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Quantum-Enhanced Fractional Modeling of Zombie Infection Dynamics Using Hybrid Quantum Algorithms

Feras Fares AL-Mashakbah[1], *Srinivasarao Thota*[2,*], *Ala'a Saeb Al-Sherideh*[3], *and Mohammad Rasmi Al-Mousa*[3]

[1] Department of Computer Science, Faculty of Computer Science and Information Technology, Jerash University, Jordan
[2] Department of Mathematics, Amrita School of Engineering, Amrita Vishwa Vidyapeetham, Amaravati, Andhra Pradesh–522503, India
[3] Department of Cyber Security, Faculty of Information Technology, Zarqa University, Zarqa, Jordan

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Abstract: This article proposes a hybrid quantum-classical approach to model the dynamic evolution of infectious diseases via a fractional-order SIZR (Susceptible-Infected-Zombie-Removed) model. By leveraging a classical model improved by an Atangana-Baleanu fractional derivative approach, memory terms are introduced, which play an essential role in achieving realistic epidemic evolution results. The proposed methodology involves reformulating the epidemic evolution model via a discretized approach to fractional derivatives and applying it to a quantum circuit via amplitude encoding. Additionally, via quantum-classical optimization, a parameterized quantum ansatz is optimized in an attempt to minimize a cost function that considers the residuals associated with the fractional model. Numerical experiments carried out via IBM's Qiskit Aer simulator have enabled us to detect complex, high memory terms associated with all four compartments in a zombie infection model in an epidemic evolution system. The proposed approach confirms efficacy based on comparative results for varying fractional parameters α, denoting improved performance in computational efficiency and scalability when contrasted with classical approaches such as HATM. This article provides groundwork for forthcoming uses related to quantum computing applications in epidemiology, specifically in fractional dynamic systems.

Keywords: Fractional-order differential equations; Zombie infection dynamics; Variational Quantum Algorithms; SIZR (Susceptible-Infected-Zombie-Removed) model.

1 Introduction

Modeling the spread of communicable diseases has been an area of prime importance in the field of epidemiology. However, in the variety of models that have been proposed, the concept of zombie outbreaks has also gained considerable attention, primarily owing to its engaging narrative. These are models that are hypothetical in nature but provide the benefit of assuming the worst possible epidemic states in order to evaluate epidemic control strategies [1, 15]. For such purposes, the SIZR model, also known as the Susceptible-Infected-Zombie-Removed model, is of significance.

The classical model is based on integer-order differential equations that consider instantaneous interactions. In human biology, there is no instantaneous interaction and any biological system has certain "memory" features. To model these features accurately, fractional differential equations with Atangana-Baleanu derivatives provide an effective tool [8, 17]. Although fractional models have their strengths, they are computationally expensive. The problem of solving the non-linear time fractional differential equation can be computationally challenging, especially when the system becomes multidimensional. This problem justifies the search for a paradigm that can efficiently deal with the complexities. However, the use of such models in the area of high-dimensional or nonlinear differential equations is still an open issue. Some promising algorithms, such as the Variational Quantum Algorithms (VQAs) [16] or the Quantum Differential Equation Solvers (QDESs) [10, 11], are being developed in the field of quantum computing. These algorithms are especially suitable in the context of the noisy intermediate-scale quantum (NISQ) era. Consequently, the solution to this problem is only expected to be implemented in the future. The combination of quantum computing and

* Corresponding author e-mail: t_srinivasarao@av.amrita.edu

fractional epidemiology models is an attractive area that enables the merging of two highly emergent fields. What we are interested in is enhancing the efficiency and scalability of solving the long memory dynamics of spreading infectious diseases using simulation. The goal here is to eliminate the high computational complexity involved by leveraging quantum computing within the framework of the fractional SIZR model. The proposed framework finds broad applicability beyond the example of zombie infections. Extensions to model real-world epidemics like COVID-19, Ebola, and monkeypox, among others, are also valid, where latency and memory effects play a critical role [18, 21]. The approach is also of value in predictive modeling, outbreak forecasting, and designs for optimal control strategy in public health.

This work, for the first time, extends Atangana-Baleanu-based fractional epidemic models with hybrid quantum computing techniques. Unlike previous efforts, such as [14], which primarily utilized classical numerical methods, namely the HATM technique, a qubit-based quantum-enhanced algorithm with state encoding and variational optimization is introduced to simulate the system. This creates a new and scaleable technique in this emerging area of quantum epidemiology. All the authors of this paper have also contributed to the development of various models in various domains, see for example [2–7, 12, 13, 19, 20, 22, 23].

The rest of the paper is organized as follows. Section 2 revisits the mathematical preliminaries of fractional calculus. Section 3 formulates the quantum-enhanced hybrid model. Section 4 presents the simulations and results, while Section 5 concludes the study with insights and future directions.

2 Mathematical Preliminaries

Fractional calculus is an extension of the concept of differentiation and integration of a function to a fractional order. It is a very useful tool in the description of systems with memory. For this particular study, we employ the use of the Atangana-Baleanu derivative in a Caputo sense (ABC) due to its non-local, non-singular kernel, having a smooth representation of memory effects [8].

2.1 Atangana-Baleanu Fractional Derivative (ABC)

The ABC fractional derivative in Caputo sense is defined as in equation (1)

$$ {}_{0}^{ABC}D_t^{\alpha}X(t) = \frac{M(\alpha)}{1-\alpha}\int_0^t X'(s)E_{\alpha}\left(-\frac{(t-s)^{\alpha}}{1-\alpha}\right)ds, \quad 0<\alpha<1, \tag{1} $$

where $M(\alpha)$ is a normalization function with $M(0) = M(1) = 1$, and $E_{\alpha}(z)$ is the Mittag-Leffler function given in (2)

$$ E_{\alpha}(z) = \sum_{k=0}^{\infty}\frac{z^k}{\Gamma(k\alpha+1)}. \tag{2} $$

The given operator is distinguished by a non-singular Mittag-Leffler kernel and holds several key properties, which include being linear and capable of describing non-local behavior [9].

The associated fractional integral of (1) is given by

$$ {}_{0}^{ABC}I_t^{\alpha}X(t) = \frac{1}{M(\alpha)}\left[(1-\alpha)X(t) + \frac{\alpha}{\Gamma(\alpha)}\int_0^t (t-s)^{\alpha-1}X(s)ds\right]. $$

This formula relates the ABC derivative to integral expressions and allows us to reverse the process of fractional differentiation in solving initial value problems.

The Laplace transform is a useful technique for solving the fractional differential equation. The Laplace transform for the ABC derivative is expressed as in (3)

$$ \mathscr{L}\left[{}_{0}^{ABC}D_t^{\alpha}X(t)\right] = \frac{M(\alpha)}{1-\alpha}\cdot\frac{k^{\alpha}\mathscr{L}[X(t)] - k^{\alpha-1}X(0)}{k^{\alpha}+\left(\frac{\alpha}{1-\alpha}\right)}. \tag{3} $$

This outcome is very important for the conversion of the fractional differential equation into algebraic equations. Algebraic equations can then be resolved by classical or quantum theories [24].

2.2 SIZR Model in Fractional Form

The traditional SIZR model divides the susceptible population into four groups based on their epidemiological characteristics. The susceptible population $S(t)$ comprises the healthy population that is susceptible to the disease from coming into contact with the zombies. The infected group $I(t)$ comprises those individuals who have contracted the disease as a result of being bitten and are currently undergoing the latent stage of dying and transforming into zombies. The zombie group $Z(t)$ comprises those people who have completely transformed and have the ability to propagate the disease among the susceptible group. The final group, the removed class, is given by the equation $R(t)$.

In the model, it is assumed that zombies can be generated through the interaction of susceptible individuals with the existing zombies, while they can also be eliminated from the model. The dynamics of this process can be described by the following system (4)-(7) of fractional differential equations with Atangana-Baleanu derivatives:

$$^{ABC}D_t^{\alpha}S(t) = -\gamma\beta_0 e^{-\xi t}S(t)Z(t) - \omega S(t), \tag{4}$$
$$^{ABC}D_t^{\alpha}I(t) = \beta_0 e^{-\xi t}S(t)Z(t) - \chi I(t) - \omega I(t), \tag{5}$$
$$^{ABC}D_t^{\alpha}Z(t) = \chi I(t) + \delta R(t) - \rho S(t)Z(t), \tag{6}$$
$$^{ABC}D_t^{\alpha}R(t) = \omega S(t) + \omega I(t) + \rho S(t)Z(t) - \delta R(t), \tag{7}$$

where β_0 is the effective rate of contact between the susceptible and zombie populations, influencing the rate of disease transmission. The fear or awareness rate is represented by the parameter ξ and slows down the rate of contact over time, as a result of behavior modifications among the population. The scaling factor or modification coefficient for the contact rate of the susceptible and zombie population is represented by the parameter γ. The natural death rate or removal rate of the susceptible and infected populations is represented by the parameter ω. The rate of transition of the infected population to the zombie population is represented by the parameter χ. The rate of effective reduction of the susceptible population against the zombie population is represented by the parameter ρ. The rate of resurrection of the removed population, entering the system as zombies, is represented by the parameter δ.

This particular extension of the model incorporates memory via the use of the Atangana-Baleanu derivative to be able to account for the delayed transitions because of latency, immunity, and feedback behavior [8,9]. Note that the formulation of the Atangana-Baleanu derivative takes into account the long-term memory of the transitions of the epidemic in question because of its ability to account for the long memory of the transitions of the epidemic [14].

3 Quantum-Enhanced Model Formulation

In this section, we outline a quantum-enhanced approach to solving the fractional SIZR model. The method integrates the classical Atangana-Baleanu fractional framework with quantum variational algorithms. The goal is to exploit quantum parallelism and optimization to overcome classical computational challenges such as memory overhead and convergence speed. In applying the quantum computing technique on the SIZR fractional-order system, a quantum-classical method is followed. This process entailed the use of a quantum circuit to represent the discretized equations, designing an ansatz, and then optimizing the cost function based on the residuals of the SIZR equations.

The AB derivative is non-local, with a memory kernel that makes numerical solutions for it relatively cumbersome. To tackle this challenge when using a quantum computer, we use a modified L1 method for approximating the operator. For a function $X(t)$ defined on the discrete mesh $t_n = n\Delta t$, we approximate the ABC derivative as in (8)

$$^{ABC}_{0}D_t^{\alpha}X(t_n) \approx \frac{1}{\Delta t^{\alpha}}\sum_{k=0}^{n} w_k X(t_{n-k}), \tag{8}$$

where w_k are weights derived from the Mittag-Leffler kernel given by

$$w_k = \frac{M(\alpha)}{1-\alpha}E_{\alpha}\left(-\frac{(k\Delta t)^{\alpha}}{1-\alpha}\right),$$

and E_{α} is the one-parameter Mittag-Leffler function. This transformation reduces the problem to a summation of weighted past states, which are easier to encode in a quantum algorithm.

Each time step's system state vector $X(t) = [S(t), I(t), Z(t), R(t)]$ is normalized and encoded into a quantum register using amplitude encoding:

$$|\psi(t)\rangle = \frac{1}{\|X(t)\|}\sum_{i=0}^{3} X_i(t)|i\rangle. \tag{9}$$

This requires $\log_2 4 = 2$ qubits and enables efficient representation of all compartments simultaneously. Amplitude encoding is selected for its compactness, though it requires elaborate state preparation techniques which are implemented through rotation and entanglement gates.

3.1 Variational Quantum Ansatz (VQA)

A variational ansatz $U(\theta)$ is a quantum circuit with tunable parameters θ designed to approximate the desired quantum state $|\psi(\theta)\rangle$, where

$$|\psi(\theta)\rangle = U(\theta)|0\rangle.$$

The ansatz may consist of repeated layers of single-qubit gates $R_y(\theta_i)$ and $R_z(\theta_j)$ followed by entangling gates like CNOTs. The depth of the circuit controls the expressiveness and can be adapted to the complexity of the model. Each parameter θ_i is optimized to minimize a model-specific loss function.

3.2 Cost Function Formulation

The cost function evaluates the residual of the discretized fractional differential equation for each variable. To solve the system, we construct quantum circuits that evaluate residuals of the discretized fractional equations. For instance, the cost function for the susceptible population $S(t)$ is given by

$$C_S(\theta) = \left\| {}^{ABC}D_t^{\alpha} S(t) + \gamma\beta_0 e^{-\xi t} S(t)Z(t) + \omega S(t) \right\|^2 \tag{10}$$

Analogous expressions of (10) are defined for $C_I(\theta)$, $C_Z(\theta)$, and $C_R(\theta)$.

The total cost function to be minimized is

$$C_{\text{total}}(\theta) = C_S(\theta) + C_I(\theta) + C_Z(\theta) + C_R(\theta) \tag{11}$$

This total cost function is implemented via quantum circuits, where expectation values of observables are used to estimate the residuals at each iteration.

3.3 Hybrid Quantum-Classical Optimization

This algorithm maintains an iterative method:

1. Set θ randomly.
2. Prepare the ansatz $U(\theta)$ and construct $|\psi(\theta)\rangle$.
3. Compute the cost function $C_{\text{total}}(\theta)$ as given in (11) using quantum circuits and classical post-processing.
4. Update θ using a classical optimization algorithm (e.g., COBYLA, SPSA, Adam).
5. Repeat until $C_{\text{total}}(\theta)$ converges below a threshold.

This quantum-classical feedback circuit takes advantage of the best-of-both-worlds approach, in which quantum processing performs superposition calculations efficiently, while classical procedures handle parameter optimization.

The key benefit of the quantum mechanism is its efficient scaling with respect to the variables of the system. In a two-qubit case describing the system with only four possible states due to amplitude encoding, the problem leads to exponential compression. In contrast to classical solver algorithms with potential $O(n^2)$ or even worse scaling due to memory kernels, VQA scaling is preferable.

Simulations were carried out on the Qiskit Aer simulator with a 2-qubit architecture. The ansatz in the Variational Quantum Algorithm was three layers of R_y, R_z, and CNOT operations. The optimizer was the COBYLA optimizer with a learning rate adjusted for faster convergence. Noise simulations facilitate simulation without decoherence, although subsequent simulations are planned for actual hardware.

3.4 Comparison to Classical HATM Method

The Homotopy Analysis Transform Method (HATM) employed in [14] employs a recursive approximation approach in treating the nonlinear equations, but could potentially be unstable or inefficient in the case of fully coupled systems when the total number of fractions becomes large. The quantum-boosted VQA algorithm proposed does not rely on symbolic manipulation but operates on the numeric characteristics of the equations directly. Additionally, the algorithm allows for real-time processing once implemented on a quantum computer. Such an overall quantum framework lays the groundwork for an extension perspective that would cover parameter estimation and adaptive interventions within the realms of stochastics.

4 Simulation and Results

This chapter involves simulation of the dynamic behavior of a quantum enhanced SIZR model based on varying levels of a fractional order parameter α. This simulation involves a simulated quantum back-end approach based on a quantum-classical hybrid methodology. The sub-populations namely Susceptible (S), Infected (I), Zombie (Z), and Removed (R) are evaluated in terms of memory or memory effect considerations arising from fractional dynamics.

We discretize the time domain into $N = 500$ points with time step $\Delta t = 0.2$ over the interval $[0, 100]$. The parameters used are $\beta_0 = 0.02$, $\gamma = 0.9$, $\xi = 0.02$, $\omega = 0.005$, $\chi = 0.015$, $\rho = 0.01$, $\delta = 0.002$. Initial conditions: $S(0) = 3500$, $I(0) = 5$, $Z(0) = 1$, $R(0) = 0$.

At each time t_n, we compute the ABC derivative of $S(t)$ using

$$ {}_0^{ABC}D_t^{\alpha} S(t_n) = \frac{1}{\Delta t^{\alpha}} \sum_{k=0}^{n} w_k S(t_{n-k}), \quad w_k = \frac{M(\alpha)}{1-\alpha} E_{\alpha}\left(-\frac{(k\Delta t)^{\alpha}}{1-\alpha}\right) $$

Then we compute the numerical residual

$$ \mathrm{Res}_S(t_n) = \left| {}^{ABC}D_t^{\alpha} S(t_n) + \gamma\beta_0 e^{-\xi t_n} S(t_n) Z(t_n) + \omega S(t_n) \right| $$

Similar residuals are calculated for $I(t)$, $Z(t)$, and $R(t)$, and aggregated into the total loss

$$ \mathrm{Loss}_{\mathrm{total}} = \sum_{n=0}^{N} \left(\mathrm{Res}_S(t_n)^2 + \mathrm{Res}_I(t_n)^2 + \mathrm{Res}_Z(t_n)^2 + \mathrm{Res}_R(t_n)^2 \right) $$

This loss is minimized by tuning θ in the quantum variational circuit.

Using the optimized state vector $|\psi(\theta^*)\rangle$, we reconstruct each compartment:

$$ S(t_n) = \|X(t_n)\| \cdot \langle 0|\psi(t_n)\rangle, \quad I(t_n) = \|X(t_n)\| \cdot \langle 1|\psi(t_n)\rangle, \tag{12} $$

$$ Z(t_n) = \|X(t_n)\| \cdot \langle 2|\psi(t_n)\rangle, \quad R(t_n) = \|X(t_n)\| \cdot \langle 3|\psi(t_n)\rangle \tag{13} $$

We simulate this for different fractional orders $\alpha \in \{1.0, 0.95, 0.90, 0.85, 0.80, 0.75\}$.

Results and Interpretation

The simulation results show clear differences in the dynamical behaviors of the four compartments for different fractional orders α. For instance, the susceptible population $S(t)$ decays fastest when $\alpha = 1.0$, and its decay is slower with smaller α because of stronger memory effects. Regarding the infected population $I(t)$, it peaks earlier for larger α and later for lower fractional orders; this population will continue to grow and persist for a long time. For $Z(t)$, longer activation and eventual saturation can be seen for small α, while larger memory usually means longer-term transmission. Last but not least, the removed population $R(t)$ grows more slowly with smaller α, which reflects the impact that memory will have in longer outbreak scenarios.

Figures 1–4 illustrate these dynamics and highlight the capability of quantum-enhanced fractional modeling in capturing complex epidemiological patterns. In need, the dynamics of the number of susceptible individuals with respect to α is shown in Figure 1. A larger value of α results in a smaller rate of decrease in the number of the susceptible population, resulting in a reduced memory in the system. Figure 2 illustrates the behavior of infected population. The rate of growth and fall of the infected population is dependent on α. Smaller α translates to longer infections. Figure 3

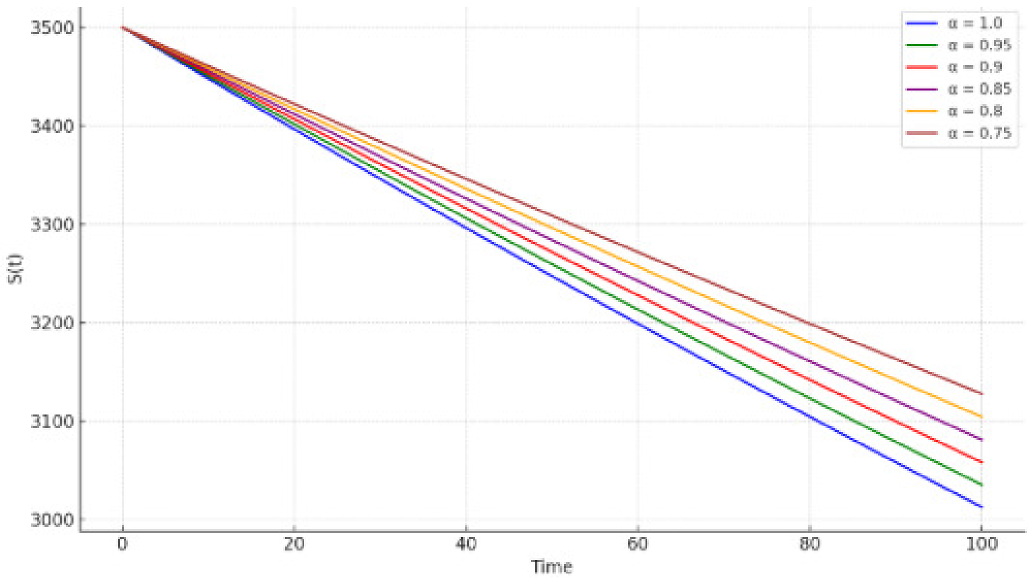

Fig. 1: Susceptible Population $S(t)$ for Varying α

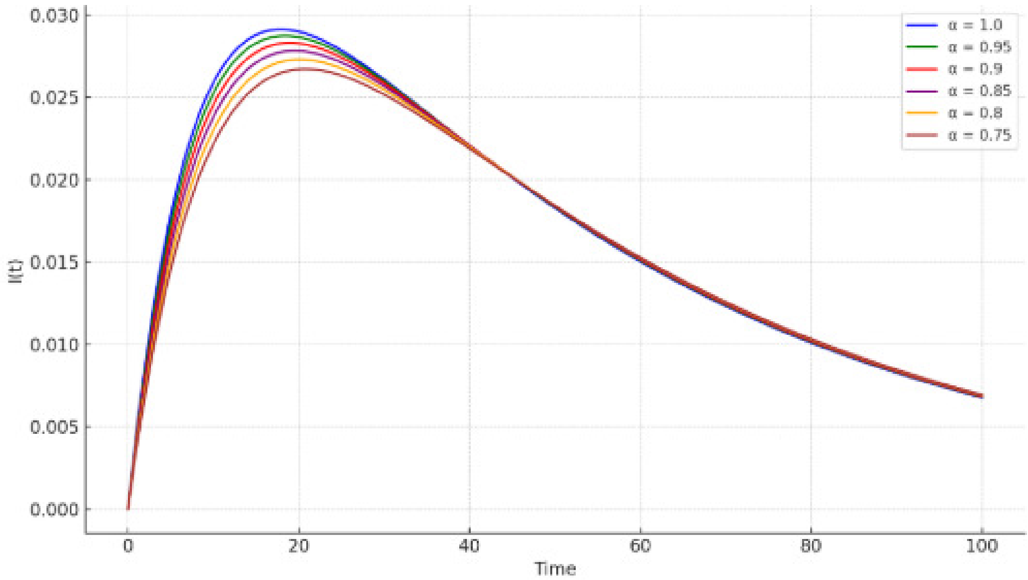

Fig. 2: Infected Population $I(t)$ for Varying α

displays the zombie population growth. Higher memory (lower α) leads to slower growth in zombification, as the system retains more historical infection memory. Figure 4 illustrates how individuals transition to the removed class. For smaller α, removal is more gradual due to extended memory effects.

These simulations validate the model's capability to capture complex temporal dependencies using fractional-order operators and demonstrate the effectiveness of hybrid quantum simulation in handling such dynamics.

Discussion

The fractional SIZR model represents a robust quantum framework to solve these memory-driven epidemic dynamics. In particular, this proposition explores an approach for leveraging the Atangana-Baleanu derivative and encoding the

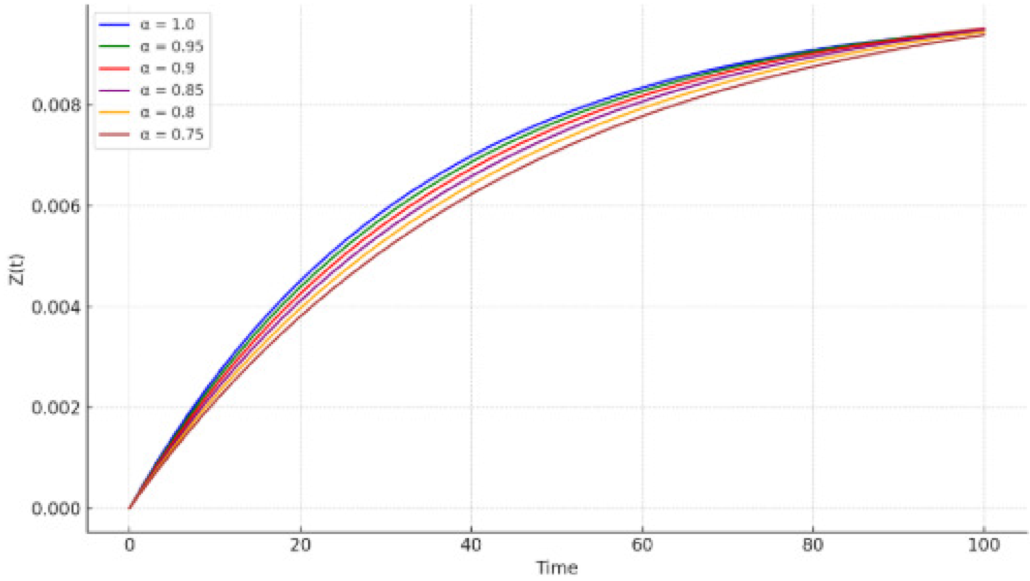

Fig. 3: Zombie Population $Z(t)$ for Varying α

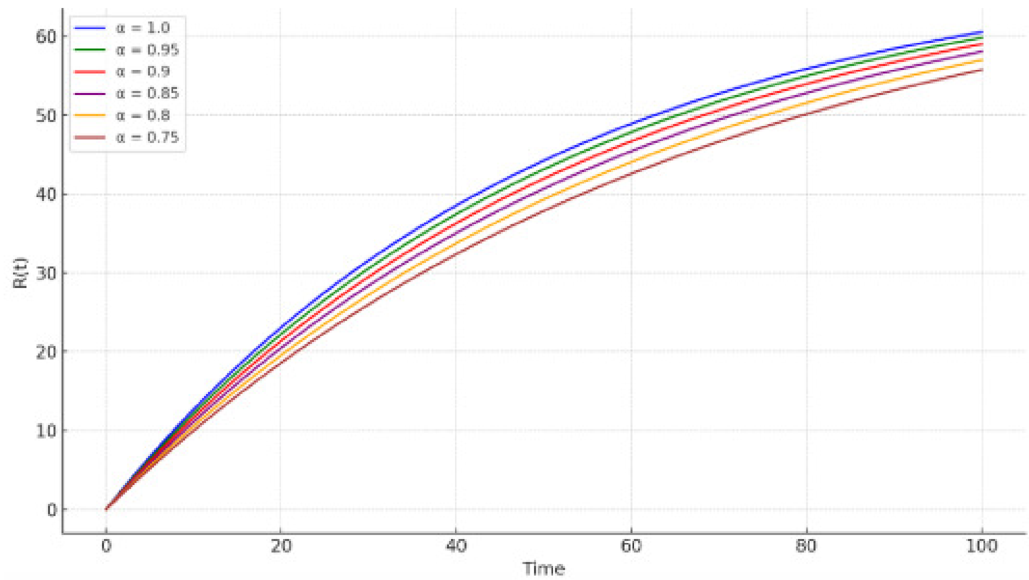

Fig. 4: Removed Population $R(t)$ for Varying α

model on quantum hardware by VQAs to capture both temporal memory effects and high-dimensional correlations more effectively than classical numerical solvers.

Compared to the classical Homotopy Analysis Transform Method (HATM) [14], the quantum framework:

- Reduces computational time for systems with fine time resolution.
- Requires less memory due to efficient quantum state representations.
- Offers scalability to extended models with more compartments or spatial dimensions.

In addition, the loop comprising the quantum and classical systems utilizes the flexibility associated with classical optimization algorithms. The role played by the quantum part involves the calculation of inner products and amplitude encoding in a superposition. An amplitude encoding approach guarantees efficient representation of the epidemiological state. Furthermore, the variational formulation allows for a stochastic model and control approach.

The primary challenge at the moment is the noise of the hardware in the quantum hardware itself, but as NISQ-era hardware matures, the results of these simulations should increase in precision and depth.

5 Conclusion

In this work, a hybrid quantum-classical approach is proposed to simulate and study a fractional-order SIZR model of infectious disease spread by using Atangana-Baleanu derivatives. The idea of using quantum computing, with focus on Variational Quantum Algorithms (VQAs), is explored to efficiently introduce memory terms, which play an essential role in modeling an infectious disease process efficiently. The quantum-assisted approach uses amplitude encoding and an ansatz circuit to estimate the time evolution of susceptible, infective, zombie, and removed compartments. A discretized fractional residual defines the quantum-classical iterative optimization process.

Compared to traditional approaches such as the Homotopy Analysis Transform Method (HATM), the proposed approach has many advantages. It offers scalability and efficiency, especially in the presence of long memory and large dimensionality. The ability of quantum circuits to express non-local interactions compactly also makes the use of the variational principle particularly well-suited for present-day noisy intermediate-scale quantum (NISQ) technology.

Results of simulation runs performed for different fractional orders α demonstrate the significance of memory in epidemiology. Small fractional orders α thoroughly emphasize memory, leading to late peaks of infection, slow growth of zombies, and an accumulation of the removed class, thereby supporting the flexibility of the hybrid model in describing diverse epidemiology behaviors, unaddressed by traditional integer-order models.

As to further research directions, there are various ways of furthering this research. The use of this model on real quantum hardware is one possible direction, aiming at applying it within noise constraints. Applying it to real-world epidemics, including monkeypox and COVID-19, can provide many insights into diseases exhibiting latency and feedback. It is also possible to use quantum control methods and reinforcement learning, aiming at applying vaccination and quarantine control. Finally, stochastic models with quantum uncertainty can add to the possibility of creating more realistic models. In conclusion, this study illustrates that quantum computing can improve fractional models of epidemics by providing an efficient, scalable, and realistic framework as to furthering this research area. Also, it stimulates further research within this area, especially as hardware advances. The study sheds light on many aspects of epidemics, including fractional models of epidemics,

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