International Journal of Advanced Biomedicine

http://dx.doi.org/10.18576/ab/090303

On Approach of Analysis of Skeletal Muscle Contraction

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Received: 12 Apr. 2024, Revised: 7 Jun. 2025, Accepted: 29 Jul. 2025.
Published online: 1 Sep. 2025.

Abstract: Prognosis of muscle contraction is an important factor in the study of the physiological characteristics of human movement. The prognosis muscle movement allows athletes and their coaches to improve the effectiveness of their training. The prognosis also helps improve the effectiveness of treatments for muscle disorders. To make the prognosis we present a model for the analysis of skeletal muscle contraction with account its deformation properties. We also introduce an analytical approach for analysis of the considered muscle contraction.

Keywords: muscle contraction; process model; analytical approach for analysis.

1 Introduction

The prognosis of muscle contraction is an important factor in the study of the physiological characteristics of human movement. Knowledge of the informative parameters of the mechanical properties of the muscle is used in medicine in the treatment of patients [1-5]. In sports, the prediction of human muscle movement helps coaches improve the effectiveness of sports training. Prognosis of muscle contraction is an important factor in the study of the physiological characteristics of human movement. The prognosis muscle movement allows athletes and their coaches to improve the effectiveness of their training. The prognosis also helps improve the effectiveness of treatments for muscle disorders. The capabilities of modern models allow conducting research and introducing correction into the treatment and training methods directly during its implementation. In this situation it is attracted an interest to take into account state of human and first of all state of muscle. This information helps to coaches to optimize their training and to doctors to optimize their treatment. In this paper we present a model for the analysis of skeletal muscle contraction, which takes into account possible deformation properties. We also present an analytical approach for analysis of the considered muscle contraction.

2 Method of solution

In this section, we consider the model of skeletal muscle contraction and analyze it. In the framework of the model under consideration, we will assume that the muscle is a locally flat object and has the structure "elastic thread - elastic-viscous substrate": it is a set of parallel threads connected to an elastic-viscous substrate. We will assume that the effective layer of tissue with depth H is reduced. A linear law of distribution along the coordinate q of the component of the displacement field normal to the muscle surface is adopted

$$U(y,z,t) = V(z,t)[1+\alpha(y,z,t)\cdot y/z], \quad (1)$$

where $U(y,z,t)$ is the normal to the muscle surface component of the displacement vector field; $V(z,t)$ is the movement of a fiber point along the Oy axis, spaced from the edge at a distance z; H is the depth of the effective layer of the substrate; y is the coordinate directed from the free surface of the muscle; z is the fiber axis coordinate; α is the empirical parameter that takes into account possible deviations of the system under consideration from ideality. The equation of transverse oscillations of a thread on an elastic-viscous substrate has the following form [6]

$$m\frac{\partial^2 V(z,t)}{\partial t^2} = \frac{\partial}{\partial z}\left[T(y,z,t)\frac{\partial V(z,t)}{\partial z}\right] - q(y,z,t), \quad (2)$$

where m is the mass of a unite of the thread; $T(y,z,t)$ it the thread tension force; $q(y,z,t)$ is the distributed shear force from the side of the substrate, directed against the axis y. Force $q(y,z,t)$ is determined through the tension in the muscle - the substrate σ, multiplied by the effective width b: $q = \sigma b$. As boundary conditions, equation (2) is supplemented by the conditions for fastening the thread

$$V(0,t) = 0,\ V(L,t) = 0, \quad (3a)$$

where L is the effective thread length. Initial conditions for the function $V(z,t)$ could be written as

$$V(z,0) = V_0,\quad \left.\frac{\partial V(z,t)}{\partial t}\right|_{t=0} = 0. \quad (3b)$$

We solve the equation (2) with conditions (3) by recently introduced method of functional corrections [7,8]. In the

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framework of the approach we transform thread tension force $T(y,z,t)$ to the following form:

$$T(y,z,t) = T_0[1+\varepsilon \cdot g(y,z,t)], \qquad (4)$$

where T_0 is the average value of the considered force, $0 \le \varepsilon < 1$, $|g(y,z,t)| \le 1$. We determine solution of the equation (2) as the following power series

$$V(z,t) = \sum_{i=0}^{\infty} \varepsilon^i V_i(z,t). \qquad (5)$$

Substitution of the considered form of solution (5) and relation (4) into equation (2) and conditions (3) as well as grouping of terms at equal powers of the parameter ε gives a possibility to obtain equations for functions $V_i(z,t)$, boundary and initial conditions for them in the following form

$$m\frac{\partial^2 V_0(z,t)}{\partial t^2} = T_0 \frac{\partial^2 V_0(z,t)}{\partial^2 z} - q(y,z,t) \qquad (6a)$$

$$m\frac{\partial^2 V_i(z,t)}{\partial t^2} = T_0 \frac{\partial^2 V_i(z,t)}{\partial^2 z} + T_0 \frac{\partial}{\partial z}\left\{ g(y,z,t)\frac{\partial V_{i-1}(z,t)}{\partial z}\right\}, i \ge 1, \qquad (6b)$$

$V_i(0,t) = 0$, $V_i(L,t) = 0$, $\left.\frac{\partial V_i(z,t)}{\partial t}\right|_{t=0} = 0$, $i \ge 0$; $V_0(z,0) = V_0$, $V_i(z,0) = 0$, $i \ge 1$. (7)

Equations (6) with conditions (7) were solve by Fourier variable separation method [9]. The considered solutions could be presented in the following form

$$V_0(z,t) = \frac{V_0 L}{\pi n}\sum_{n=0}^{\infty}\left[(-1)^n - 1\right]\sin\left(\frac{\pi n z}{L}\right)\sin\left(\sqrt{\frac{T_0}{m}}\frac{t}{L}\right) -$$

$$-\frac{T_0}{m}\sum_{n=0}^{\infty}\sin\left(\frac{\pi n z}{L}\right)\sin\left(\sqrt{\frac{T_0}{m}}\frac{t}{L}\right)\int_0^L q(y,z,t)\sin\left(\frac{\pi n z}{L}\right)dz, \qquad (8)$$

$$V_i(z,t) = -\frac{\pi n}{L}\frac{T_0}{m}\sum_{n=0}^{\infty}\sin\left(\frac{\pi n z}{L}\right)\sin\left(\sqrt{\frac{T_0}{m}}\frac{t}{L}\right) \times$$

$$\times \int_0^L \left\{ g(y,z,t)\frac{\partial V_{i-1}(z,t)}{\partial z}\right\}\cos\left(\frac{\pi n z}{L}\right)dz, \; i \ge 1. \qquad (9)$$

Spatio-temporal distributions of the movement of a fiber point along the Oy axis was analyzed analytically by using the second-order approximation in the framework of the method of function corrections. The approximation is usually enough good approximation for to make qualitative analysis and to obtain some quantitative results. All obtained results have been checked by comparison with results of numerical simulations.

3 Discussion

In this section, we will analyze the spatio-temporal distribution of the fiber point displacement along the *Oy* axis. Figures 1, 2 and 3 show typical dependences of the considered distribution on the coordinate during fiber compression for various values of the external force q. Increasing of the number of figures corresponds to increasing of the considerate force (curves with larger maximum correspond to analytical results, curves with smaller maximum correspond to numerical results). An increase in this force corresponds to fiber elongation. A similar result was obtained when analyzing the change in fiber over time. However, excessive external force can cause fiber breakage. The maximum external force is determined individually.

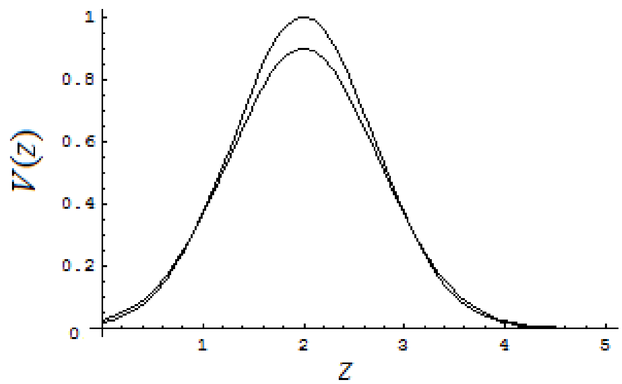

Fig. 1: Typical dependence of the distribution of the fiber point displacement along the *Oy* axis for fixed value of the external force q. Curve with larger maximum corresponds to analytical results. Curve with smaller maximum corresponds to numerical results.

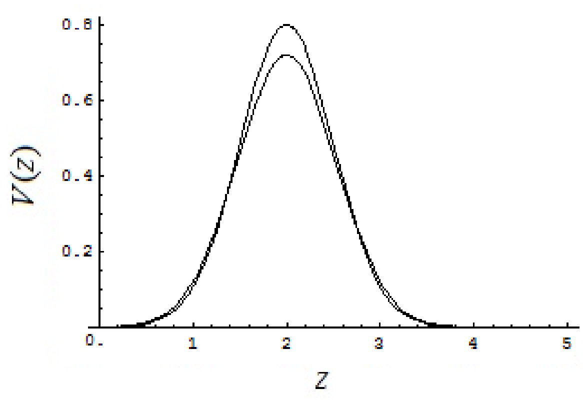

Fig. 2: Typical dependence of the distribution of the fiber point displacement along the *Oy* axis for fixed value of the external force q, which is smaller, than external force on the figure 1. Curve with larger maximum corresponds to analytical results. Curve with smaller maximum corresponds to numerical results.

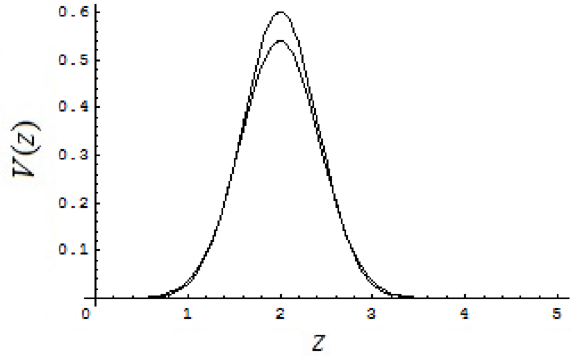

Fig. 3: Typical dependence of the distribution of the fiber point displacement along the *Oy* axis for fixed value of the external force *q*, which is smaller, than external force on the figure 2. Curve with larger maximum corresponds to analytical results. Curve with smaller maximum corresponds to numerical results

4 Conclusion

In this paper, we propose a model for the analysis of skeletal muscle contraction, which takes into account its deformation properties. We analyzed the considered model. We introduce an analytical approach for analysis of the considered muscle contraction.

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