Journal of Statistics Applications & Probability
An International Journal

http://dx.doi.org/10.18576/jsap/110111

Stress-Strength Reliability from Odd Generalized Exponential-Exponential Distribution with Censored Data

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Received: 18 Dec. 2020, Revised: 20 Feb. 2021, Accepted: 2 May 2021
Published online: 1 Jan. 2022

Abstract: Reliability and survival analysis is important in lifetimes of the units in an experiment which depend on modeling of events which depend on time, systems, components, or random variables. This paper proposes an inferences model for a simple stress-strength model with Type-II censored sample. This case is studied in our paper with Odd Generalize Exponential-Exponential distribution (OGEE), by using the point and interval estimation parameters of (OGEE) distribution for type-II censored . have been studied and compared with Mean square errors to decide which method is more suitable for studying the reliability with the model of stress-strength. The data were generated by using the Monte Carlo study of simulated samples.

Keywords: Odd generalize exponential-exponential distribution; type-II censored sample; stress- strength model, maximum likelihood function

1 Introduction

The problem of estimating and testing the reliability based on stress-strength modeling, $R = (X < Y)$ is a measure of component reliability when it is subjected to random stress X and has strength Y. In this context, R can be considered as a measure of system performance and naturally arise in electrical and electronic systems. Other interpretation can be that, the reliability,R, of the system is the probability that the system is strong enough to overcome the stress imposed on it. The component failing if and only if at any time the applied stress is greater than its strength. Other applications for the reliability parameter exists when X and Y have different interpretation, such as when Y is the response for a control group and X is the response for the treatment group.

Many authors have studied the stress-strength parameter [1] they consider the statistical inferences of the unknown parameters of a Weibull distribution when the data are Type-I censored. It is well known that the maximum likelihood estimators do not always exist, and even when they exist, they do not have explicit expressions. They propose a simple fixed point type algorithm to compute the maximum likelihood estimators, when they exist. They also propose approximate maximum likelihood estimators of the unknown parameters, which have explicit forms. Barbiero [2] studied statistical inference for the reliability of stress-strength models when stress and strength are independent Poisson random variables. Also, [3–11] discussed the different distributions for stress-strength model.

This paper tends to estimate stress- strength model $R = (Y < X)$ where strength and strength are two independent Type-II with OGEE distribution. Assume those scale parameters are known. The importance of OGEE distribution is the flexibility in modeling lifetime data for better representation of the phenomenon contained in the data set. For more information on OGEE distribution, see [12]. According to paper [12], the new distribution OGEE can be represented for its Pdf and Cdf.

$$f(x) = \lambda\theta e^{\theta x} e^{-\lambda(e^{\theta x}-1)}, \tag{1}$$

$$F(x) = 1 - e^{-\lambda(e^{\theta x}-1)}, \qquad x > 0, \quad \lambda > 0, \quad \theta < \infty. \tag{2}$$

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The maximum likelihood estimate and exact confidence interval of R is derived. Besides, Bayes estimator of R is derived, and all of these estimators are obtained based on mean square errors. The paper is organized as follows. In Section (2), system reliability. Maximum likelihood estimator is shown in Section (3), exact C.I is shown in Section (4), the Bayes estimator is shown in Section (5) and Bootstrap C.I are obtained in Section (6). For simulation, the studies' proposal is shown in Section (7). Tables are represented in Section (8). Finally, conclusions appear in Section (9).

2 System Reliability R

Let X and Y be two independent OGEE random variables with parameters (λ_1, θ) and (λ_2, θ), respectively. Thee reliability of system is defined as follows:

$$R = P(Y < X) = \int_0^\infty P[Y < X|Y = y]dy,$$

$$R = \int_0^\infty 1 - e^{-\lambda_1(e^{\theta x}-1)}\lambda_2\theta e^{x\theta}e^{-\lambda_2(e^{\theta x}-1)}dx = \frac{\lambda_2}{\lambda_1+\lambda_2}. \tag{3}$$

3 Maximum Likelihood Estimator

Assume that two independent random samples $X_1, X_2, \cdots, X_n$ and $Y_1, Y_2, \cdots, Y_n$ are observed from OGEE $\sim (\lambda_1, \theta)$ and OGEE $\sim (\lambda_2, \theta)$. respectively. The likelihood function of λ_1 and λ_2 for the observed samples is:

$$L(X_1,X_2,\cdots,X_n,Y_1,Y_2,\cdots,Y_m;\lambda_1,\lambda_2\theta) = \frac{n!}{(n-r_1)!}\prod_{i=0}^{r_1}\lambda_1\theta e^{\theta x_i}e^{-\lambda_1(e^{\theta x_i}-1)}\left[e^{-\lambda_1(e^{\theta x_{r_1}}-1)}\right]^{n-r_1}$$
$$\frac{m!}{m-r_2}\prod_{j=0}^{r_2}\lambda_2\theta e^{\theta y_j}e^{-\lambda_2(e^{\theta y_j}-1)}\left[e^{-\lambda_2(e^{\theta y_{r_2}}-1)}\right]^{m-r_2}.$$

The estimations $\hat{\lambda}_1$ and $\hat{\lambda}_2$ of the parameters λ_1 and λ_2, respectively, can be obtained as the solution the the likelihood equations

$$\frac{\partial l}{\partial \lambda_1} = \frac{(r_1+1)}{\lambda_1} - e^{\theta\sum_{i=0}^{r_1}x_i} + (r_1+1) - (n-r_1)(r_1+1)\left(e^{\theta x_{r_1}}+1\right) = 0. \tag{4}$$

And

$$\frac{\partial l}{\partial \lambda_2} = \frac{(r_2+1)}{\lambda_2} - e^{\theta\sum_{j=0}^{r_2}x_i} + (r_2+1) - (m-r_2)(r_2+1)\left(e^{\theta y_{r_2}}+1\right) = 0. \tag{5}$$

From Equations (4) and (5), the estimators $\hat{\lambda}_{1\,\mathrm{MLE}}$ and $\hat{\lambda}_{2\,\mathrm{MLE}}$ are given by

$$\hat{\lambda}_{1\,\mathrm{MLE}} = \frac{(r_1+1)}{e^{\theta\sum_{i=0}^{r_1}x_i} - (r_1+1) + (n-r_1)(r_1+1)(e^{\theta x_{r_1}}+1)},$$

$$\hat{\lambda}_{2\,\mathrm{MLE}} = \frac{(r_2+1)}{e^{\theta\sum_{j=0}^{r_2}y_j} - (r_2+1) + (m-r_2)(r_2+1)(e^{\theta y_{r_2}}+1)},$$

Once the estimators $\hat{\lambda}_{1\mathrm{MLE}}$ and $\hat{\lambda}_{2\mathrm{MLE}}$, are derived and using the invariance property of the MLEs in (3), the MLE of R denoted as $\hat{R}_{\mathrm{MLE}}$ becomes

$$\hat{R}_{\mathrm{MLE}} = \frac{\frac{(r_2+1)}{e^{\theta\sum_{j=0}^{r_2}y_j}-(r_2+1)+(m-r_2)(r_2+1)(e^{\theta y_{r_2}}+1)}}{\frac{(r_1+1)}{e^{\theta\sum_{i=0}^{r_1}x_i}-(r_1+1)+(n-r_1)(r_1+1)(e^{\theta x_{r_1}}+1)}+\frac{(r_2+1)}{e^{\theta\sum_{j=0}^{r_2}y_j}-(r_2+1)+(m-r_2)(r_2+1)(e^{\theta y_{r_2}}+1)}} \tag{6}$$

4 Asymptotic distribution and confidence interval of R

based on the asymptotic, see [13, 14]. The general conditions of the MLEs of $\hat{\lambda}_1$ and $\hat{\lambda}_2$ distribution of the MLEs immediately follows from the Fisher information matrix of λ_1 and λ_2. That is, as $n,m \to \infty$ and $\frac{n}{m} \to k$, where $0 < k < 1$, it follows that:

$$\left[\sqrt{n}(\hat{\lambda}_1 - \lambda_1), \sqrt{m}(\hat{\lambda}_2 - \lambda_2)\right] \xrightarrow{D} N_2(0, \delta(\lambda)),$$

where

$$\delta(\lambda) = I^{-1}(\lambda) = \begin{pmatrix} I_{11} & I_{12} \\ I_{21} & I_{22} \end{pmatrix}^{-1}, \tag{7}$$

and the matrix $I(\lambda)$ is the Fisher information matrix of the parameter vector $\lambda = (\lambda_1, \lambda_2)$, and the ij^{th} element is given by the second partial derivatives $I_{ij} = \frac{\partial^2 \ln L(\lambda)}{\partial \lambda_1 \partial \lambda_2}$, $i, j = 1, 2$. From the asymptotic properties of the MLEs of λ_1 and λ_2, one can easily get,

$$\sqrt{n}(\hat{R} - R) = \sqrt{n}\left(\frac{\hat{\lambda}_2}{\hat{\lambda}_1 + \hat{\lambda}_2} - \frac{\lambda_2}{\lambda_1 + \lambda_2}\right) \xrightarrow{D} N_2(0, \sigma^2),$$

where

$$\sigma^2_{\hat{R}_1} = \frac{(r_1+1)(r_2+1)(\lambda_1+\lambda_2)^2}{\lambda_1^2 \lambda_2^2}. \tag{8}$$

A $(1-\alpha)100\%$ approximate confidence interval of R can be constructed based on the asymptotic results obtained. This asymptotic confidence interval is given by

$$\hat{R} \pm Z_{1-\frac{\alpha}{2}} \hat{\sigma}, \tag{9}$$

where $\hat{\sigma}$ is the asymptotic standard deviation of $\hat{R}$.

5 Bayesian estimation of R

In this section, the Bayes estimator of R denoted as R_B is obtained with non-informative prior, where the equation to find fisher information as follow:

$$I(\lambda_1) = -E\left(\frac{\partial^2 \log(\lambda_1)}{\partial \lambda_1^2}\right). \tag{10}$$

The Fisher information measures the sensitivity of an estimator. Jeffrey's [9] be considered as a prior for the likelihood function $L(\theta)$. The Jeffrey's prior is justified on the grounds if its invariance under parameterization according to [15]. Then, the prior distribution for λ_{1B} and λ_{2B} respectively

$$\pi(\lambda_{1B}) \propto \frac{1}{\lambda_{1B}} \qquad \text{and} \qquad \pi(\lambda_{2B}) \propto \frac{1}{\lambda_{2B}}. \tag{11}$$

Based on the above assumptions and from Equation (11), the joint density of the data λ_{1B} and λ_{2B} cab be obtained as

$$L(X_1, X_2, \cdots, X_n, Y_1, Y_2, \cdots, Y_m, \lambda_{1B}) = L(X_1, X_2, \cdots, X_n, Y_1, Y_2, \cdots, Y_m, \lambda_{1B})\pi(\lambda_{1B})$$

and

$$L(X_1, X_2, \cdots, X_n, Y_1, Y_2, \cdots, Y_m, \lambda_{2B}) = L(X_1, X_2, \cdots, X_n, Y_1, Y_2, \cdots, Y_m, \lambda_{2B})\pi(\lambda_{2B}).$$

Therefore, the posterior density of these data, λ_{1B} and V_{2B} given the data can be obtained as follows:

$$\pi^*(\lambda_{1B}) = L\left(\frac{\lambda_{1B}}{X_1, X_2, \cdots, X_n, Y_1, Y_2, \cdots, Y_m}\right) = \frac{L(X_1, X_2, \cdots, X_n, Y_1, Y_2, \cdots, Y_m, \lambda_{1B})\pi(\lambda_{1B})}{\int_0^1 L(X_1, X_2, \cdots, X_n, Y_1, Y_2, \cdots, Y_m, \lambda_{1B})\pi(\lambda_{1B}) d\lambda_{1B}}$$

and

$$\pi^*(\lambda_{2B}) = L\left(\frac{\lambda_{2B}}{X_1, X_2, \cdots, X_n, Y_1, Y_2, \cdots, Y_m}\right) = \frac{L(X_1, X_2, \cdots, X_n, Y_1, Y_2, \cdots, Y_m, \lambda_{2B})\pi(\lambda_{2B})}{\int_0^1 L(X_1, X_2, \cdots, X_n, Y_1, Y_2, \cdots, Y_m, \lambda_{2B})\pi(\lambda_{2B}) d\lambda_{2B}}. \tag{12}$$

From Equation (12), the posterior density of these data λ_{1B} and λ_{2B} are given by:

$$\pi^*(\lambda_{1B}) = \frac{\lambda_1^{r_1} e^{-\lambda_1\left\{e^{\theta\sum_{i=0}^{r_1} x_i}+(r_1+1)-(r_1+1)(n-r_1)\left[e^{\theta x_{r_1}}+1\right]\right\}} \times \left\{e^{\theta\sum_{i=0}^{r_1} x_i}+(r_1+1)-(r_1+1)(n-r_1)\left[e^{\theta x_{r_1}}+1\right]\right\}^{r_1}}{\Gamma(r_1+1)} \tag{13}$$

$$\pi^*(\lambda_{2B}) = \frac{\lambda_2^{r_2} e^{-\lambda_2\left\{e^{\theta\sum_{j=0}^{r_2} y_j}+(r_2+1)-(r_2+1)(m-r_2)\left[e^{\theta y_{r_2}}+1\right]\right\}} \times \left\{e^{\theta\sum_{j=0}^{r_2} y_j}+(r_2+1)-(r_2+1)(m-r_2)\left[e^{\theta y_{r_2}}+1\right]\right\}^{r_2}}{\Gamma(r_2+1)} \tag{14}$$

From Equations (13) and (**??**), the estimators $\hat{\lambda}_{1B}$ and $\hat{\lambda}_{2B}$ are given by

$$\begin{aligned}\hat{\lambda}_{1B} &= E\left(\frac{\lambda_1}{X_1,X_2,\cdots,X_n,Y_1,Y_2,\cdots,,Y_m}\right) = \int_0^\infty \lambda_1\pi^*(\lambda_{1B})d\lambda_1,\\ \hat{\lambda}_{2B} &= E\left(\frac{\lambda_2}{X_1,X_2,\cdots,X_n,Y_1,Y_2,\cdots,,Y_m}\right) = \int_0^\infty \lambda_2\pi^*(\lambda_{2B})d\lambda_2.\end{aligned} \tag{15}$$

Once the estimators $\hat{\lambda}_{1B}$ and $\hat{\lambda}_{2B}$, are derived and using the invariance property of the MLEs in (3), the MLE of R denoted as $\hat{R}_B$ becomes

$$\hat{R}_B = frac\hat{\lambda}_{2B}\hat{\lambda}_{1B}+\hat{\lambda}_{2B}. \tag{16}$$

Use the estimators $\hat{\lambda}_{1B}$ and $\hat{\lambda}_{2B}$ to estimate the bootstrap sample $X_1^*,X_2^*,\cdots,X_n^*$ and $Y_1^*,Y_2^*,\cdots,Y_m^*$ the compute the estimated value of $\hat{R}_B$ by Bayes which shown in (16).
Calculate the bootstrap MSE by

$$\widehat{\text{MSE}}_{BB} = \frac{1}{N}\sum_{i=1}^{N}(\bar{R}^(j)-\bar{R}_B).$$

The asymptotic $(1-\alpha)100\%$ confidence interval is given by

$$\left(\bar{R}-Z_{\frac{\alpha}{2}}\sqrt{\widehat{\text{MSE}}_{BB}},\bar{R}+Z_{\frac{\alpha}{2}}\sqrt{\widehat{\text{MSE}}_{BB}}\right). \tag{17}$$

6 Simulation study

In this section some numerical experiments reported and performed by using MATHCAD PROGRAM 2001, to evaluate the behavior of the proposed methods for different effective sample sizes, different sampling schemes and different parameter values . The performances of the MLEs and the Bayes estimates are compared in terms of biases , mean squares errors (MSEs) loss function . Bayes estimates, are computed based on non-informative . Monte Carlo simulation is performed to test the behavior of the proposed estimators.

The simulations are based on 10000 replications and the results are presented in Tables (1) for MLE. In Table (2) for Bayes estimators and in Table (3), both CI and B-CI using Equations (9) and (17), respectively.

The results are shown in Table (1). All simulations are based on the following sample sizes; n and $m = 5,10,15,25$ and 50 where λ_1 and $\lambda_2 = 0.1,0.2,0.5,0.001,0.06$, respectively. Table (2) shows the results of Bayes estimation of R.

7 Tables

Table 1: MI method estimation of $\hat{R}_{\text{MLE}}$

(n,m)	λ_1	λ_2	$\hat{\lambda}_{1\text{MLE}}$	$\hat{\lambda}_{2\text{MLE}}$	R	$\hat{R}_{\text{MLE}}$	Bias	MSE
(5,5)	0.1	0.2	0.092	0.205	0.666	0.619	0.0046462	0.022
	0.5	0.5	0.545	0.555	0.500	0.524	0.1550000	0.569
	0.001	0.06	0.001202	0.66	0.983	0.950	−0.004642	0.001262
(5,10)	0.1	0.2	0.11	0.192	0.666	0.616	0.002212	0.015
	0.5	0.5	0.495	0.495	0.5	0.655	0.121	0.062
	0.001	0.06	0.001226	0.062	0.983	0.951	−0.002962	0.001144
(10,10)	0.1	0.2	0.105	0.211	0.666	0.626	0.012	0.012
	0.5	0.5	0.542	0.542	0.5	0.562	0.125	0.062
	0.001	0.06	0.00102	0.062	0.983	0.952	−0.001512	0.00102
(15,15)	0.1	0.2	0.099	0.210	0.666	0.624	0.019	0.006251
	0.5	0.5	0.529	0.502	0.500	0.559	0.126	0.049
	0.001	0.06	0.0011	0.062	0.983	0.952	−0.001215	0.001006
(15,25)	0.1	0.2	0.101	0.206	0.666	0.622	0.009021	0.0056092
	0.5	0.5	0.511	0.522	0.5	0.552	0.106	0.02
	0.001	0.06	$1.026e^{-2}$	0.062	0.983	0.952	−0.001456	0.001002
(25,25)	0.1	0.2	0.102	0.219	0.666	0.625	0.012	0.00511
	0.5	0.5	0.454	0.529	0.5	0.552	0.115	0.021
	0.001	0.06	0.00102	0.062	0.983	0.954	−0.000926	0.0009696
(50,50)	0.1	0.2	0.102	0.202	0.666	0.621	0.016	0.00252
	0.5	0.5	0.504	0.505	0.5	0.566	0.104	0.019
	0.001	0.06	0.00101	0.061	0.983	0.954	−0.00106	0.000991

Table 2: Bayes estimation of R

(n,m)	λ_1	λ_2	R	$\hat{R}_B$	Bayes with MSE	
					Bias	MSE
(5,5)	0.1	0.2	0.666	0.61	−0.059	0.045
	0.001	0.06	0.983	0.966	−0.00522	0.00129
(5,10)	0.1	0.2	0.666	0.615	−0.051	0.026
	0.001	0.06	0.983	0.965	−0.00629	0.00125
(10,10)	0.1	0.2	0.666	0.611	−0.025	0.016
	0.2	0.1	0.333	0.225	−0.125	0.021
	0.001	0.06	0.983	0.95	−0.004254	0.001064
(15,15)	0.1	0.2	0.666	0.611	−0.025	0.016
	0.2	0.1	0.333	0.266	0.102	0.016
	0.001	0.06	0.983	0.95	−0.004254	0.001064
(15,25)	0.1	0.2	0.666	0.615	−0.009621	0.006696
	0.2	0.1	0.333	0.266	−0.102	0.015
	0.001	0.06	0.983	0.952	−0.002466	0.001005
(15,25)	0.1	0.2	0.666	0.61	−0.0002666	0.005291
	0.2	0.1	0.333	0.251	−0.069	0.005621
	0.001	0.06	0.983	0.952	−0.002055	0.0009949
(15,25)	0.1	0.2	0.666	0.614	0.005201	0.00292
	0.2	0.1	0.333	0.224	−0.046	0.004
	0.001	0.06	0.983	0.952	−0.001292	0.0009504

Table 3: MI method estimation of $\hat{R}_{\text{MLE}}$

(n,m)	R	CI	$B-\text{CI}$	(n,m)	R	CI	$B-\text{CI}$
$(5,10)$	0.666	(0.612, 0.676)	(0.613, 0.699)	$(15,25)$	0.666	(0.603, 0.669)	(0.612, 0.679)
	0.333	(0.255, 0.355)	(0.252, 0.393)		0.333	(0.245, 0.351)	(0.255, 0.399)
	0.983	(0.950, 0.999)	(0.950, 0.991)		0.983	(0.954, 0.994)	(0.941, 0.996)
$(10,10)$	0.666	(0.612, 0.683)	(0.609, 0.676)	$(25,25)$	0.666	(0.610, 0.676)	(0.612, 0.689)
	0.333	(0.264, 0.395)	(0.266, 0.391)		0.333	(0.265, 0.369)	(0.246, 0.366)
	0.983	(0.953, 0.996)	(0.945, 0.999)		0.983	(0.954, 0.993)	(0.940, 0.996)
$(15,15)$	0.666	(0.606, 0.686)	(0.604, 0.685)	$(50,50)$	0.666	(0.613, 0.690)	(0.632, 0.694)
	0.333	(0.265, 0.369)	(0.264, 0.369)		0.333	(0.269, 0.363)	(0.250, 0.361)
	0.983	(0.953, 0.995)	(0.940, 0.995)		0.983	(0.955, 0.990)	(0.939, 0.996)

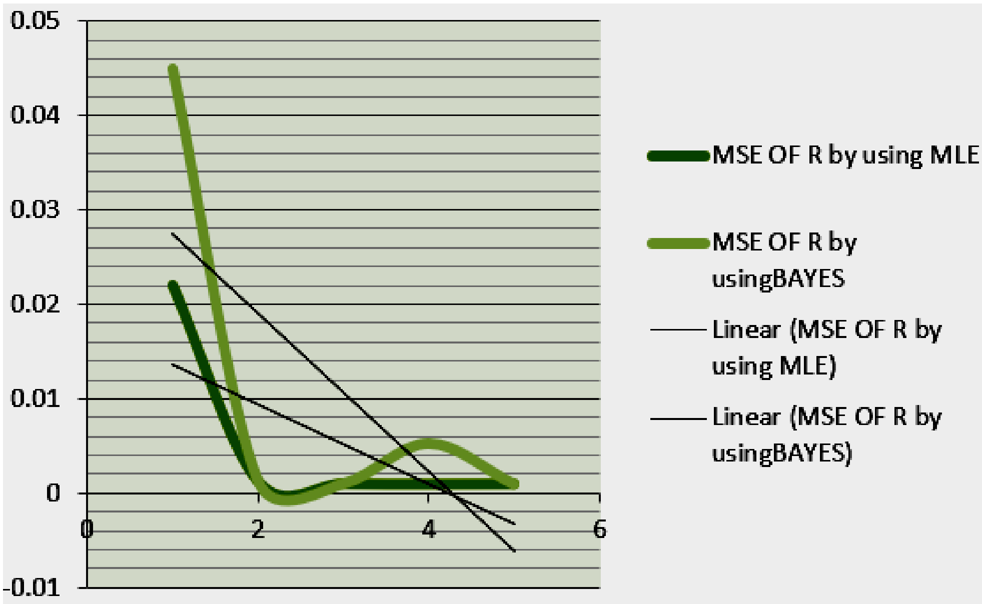

Fig. 1: Comparing between MSEs of estimated values of R by using MLE method and Bayes method.

8 Conclusion

In this paper, the problem of estimating $R = P(X < Y)$ for the OGEE distribution was studied, the results were tabulated in Tables 1, 2 and 3, we observed that :

1. The MSE of the estimates of R decreases as the increases for (n,m).
2. The performance of the Bayes estimators based on MSEs are better than MSEs of MLE.
3. The average lengths of all intervals decrease as (n,m) increases.
4. The length of asymptotic confidence interval of MLE is smaller than the Boot-strap Bayes confidence intervals.

Acknowledgement

The authors would like to thank Professor Haroun Barakat and Professor Abdullah Abdel Fattah for their contributions to the research. They also want to thank both editors and reviewers.

Conflict of interest

The authors declare that they have no conflict of interest.

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