_{34}^{5}(t))\copyright Av_4(t), \quad (11)$$

$$Av_4(t) = M_4(t) + (q_{43}^{C_4}(t) + q_{43}^{6}(t))\copyright Av_3(t) + q_{47}^{C_4}(t)\copyright Av_7(t), \quad (12)$$

$$Av_7(t) = M_7(t) + q_{72}(t)\copyright Av_2(t) + q_{74}(t)\copyright Av_4(t). \quad (13)$$

where,

$M_0(t) = M_2(t) = \overline{F_1}(t)\ \overline{G_1}(t)$,

$M_1(t) = M_7(t) = \overline{F_2}(t)\ \overline{G_2}(t)$,

$M_3(t) = \overline{F_2}(t)\ \overline{K}(t) + \overline{F_2}(t)\ K(t)\ \overline{G_2}(t) + \overline{F_2}(t) \int_0^t G_2(x)\, dK(x)$,

$M_4(t) = \overline{F_1}(t)\ \overline{K}(t) + \overline{F_1}(t)\ K(t)\ \overline{G_1}(t) + \overline{F_1}(t) \int_0^t G_1(x)\, dK(x)$.

Taking the Laplace transform of equations (8)-(13) and solving for $Av_0^*(t)$, then we get the steady state availability of the system $Av_0(t)$ in the form,

$$Av_0 = \lim_{s\to 0} sAv_0^*(s) = \frac{N_2}{D_2}, \quad (14)$$

where,

N_2 = {μ_0 + P_{01} μ_0} {(1-$P_{47}^{C_4}$ P_{01})(1-$P_{32}^{C_3}$ P_{23}) - (1- P_{27} P_{72})(P_{43}^{6} + $P_{43}^{C_4}$)($P_{34}^{C_3}$ + P_{34}^{5}) - P_{23} $P_{47}^{C_4}$ P_{72} ($P_{34}^{C_3}$ + P_{34}^{5}) - $P_{32}^{C_3}$ P_{27} P_{74} (P_{43}^{6} + $P_{43}^{C_4}$) - P_{27} P_{72}}+ μ_2 {P_{01} P_{12} (1-$P_{47}^{C_4}$ P_{74} - (P_{43}^{6} + $P_{43}^{C_4}$) ($P_{34}^{C_3}$ + P_{34}^{5})) + P_{03} ($P_{32}^{C_3}$ (1- $P_{47}^{C_4}$ P_{74}) + $P_{47}^{C_4}$ P_{72} (P_{34}^{5} + $P_{34}^{C_3}$)) + P_{01} P_{14} ($P_{32}^{C_3}$ (P_{43}^{6}) + $P_{43}^{C_4}$) + $P_{47}^{C_4}$ P_{72})} + {μ_3 + ω_1 + ω_2} {P_{01} P_{12} (P_{23} (1-P_{74} $P_{47}^{C_4}$) + P_{27} P_{74} (P_{43}^{6} + $P_{43}^{C_4}$)) + P_{03} (1- P_{27} P_{72} - $P_{47}^{C_4}$ P_{74}) + P_{01} P_{14} ((1-P_{27} P_{72})(P_{43}^{6} + $P_{43}^{C_4}$) + P_{23} P_{72} $P_{47}^{C_4}$)} + {μ_4 + ω_3 + ω_4 } { P_{01} P_{12} (P_{27} P_{74} + P_{23} ($P_{34}^{C_3}$ + P_{34}^{5})) + P_{03} ((1-P_{27} P_{72})(P_{34}^{5}) + $P_{34}^{C_3}$) + P_{27} P_{74} $P_{32}^{C_3}$) + P_{01} P_{14} (1- $P_{32}^{C_3}$ P_{23} - P_{27} P_{72}) + μ_5 {P_{01} P_{12} (P_{27} (1 - (P_{43}^{6} + $P_{43}^{C_4}$) (P_{34}^{5} + $P_{34}^{C_3}$)) + P_{23} $P_{47}^{C_4}$ (P_{34}^{5} + $P_{34}^{C_3}$)) + P_{03} (P_{27} $P_{32}^{C_3}$ + $P_{47}^{C_4}$ (P_{34}^{5} + $P_{34}^{C_3}$)) + P_{01} P_{14} (1 - $P_{32}^{C_3}$ P_{23} - P_{27} P_{72})},

Table 1: Effect of $\lambda_2, \alpha_1, \alpha_2, \mu$ on MTSF when $\lambda_1 \in \{0.1, \cdots, 0.5\}$.

λ_1	MTSF		
	A	*B*	*C*
0.1	672.16	366.594	118.65
0.2	315.205	160.945	40.6057
0.3	186.98	95.253	23.3281
0.4	126.401	65.5772	16.7451
0.5	93.0855	49.4803	13.395

Table 2: Effect of $\lambda_2, \alpha_1, \alpha_2, \mu$ on Av(∞) when $\lambda_1 \in \{0.1, \cdots, 0.5\}$.

λ_1	Av(∞)		
	A	*B*	*C*
0.1	0.999134	0.990773	0.983274
0.2	0.998012	0.986145	0.949213
0.3	0.996468	0.981892	0.911351
0.4	0.994614	0.977581	0.875892
0.5	0.992545	0.97319	0.844388

$D_2 = m_2\ (P_{32}^{C_3} + P_{47}^{C_4}\ P_{72}$ - $P_{32}^{C_3}\ P_{47}^{C_4}$) + $m_3\ (P_{23} + P_{27}\ P_{74}\ P_{43}^{6} + P_{43}^{C_4}\ P_{27}\ P_{74}$ - $P_{23}\ P_{74}\ P_{47}^{C_4}$) + $m_4\ (P_{23}\ P_{72}\ P_{34}^{5} + P_{34}^{C_3}\ P_{23}\ P_{72} + P_{74}$ - $P_{23}\ P_{74}\ P_{32}^{C_3}$) + $m_7\ (P_{47}^{C_4} + P_{32}^{C_3}\ P_{27}$ - $P_{32}^{C_3}\ P_{47}^{C_4}$),

$m_2 = m_{23} + m_{27}$, $m_3 = m_{32}^{C_3} + m_{34}^{C_3} + m_{34}^{5}$, $m_4 = m_{43}^{C_4} + m_{47}^{C_4} + m_{43}^{6}$, $m_7 = m_{72} + m_{74}$.

7 Numerical analysis and discussion

In this section, some of the results are obtained from above sections and are illustrated with a numerical example, we assume that

$f_i(t) = \lambda_i^2$ t $e^{-\lambda_i t}$, $\lambda_i > 0$;
$k(t) = \mu^2$ t $e^{-\mu t}$, $\mu > 0$;
$g_i(t) = \alpha_i^2$ t $e^{-\alpha_i t}$, $\alpha_i > 0$; for all i = 1,2.

The result shown in Table 1 presents the mean time to system failure of the system computed by varying its failure rate (λ_1) from 0.1 to 0.5 and change other parameters as λ_2 = 0.4,0.5,0.7. α_1 = 1.2,0.8,0.5. α_2 = 0.9,0.6, 0.3 and μ = 1.1,0.9,0.3 the mean time to system failure of the system decrease with increasing of λ_1 and λ_2. The mean time to system failure of the system increase with increasing of α_1, α_2 , μ . These results are shown in Fig 2. The steady-state availability of the system has been calculated by varying the failure rate (λ_1) from 0.1 to 0.5 and change other parameters as λ_2 = 0.4,0.5,0.7. α_1 = 1.2,0.8,0.5. α_2 = 0.9,0.6, 0.3 and μ = 1.1,0.9,0.3. The results are shown in Table 2 and Fig 3. . It seems that the steady-state availability of the system decrease with increasing of λ_1 and λ_2. The steady-state availability of the system increase with increasing of α_1, α_2 , μ .

where $A = \{\alpha_1 = 1.2, \alpha_2 = 0.9, \lambda_2 = 0.4, \mu = 1.1\}$, $B = \{\alpha_1 = 0.8, \alpha_2 = 0.6, \lambda_2 = 0.5, \mu = 0.9\}$, $C = \{\alpha_1 = 0.5, \alpha_2 = 0.3, \lambda_2 = 0.7, \mu = 0.3\}$.

where $A = \{\alpha_1 = 1.2, \alpha_2 = 0.9, \lambda_2 = 0.4, \mu = 1.1\}$, $B = \{\alpha_1 = 0.8, \alpha_2 = 0.6, \lambda_2 = 0.5, \mu = 0.9\}$, $C = \{\alpha_1 = 0.5, \alpha_2 = 0.3, \lambda_2 = 0.7, \mu = 0.3\}$.

7.1 Sensitivity analysis and relative sensitivity analysis

In this subsection, we calculate the sensitivity analysis and relative sensitivity analysis of MTSF and steady-state availability with respect to one of system parameters κ where κ = α_1 , α_2, λ_1, λ_2 and μ

Table 3: Sensitivity and relative sensitivity of MTSF with respect to $\lambda_1, \lambda_2, \alpha_1, \alpha_2$ and μ

ϕ_{λ_1}	ϕ_{λ_2}	ϕ_{α_1}	ϕ_{α_2}	ϕ_{μ}
-835.84	-685.288	-1.31441	47.6518	269.616
σ_{λ_1}	σ_{λ_2}	σ_{α_1}	σ_{α_2}	σ_{μ}
-1.3411	-1.46601	-0.00844	0.229364	1.58614

7.1.1 Sensitivity analysis and relative sensitivity analysis for MTSF

Defined (7) with respect to κ, we obtain.

$$\phi_\kappa = \frac{\partial MTSF}{\partial \kappa}, \tag{15}$$

where κ = α_1 , α_2, λ_1, λ_2 and μ. The relative sensitivity analysis of MTSF is defined as the percentage change that resulting from the percentage change in one of system parameters κ.

$$\sigma_\kappa = \phi_\kappa\left(\frac{\kappa}{MTSF}\right), \tag{16}$$

We first perform the sensitivity and the relative sensitivity analysis of MTSF with respect to different system parameters α_1 , α_2, λ_1, λ_2 and μ. respectively. We will see how effect on MTSF about system parameters. Numerical results of the sensitivity and the relative sensitivity analysis of MTSF are shown in Table 3 when λ_1 = 0.3, λ_2 = 0.4, α_1 = 1.2, α_2 = 0.9, and μ = 1.1. The order of magnitude of the effect on MTSF about system parameters can be determined by the absolute value in Table 3. Therefore, the order of magnitude of the sensitivity to the MTSF is $\lambda_1 > \lambda_2 > \mu > \alpha_2 > \alpha_1$. Moreover, the order of magnitude of the relative sensitivity to the MTSF is $\lambda_1 > \lambda_2 > \mu > \alpha_2 > \alpha_1$ when λ_1 = 0.3, λ_2 = 0.4, α_1 = 1.2, α_2 = 0.9 and μ = 1.1.

7.1.2 Sensitivity analysis and relative sensitivity analysis for steady-state availability

We perform the sensitivity analysis of changes in steady-state availability with respect to one of system parameters

Table 4: Sensitivity and relative sensitivity of Av(∞) with respect to $\lambda_1, \lambda_2, \alpha_1, \alpha_2$ and μ

π_{λ_1}	π_{λ_2}	π_{α_1}	π_{α_2}	π_{μ}
-0.0172	-0.00397	0.015141	0.022354	-0.02869
ψ_{λ_1}	ψ_{λ_2}	ψ_{α_1}	ψ_{α_2}	ψ_{μ}
-0.0052	-0.00159	0.01824	0.02019	-0.03167

κ where κ = α_1 , α_2, λ_1, λ_2 and μ. Differentiating (14) with respect to κ, we obtain

$$\Pi_\kappa = \frac{\partial Av(\infty)}{\partial \kappa}, \tag{17}$$

The relative sensitivity of steady-state availability is defined as the percentage change that resulting from the percentage changes in one of system parameters κ.

$$\psi_\kappa = \Pi_\kappa\left(\frac{\kappa}{Av(\infty)}\right), \tag{18}$$

Numerical results are provided to illustrate the sensitivity of steady-state availability with respect to system parameters. Table 4 show that sensitivity and relative sensitivity of steady-state availability for the base case with respect to one of system parameters κ where κ where κ = α_1 , α_2, λ_1, λ_2 and μ. when λ_1 = 0.3, λ_2 = 0.4, α_1 = 1.2, α_2 = 0.9, and μ = 1.1, respectively. Therefore, the order of magnitude of the sensitivity to the steady-state availability is $\mu > \alpha_2 > \lambda_1 > \alpha_1 > \lambda_2$ Moreover, the order of magnitude of the relative sensitivity to the steady-state availability is $\mu > \alpha_2 > \alpha_1 > \lambda_1 > \lambda_2$ when λ_1 = 0.3, λ_2 = 0.4, α_1 = 1.2, α_2 = 0.9, and μ = 1.1

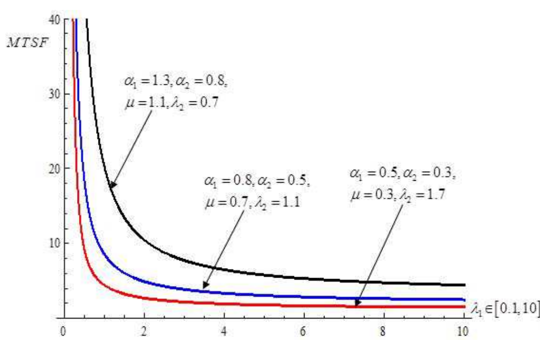

Fig. 2: Effect of $\lambda_2, \alpha_1, \alpha_2, \mu$ on MTSF

8 Conclusions

In this paper, A regenerative point technique is used to derive the mean time to system failure, MTSF, and steady-state availability, Av(∞), of a system consisting of two non-identical units and repairman. The effect of the system parameters λ_1, λ_2, α_1, α_2, and μ has been discussed. It has been found that the failure rate shows a strong effect on the system than that of the other parameters with respect to mean time to system failure. While the repair rate shows a strong effect on the system than that of the other parameters with respect to steady-state availability. We have the same results, using the sensitivity analysis and relative sensitivity analysis in specific values of the system parameters.

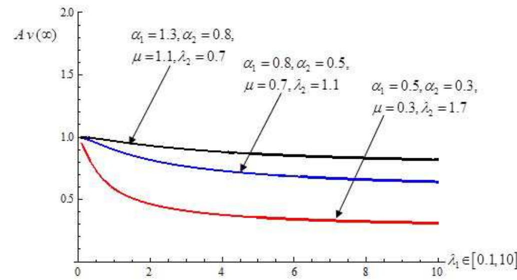

Fig. 3: Effect of $\lambda_2, \alpha_1, \alpha_2, \mu$ on Availability

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